Event generators for collider physics

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Introduction

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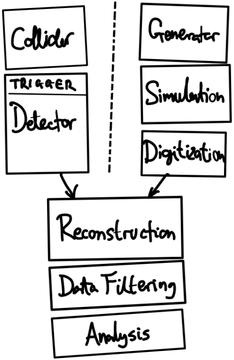
“Evgen” is simulation of the fundamental particle-collision process

- Ubiquitous in HEP, from experiment design to interpretation
- I’ll focus on fully differential “shower+hadronisation” gens (SHGs): those that make “realistic” events that can be fed to a detector simulation
- These are serious theory tools ⇒ the link between “hardcore theory” and experiment. Precision (and CPU cost) have rocketed in the last decade
- But often treated as *black boxes*… rarely a good idea. A little extra understanding can go a long way, so let’s see what we can do!



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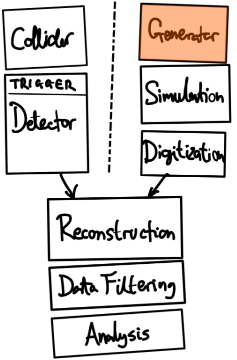


What is an event generator?

- "Event generator" is an overloaded phrase: many pheno people refer to partonic MC integrators as event generators.
- For experiment purposes a real EG produces *exclusive* events
  - Realistic final-particle multiplicities & composition, like real data
  - Fortunately HEP final-states *really* can be described in full detail
- Correlations are not easily fakeable, e.g. from sampling data distributions: *microscopic models* produce best and most richly structured phenomena
  - i.e. event generators are based on fundamental QFT
  - but *approximately*: can't explicitly calculate full-multiplicity processes
- Since QCD is the strongest force, QCD effects usually dominate MC physics. But this is changing, cf. precision era and large EW corrections for future colliders, for both lepton and hadron beams.

Shower & hadronisation event generators (SHGs)

What's an SHG good for? Depends who you ask!

- **Experimentalists:** design of colliders, detectors & analyses, background estimation, signal estimation, pile-up estimation, unfolding. . . ∼**everything!**
- **Theory/pheno:** dressing parton level calculations to make them more realistic ("easily" include effects that aren't the focus of the study e.g. decays or UE); constraining BSM models by "recasting" experimental data
- **Generator authors:** understanding (how to work with) QCD – both perturbative and non-perturbative; enabling both the above

SHGs often take in partonic events via *LHE format* (though this is evolving), and output full particle-level events via *HepMC format(s)*

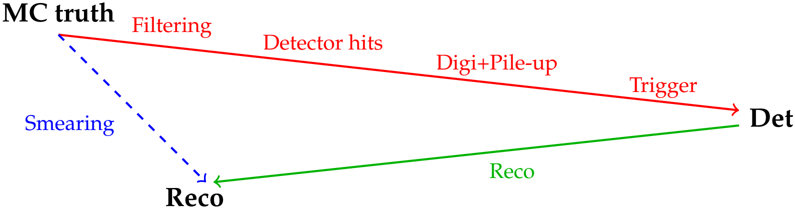
A selective list

- **Partonic subprocess generators (used as SHG input via LHE):**
  - Multi-leg LO: MadGraph5, Sherpa, ~~AlpGen~~
  - NLO (+ multileg): POWHEG-BOX, Sherpa, OpenLoops, MadGraph5-aMC@NLO, MCFM
  - Specialist processes: HEJ, Prophecy4f, WHiZard, Protos
- **Main general-purpose event gens:**
  - C++: Sherpa, Herwig 7, Pythia 8
  - FORTRAN: ~~FHerwig~~ and ~~FPythia~~
- **Afterburners:**
  - EvtGen, Photos, Tauola, ~~Jimmy~~
- **Specialist all-in-one:**
  - Min bias & air showers: PHOJET, EPOS, QGSJET, SYBILL
  - Heavy ion: HIJING, HYDJET, Starlight, Angantyr
- **Utilities:**
  - LHAPDF, HepMC

Evgen in experiment data-processing

Typical experimental use of generators is to feed their output into a detector sim, e.g. based on Geant 4.

