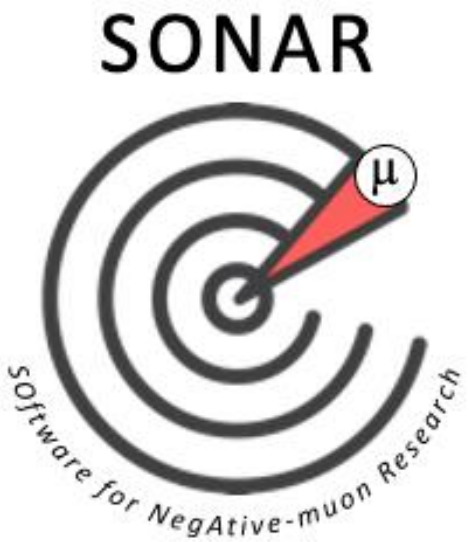
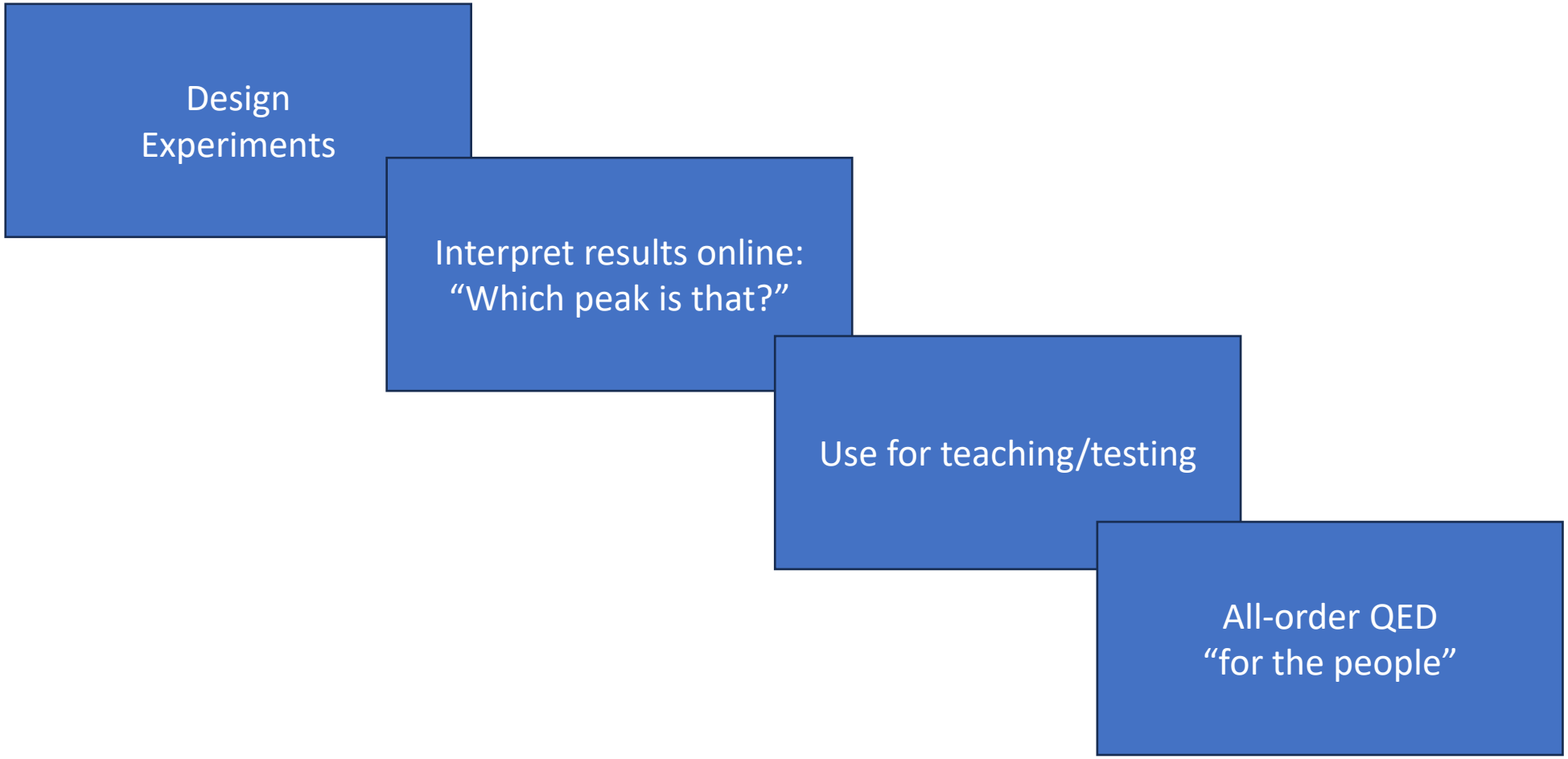
Increasing Capabilities of MuDIRAC



Ben Ohayon
Technion IIT
(QUARTET & Ref-Rad collaborations)

Energy calculations in MuDirac:



Status:

Mass: Atomic mass AME

RMS Radius: Angeli & Marinova

Nuclear model: Point
Sphere
Fermi-2.3fm

Wishlist 1: enable input of nuclear mass

Why change nuclear mass:

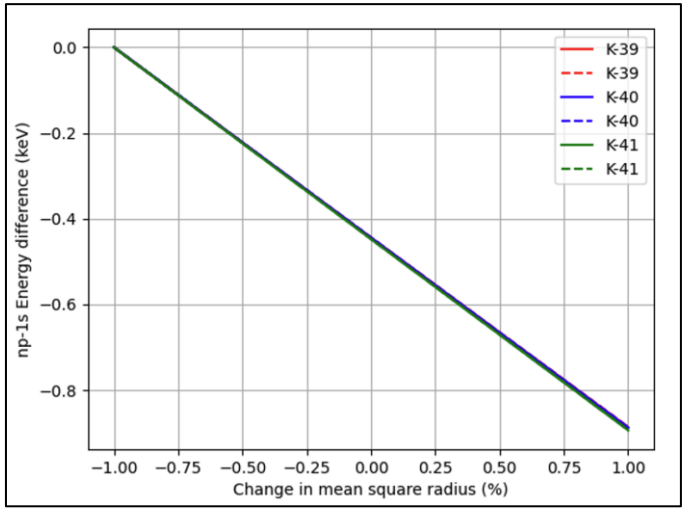
- Implement correct mass (nuclear vs. atomic)
- Test recoil corrections by taking M->infinity

	Status:	**Wishlist**:
Mass:	Atomic mass AME	Light nuclei (proton, deuteron,…) mass from here The rest from: Nuclear mass $\approx Atomic\ mass - Z \times m_e$ **User input** mass (useful also to take to ∞)
RMS Radius:	Angeli & Marinova	
Nuclear model:	Point Sphere Fermi-2.3fm	

Wishlist 2: enable input of nuclear radius and model

Why change radius:

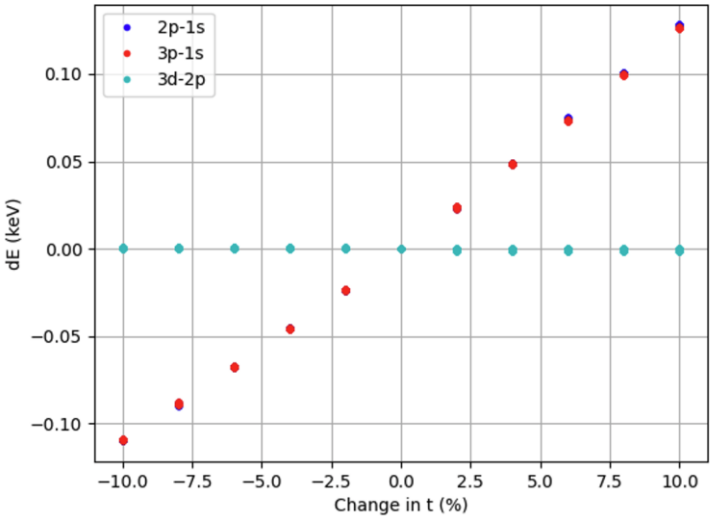
- Implement updated radii from literature
- Test sensitivity: $\frac{dE}{dR^2}$ ⟷



- Add other effective contact terms (e.g. Darwin-Foldi)

Why change nuclear model:

- Implement measured models (mostly from VJV1987)
- Check model-dependency of results (e.g. scan Fermi skin parameter “t”) ⟷



- Compare with analytical perturbative calculations (usually **gaussian** and **exponent**)

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Nuclear model:	Point Sphere Fermi-2.3fm	

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Nuclear model:	Point Sphere Fermi-2.3fm	Gaussian and exponent (for comparing with literature) **User input** fermi skin thickness parameter Measured models (De-Vries 1987)

Example on usefulness of scanning M and light-speed:

$$\Delta E_{VP} = E\ (point, Eehling{:}True) - E\ (point, Eehling{:}False)$$

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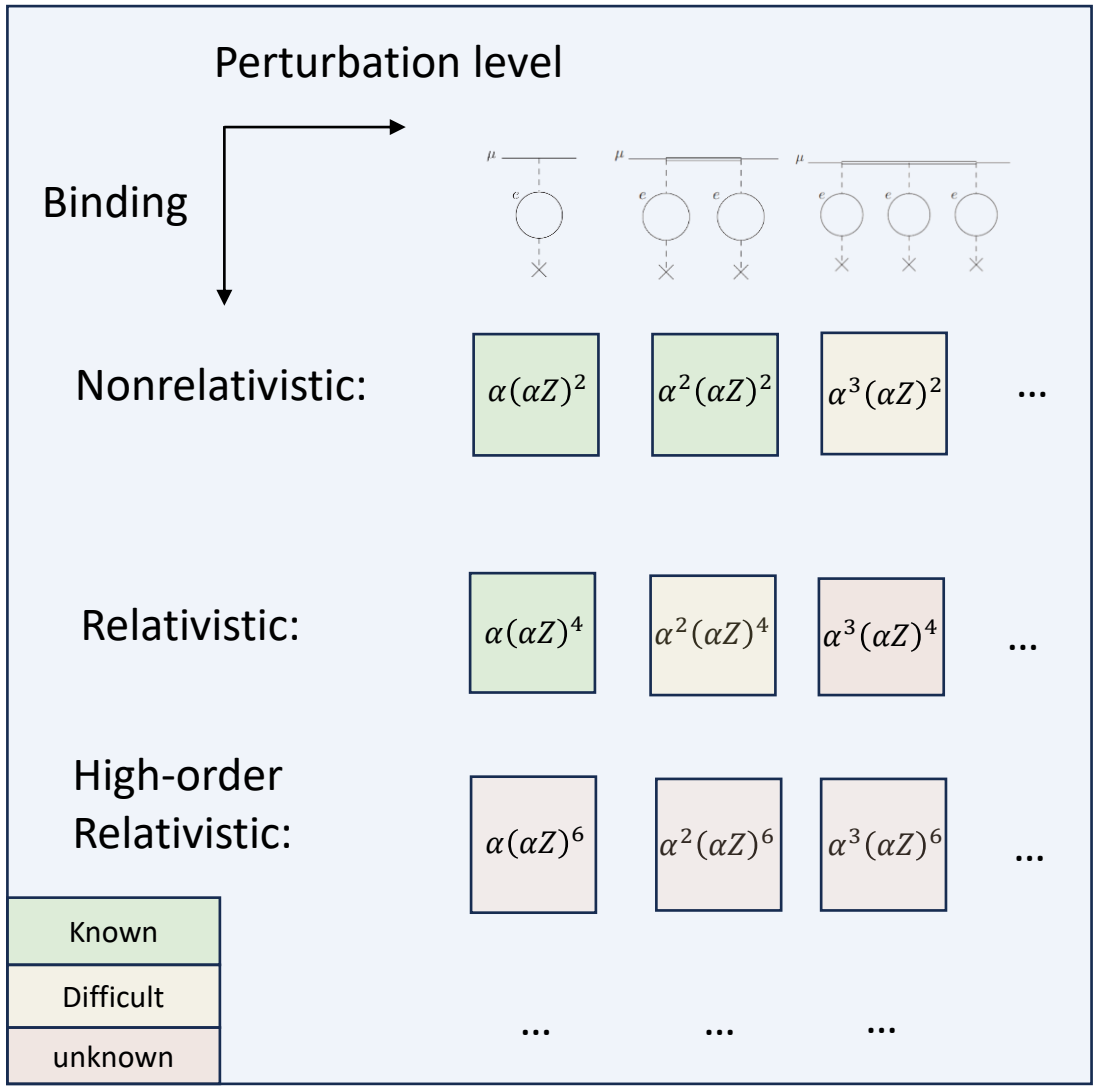
What is inside ΔE_{VP}:



