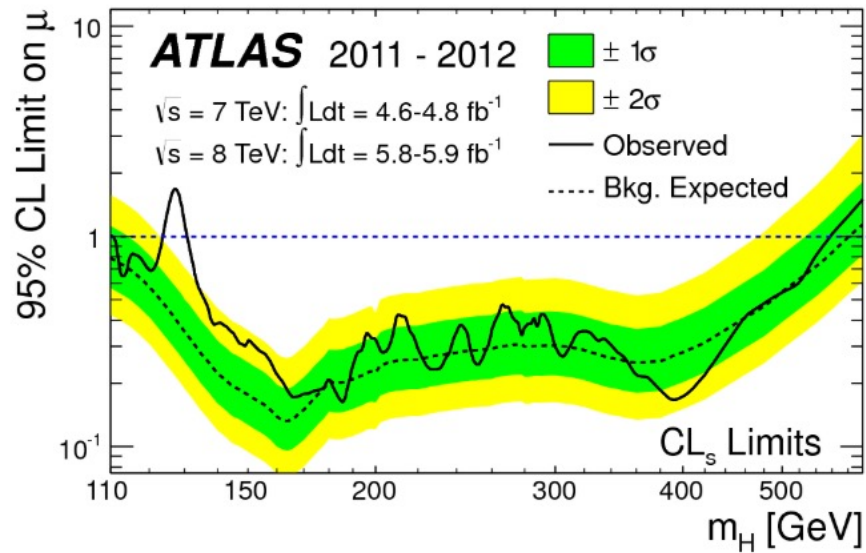


$\mu$ : parameters of interest. Held constant for *conditional fits*, float for *unconditional fits*

$\theta$ : nuisance parameters. Floating in both conditional and unconditional fits

$\alpha$ : prespecified parameters. Held constant in both conditional and unconditional fits