Then apply the same reconstruction + analysis as for data:

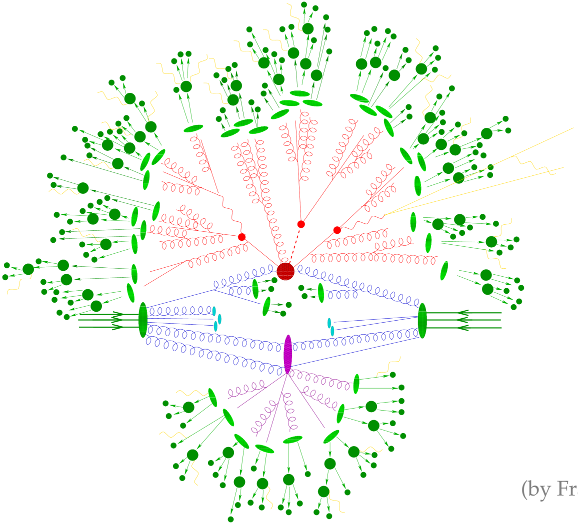


The generator bit of this chain was historically cheap: CPU/memory requirements *much* less than detector geometry + B-field stepping + material interaction + secondaries. Not true these days! ↗ ↗

Generator capabilities, complexity & CPU demands can now be very high: process complexity ⇒ inefficiency

Anatomy of an SHG

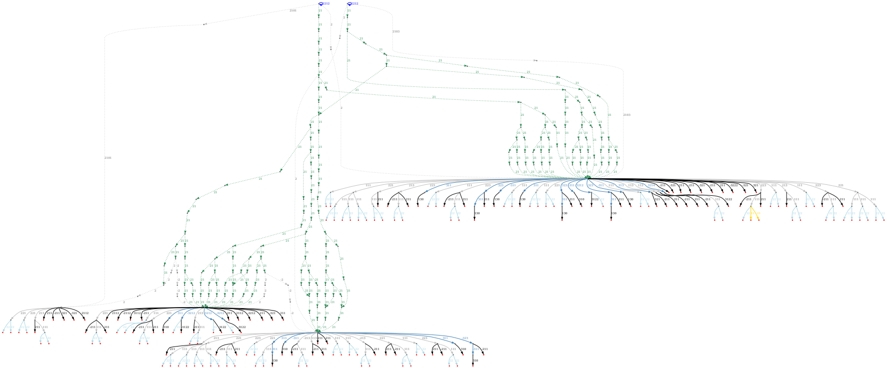
Computed outward, starting from the hard-scattering matrix element, and evolving back to the beams (PDFs) and forward to the final state:



(by Frank Krauss)

A “real” MC event

Here’s an MC event graph showing all the particles (lines) and interactions (vertices) in a Pythia *event record*.

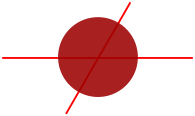


Guess the event type?

2212=proton, 1–6=quarks, 11–16=leptons, 21–25=gluon/photon/Z/W/H

Matrix elements

SHG step-by-step



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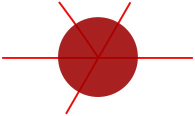


Matrix elements and phase-space

Cross-sections for a scattering subprocess $ab \to n$ computed in collinear factorization, with *factorization scale* μ_F:

$$\begin{aligned}\sigma &= \sum_{a,b} \iint_0^1 \mathrm{d}x_a \mathrm{d}x_b \int f_a^{h_1}(x_a,\mu_\mathrm{F})\, f_b^{h_2}(x_b,\mu_\mathrm{F})\, \mathrm{d}\hat{\sigma}_{ab\to n}(\mu_\mathrm{F},\mu_\mathrm{R}) \\ &= \sum_{a,b} \iint_0^1 \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}\Phi_n\, f_a^{h_1}(x_a,\mu_\mathrm{F})\, f_b^{h_2}(x_b,\mu_\mathrm{F}) \\ &\qquad \times \frac{1}{2\,\hat{s}} |\mathcal{M}_{ab\to n}|^2(\Phi_n;\mu_\mathrm{F},\mu_\mathrm{R})\,,\end{aligned}$$

This is the core of all event generation: a combined *integral* of PDFs and partonic matrix element over phase space in $x_{a,b}$ and Φ.

The “MC” comes in because the integral is done by Monte Carlo sampling in $4(n-1)+2$ phase-space dimensions: error reduces as $\sqrt{\text{samples}}$, rather than degrading with N_dim.

ME integration & generation

Event generation is preceded by an integration phase to evaluate total cross-section and map the $d\hat{\sigma}$ structure in ME phase-space

Naïve sampling doesn't get far: easy to spend all CPU on *atypical points* which are invalidated if and when a sample hits the typical set.