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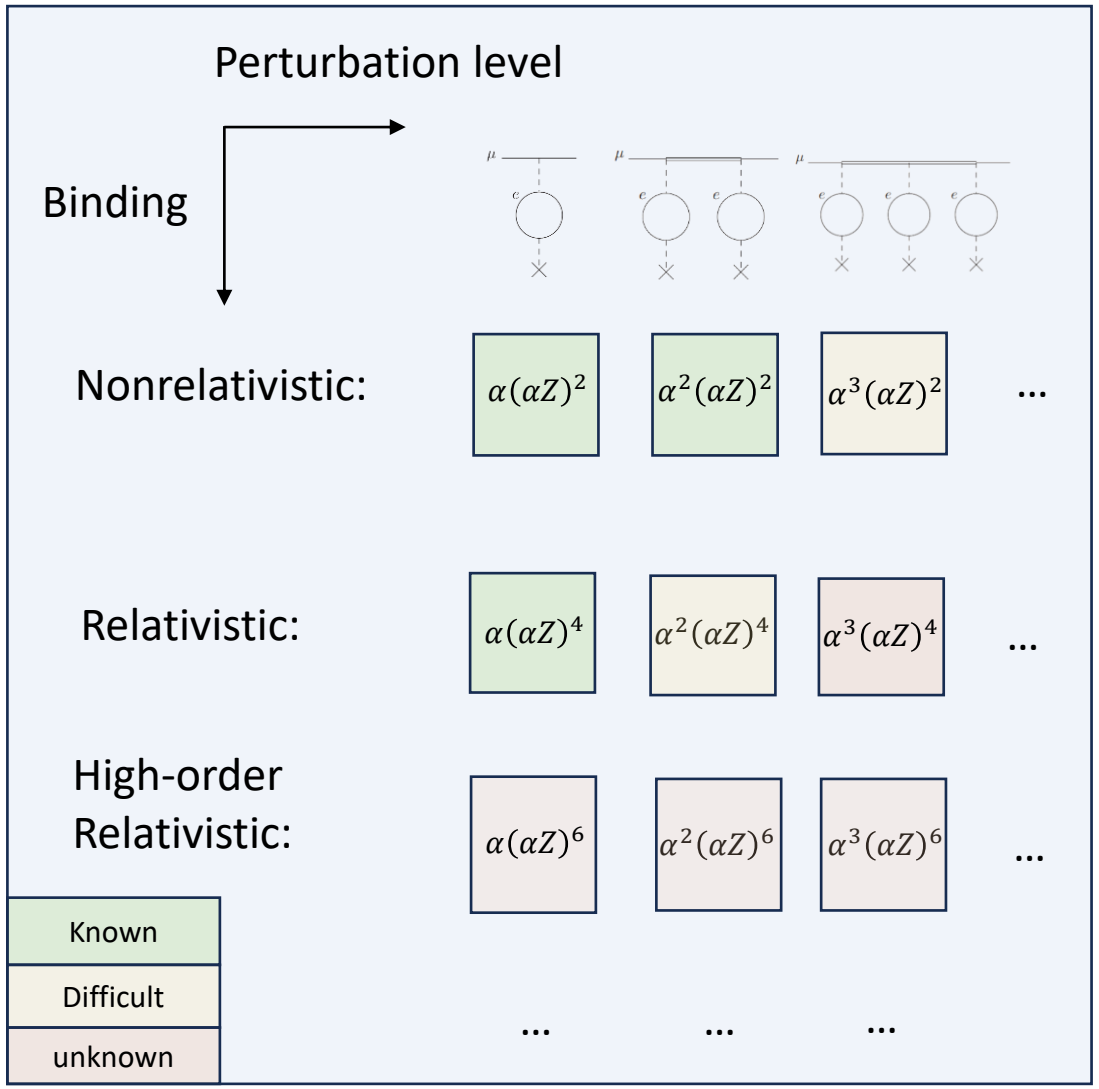

- Missing relativistic-recoil beyond reduced mass.
  Isolate by taking **M->infinit**y and complement with perturbation theory
- Isolate nonrelativistic terms by taking **C->infini**ty.
- Isolate high-order remainder by subtracting first terms from perturbation theory.

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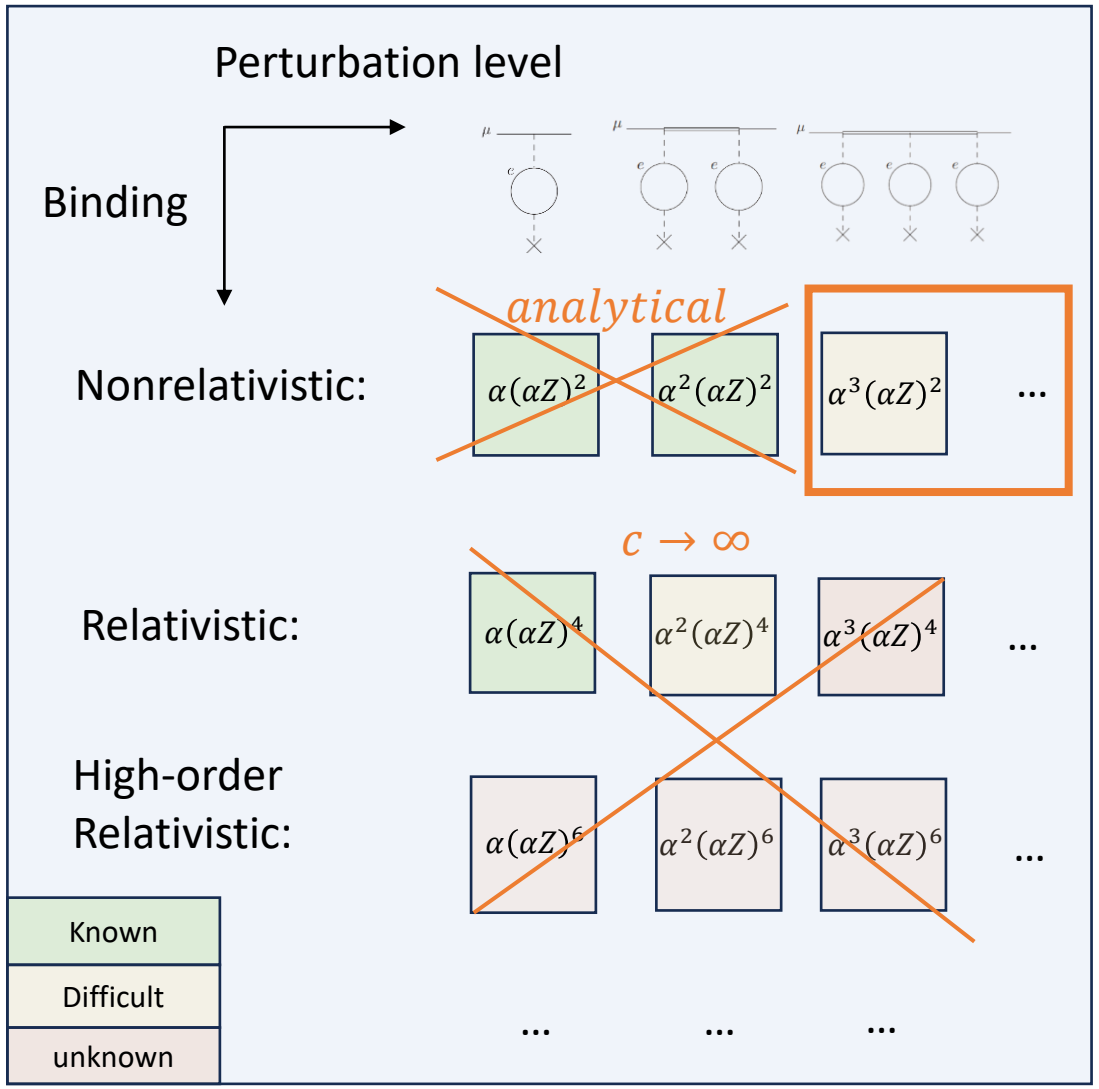

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Nuclear model:	Point Sphere Fermi-2.3fm	Gaussian and exponent (for comparing with literature) **User input** fermi skin thickness parameter Measured models (De-Vries 1987)
Light speed / Alpha :	Regular	“infinity” (removing relativistic effects)

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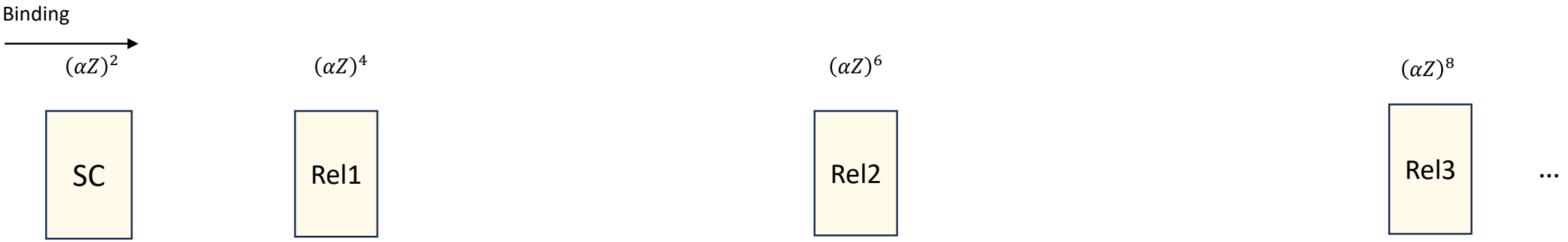
RMS Radius:	Angeli & Marinova	Regular updates for example: • H to ^{4}He from here • ^{6}Li to ^{13}C from here • … **User input** RMS radius (highly useful) Future: MuDirac+Data=updated Radius + More digits (for individual energies)
Nuclear model:	Point Sphere Fermi-2.3fm	Gaussian and exponent (for comparing with literature) **User input** fermi skin thickness parameter Measured models (De-Vries 1987)
Light speed / Alpha :	Regular	"infinity" (removing relativistic effects)

Adding more QED terms

What we have: Dirac-Coulomb, point, no VP:



