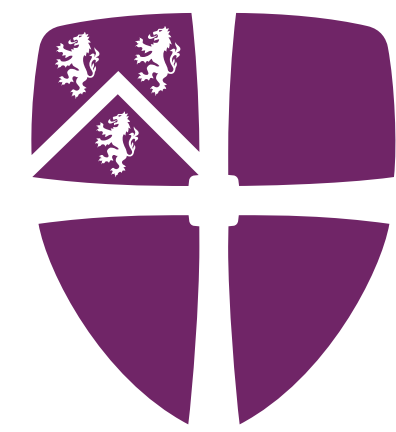


IBM Q



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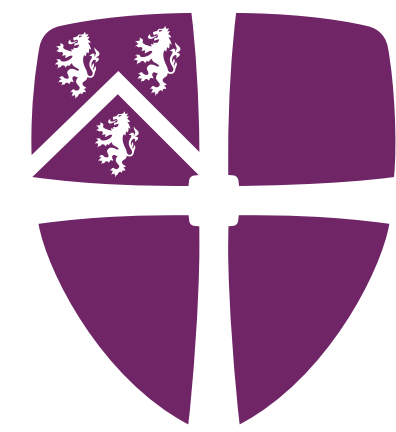


Quantum Computing for Particle Physics

Simon Williams

Rutherford Appleton Laboratory,
7th February 2024

IBM Q

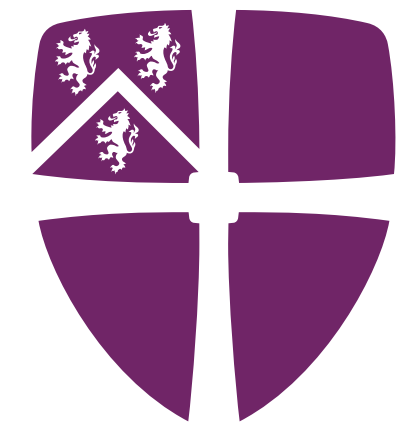


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- Quantum Computing - The Power of the Qubit
 - The Quantum Walk
- Why are we interested in High Energy Physics?
 - Event generation in high energy collisions
- Quantum Parton Showers
- Track Finding via Quantum Template Matching

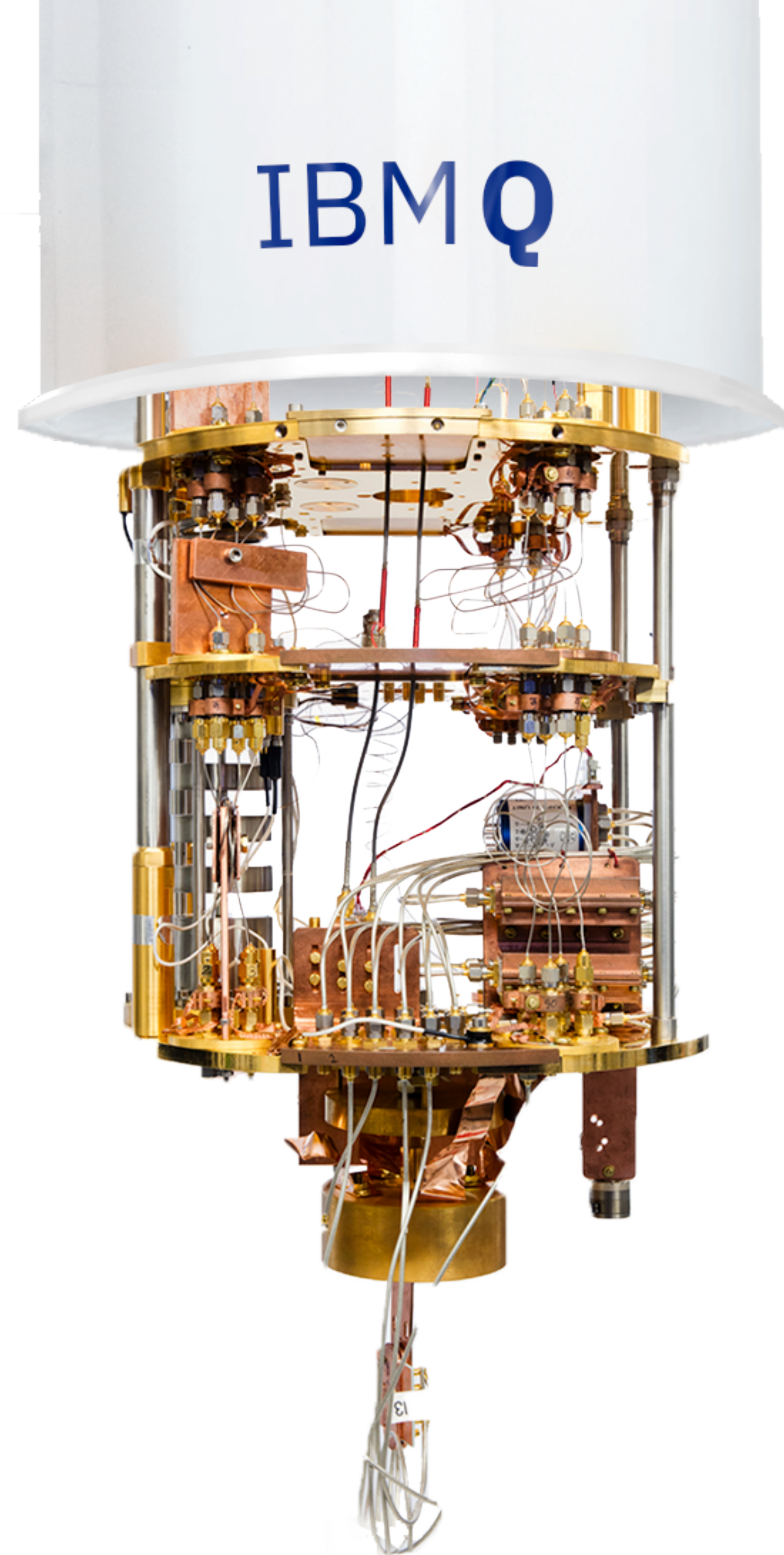
IBMQ



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Quantum Computing The Power of the Qubit



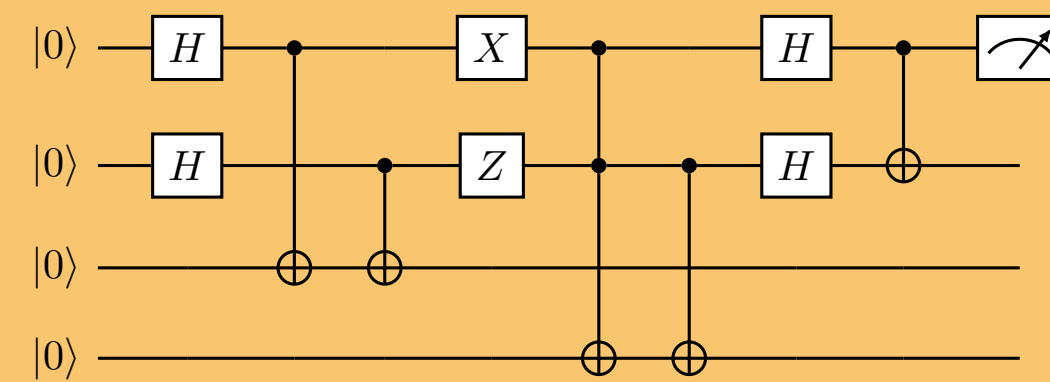
Quantum Computing - The Power of the Qubit!



“Nature is quantum [...] so if you want to simulate it, you need a quantum computer”
- Richard Feynman (1982)

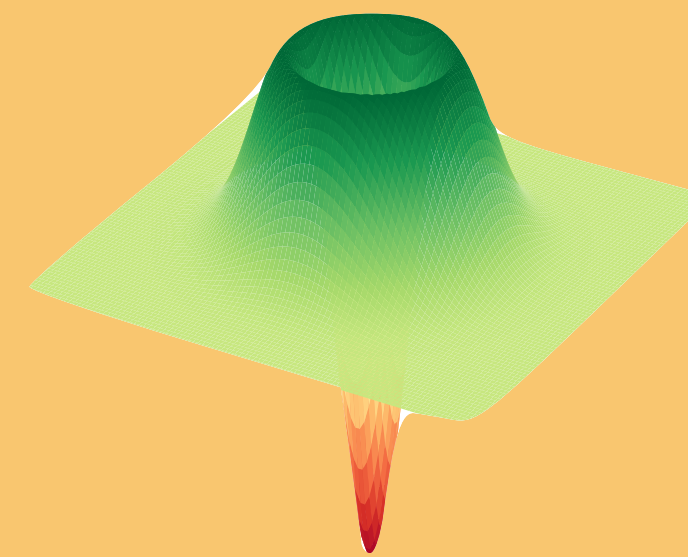
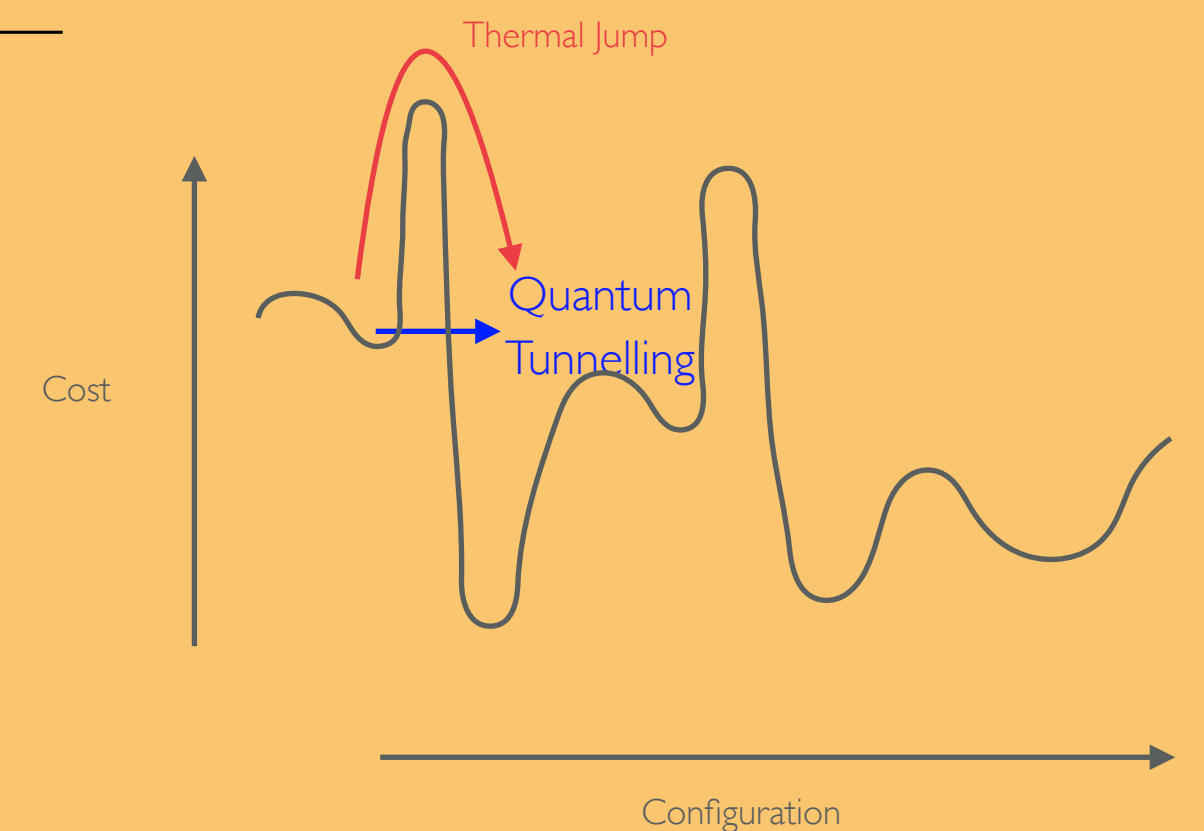
Quantum Computing has had a lot of successes since - most recently with Shor and Deutsch winning the **Breakthrough Prize** and the **2022 Nobel Prize** going to Quantum Information

Types of Quantum Device:



Superconductor
Quantum Computing

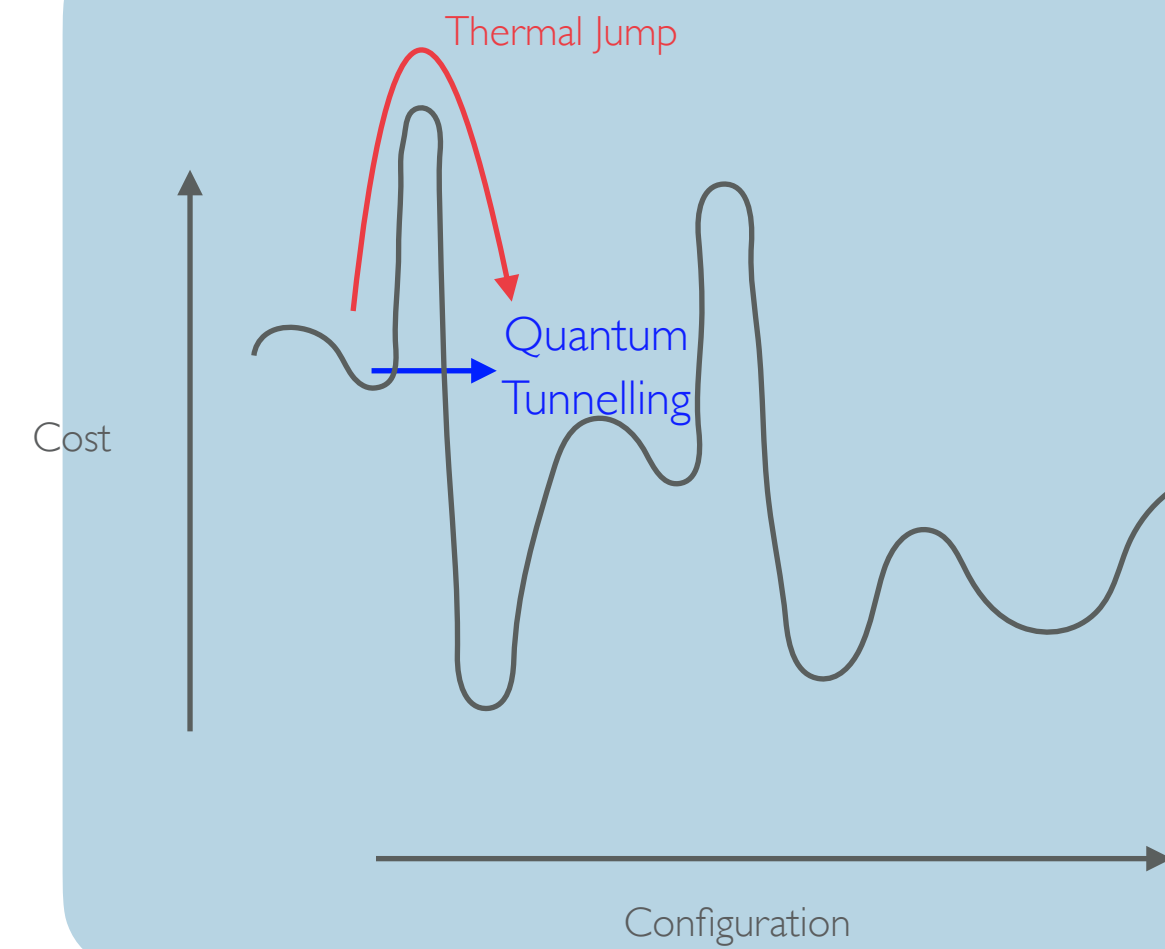
Quantum Annealing



Photonic Devices

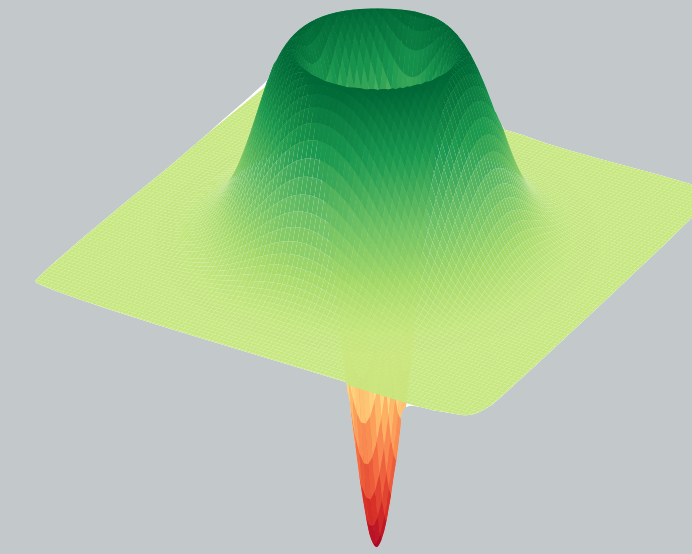
Types of Quantum Computing Devices

Quantum Annealing



$$H(\sigma) = - \sum_{i,j} J_{ij} \sigma_i \sigma_j - \mu \sum_j h_j \sigma_j$$

Photonic Quantum Devices



Type of gate quantum computing, manipulating photon states

Advantages:

- Well suited to optimisation problems

Disadvantages:

- Uncontrollable, noisy devices
- Not universal devices

Advantages:

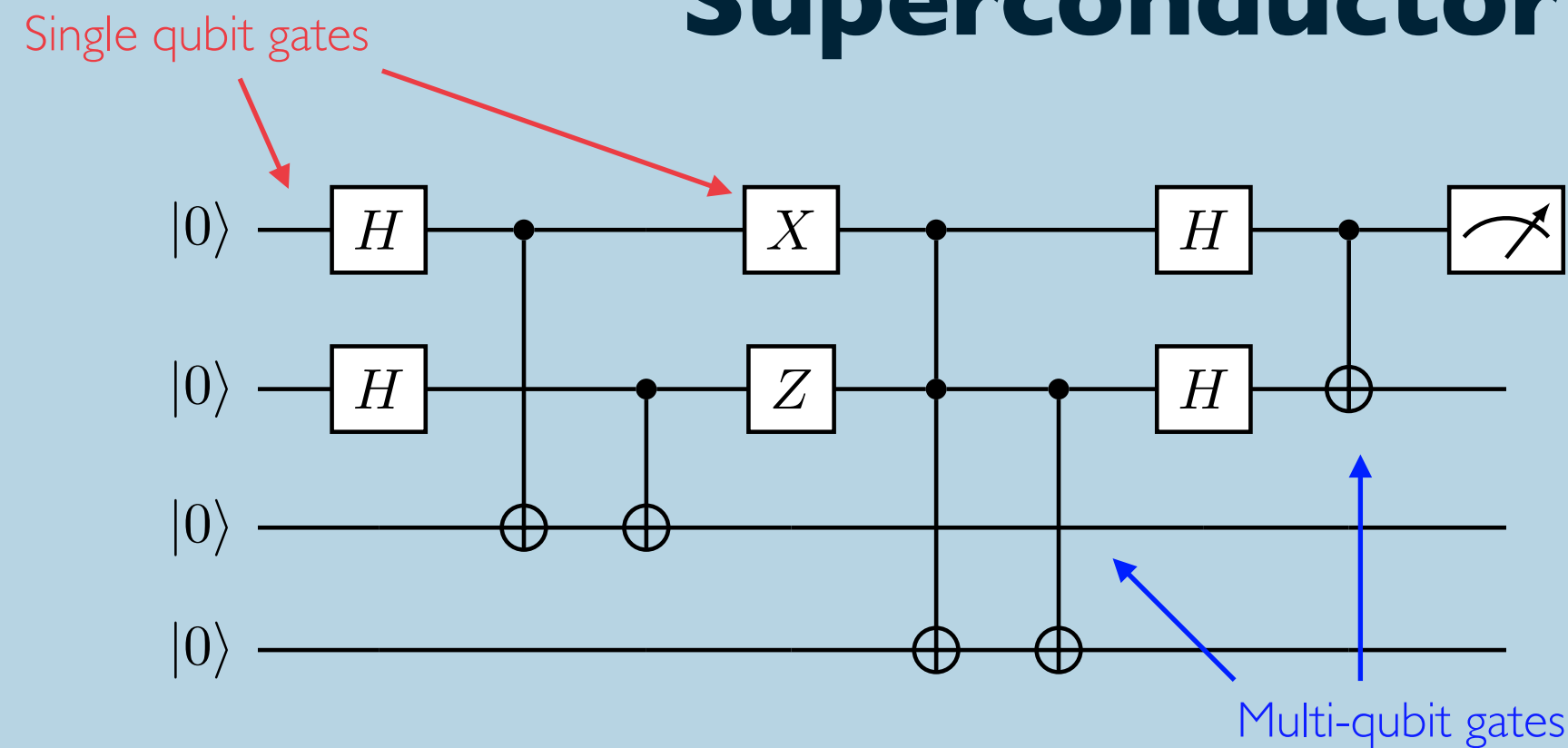
- Continuous variable devices
- Only weak interactions with environment

Disadvantages:

- All states must be Gaussian

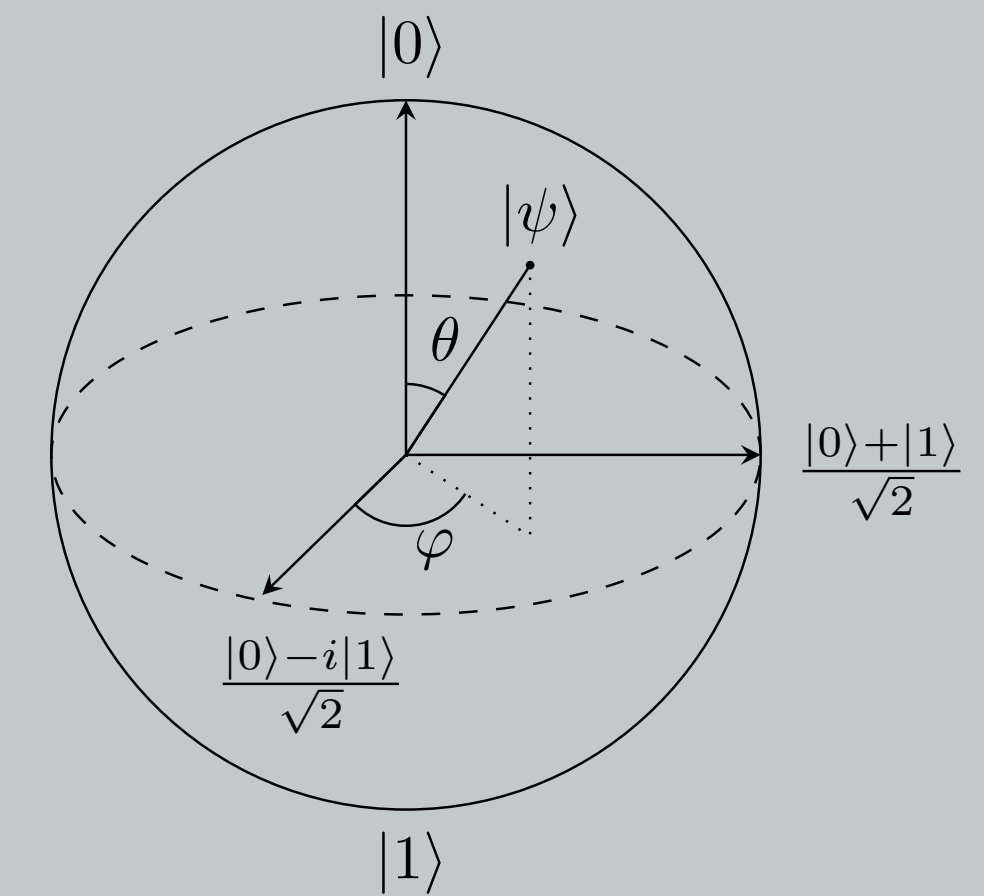
Types of Quantum Computing Devices

Superconductor QCs



Qubit model:

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$



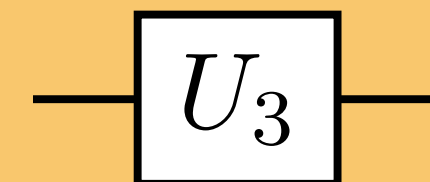
Advantages:

- Highly controllable qubits
- Universal computation

Disadvantages:

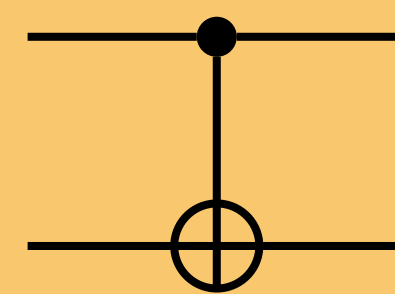
- Small number of qubits, not very fault tolerant

Single qubit gates:



$$U_3 |0\rangle \rightarrow \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$

Multi-qubit gates:

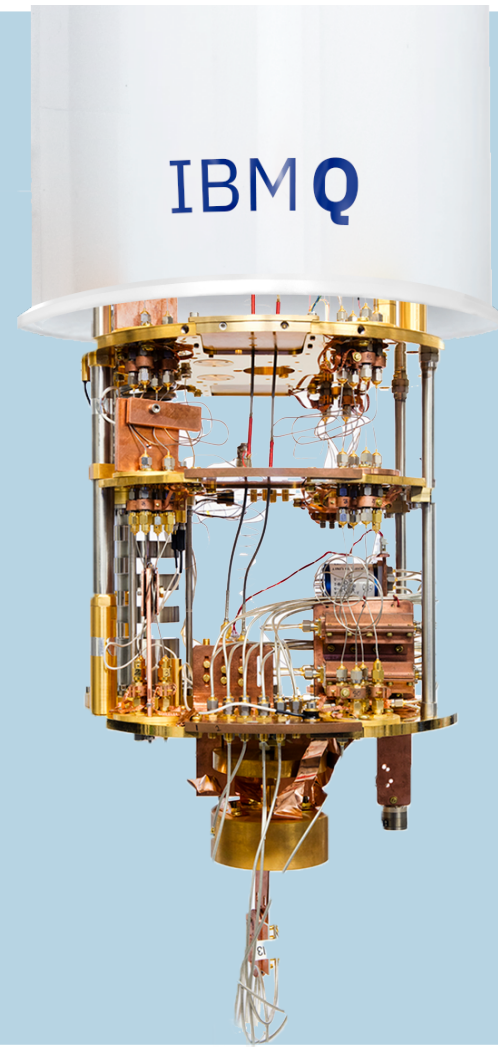


$$\begin{aligned} \text{CNOT } |00\rangle &\rightarrow |00\rangle, \text{CNOT } |10\rangle \rightarrow |11\rangle, \\ \text{CNOT } |01\rangle &\rightarrow |01\rangle, \text{CNOT } |11\rangle \rightarrow |10\rangle \end{aligned}$$

Noisy Intermediate-Scale Quantum Devices

NISQ devices:

No continuous quantum error correction, prone to large noise effects from environment.



Quantum errors:

Multiqubit qubit gates: CNOT gates have higher associated errors than single qubit gates.

SWAP errors: SWAP operations require 3 CNOT gates

T1 times: The time it takes for an excited qubit to decay back to the ground state.

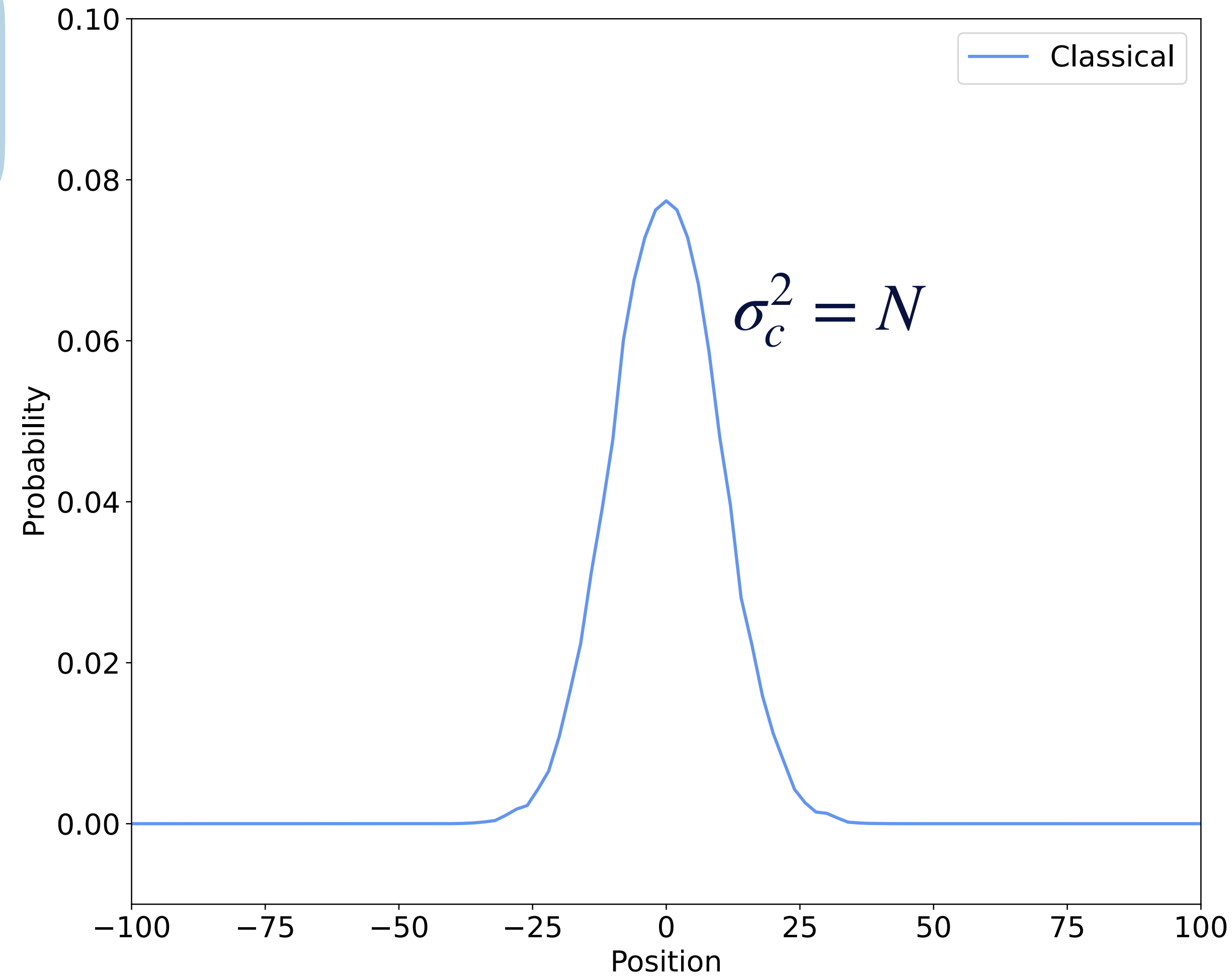
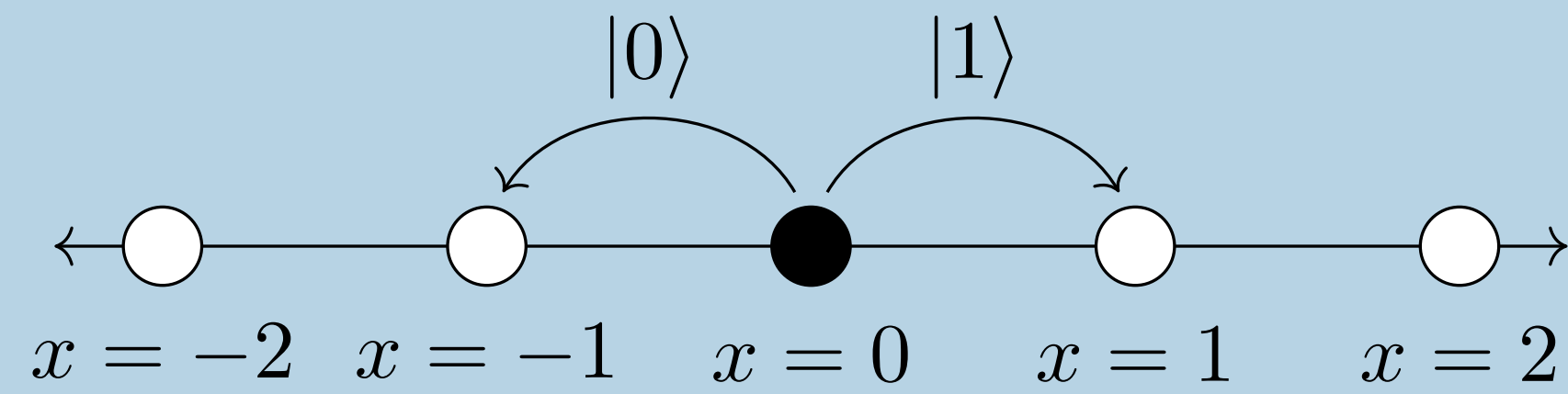
Circuit depth! - Compact circuits needed!

Transpilation:

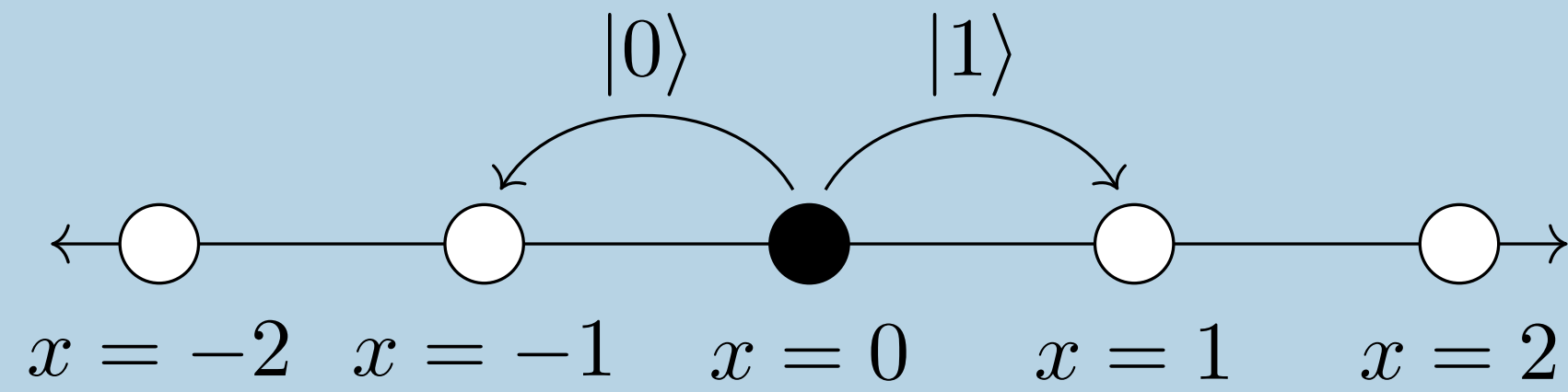
Loading the circuit onto the backend, transpilation can be used to optimise the circuit: **qubit and coupling mapping, noise models, etc.**