### Hypothesis space: $m_H$ vs $\mu$

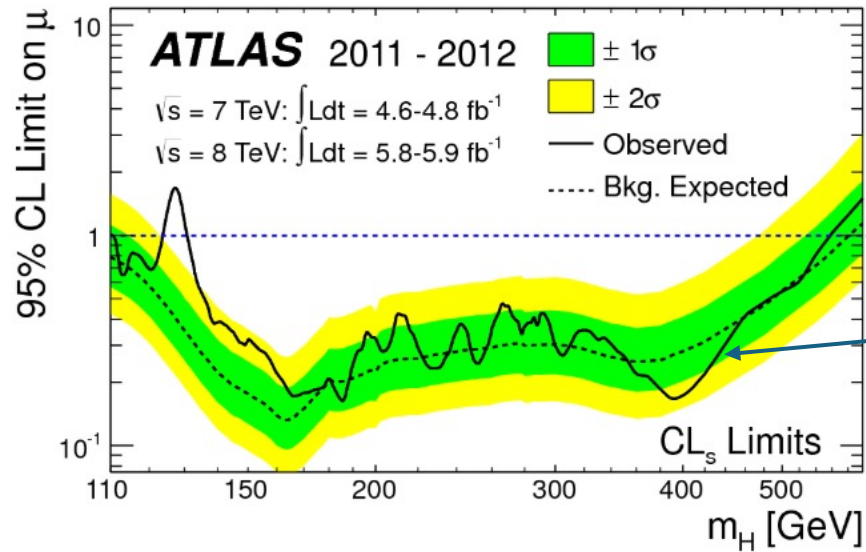


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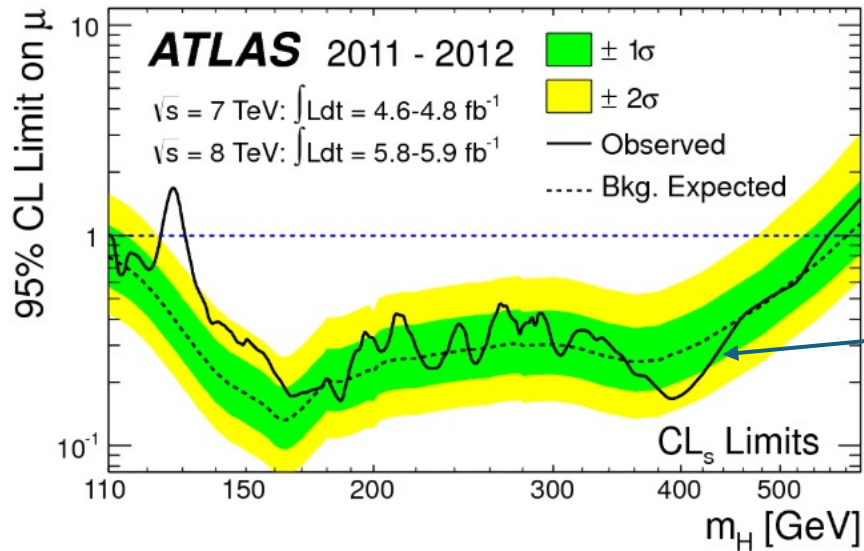
$\theta$ : nuisance parameters. Floating in both conditional and unconditional fits

$\alpha$ : prespecified parameters. Held constant in both conditional and unconditional fits

Contour where  $p_{\text{CLs}}(\text{obs})=0.05$

$$p_{\text{CLs}}(\text{obs}) = p_{\text{null}}(\text{obs}) / p_{\text{alt}}(\text{obs})$$

### Hypothesis space: $m_H$ vs $\mu$



Contour where  $p_{\text{CLs}}(\text{obs})=0.05$

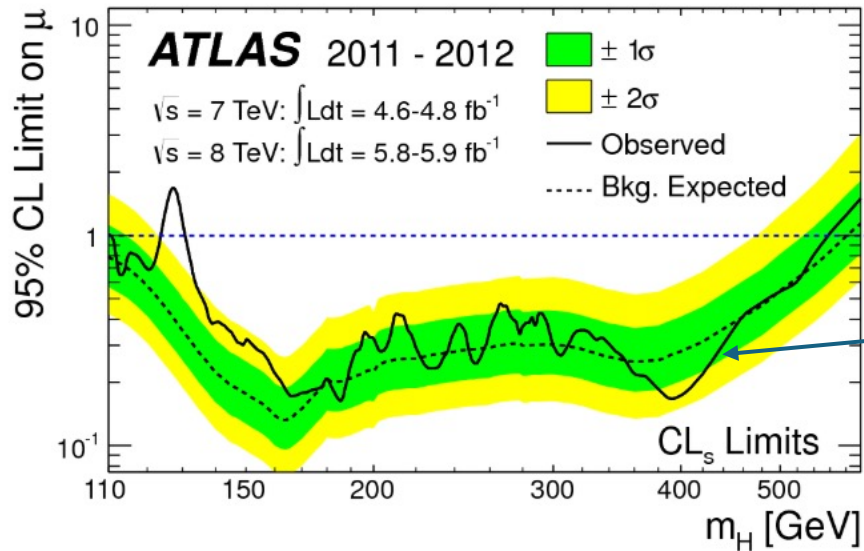
$$p_{\text{CLs}}(\text{obs}) = p_{\text{null}}(\text{obs}) / p_{\text{alt}}(\text{obs})$$

#### Null and Alt hypothesis for exclusion hypothesis testing

Null hypothesis: the parameter values set equal to the ones that define the hypothesis point

Alt hypothesis: same as null hypothesis except some parameter (usually poi) set to different value (usually 0)

### Hypothesis space: $m_H$ vs $\mu$



Contour where  $p_{\text{CLs}}(\text{obs})=0.05$

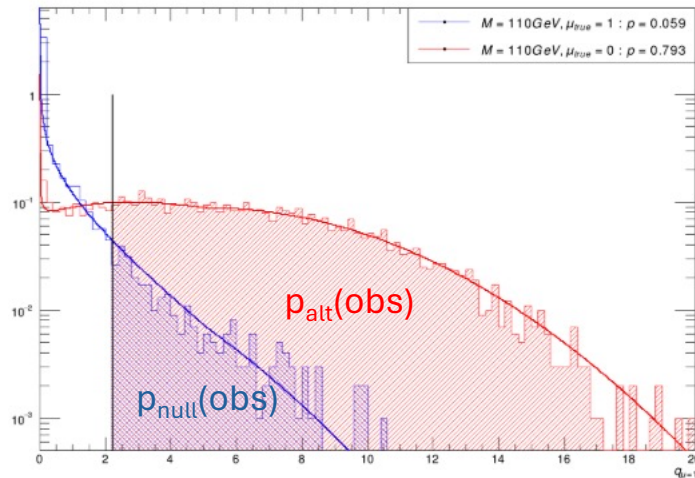
$$p_{\text{CLs}}(\text{obs}) = p_{\text{null}}(\text{obs}) / p_{\text{alt}}(\text{obs})$$