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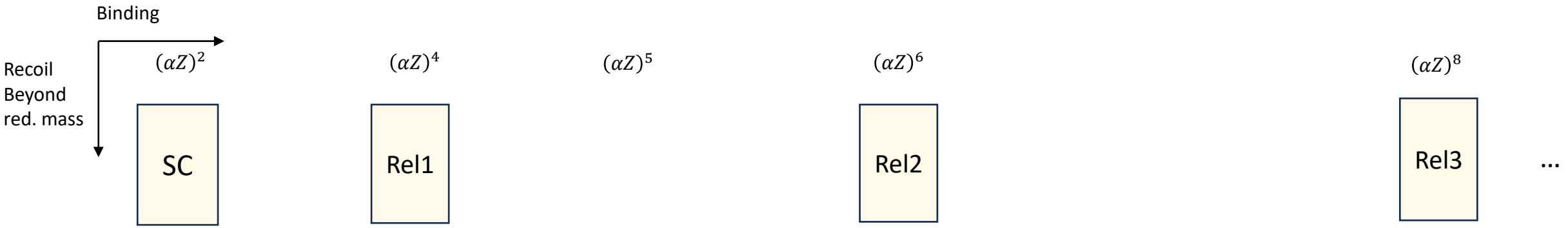
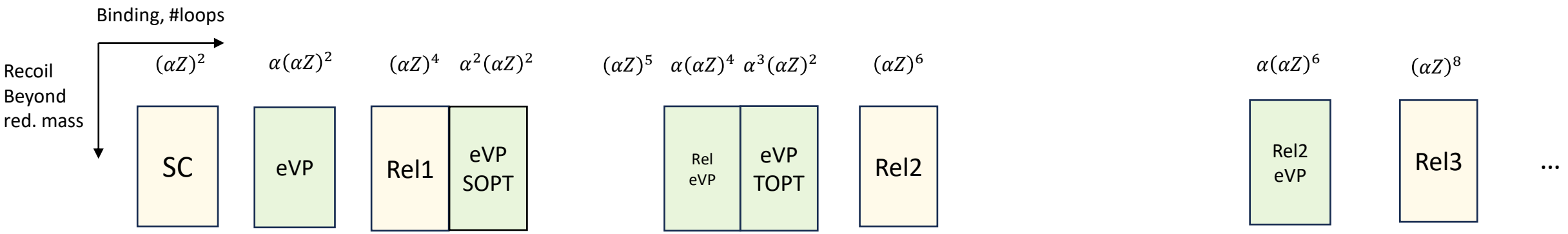


Missing relativistic recoil:

What we have: Dirac-Coulomb-Uehling, point:



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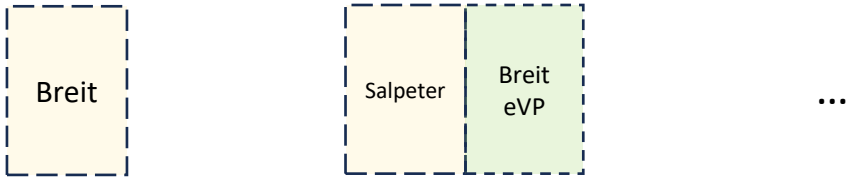
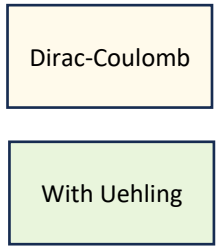
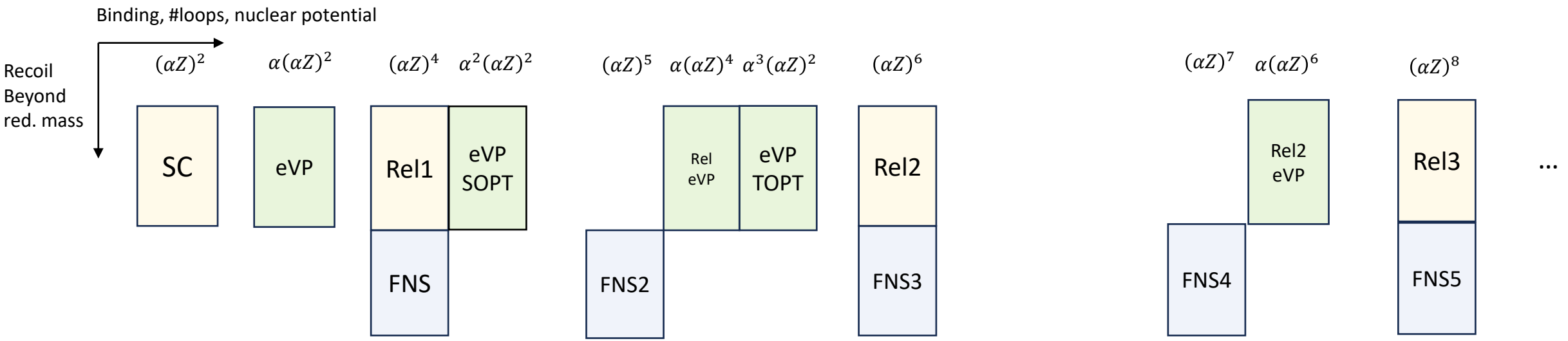




What we have: Dirac-Coulomb-Uehling, finite-size:



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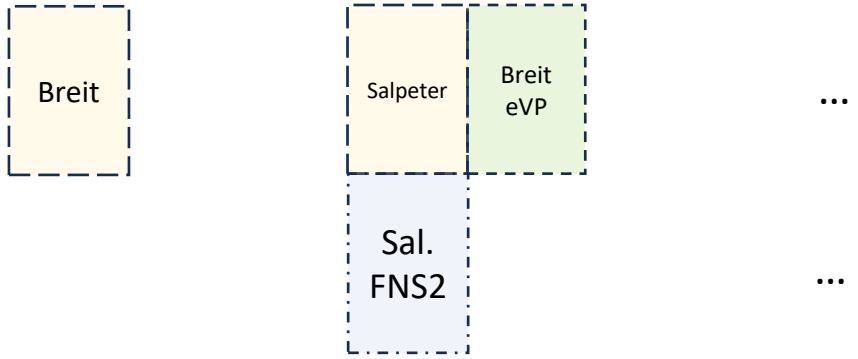

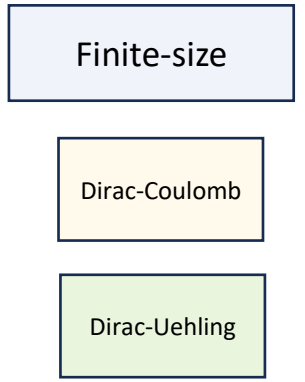

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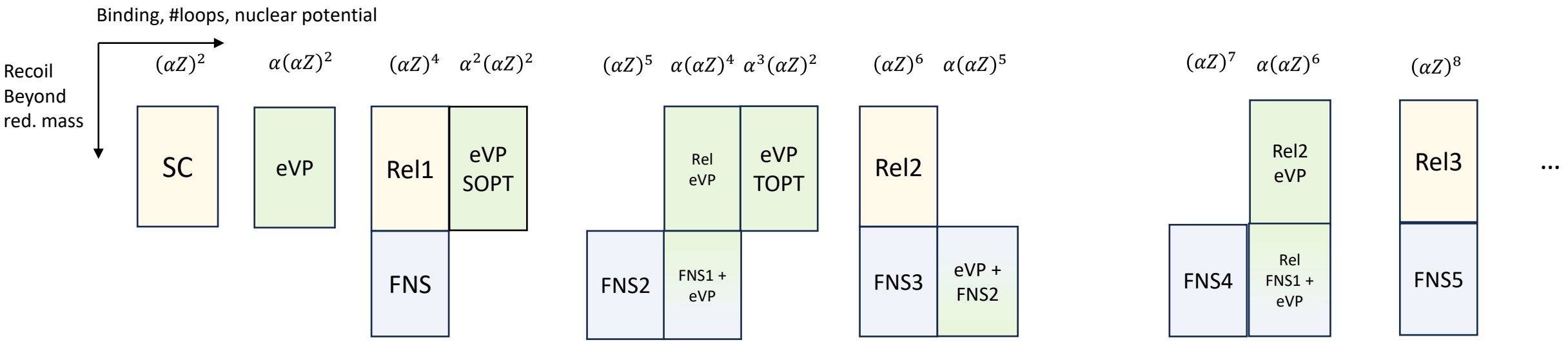

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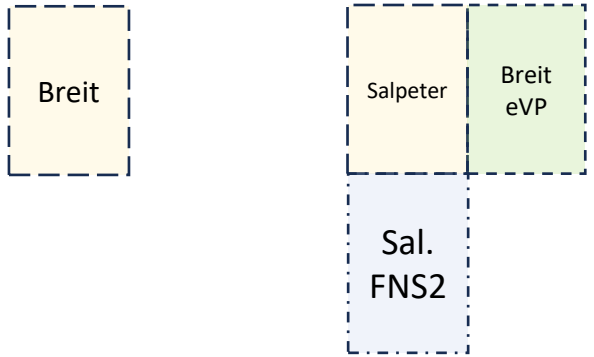

Finite-size

Dirac-Coulomb

Dirac-Uehling

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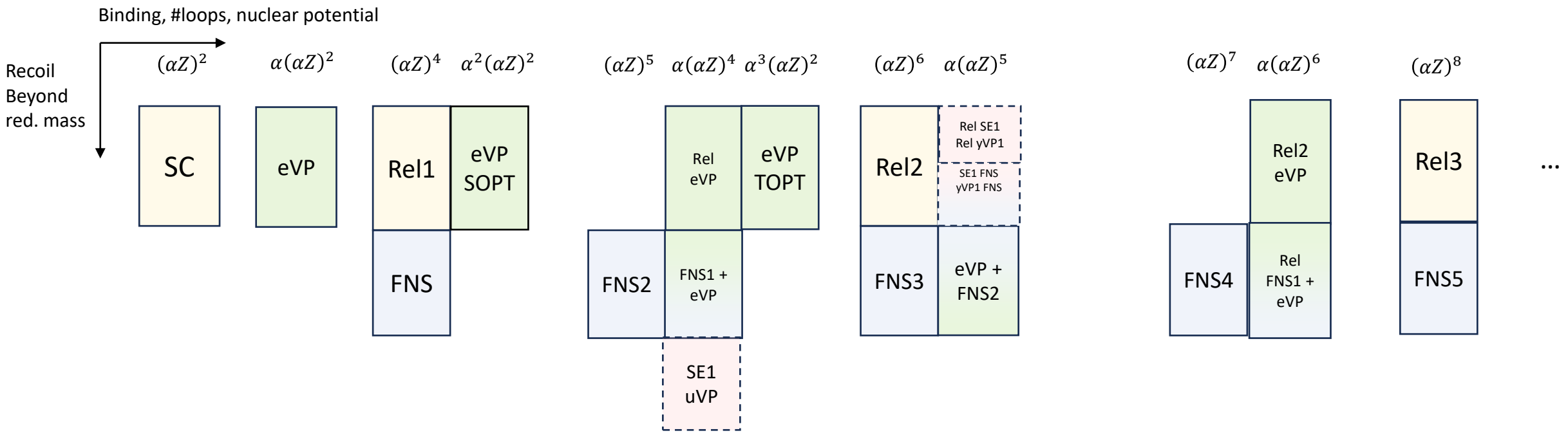

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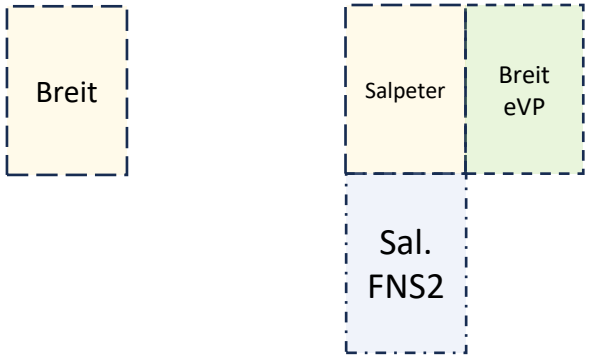