The Quantum Walk

The Quantum Walk

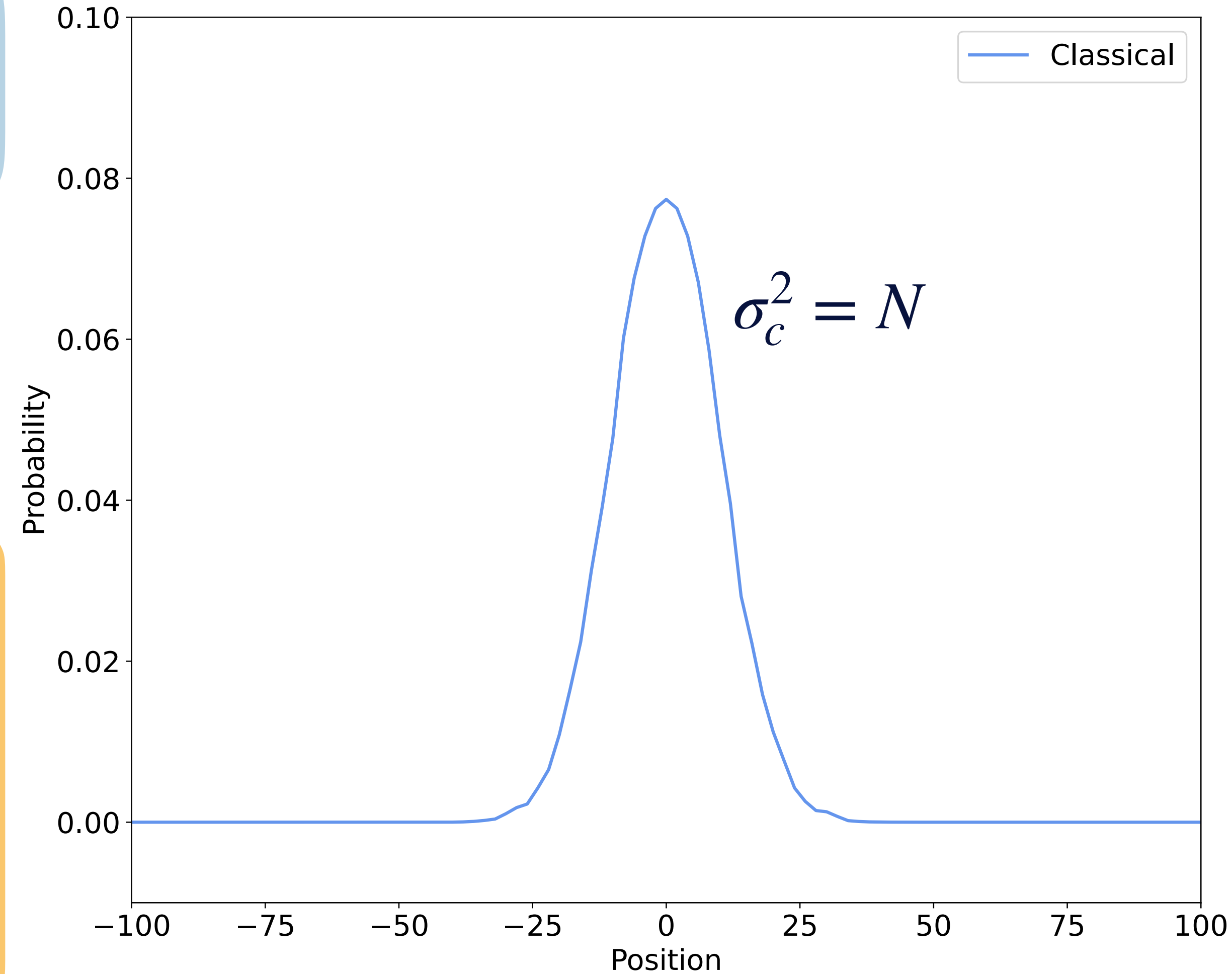


The Quantum Walk

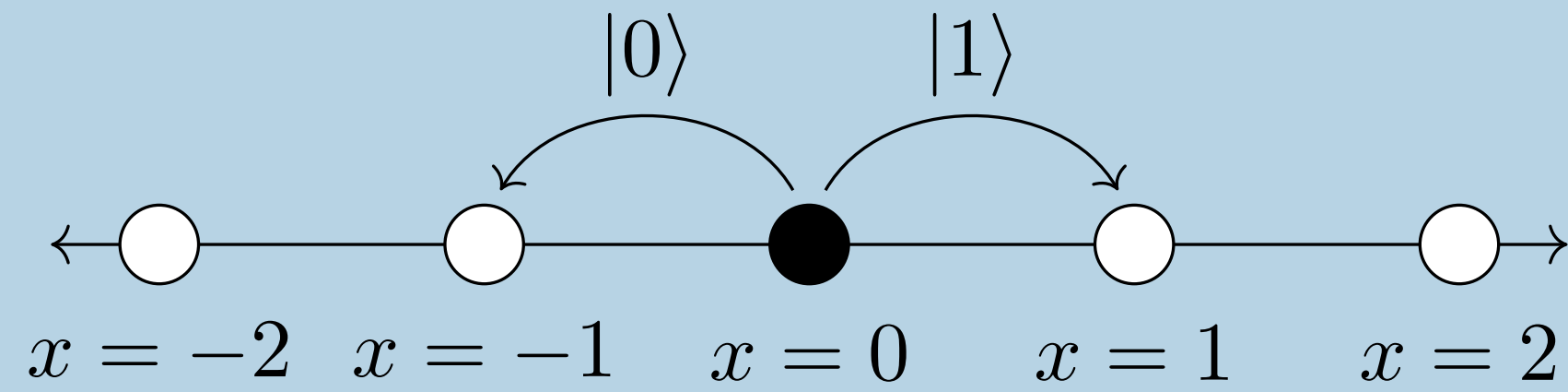


Coin
Operation:

$$C|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



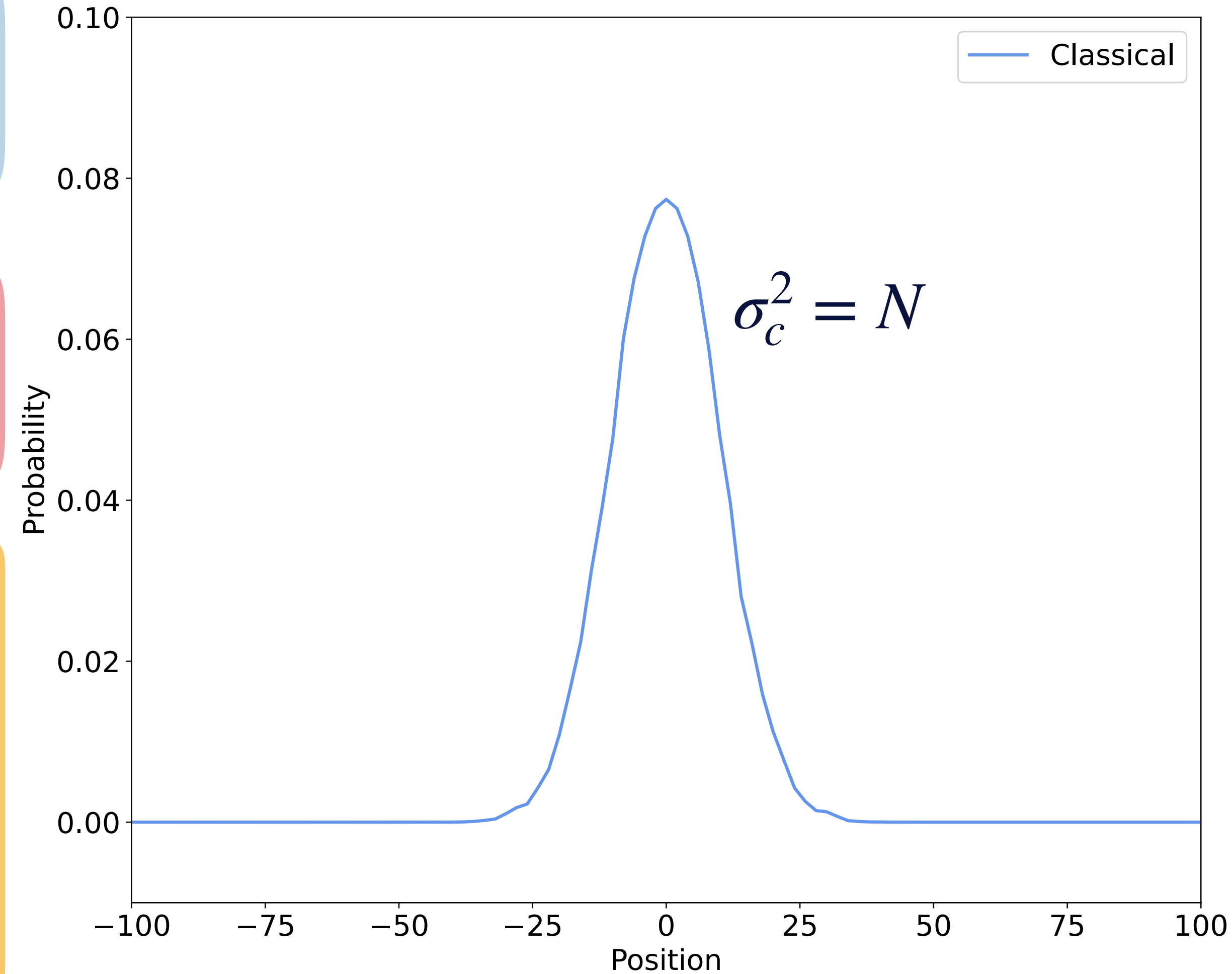
The Quantum Walk



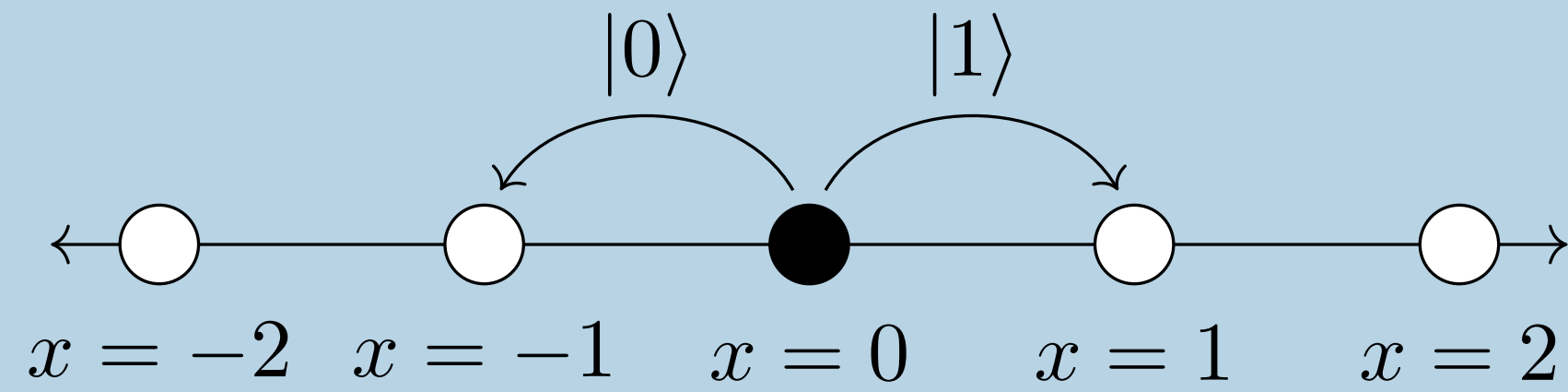
$$\left. \begin{aligned} \mathcal{H}_P &= \{ |i\rangle : i \in \mathbb{Z} \} \\ \mathcal{H}_C &= \{ |0\rangle, |1\rangle \} \end{aligned} \right\} \mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_P$$

Coin Operation:

$$C|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$



The Quantum Walk



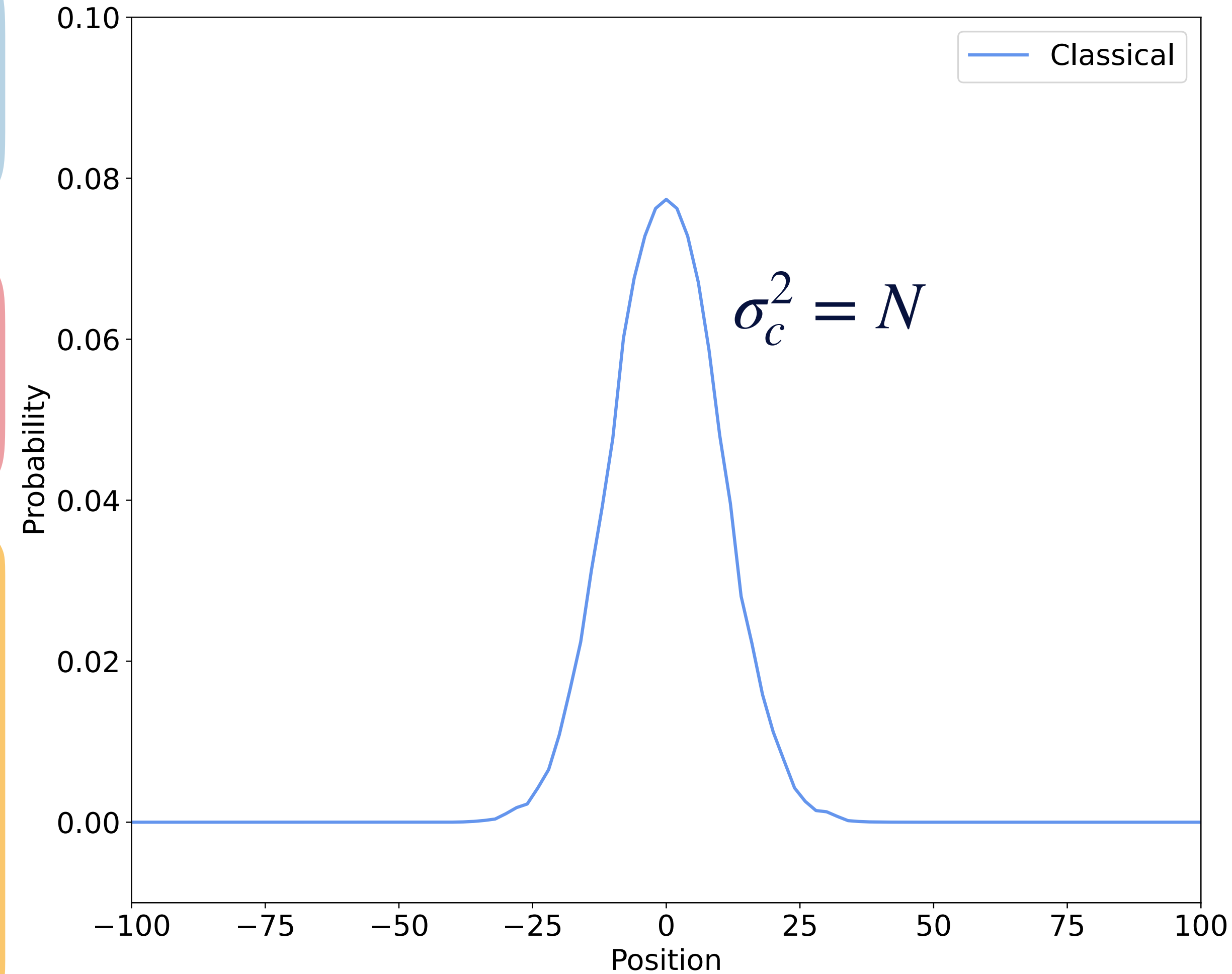
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Unitary Transformation:

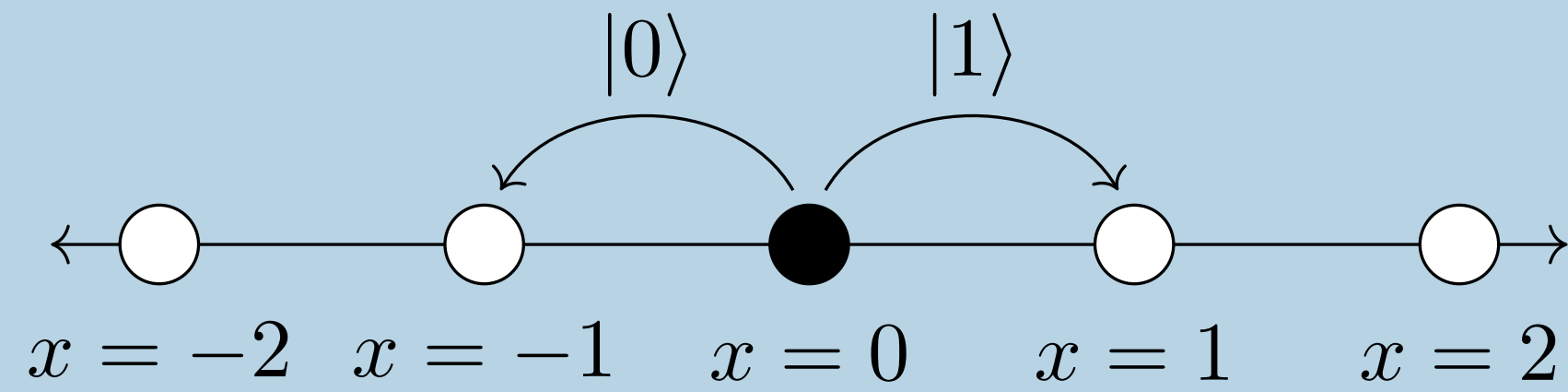
$$U = S \cdot (C \otimes I)$$

Coin Operation:

$$C|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$



The Quantum Walk



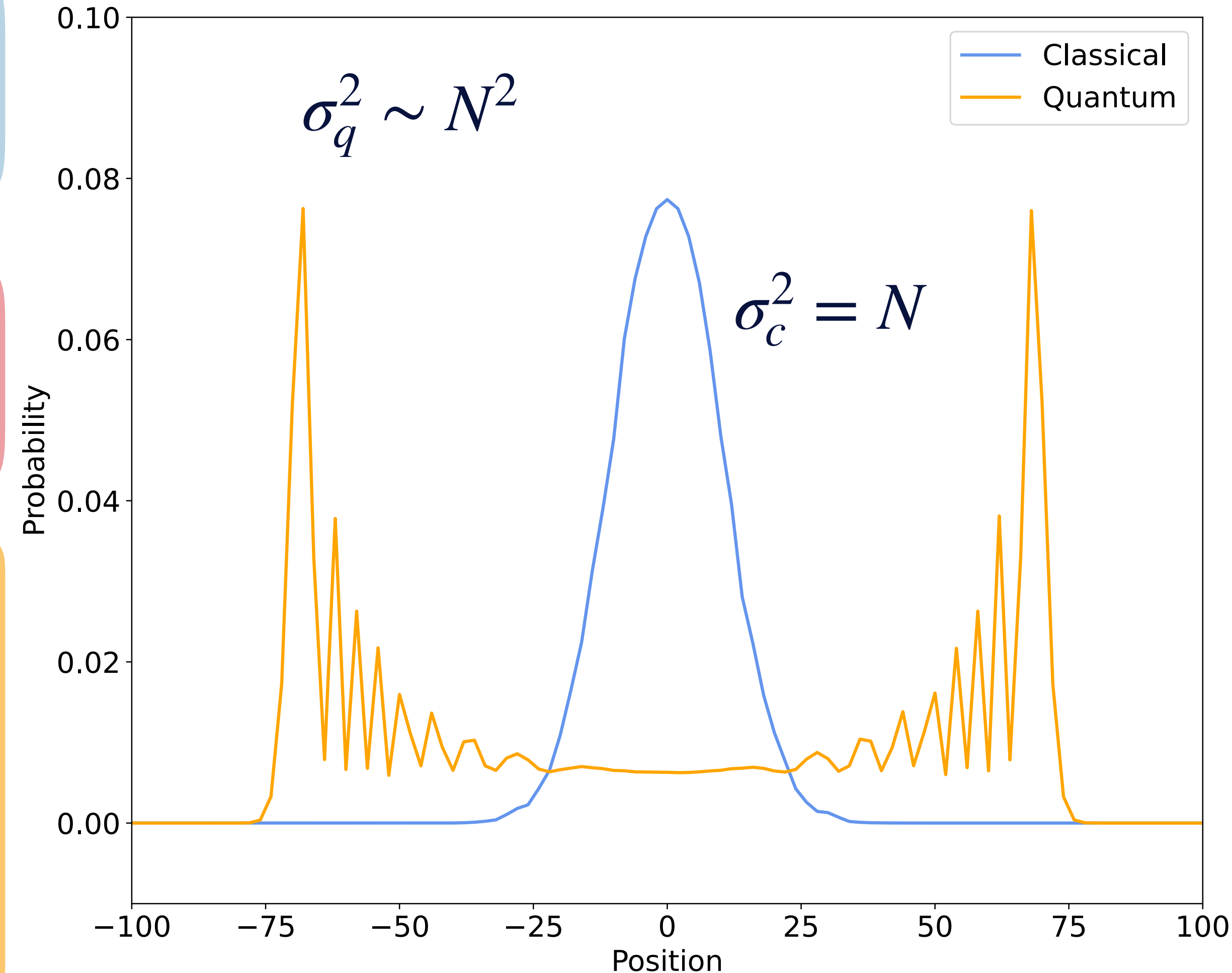
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Unitary Transformation:

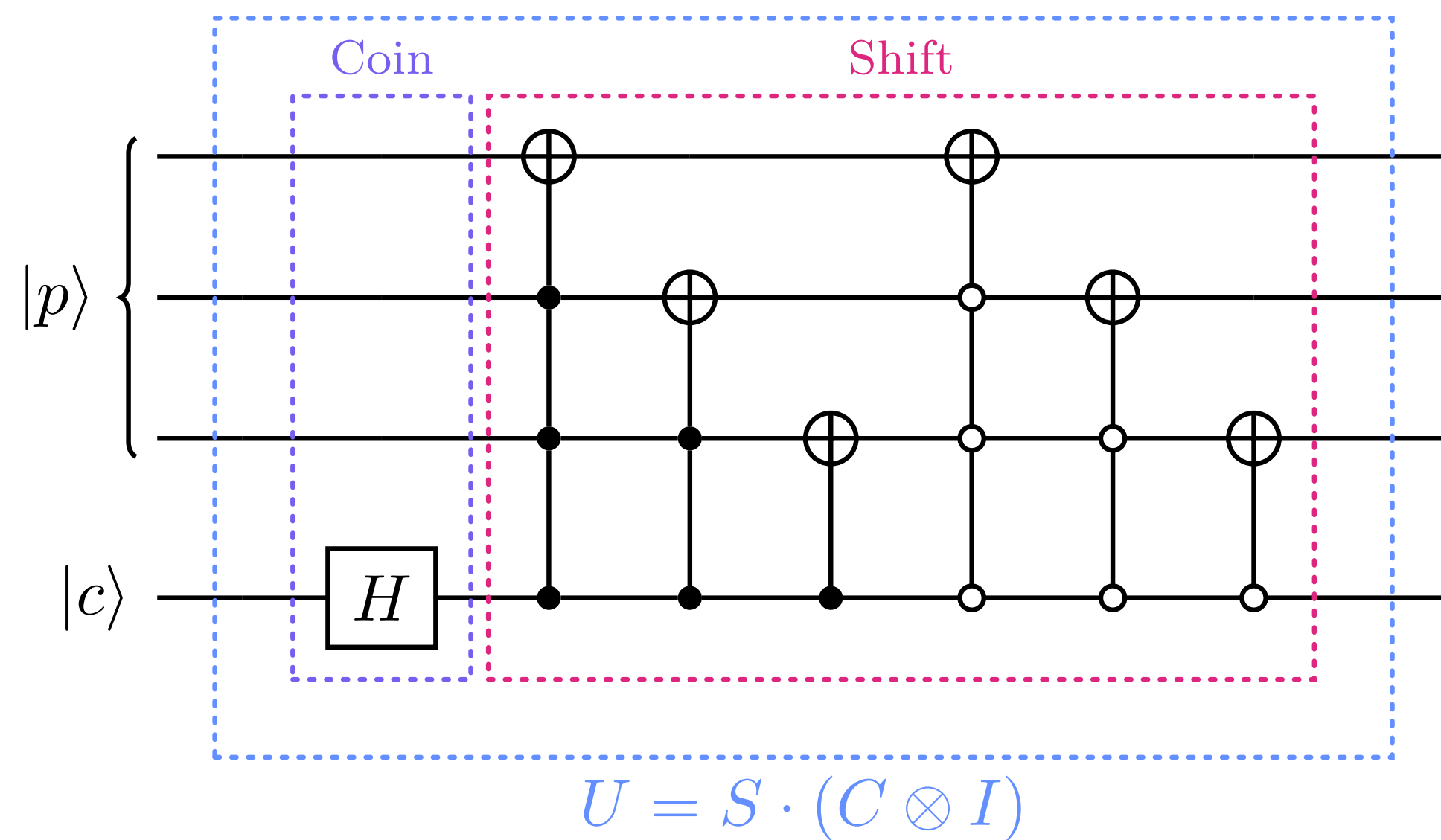
$$U = S \cdot (C \otimes I)$$

Coin Operation:

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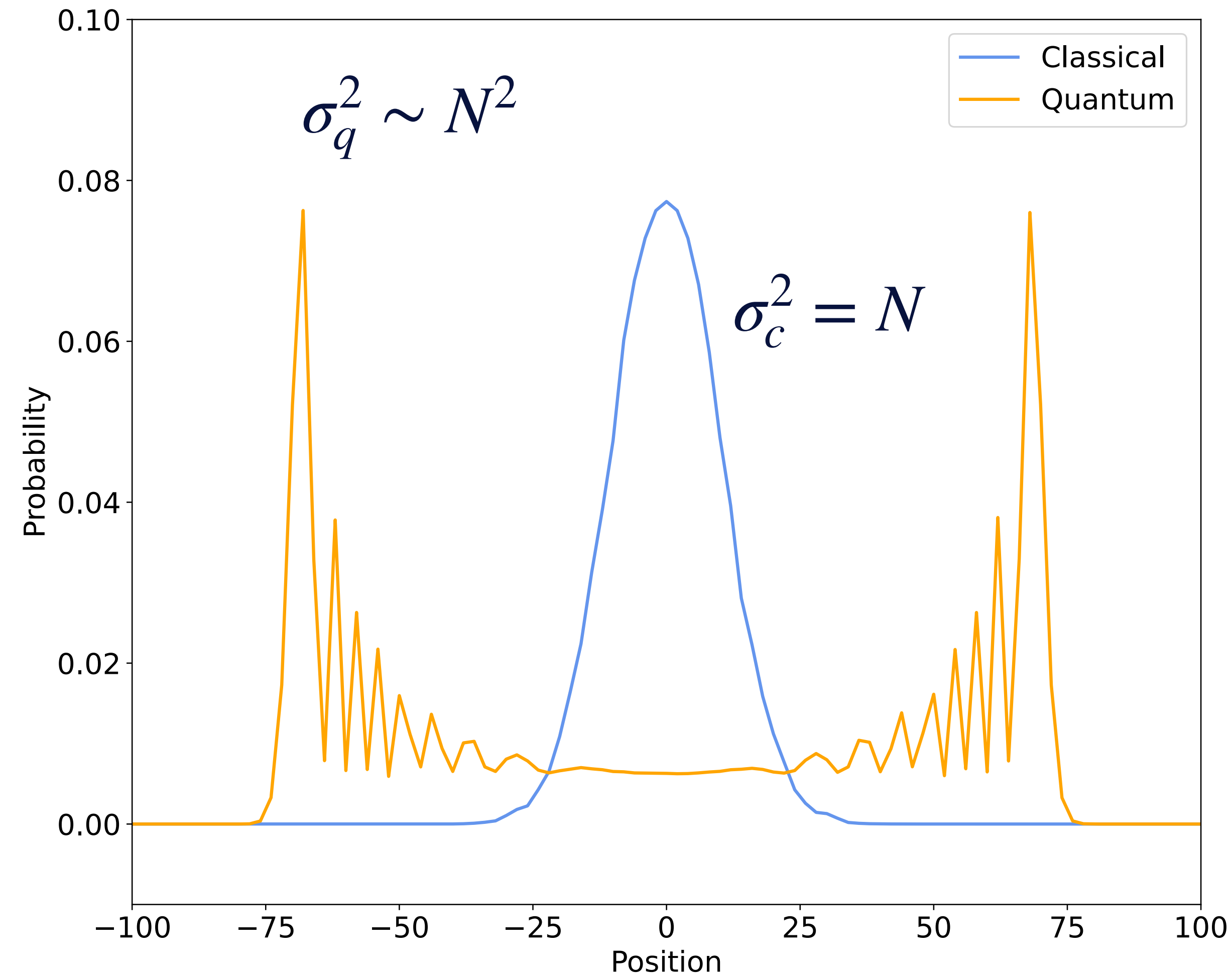


The Quantum Walk



Circuit depth of a quantum walk grows **linearly** with the number of steps

Suitable **quantum circuit architecture** for **NISQ** era devices



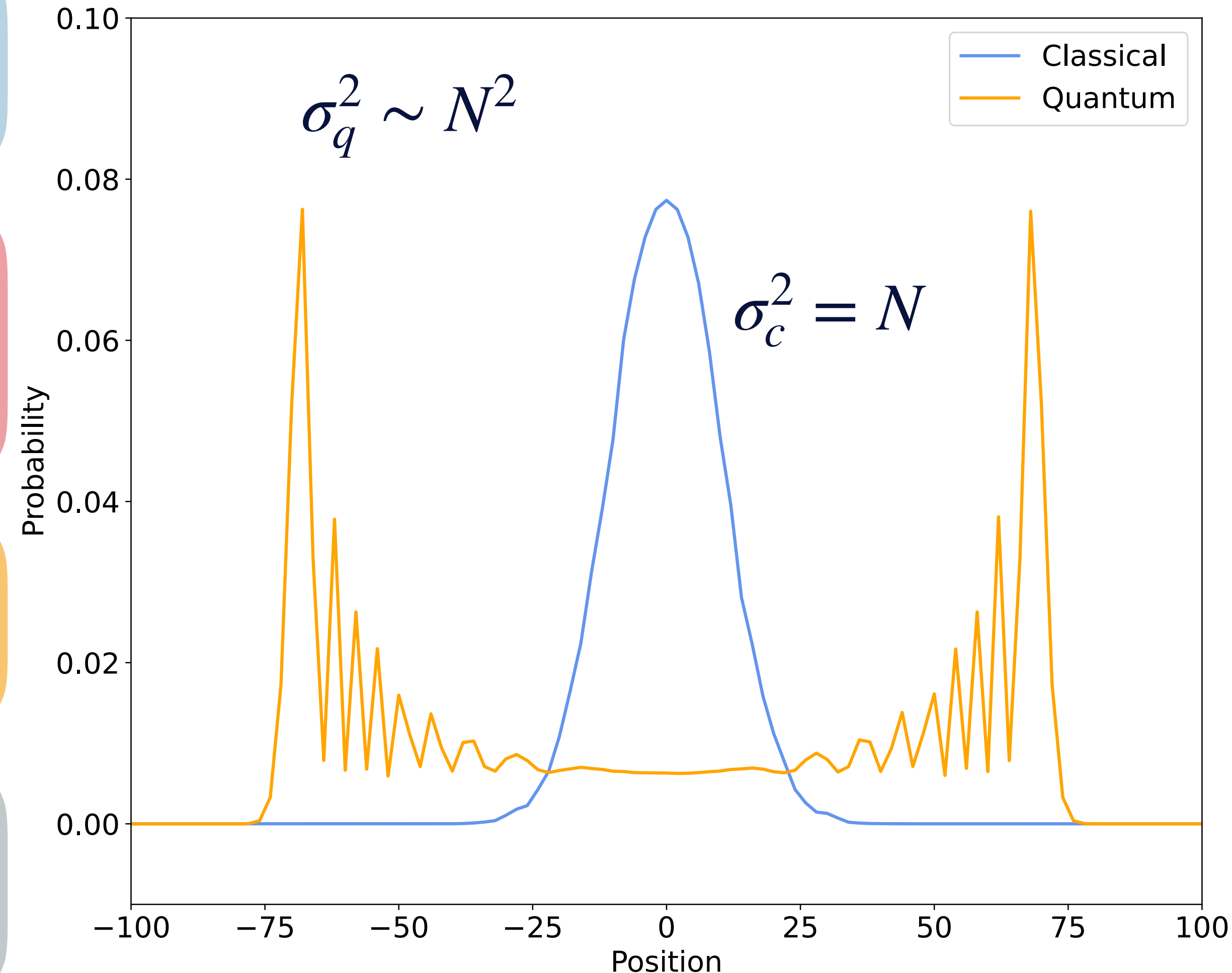
Speed up via Quantum Walks

Quantum Walks have long be conjectured to achieved at least **quadratic speed up**

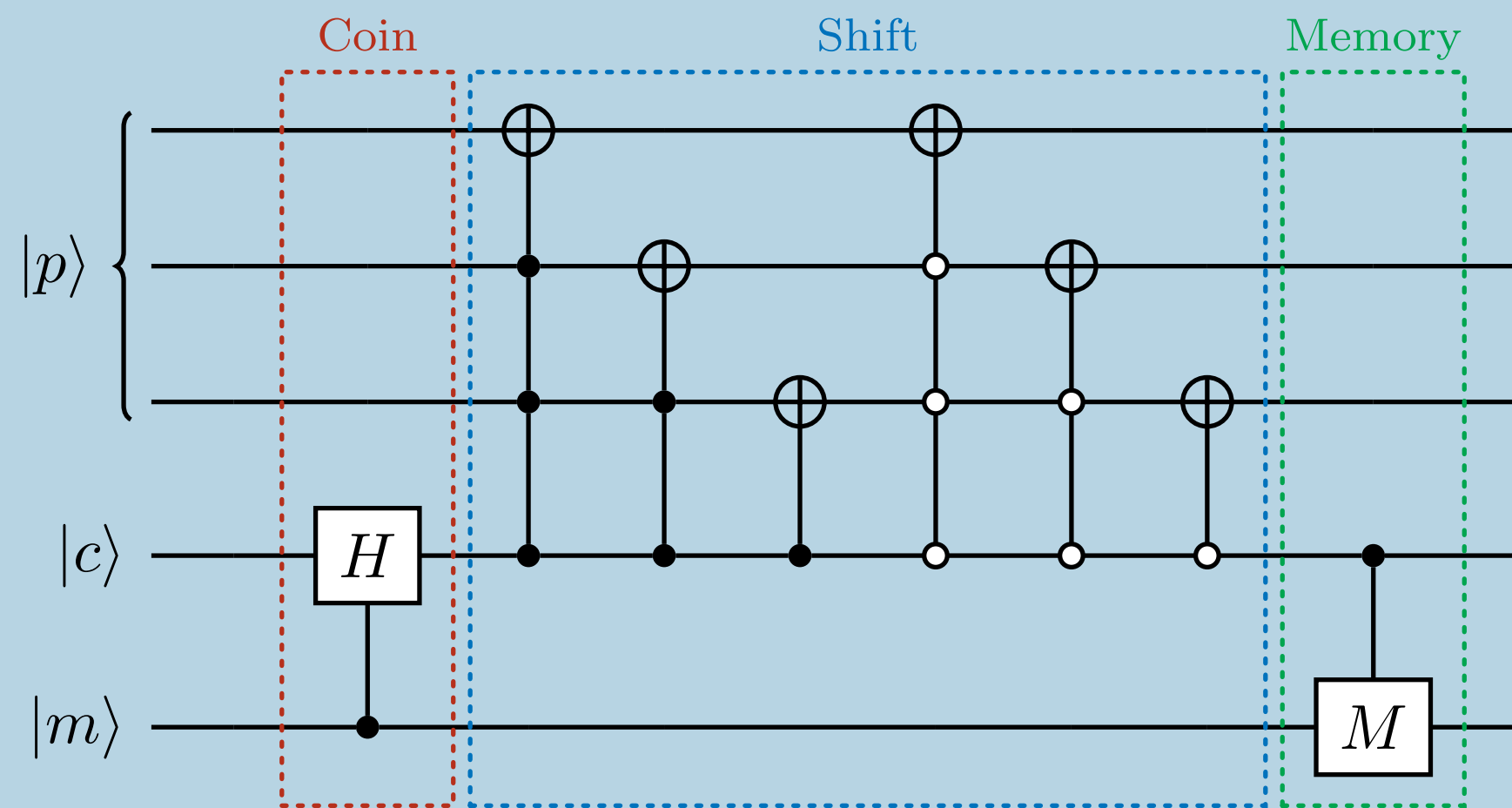
Szegedy Quantum Walks have been proven to achieve quadratic speed up for **Markov Chain Monte Carlo**

This has been proven under the condition that the MCMC algorithm is **reversible and ergodic**

Work is ongoing to prove this is true for all QWs, but latest upper limits are on par with classical RW



Quantum Walks with Memory



Qubit model:

Augment system further by adding an additional memory space

$$\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_C \otimes \mathcal{H}_M$$

Advantages:

- Arbitrary dynamics
- Classical dynamics in unitary evolution

Disadvantages:

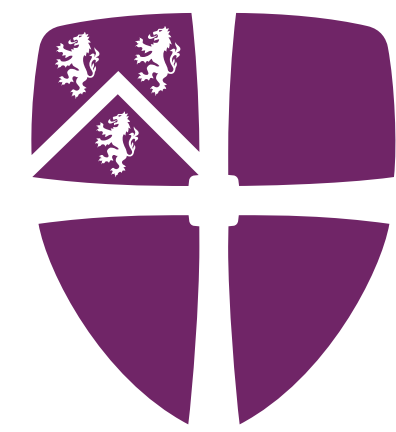
- Tight conditions on quantum advantage

Quantum Parton Showers:

Quantum Walks with memory have proven to be very useful for quantum parton showers.