### Null and Alt hypothesis for exclusion hypothesis testing

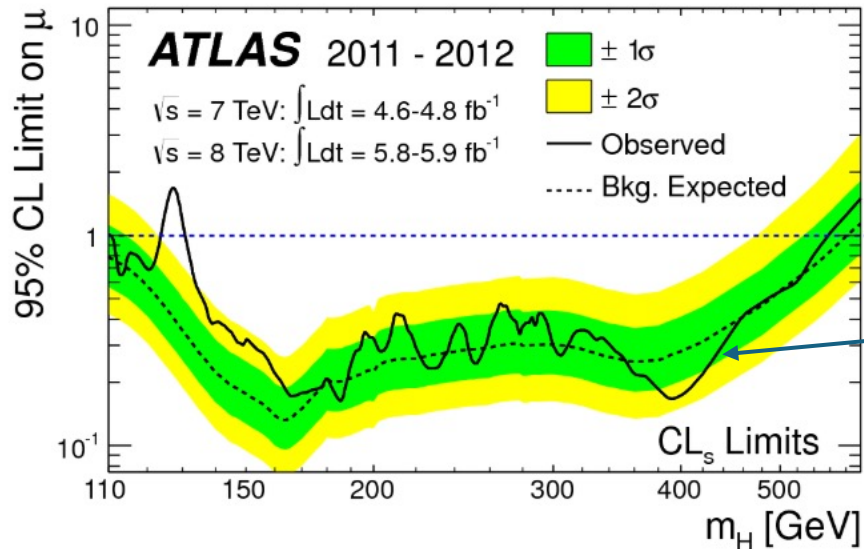
Null hypothesis: the parameter values set equal to the ones that define the hypothesis point

Alt hypothesis: same as null hypothesis except some parameter (usually  $\mu$ ) set to different value (usually 0)

At any hypothesis point in the hypothesis space, can get create test-statistic distributions under the null and alternative hypothesis, and integrate to obtain the p-value



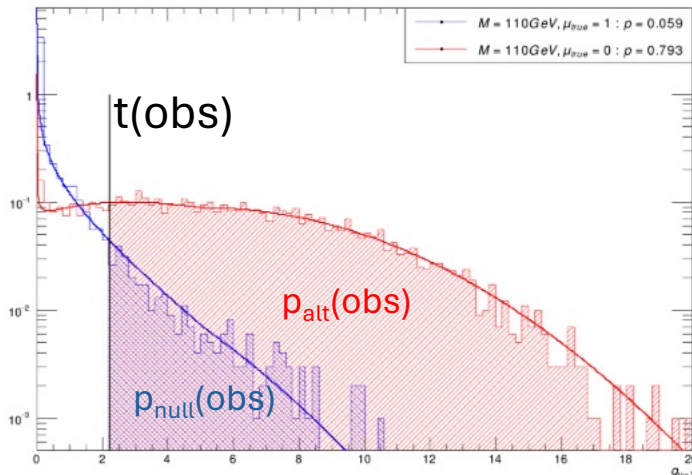
### Hypothesis space: $m_H$ vs $\mu$



Contour where  $p_{CLs}(obs)=0.05$

$$p_{CLs}(obs) = p_{null}(obs) / p_{alt}(obs)$$

At any hypothesis point in the hypothesis space, can get create test-statistic distributions under the null and alternative hypothesis, and integrate to obtain the p-value



$\mu$ : parameters of interest. Held constant for *conditional fits*, float for *unconditional fits*

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### Null and Alt hypothesis for exclusion hypothesis testing

Null hypothesis: the parameter values set equal to the ones that define the hypothesis point

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Test statistic (normalized) distribution, for toys produced from model with parameter values  $\mu', \theta', \alpha'$