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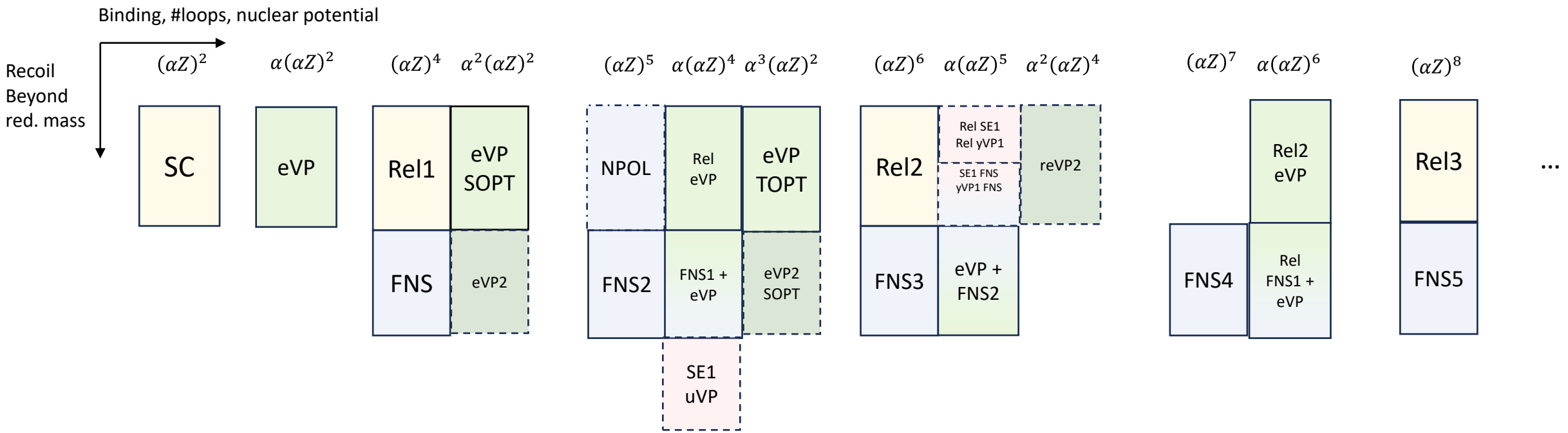

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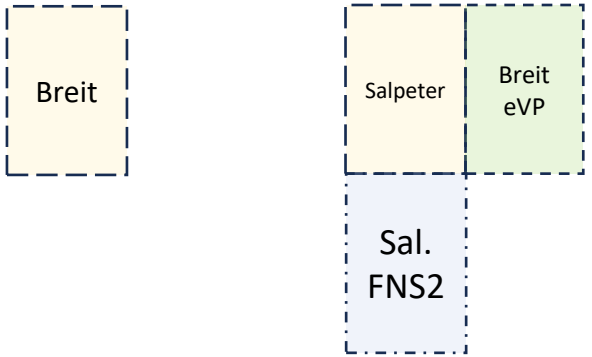

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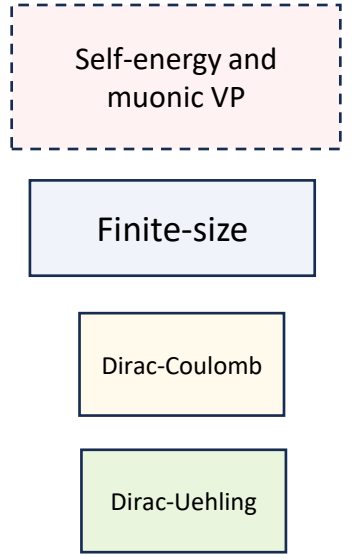

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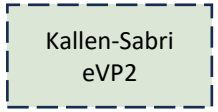

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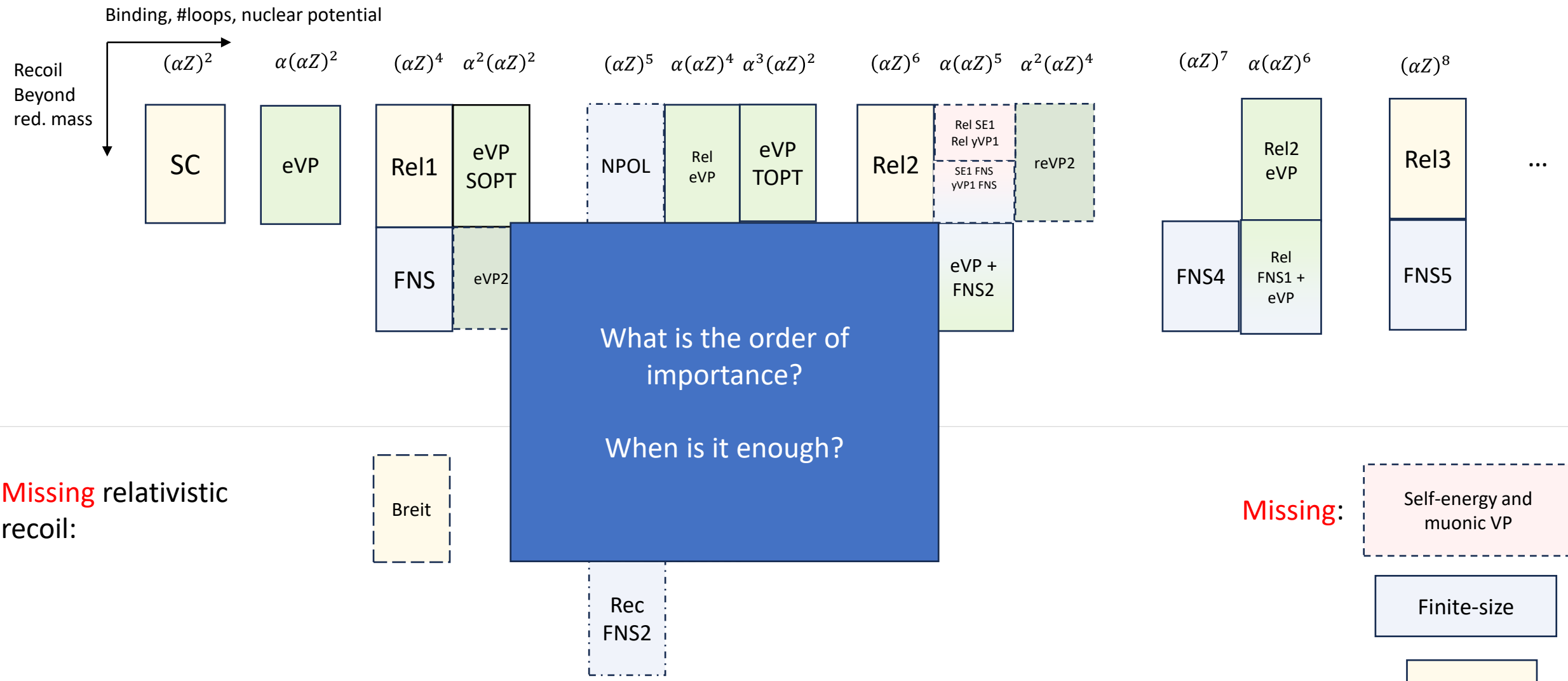
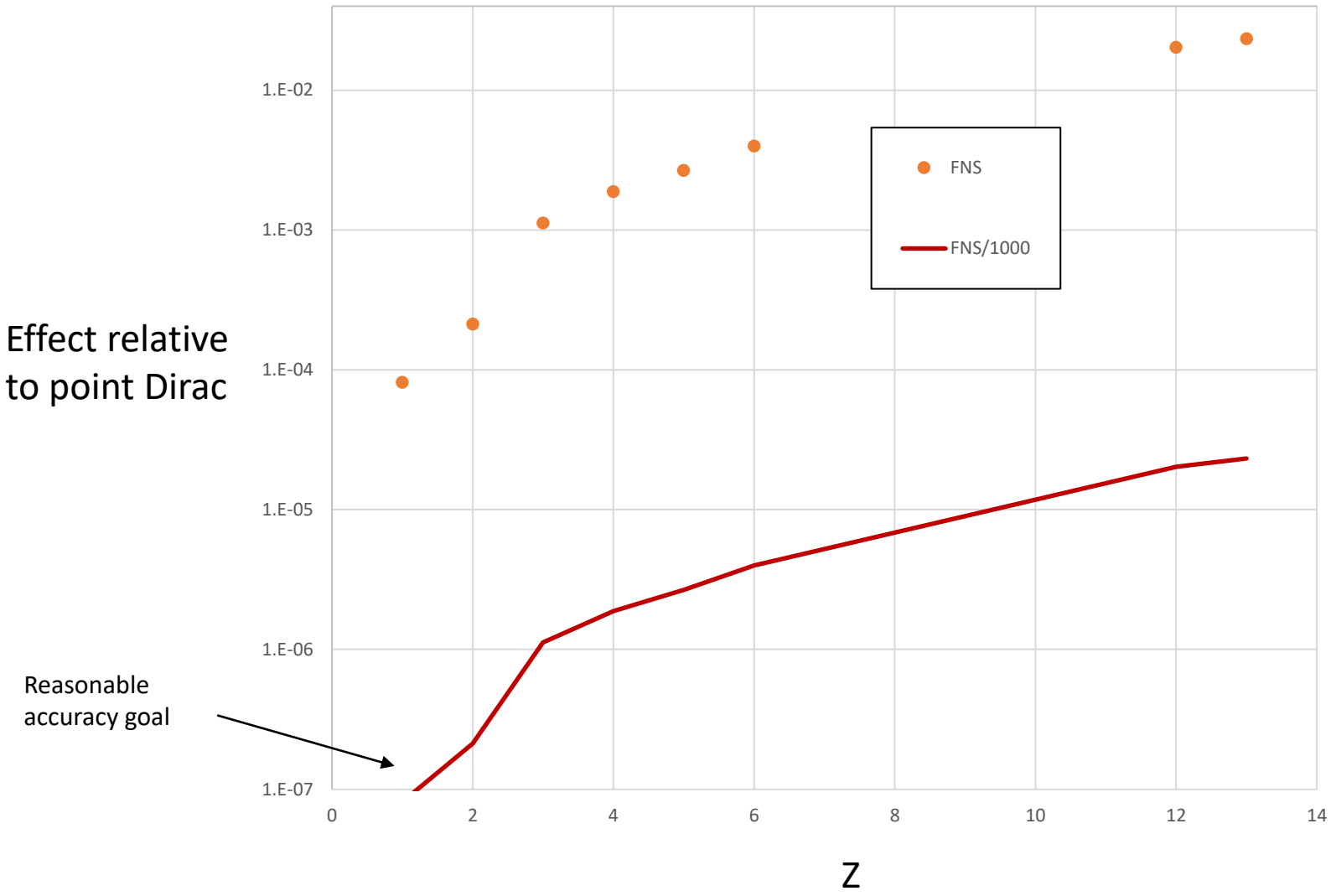
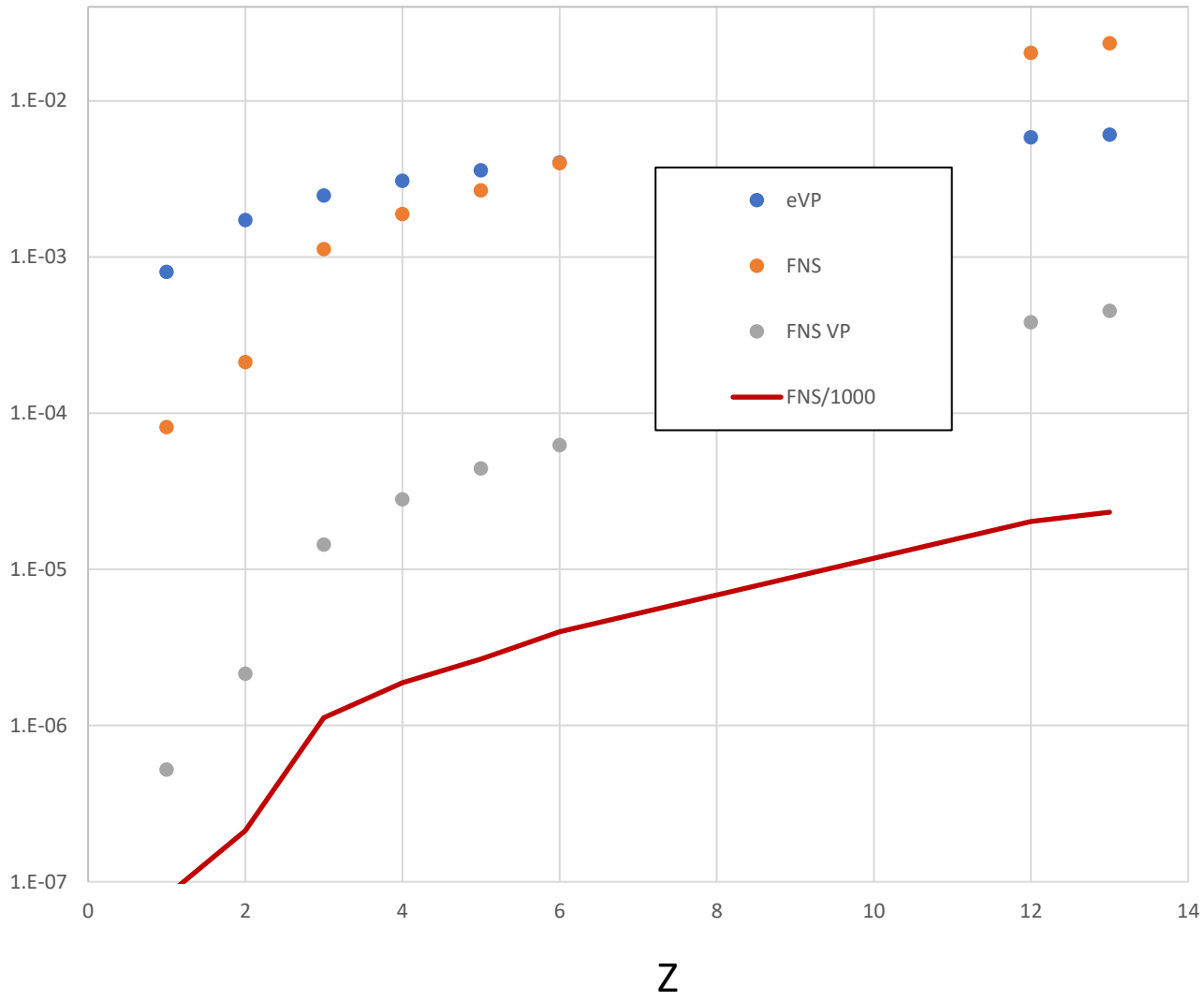