A better strategy:

- Jacobian-transform phase space to remove divergent structures;
- But there are *many* characteristic divergences in matrix elements ⇒ multi-channel integration: one per divergence, cf. MG5
- Use standard sampling techniques, or adaptive sampling algorithms — even ML — to "learn the space"
- It's still slow, and these methods are very far from perfect. The #1 MC production challenge is phase-space efficiency in NLO multijet processes

Integration can take *months*! Massively parallel computations becoming normal. Typically save integration results as "gridpack" files for batched event generation.

(Far) beyond LO

We are long past the point where SHGs only handle lowest-order partonic subprocesses (sometimes enhanced with LO *ME corrections* for the first parton-shower emission).

Extra partonic emissions at tree-level increase the final state multiplicity and change the event kinematics directly. Automated by LO *merging and matching schemes* like MLM, CKKW(-L), etc.

Breyond tree-level is more involved. An NLO cross-section has 3 parts:

$$\mathrm{d}\sigma^{\mathrm{NLO}} = \mathrm{d}\tilde{\Phi}_n \left[\mathcal{B}(\tilde{\Phi}_n) + \alpha_\mathrm{s}\mathcal{V}(\tilde{\Phi}_n)\right] + \mathrm{d}\tilde{\Phi}_{n+1}\alpha_\mathrm{s}\mathcal{R}(\tilde{\Phi}_{n+1})$$

But infrared divergences occur in both the $\mathcal{R}$eal emission and $\mathcal{V}$irtual correction parts – i.e. in different Φ dimensionalities.

Bloch–Nordsieck / KLN theorems: for infra-red-safe observables, these divergences must cancel. cf. ME squaring

(Far) beyond LO

Subtraction: use universal splitting kernel $\mathcal{S}$ which encodes real emission divergence structure so $\mathcal{R} - \mathcal{B} \otimes \mathcal{S}$ is finite → computable:

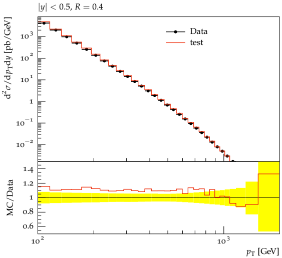
$$\begin{aligned}\sigma^{\mathrm{NLO}} = \int_n \mathrm{d}\tilde{\Phi}_n^{(4)}\,\mathcal{B} \quad &+ \quad \alpha_s \int_{n+1} \mathrm{d}\tilde{\Phi}_{n+1}^{(4)} \left[\mathcal{R} - \mathcal{B} \otimes \mathcal{S}\right] \\ &+ \quad \alpha_s \int_n \mathrm{d}\tilde{\Phi}_n^{(D)} \left[\tilde{\mathcal{V}} + \mathcal{B} \otimes \int_1 \mathrm{d}\Phi_1^{(D)} \mathcal{S}\right],\end{aligned}$$

Many NLO ME calculators, but only a few automated ones. aMC@NLO and Sherpa fully automated; POWHEG-BOX is a framework to assist manual implementation.

Virtual terms from dedicated calculators, e.g. BlackHat/OpenLoops/NJETS/GoSAM via BLHA interface. Technically solved: processes like $W + 5$ jets or fully decayed $t\bar{t}$ and single-top are possible at NLO… *if you can spare the integration time!*

Biased event generation and weights

- For physics purposes, we want a flat distribution of event statistics across observables
- But many distributions fall fast: if we wait for an unbiased generator to produce a TeV-scale jet, we need to make as many events as the LHC does! *At sim-reco level…*
- Neat trick: bias the sampling to produce events not from a physical distribution but from an "enhanced" one, e.g. $\hat{p}_T^4 \cdot \mathrm{d}\hat{\sigma}(\Phi)/\mathrm{d}\omega$
- Experiments often also create piece-wise "sliced" samples with matched min and max cuts on subprocess $\hat{p}_T$



Distributions like this are hard to make without biasing

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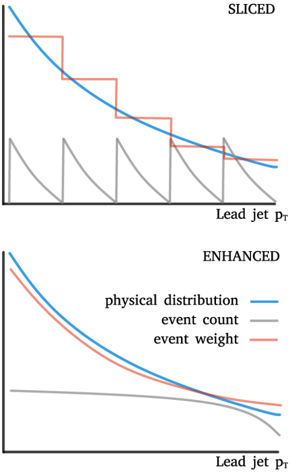
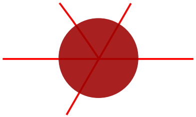