K. Bepari, S. Malik, M. Spannowsky and SW, **Phys. Rev. D 106 (2022) 5, 056002**

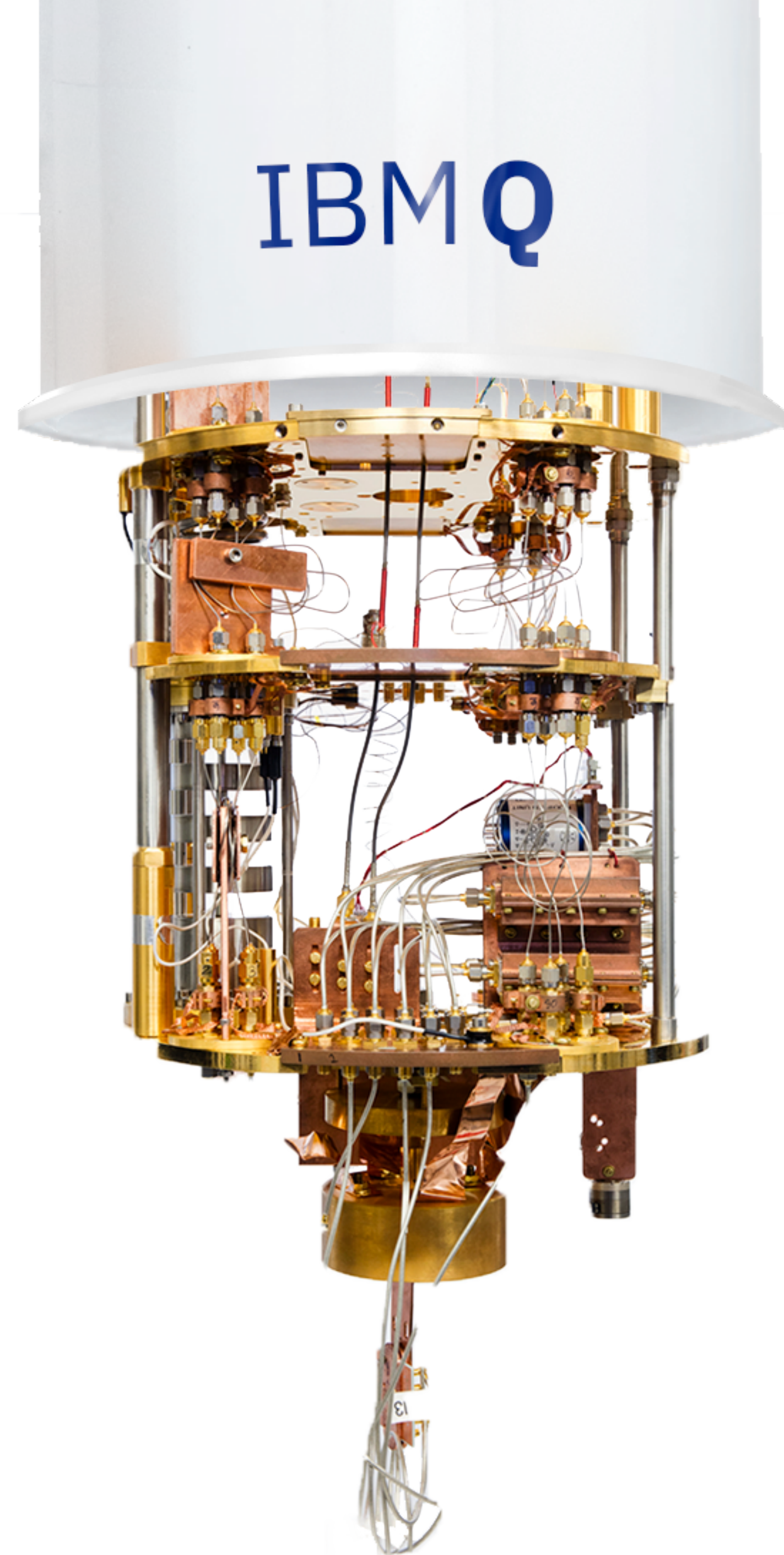
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**Why are we interested in High Energy
Physics?**



Event Generation - What's the problem?

SciPost Phys. Codebases 8 (2022)

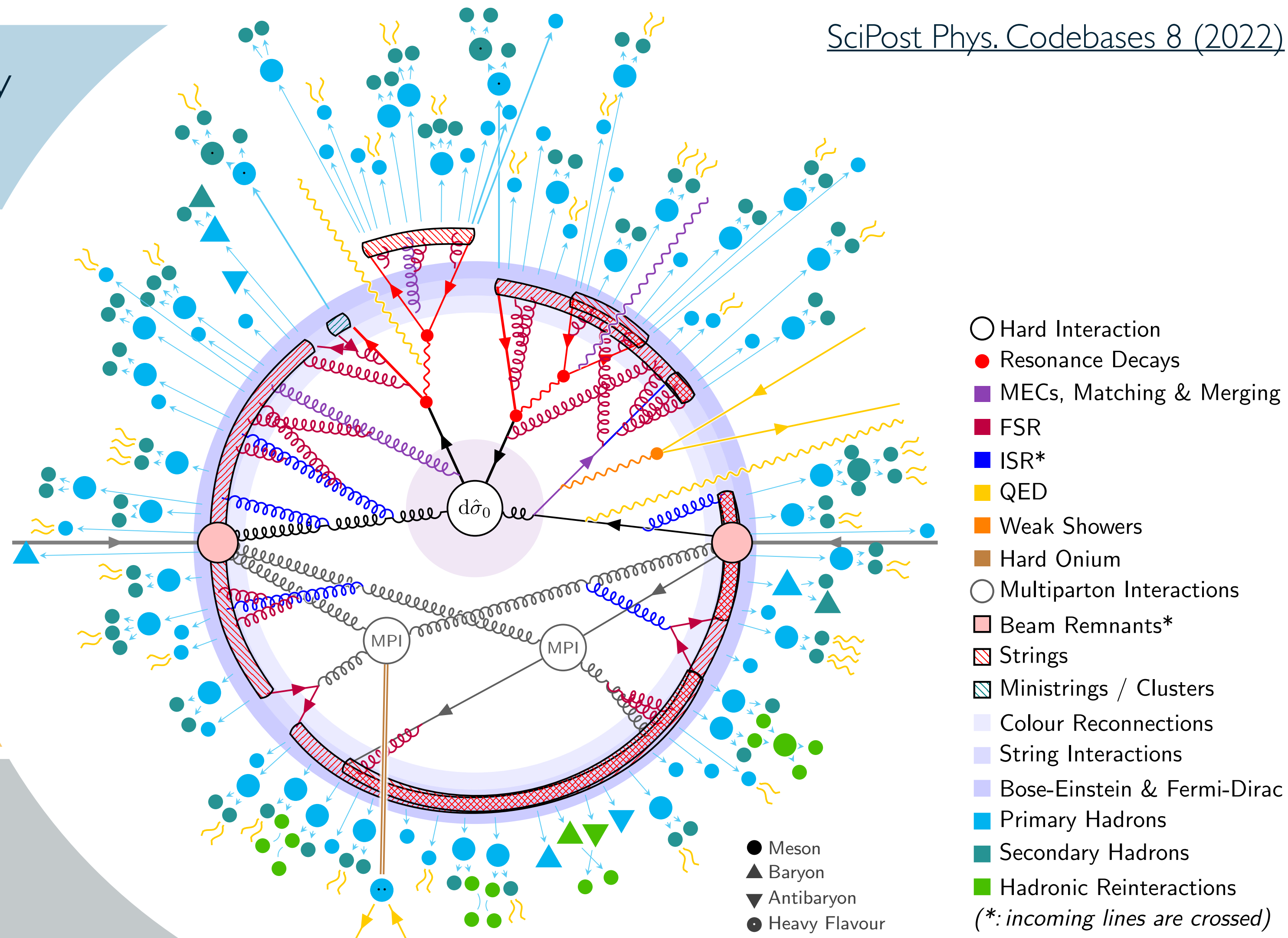
Typical hadron-hadron collisions are highly complex resulting in $O(1000)$ particles

The theoretical description of collision events is **highly complex**

Monte Carlo Event

Generators have been the most successful approach to simulating particle collisions

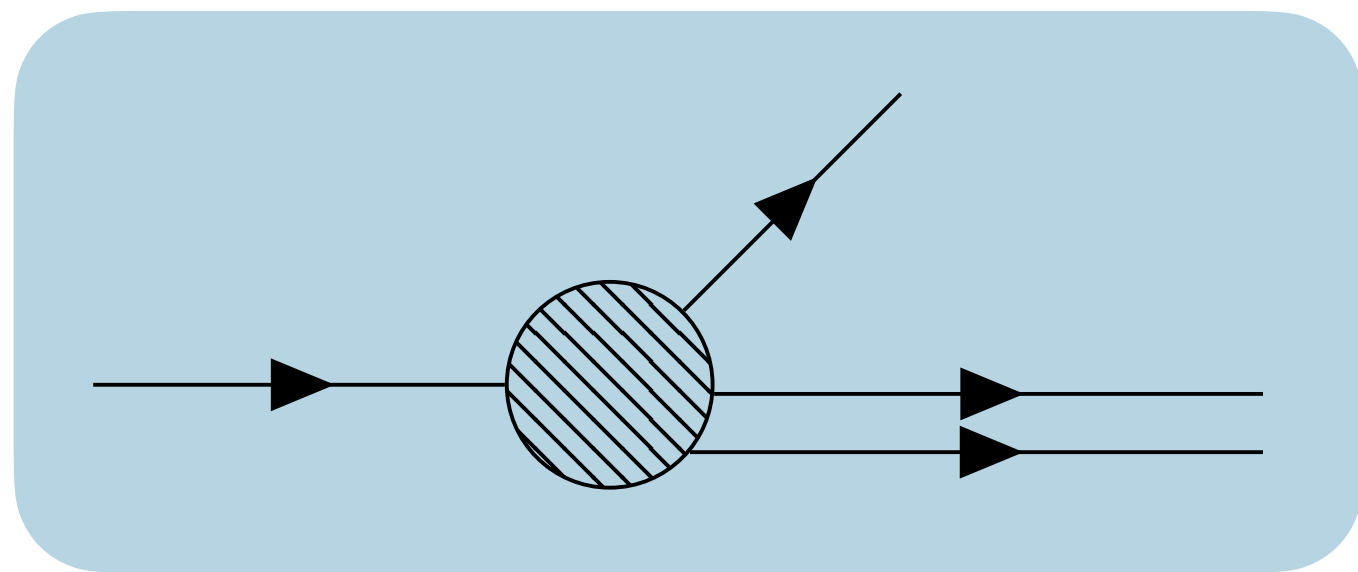
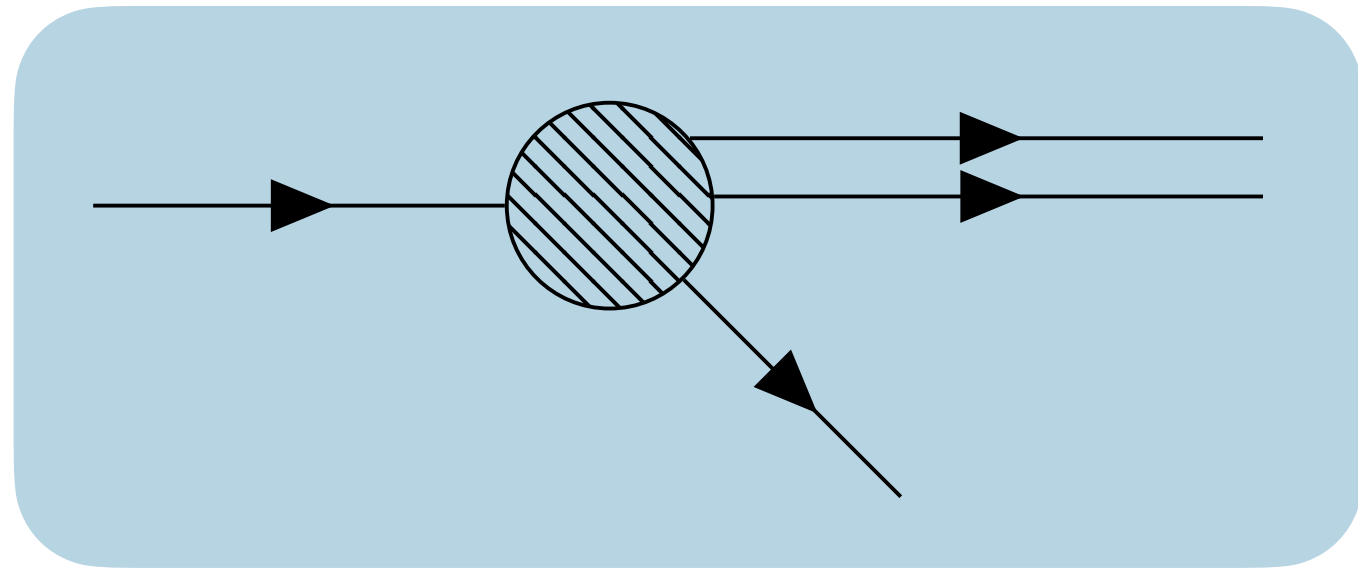
MC Event Generators exploit **factorisation theorems** in QCD



Event Generation - What's the problem?