Missing contributions (1S state):



Missing contributions (1S state):

Effect relative to point Dirac



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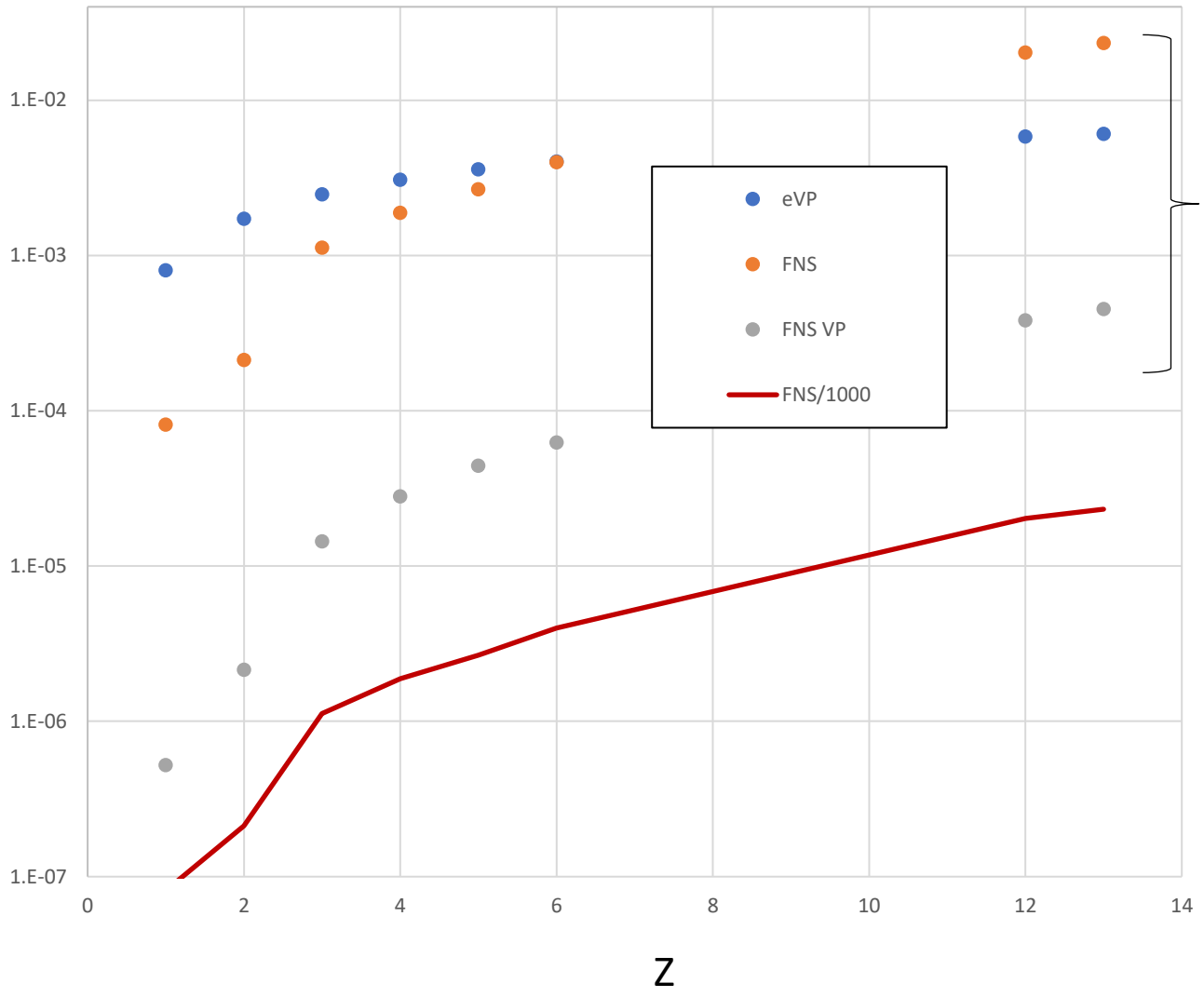

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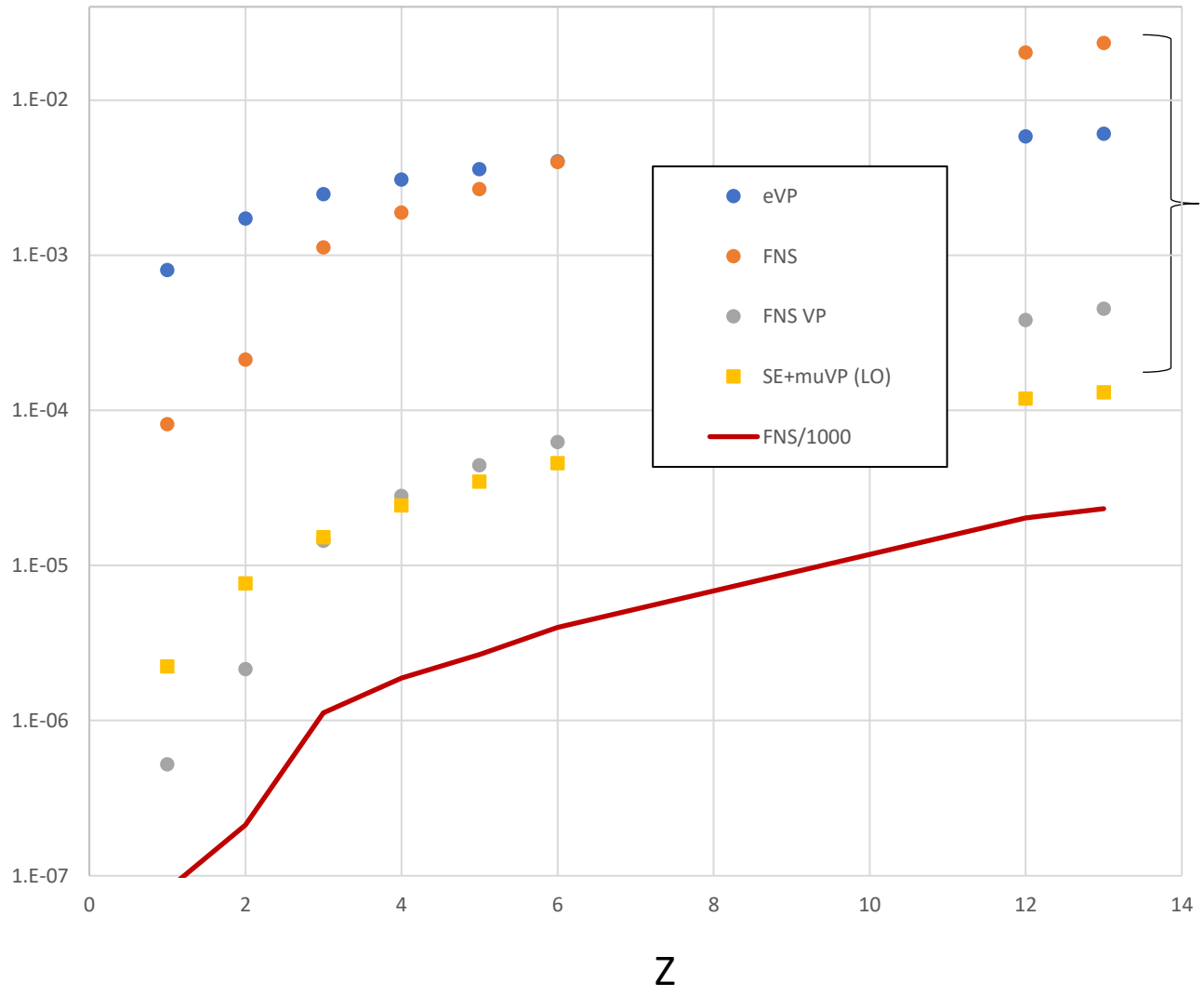

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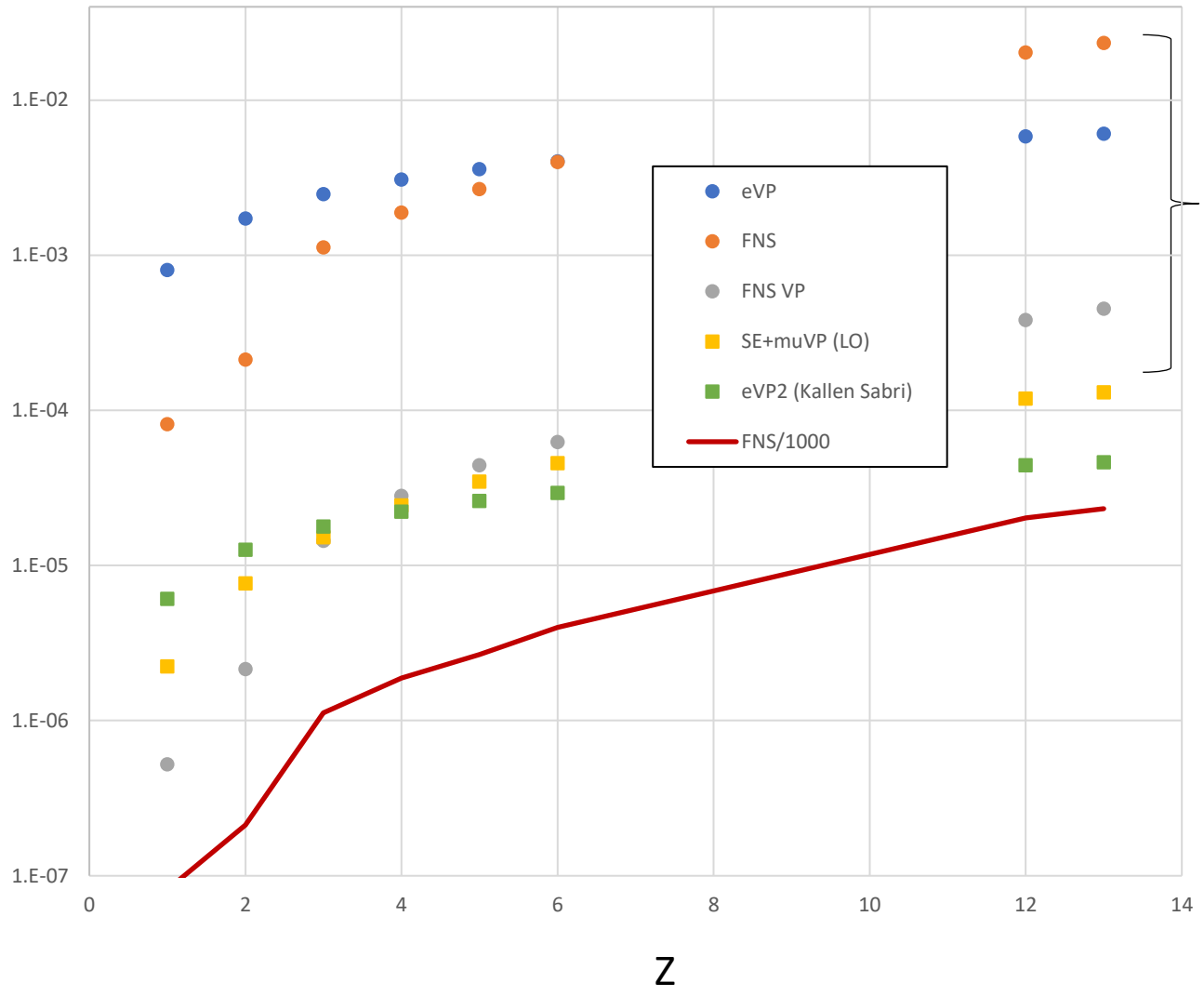