Parton showers

SHG step-by-step



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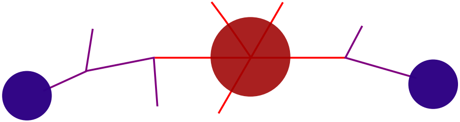

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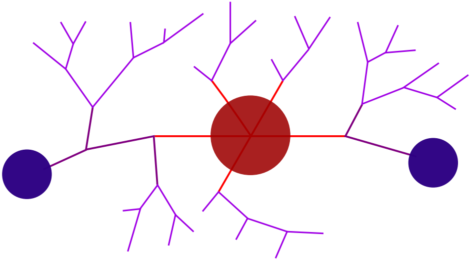


Splitting functions and resummation

Limited parton–jet or parton–hadron duality: can compare *fixed-order partonic events* to data for IR-safe observables.

Soft and collinear phase-space, $1\ \mathrm{GeV} < p_\mathrm{T} \lesssim 20\ \mathrm{GeV}$, gets large *resummation corrections* from multiple QCD emissions.

Analytic resummation is process-specific. But factorizes in the collinear limit: cross-section for process σ_0 with parton i to be accompanied by a collinear parton j with momentum fraction z:

$$\mathrm{d}\sigma \approx \sigma_0 \sum_{\text{partons},i} \frac{\alpha_\mathrm{s}}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2}\, \mathrm{d}z\, P_{ji}(z,\phi)\mathrm{d}\phi$$

where θ is the angle between i and j and

$$\begin{aligned} P_{qq}(z) &= C_F\,\tfrac{1+z^2}{1-z}, & P_{gq}(z) &= C_F\,\tfrac{1+(1-z)^2}{z}, \\ P_{gg}(z) &= C_A\,\tfrac{z^4+1+(1-z)^4}{z(1-z)}, & P_{qg}(z) &= T_R(z^2+(1-z)^2) \end{aligned}$$

These are the spin-averaged QCD collinear *splitting functions*, or *DGLAP kernels*. Divergences as $\theta, z \to 0 \Rightarrow$ **soft & collinear emissions dominate**

Parton showers

Parton showers are Markov chain algorithms based on the QCD splitting functions → process-independent, approximate resummation. Creates parton multiplicities ∼ realistic hadron multiplicities.

From splitting functions can calculate *probability of no emission* between scales Q and q (setting an IR cutoff for resolvability & perturbativity):

$$\Delta_i(Q^2, q^2) = \exp\left\{ -\sum_j \int_{q^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_\mathrm{s}}{2\pi} \int_{q^2/k^2}^{1-q^2/k^2} \mathrm{d}z \, P_{ji}(z) \right\}$$

The famous *Sudakov form factor*. Can be inverted to generate a random parton emission with physical k distribution from a random number

Split into *initial-state* (ISR) and *final-state* (FSR) showers in SHGs:

1. ISR: generate high-p_T extra emissions on the incoming parton legs back to the proton (using *backward evolution*);
2. FSR: dress all final-state partons with a forward evolution down to the QCD perturbative cutoff Λ_QCD

Motivating the Sudakov

Integration is the natural continuous limit of a sum:

$$\sum \quad \rightarrow \quad \int$$

What's the equivalent continuum limit of a *product*?

$$\prod \quad \rightarrow \quad ???$$

Motivating the Sudakov

Integration is the natural continuous limit of a sum:

$$\sum \quad \to \quad \int$$

What's the equivalent continuum limit of a *product*?

$$\prod = \exp\left\{\sum \ln\right\} \to \exp\left\{\int \ln\right\}$$

Every d*t* scale-step has a differential probability of no-emission; the Sudakov *exponentiation* multiplies them together.

More about showers

- In practice it's a bit more complex, with running α_s (and carefully chosen running scale), spin effects, quark masses, etc.
- Any evolution variable $k^2 \propto \theta^2$ is permitted in the collinear limit and will resum the divergence.
  - But some are better than others: colour-coherence effects suppress emissions outside the previous emission cone.
  - Quantum effect reproduced by θ-ordering and p_T-ordering, but not virtuality. All modern generators enforce a degree of colour coherence.
- Compared to the ME, Markov process → shower misses correlations.