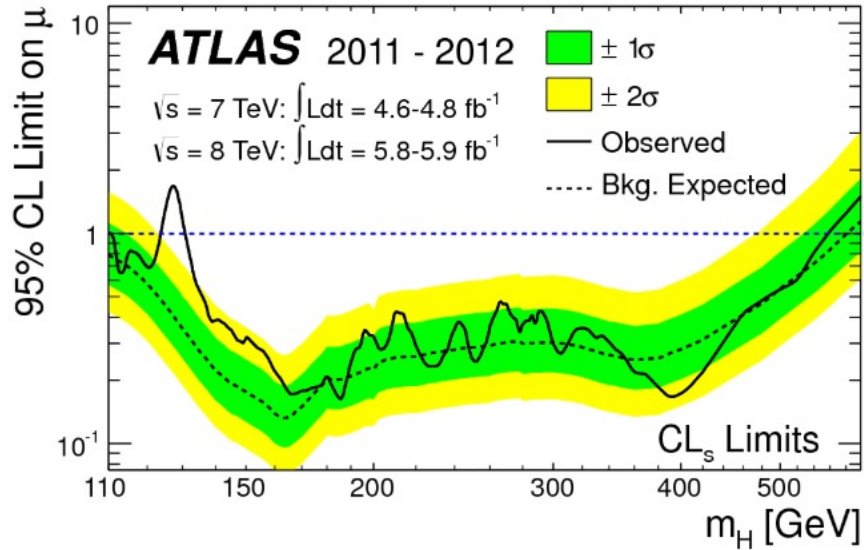
$$p_X(D) = \int_{t(D)}^{\infty} f(t | \mu' = \mu_X, \theta' = \hat{\theta}(obs | \mu = \mu_X), \alpha' = \alpha_X) dt$$

Test statistic evaluated for dataset D

Required property of test statistic: increasing value means increasing incompatibility with the null hypothesis

# What we need ...

1. Way to calculate an appropriate test statistic at any given hypothesis point, for any given dataset
  - Profile log-likelihood ratio has the required property (increasing value for decreasing compatibility with null hypothesis)
2. Way to generate toy datasets for a given set of parameter values
3. Use those toy datasets with the test statistic function to build the null and alt hypothesis test statistic distributions at each hypoPoint
4. Scan the hypoSpace (efficiently) to find the  $p_{\text{CLS}}=0.05$  contour



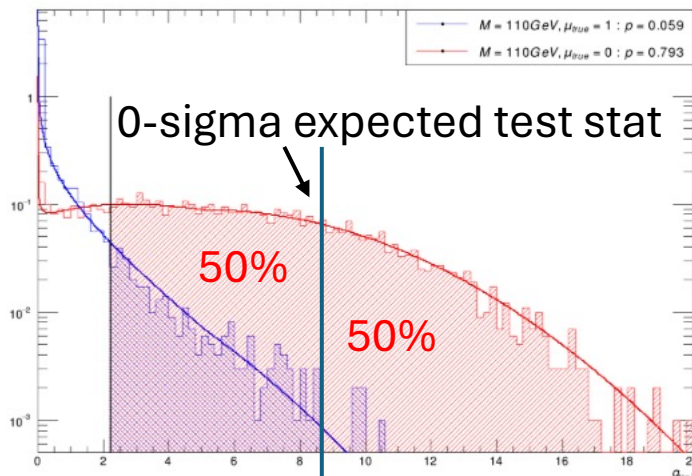
Contour where  $p_{\text{CLs}}(\text{exp-Nsigma})=0.05$

$$p_{\text{CLs}}(\text{exp-Nsigma}) = p_{\text{null}}(\text{exp-Nsigma}) / p_{\text{alt}}(\text{exp-Nsigma})$$

What is an N-sigma expected dataset?

0-sigma expected dataset would be asimov dataset corresponding to  $\mu=0$  hypothesis.

But other ones not well-defined ...



Better definition:

$$p_{\text{alt}}(\text{exp-Nsigma}) = \Phi(N) \leftarrow \text{gaussian CDF}$$

$$\text{e.g. } p_{\text{alt}}(\text{exp-0sigma}) = 0.5$$

This then defines an N-sigma expected test-statistic value.

Measure  $p_{\text{null}}(\text{exp-Nsigma})$  from the blue histogram, integrating from the N-sigma expected test statistic value.