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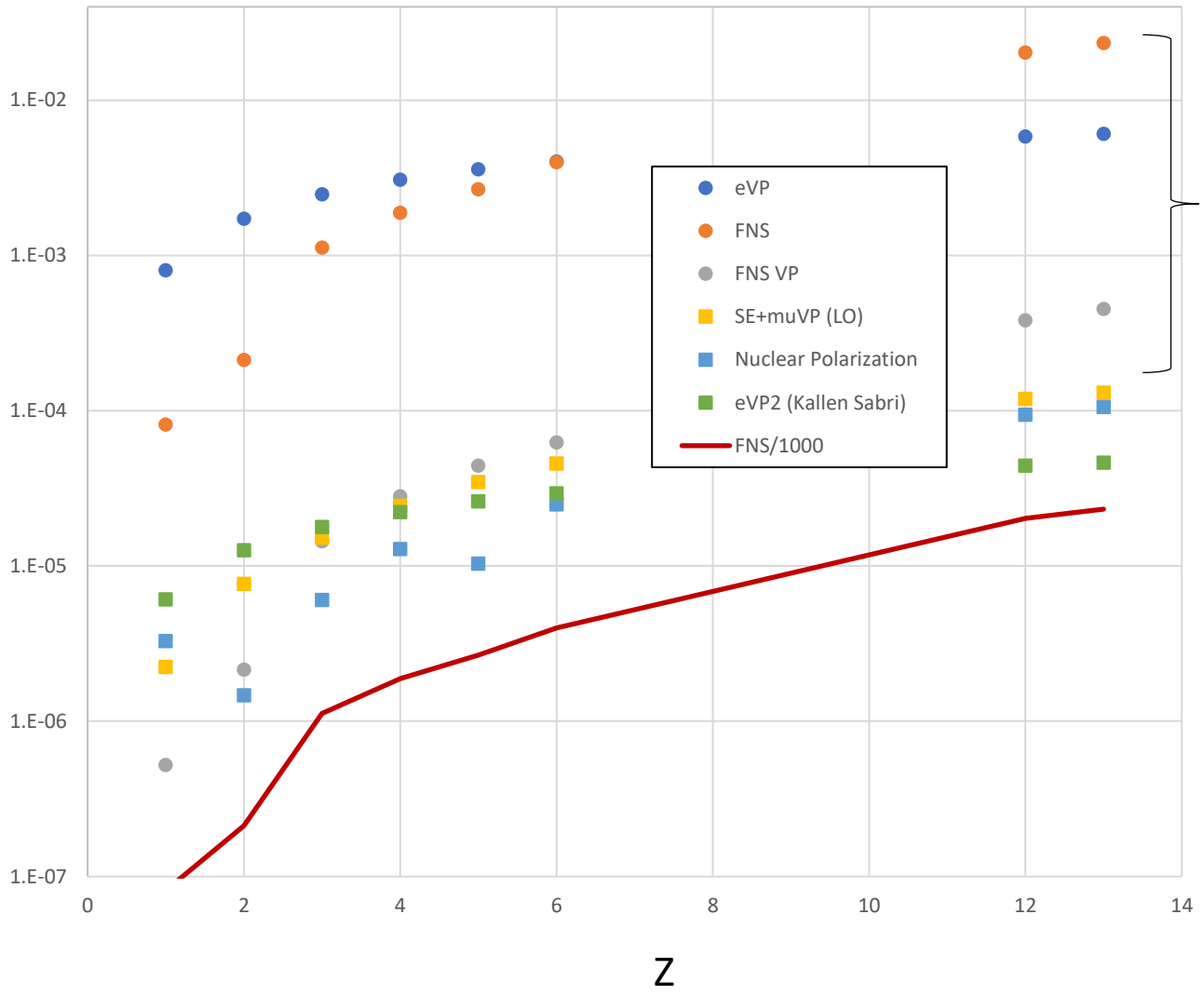

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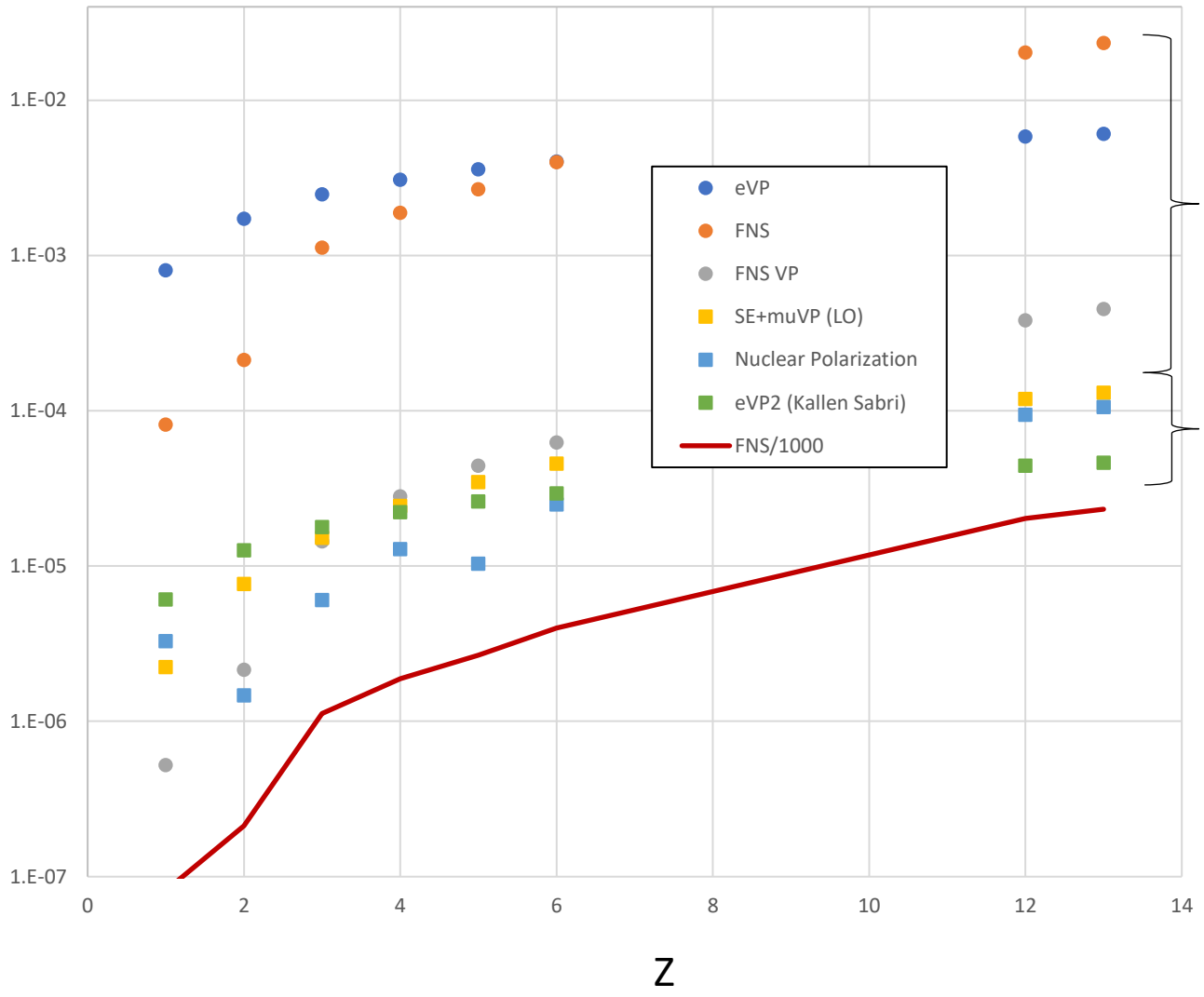

Implemented in MuDirac

I suggest for 2.0
(x10 accuracy)

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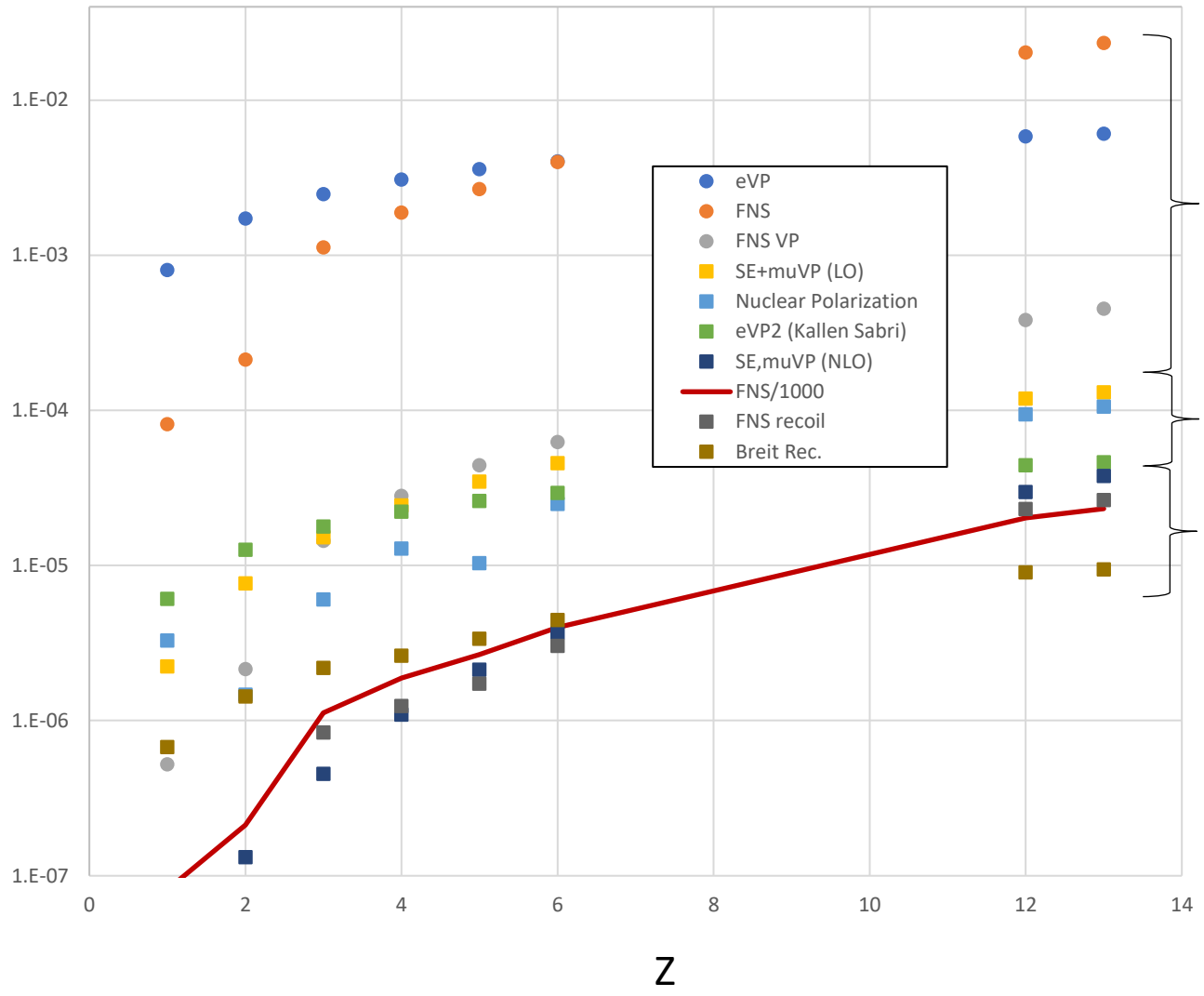