More about showers

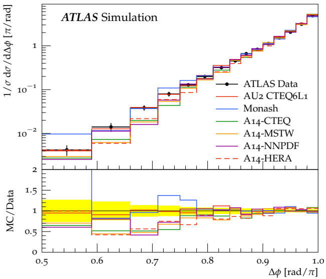
- Initial state shower adds complication
  - Emissions modifying initial partons need to change the PDF x and flavour used ⇒ ISR Sudakovs include a PDF ratio term $\frac{x/z f_j(x/z, k^2)}{x f_i(x, k^2)}$ to ∼cancel up the chain
  - Forward-evolving from the hadron to find a consistent hard process configuration would be hopelessly inefficient ⇒ backward evolution.
- Actually, $1 \to 2$ showers have problems:
  - Can't have finite relative p_T and real, on-shell partons since $1 \to 2$ violates Lorentz symmetry ⇒ *reshuffling*
  - Much modern activity uses $2 \to 3$ dipole showers and higher variants – also for NLO subtraction compatibility: CSS, MatchBox, DIRE, Vincia, …

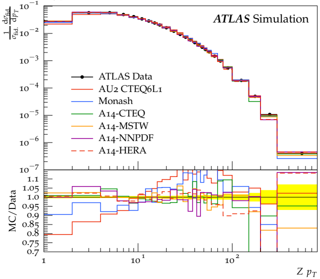
Forward evolution of spacelike shower, but more fun

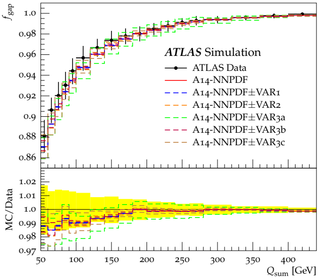
Shower observables

ISR: extra jets, jet distributions, $Z\ p_T$, gaps

NB. distinctly not collinear!


Dijet azimuthal decorrelations for $210 < p_\perp^{\max}/\mathrm{GeV} < 260$


$Z\ p_T$ “dressed” muons

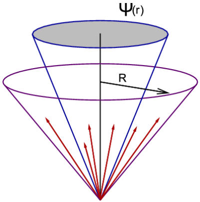

Gap fraction vs. Q_{sum} for veto region: $|y| < 0.8$

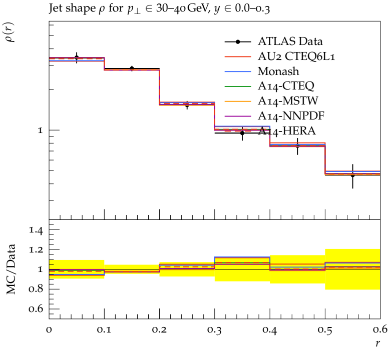
Shower observables

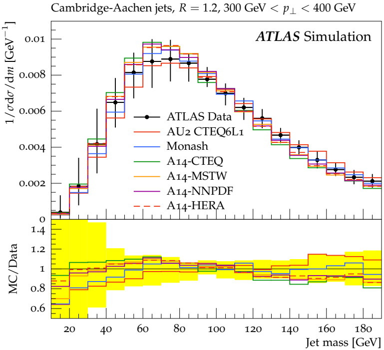
FSR: jet shapes, jet masses

i.e. adding structure to the parton↔jet duality

As jet scale increases, jets become increasingly collimated




Jet shape ρ for $p_\perp \in$ 30–40 GeV, $y \in$ 0.0–0.3


Cambridge-Aachen jets, $R = 1.2$, 300 GeV $< p_\perp <$ 400 GeV

ME–shower interfacing: “matching”

***Really* not enough space to do this topic justice: huge developments in last 20 years**

Issues are almost always because of double counting when the shower is used: an n-leg ME with parton shower *contains* the $n+1, \ldots$ terms.

To improve on the Born+shower approximation, need to **remove overlap.**

For LO multi-leg: MLM and CKKW schemes both designed to replace the shower’s collinear splitting functions with proper matrix elements in the relevant (hard) phase-space

Phase-space slicing definitions took 10 years to iterate to better control. Introduces *merging scales*, which need to be chosen to minimise observable sensitivity: not “fire and forget” generation

Matching at NLO

Natural to go beyond tree-level matching: completely consistent NLO includes the Born, one-loop, and one-real emission amplitudes and all their interferences, *and* smoothly matches the real emissions between shower and ME.

Main benefit: NLO *scale stability*. Normalisation could always be taken from partonic highest-tech integrators, but (1-emission) shapes now also stable without disrupting formal accuracy of parton shower
⇒ ***rough* rule: shapes from real, normalisation from virtual**

- MC@NLO method: extension of fixed-order subtraction to use shower-specific splitting functions: process-specific, "some" negative weights.
- POWHEG[1] method: "NLO matrix-element correction". Closer to all-positive weights, and shower-independent.
  Convenience ⇒ large uptake.