Z

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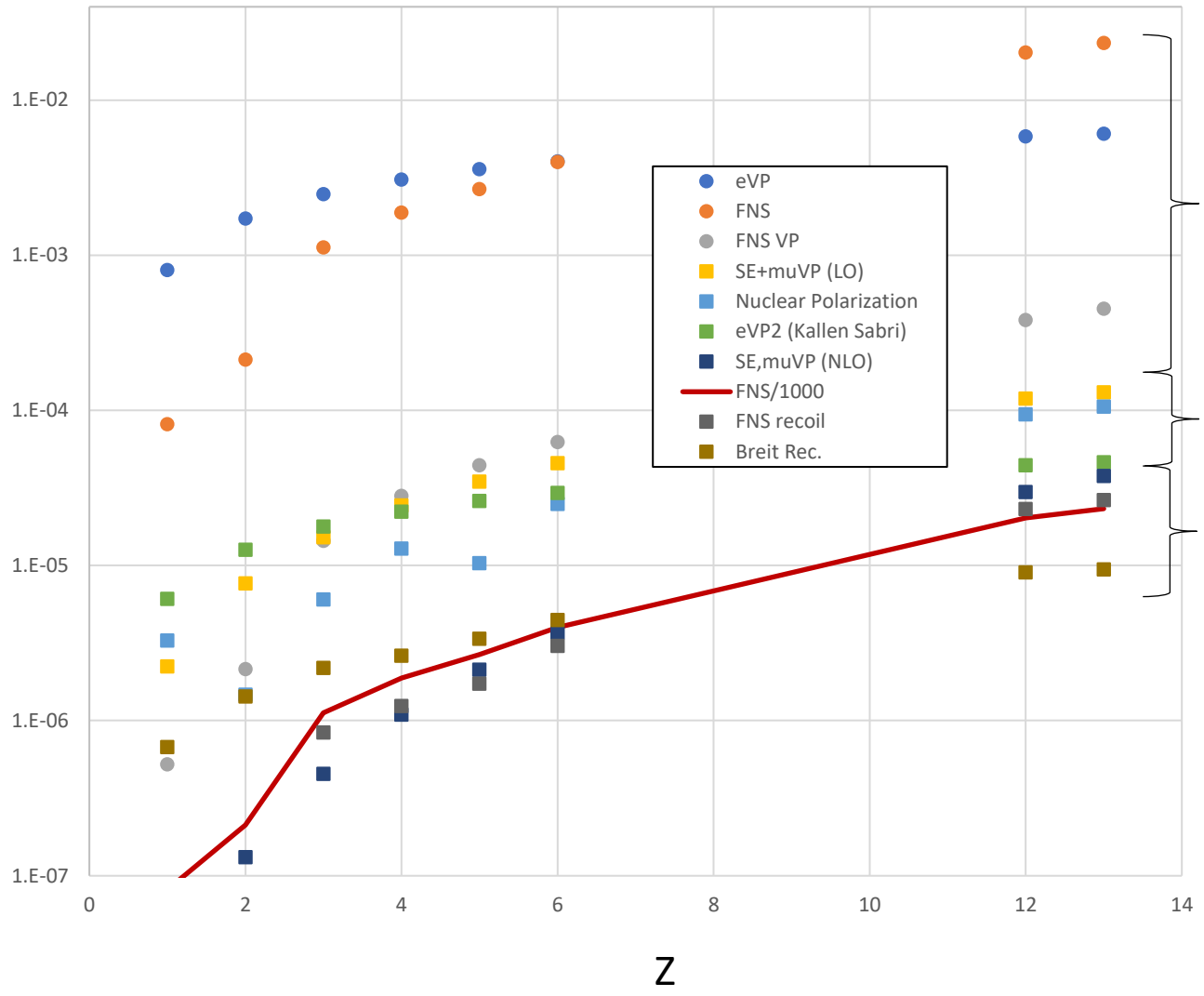
Future

Missing contributions (1S state):

Effect relative to point Dirac



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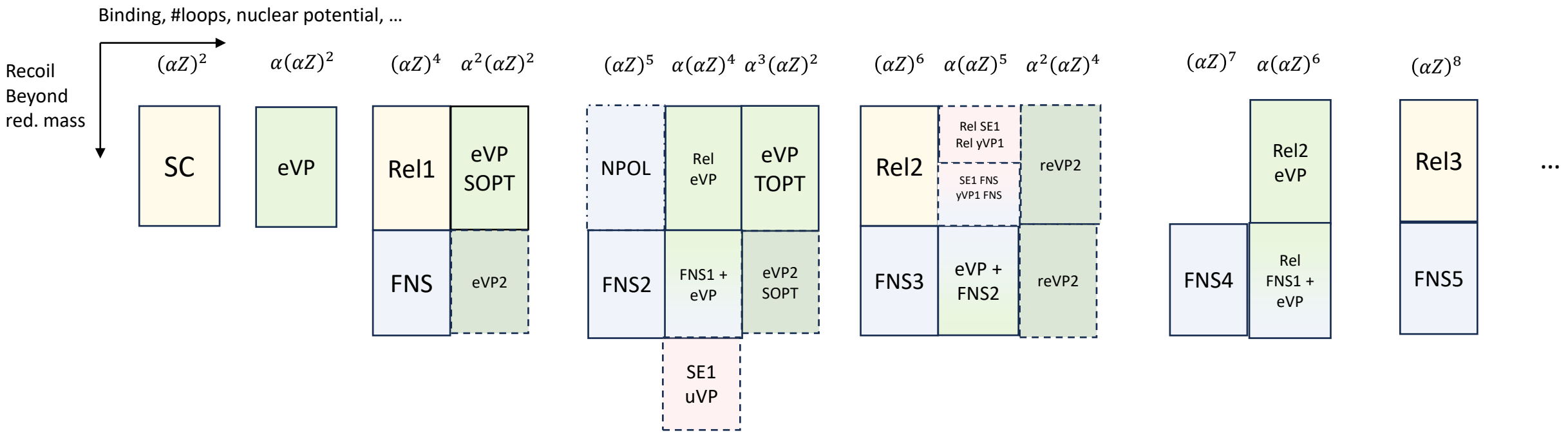
Future

Light systems are more difficult than medium mass systems!

More QED, where to start?



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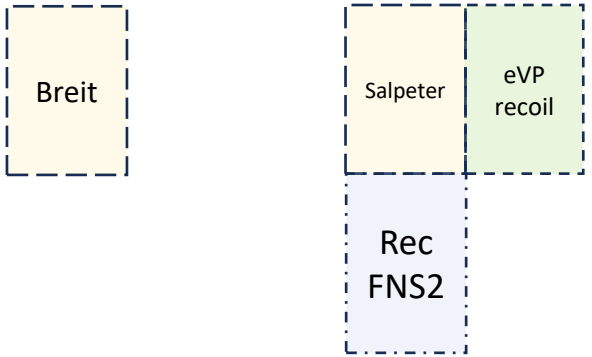

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Finite-size

Dirac-Coulomb

Missing:

Kallen-Sabri eVP2

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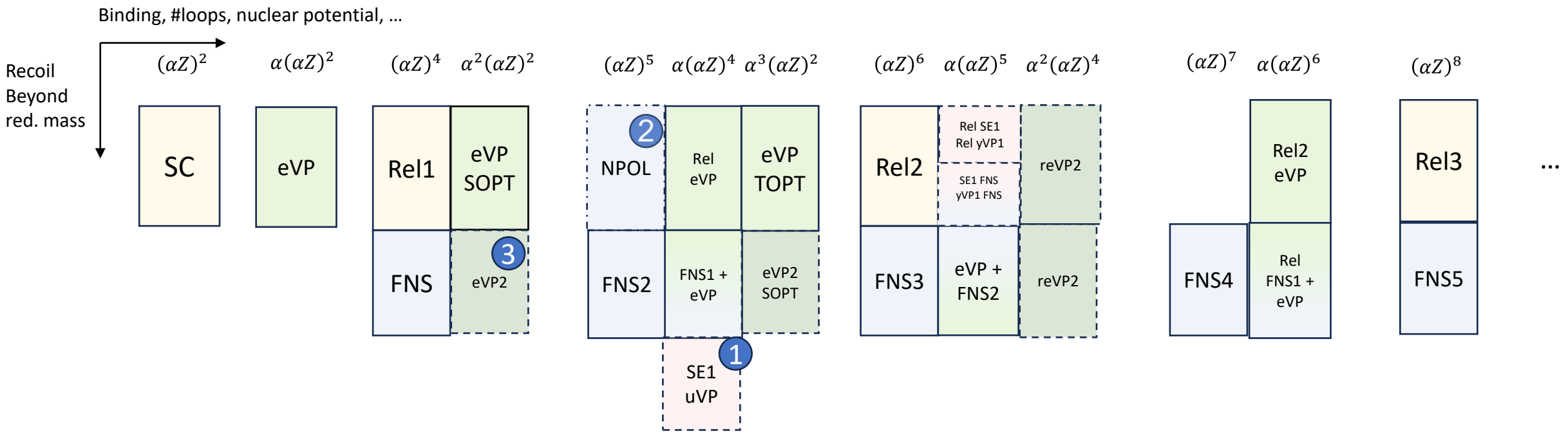

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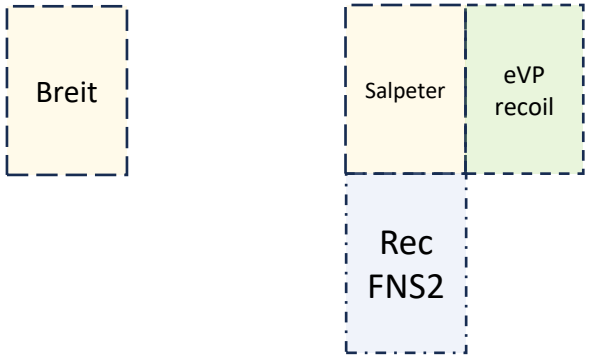