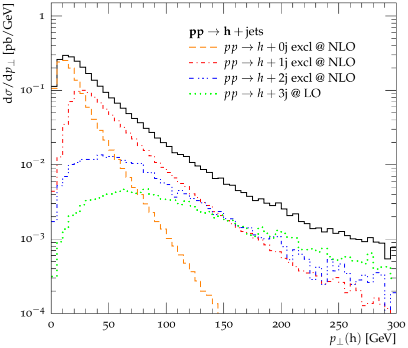
---

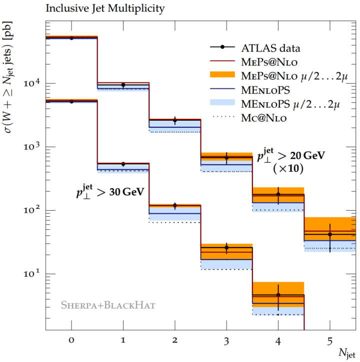
[1] Not, not, *not* 'PowHeg'!!! 'PoWHEG', if you must.

State of the art: NLO matching + merging

Now very impressive situation: automatic generation & merging of many NLO and LO multi-leg + shower in MEPS@NLO (Sherpa) and FxFx (MG5-aMC@NLO)

Bookkeeping *tour de force!* And at huge CPU cost in *unweighting*





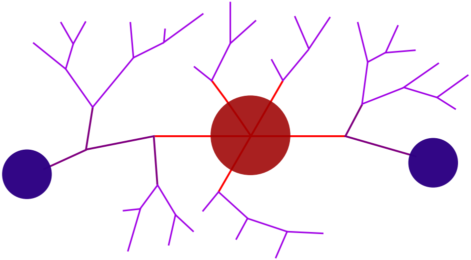
Increasingly NLO with EW corrections. NNLO-PS sometimes possible.

Which features are worth the CPU/opportunity cost?

Non-perturbative stuff

(that we wish wasn't there…)

SHG step-by-step



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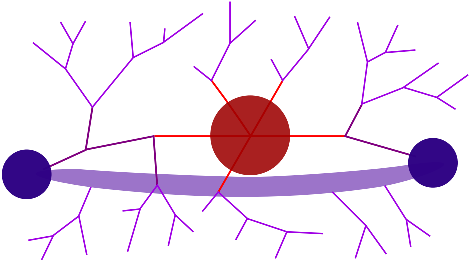

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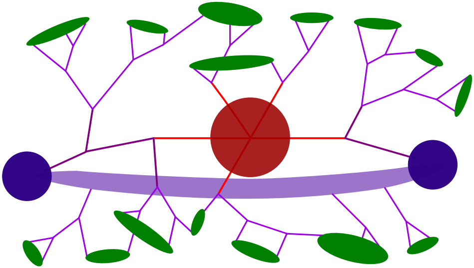

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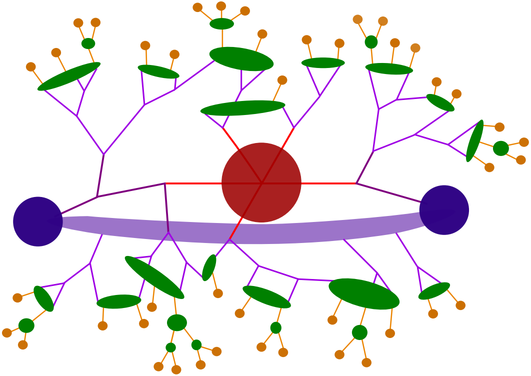
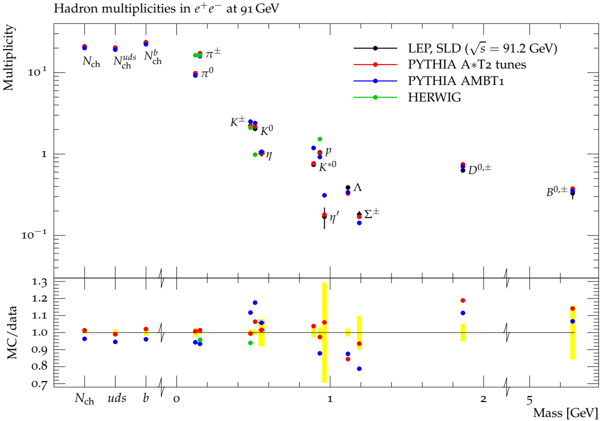