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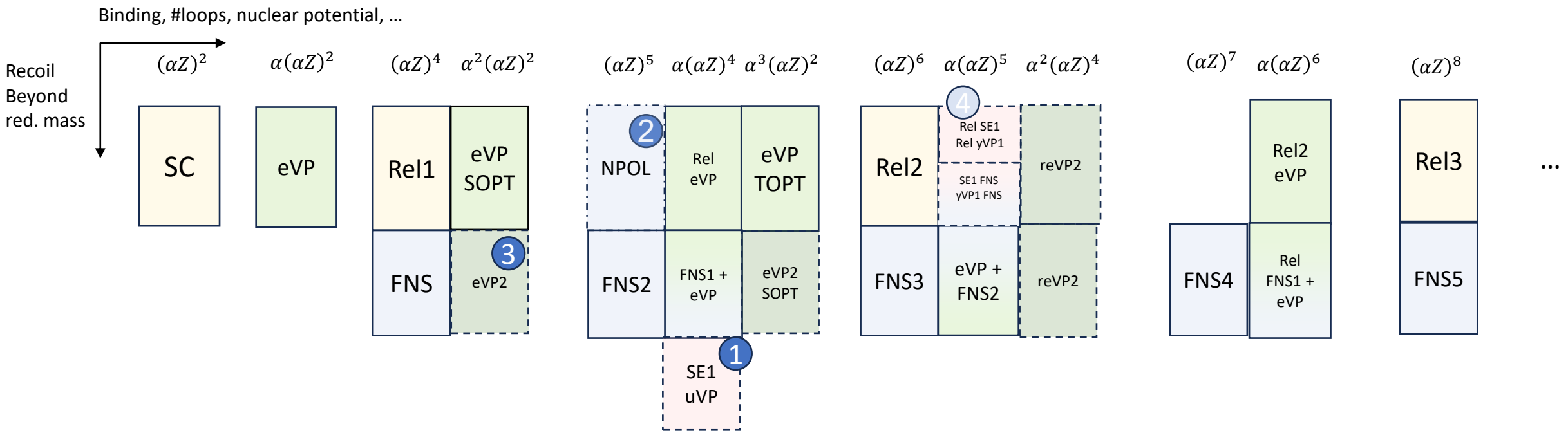

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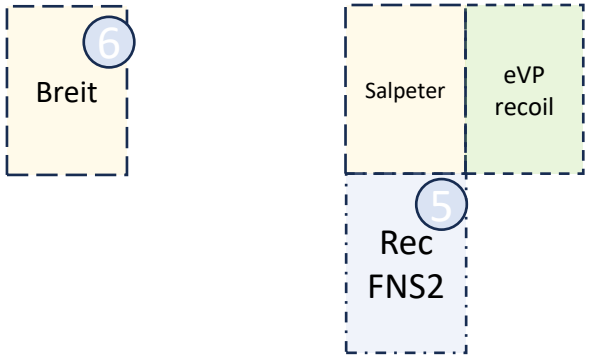

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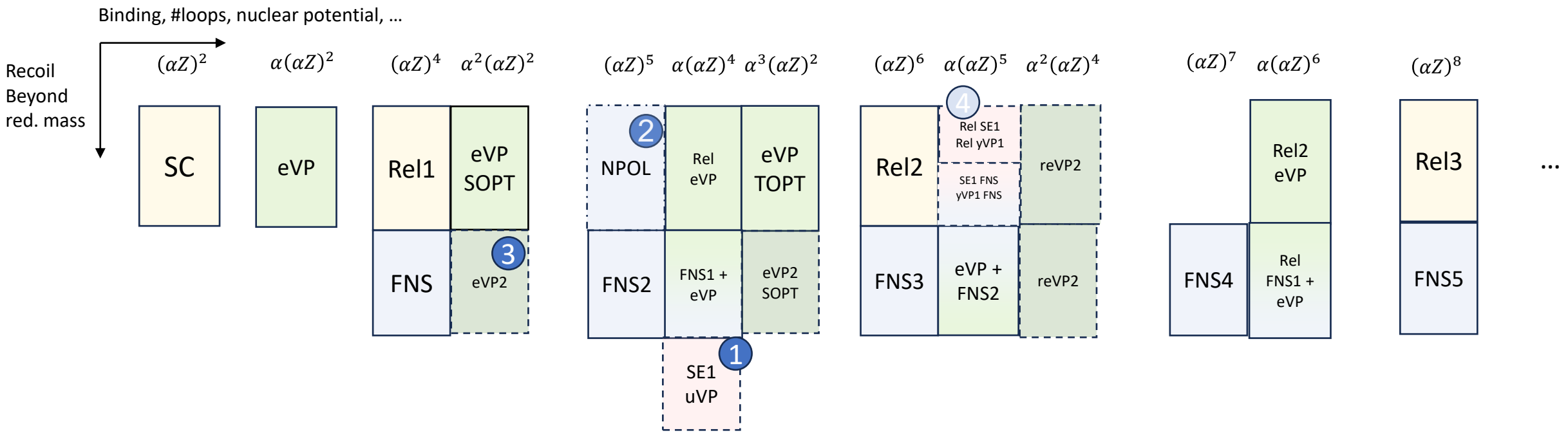

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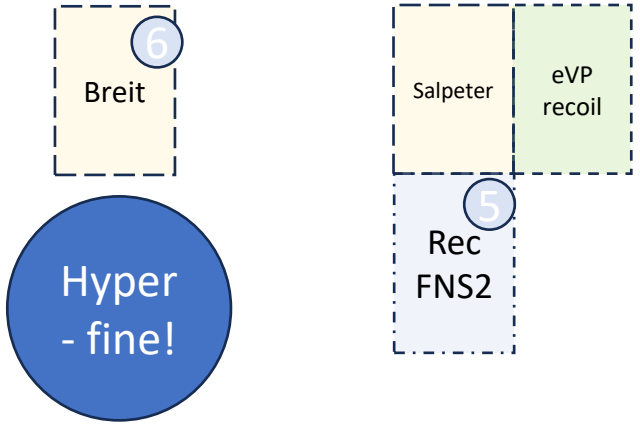

Missing:

- Self-energy and muonic VP
- Finite-size
- Dirac-Coulomb
- Dirac-Uehling

Missing:

- Kallen-Sabry eVP2

Adding the missing pieces:

1a. Leading-order self-energy corrections:

Simple equation valid for muonic atoms:

$$\Delta E(n\mathrm{S}) = \frac{\alpha}{\pi}\frac{(Z\alpha)^4 m}{n^3}\left(\frac{\mu}{m}\right)^3\left[\left(\frac{4}{3}\ln\left[\frac{m}{\mu}\frac{1}{(Z\alpha)^2}\right]+\frac{10}{9}\right)-\frac{4}{3}\ln k_0(nS)\right]$$

$$\Delta E(n\ell_j)\big|_{\ell\neq 0} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4 m}{n^3}\left(\frac{\mu}{m}\right)^3\left(-\frac{1}{2\kappa(2\ell+1)}\frac{m}{\mu}-\frac{4}{3}\ln k_0(n\ell)\right)$$

Calculate Bethe logs, or lookup table:

Bethe Logarithms $\ln k_0(n\ell)$ for $n \le 12$ and $\ell \le 4$

	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
	$\ln k_0 \times 10^{-1}$	$\ln k_0 \times 10^{1}$	$\ln k_0 \times 10^{2}$	$\ln k_0 \times 10^{2}$	$\ln k_0 \times 10^{2}$
$n = 1$	0.298 412 856	—	—	—	—
$n = 2$	0.281 176 989	-0.300 167 086	—	—	—
$n = 3$	0.276 766 361	-0.381 902 294	-0.523 214 814	—	—
$n = 4$	0.274 981 184	-0.419 548 946	-0.674 093 888	-0.173 366 148	—
$n = 5$	0.274 082 373	-0.440 346 956	-0.760 075 126	-0.220 216 838	-0.077 209 890
$n = 6$	0.273 566 421	-0.453 121 977	-0.814 720 396	-0.250 217 976	-0.096 279 743
$n = 7$	0.273 242 913	-0.461 551 773	-0.851 922 329	-0.270 909 573	-0.109 447 274
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$n = 12$	0.272 617 934	-0.479 171 116	-0.934 295 399	-0.318 351 910	-0.140 146 873

Adding the missing pieces:

1a. Leading-order self-energy corrections:

Simple equation valid for muonic atoms:

$$\Delta E(n\mathrm{S}) = \frac{\alpha}{\pi}\frac{(Z\alpha)^4 m}{n^3}\left(\frac{\mu}{m}\right)^3\left[\left(\frac{4}{3}\ln\left[\frac{m}{\mu}\frac{1}{(Z\alpha)^2}\right]+\frac{10}{9}\right)-\frac{4}{3}\ln k_0(nS)\right]$$

$$\Delta E(n\ell_j)|_{\ell\neq 0} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4 m}{n^3}\left(\frac{\mu}{m}\right)^3\left(-\frac{1}{2\kappa(2\ell+1)}\frac{m}{\mu}-\frac{4}{3}\ln k_0(n\ell)\right)$$

1b. Leading-order muon VP:

- Option 1: Replace 10/9 with 38/45 above (only valid for muonic atoms)
- Bonus: Hadron-VP is muon-VP times 0.675(16) (only valid for muonic atoms)
- Option 2: change mass in Euhling potential (valid also for e.g. antiprotonic atoms)

Calculate Bethe logs, or lookup table:

Bethe Logarithms $\ln k_0(n\ell)$ for $n \le 12$ and $\ell \le 4$

	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
	$\ln k_0 \times 10^{-1}$	$\ln k_0 \times 10^{1}$	$\ln k_0 \times 10^{2}$	$\ln k_0 \times 10^{2}$	$\ln k_0 \times 10^{2}$
$n=1$	0.298 412 856	—	—	—	—
$n=2$	0.281 176 989	-0.300 167 086	—	—	—
$n=3$	0.276 766 361	-0.381 902 294	-0.523 214 814	—	—
$n=4$	0.274 981 184	-0.419 548 946	-0.674 093 888	-0.173 366 148	—
$n=5$	0.274 082 373	-0.440 346 956	-0.760 075 126	-0.220 216 838	-0.077 209 890
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- Need look-up table with values from literature. Also new initiatives at Mainz (Bacca, Gorschteyn, Orishkina, …)
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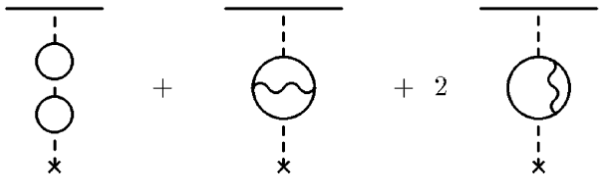
3. Kallen-Sabri potential

- Same as was done with Euhling.



Example: Ongoing reevaluation of ^{27}Al radius

TABLE V. Contributions to the energy levels of μ^{27}Al, eV. Considered accuracy goal is 5 eV.

	Order	^{1}s	[7]	^{2}p$_{1/2}$
Dirac	$(\alpha Z)^{2+}$	474515		118696
Breit Recoil	$(\alpha Z)^4$	4		0.
Sum		474519		118969
Recoil				
FNS	$(\alpha Z)^5$	13(5)		0
Salpeter	$(\alpha Z)^5$	−4		0.
Sum		9(5)	10(2)	0.
VP1				
NR	$\alpha(\alpha Z)^2$	2855		265
Rel.	$\alpha(\alpha Z)^4$	21		3
SOPT	$\alpha^2(\alpha Z)^2$	9		1
FNS	$\alpha(\alpha Z)^{4+}$	-214()		1
Sum		2671	2675	269
VP2				
LO	$\alpha^2(\alpha Z)^2$	22		2
FNS	$\alpha^2(\alpha Z)^4$	−2(2)		
Sum		20(2)	18	2
VP3	$\alpha^3(\alpha Z)^2$	→	−2	
SE1+μVP1				
LO	$\alpha(\alpha Z)^4$	−62	−61	0.
NLO	$\alpha(\alpha Z)^5$	**−18**		0.
FNS	$\alpha(\alpha Z)^5$	**−17(2)**		0.
HVP	$\alpha(\alpha Z)^4$	4		0.
QED tot		...()	2640(2)	271
NPOL		55(*10*)	45	
FNS				
FNS/r_c^2	$(\alpha Z)^{4+}$	−1234.5/fm^2		−0.6/fm^2

Calculated with MuDirac !

Summary

- MuDIRAC very useful tool: Pedagogical, fundamental, QED, …
- Wish to scan more inputs (mass, radius, model, light-speed,…)
- Wish to add QED: Self-energy, Muon VP, VP2, …
- Add a nuclear polarization (look-up table+scaling)
- At some point, *invert* logic MuDIRAC3.0+Experiment=improved radii

Summary

- MuDIRAC very us QED, …
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- At some point, *invert* logic MuDIRAC3.0+Experiment=improved radii

A lot of work, but **deterministic** and **modular**:

well-suited for community effort!