Hadronisation

- At scales below shower cutoff $Q_0 \sim \mathcal{O}(\Lambda_{\text{QCD}})$, confinement means that physics is non-perturbative. **Source of most tuning params**
- Observe limited transverse momenta and Q^2-independent energy fractions: most quantum number flow done by the shower fragmentation, so hadronisation can be ~ localised
- Two main modern hadronisation models: *Lund string and cluster*

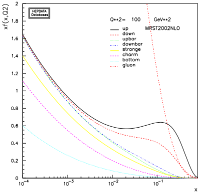


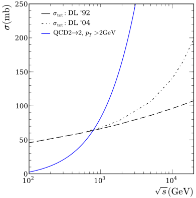
Hadronisation models

- **Lund string (Pythia):**
  - Inspired by *linear scaling of QCD potential* at large distances
  - Break colour strings to produce new quark pairs; gluons form kinks in strings
  - Lorentz invariance and LR-symmetry give Lorentz invariant *Lund symm frag function*
  - Kinematics well-described, but flavour – esp. baryons – not natural
- **Cluster hadronisation (Herwig, Sherpa):**
  - *Colour preconfinement*, seen in colour-connected neighbour parton mass spectrum
  - Non-pert $\rightarrow q\bar{q}$, then cluster colour singlets: requires finite gluon *constituent mass*
  - Clusters treated as *meson resonances*
- Both models (except Sherpa) also contain *colour reconnection* heuristics
- **Tuning!**

Underlying event

HERA data show inclusive jet cross-section rising strongly with energy due to low-x PDFs (esp. gluon)
$\Rightarrow$ unitarity violation





- Eikonal models interpret the bottom-up to top-down σ ratio as mean number of *multiple-parton interactions (MPI)* $\Rightarrow$ sample Poisson to make n pQCD subprocesses
- Hadron impact parameter $\sim 1/Q \Rightarrow$ transverse overlap also important
- Low hard-process scale $Q \Rightarrow$ low overlap & low n: "minimum bias" cf. pile-up
- High hard-process scale $Q \Rightarrow$ total overlap & high n: *pedestal effect* $\rightarrow$ "underlying event"
- Extra details: $\hat{p}_T$ cutoff/screening, proton overlap form factor, colour reconnection. **Tuning!**

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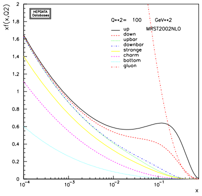
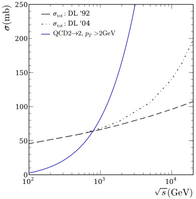
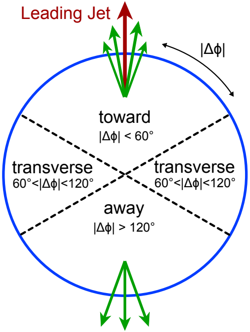
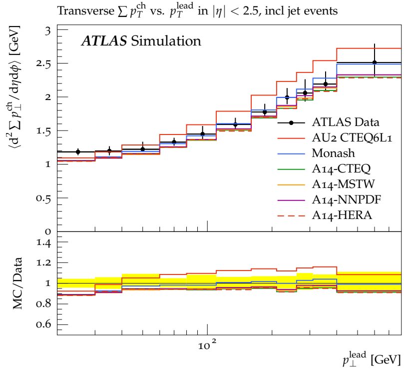
HERA data show inclusive jet cross-section rising strongly with energy due to low-x PDFs (esp. gluon)
$\Rightarrow$ unitarity violation



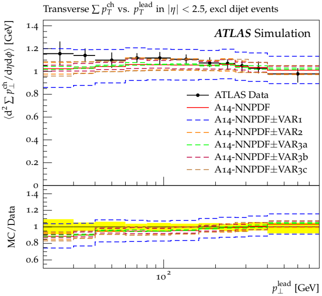






MC tuning

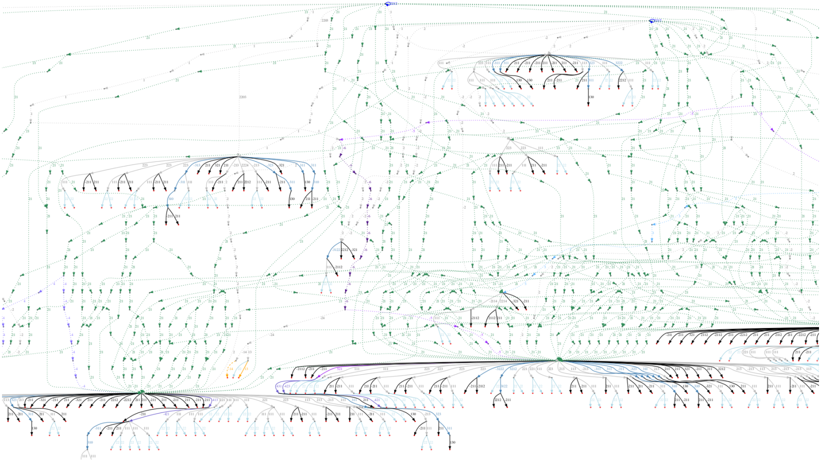
- Freedom to describe data with generator models, via the *ad hoc* and beyond-fixed-order components: MPI, hadronisation, ~ showers
- Need to be careful! A pragmatic trick at LO may backfire spectacularly when a better ME is added. Knowing the *limits* of a generator configuration is important
- A global view is crucial: one number/distribution can always be overtuned at cost of others
- Rivet & Professor/Apprentice tools used to build tunes & *eigentunes*



Using MC events…wisely

Example SHG event

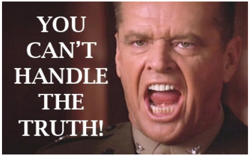
(Part of) a single-top HepMC event graph:

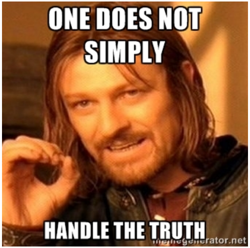


Ouch! How to safely *use* this??

Caveats on event record interpretation

- **The SHG generator (or LHE) event record is often called "truth" – a dangerous phrase.**
- We're doing quantum mechanics: there is no unambiguous truth!
  ⇒ event records are half-physics, half-debug-info…and zero indication of amplitude interference
- It gets worse: kinematic frames aren't defined (until the final-state) & momentum isn't necessarily conserved at vertices!
- **BEWARE!!**





Caveats on event record interpretation

- That said, like all good myths, there is a core of truth to the widespread physical interpretation of event records
- And sometimes precision EW or PDF theorists will request correction to partonic level rather than forward-folding of their calculations, e.g. "Born-level Z".

  NB. expts don't have to say "yes"!
- First **think about the physics** – e.g. is there a real distinction between hard photons and shower photons? Good discipline/introspection anyway!
- And first try to do what you need directly from the physical hadrons etc. See Rivet ↗ & ATLAS PUB note on safe truth observables ↗

Apply brain!

Conclusions

Summary

- Event generators are super-, super-important for LHC physics
- And demands are only increasing: we demand processes and levels of data description (and predictivity) that would have been laughed at pre-LHC
- Both experiment and theory owe a great deal to the few phenomenologists who've provided us with these codes
- SHGs based on a core of perturbative QCD (& EW) of increasing sophistication. And increasing CPU cost…
- Wrapped with perturbative iterated parton showers ⇒ resum logs & generate a good approximation to "real" final-states. Plus pheno models and tuning for the stuff we don't understand *ab initio*
- **Follow-up material:**
  MCnet review – arXiv:1101.2599
  Hoeche review – arXiv:1411.4085
  "Practical Collider Physics" – AB, White, White
  "QCD & Collider Physics" – Ellis, Stirling, Webber
  "QCD" – Dissertori, Knowles, Schmelling
  MCnet summer schools: 2018 2019 2022

Some hands-on exercises

Being able to run event generators yourself (rather than just using experiment samples) is a big advantage.

- Get Pythia 8 and generate QCD dijet events with ME $\hat{p}_T$ cutoff of 50 GeV. Analyse with Rivet's `MC_JETS` analysis. What does it look like if you change the cutoff? How about the shower α_s?
- Get the POWHEG-BOX heavy-flavour (hvq) process and generate $t\bar{t}$ events. Shower with Pythia and compare to top-pair production in LO Pythia using also the `MC_TTBAR` analysis: differences in normalisation? shapes?
- Get MG5 and generate a `p p > z > mu+ mu-` process with Pythia showering. Use `add process` to merge this with $+1, 2$ jets MEs. Analyse with Rivet or MadAnalysis5. What changes?
- Generate a BSM process, e.g. SUSY gluino pair production in Pythia's built-in processes (you will need an SLHA file, e.g. from an experiment publication), or a FeynRules UFO model like SMEFTsim ↗ in MG5

The MCnet hepstore ↗ Docker images may be useful