Event Generation - What's the problem?

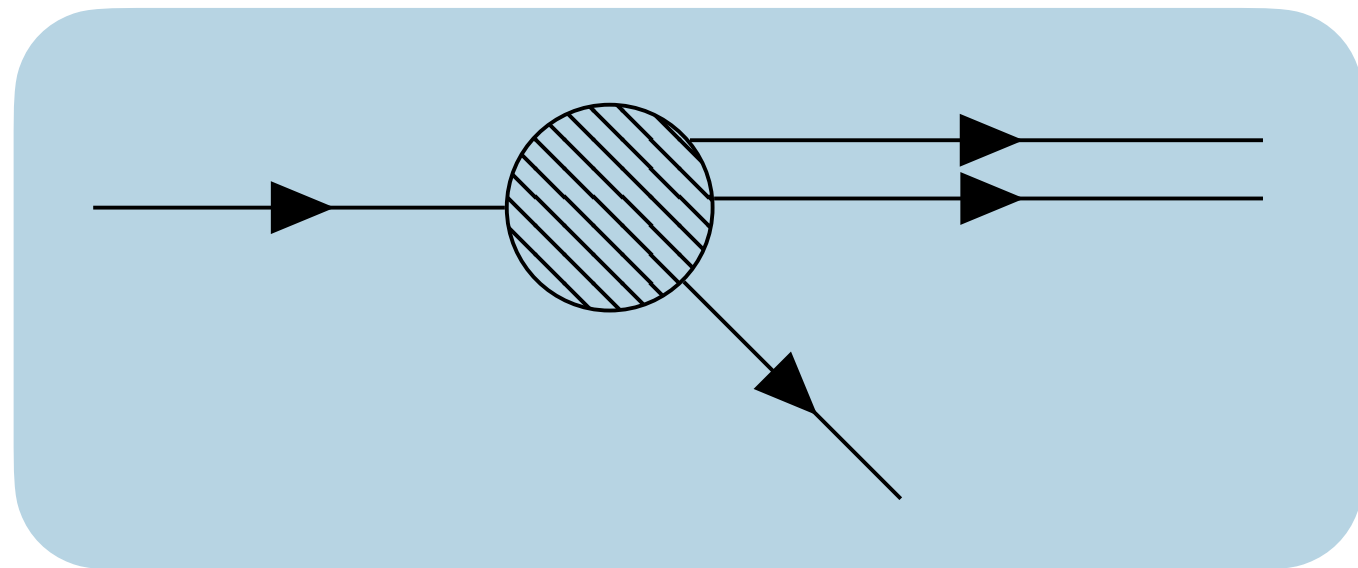
Parton Density Functions



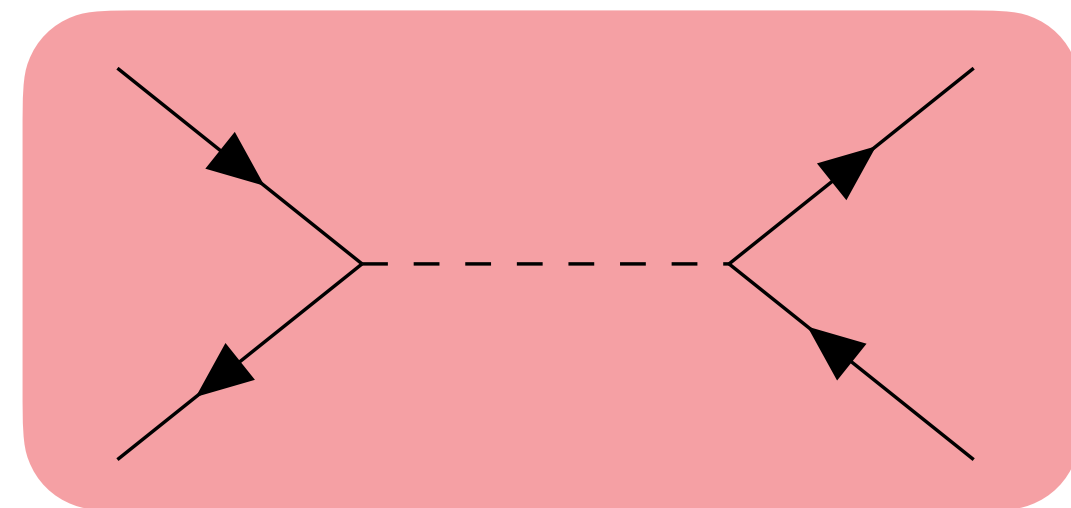
[Phys. Rev. D 103, 034027](#)

Event Generation - What's the problem?

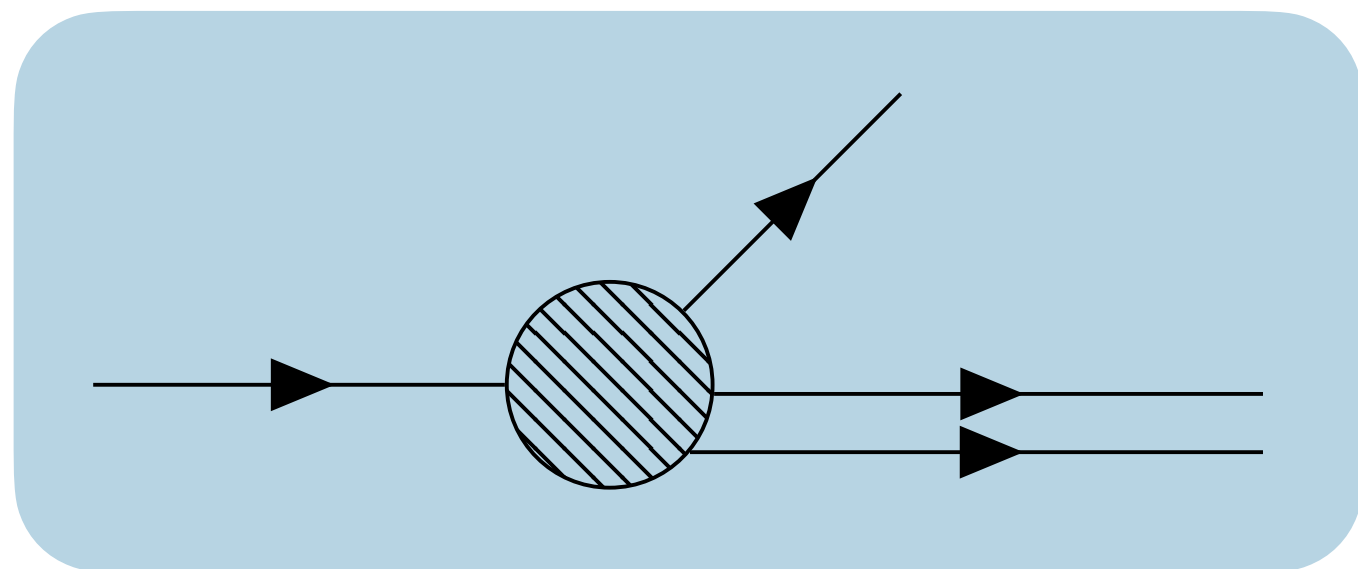
Parton Density Functions



Hard Process



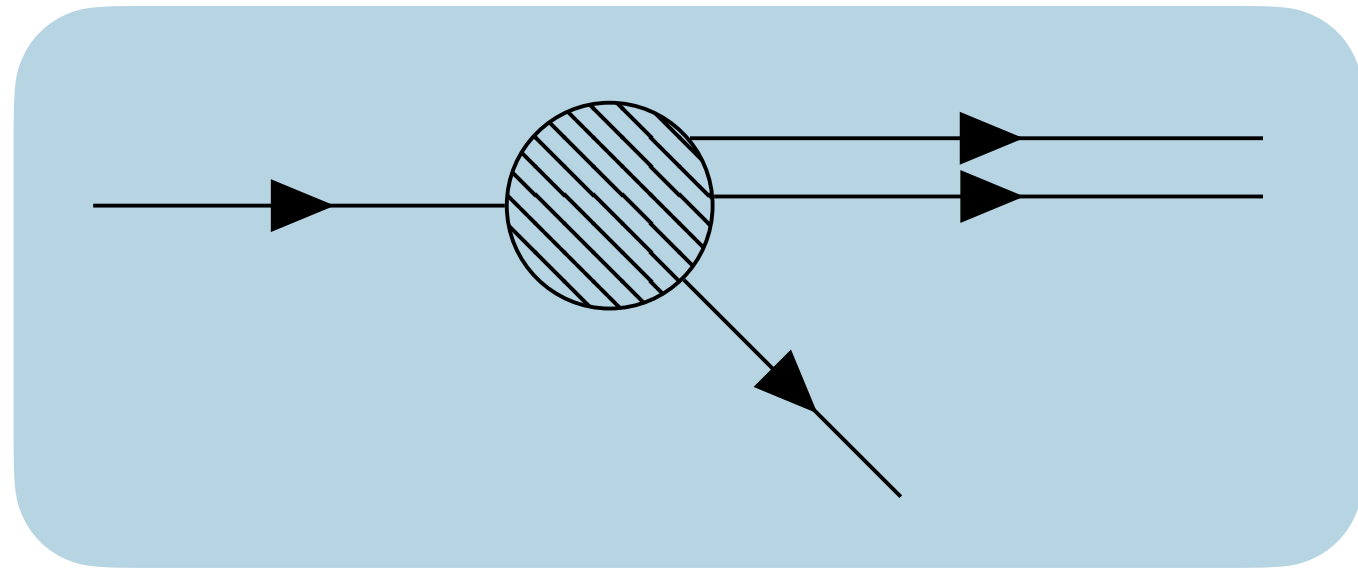
[Phys. Rev. D 103, 076020](#)



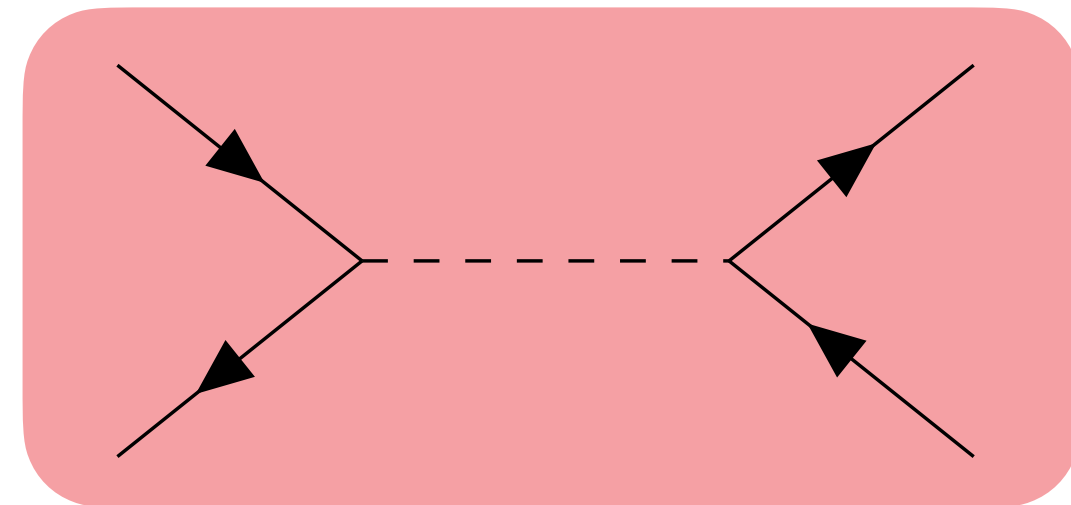
[Phys. Rev. D 103, 034027](#)

Event Generation - What's the problem?

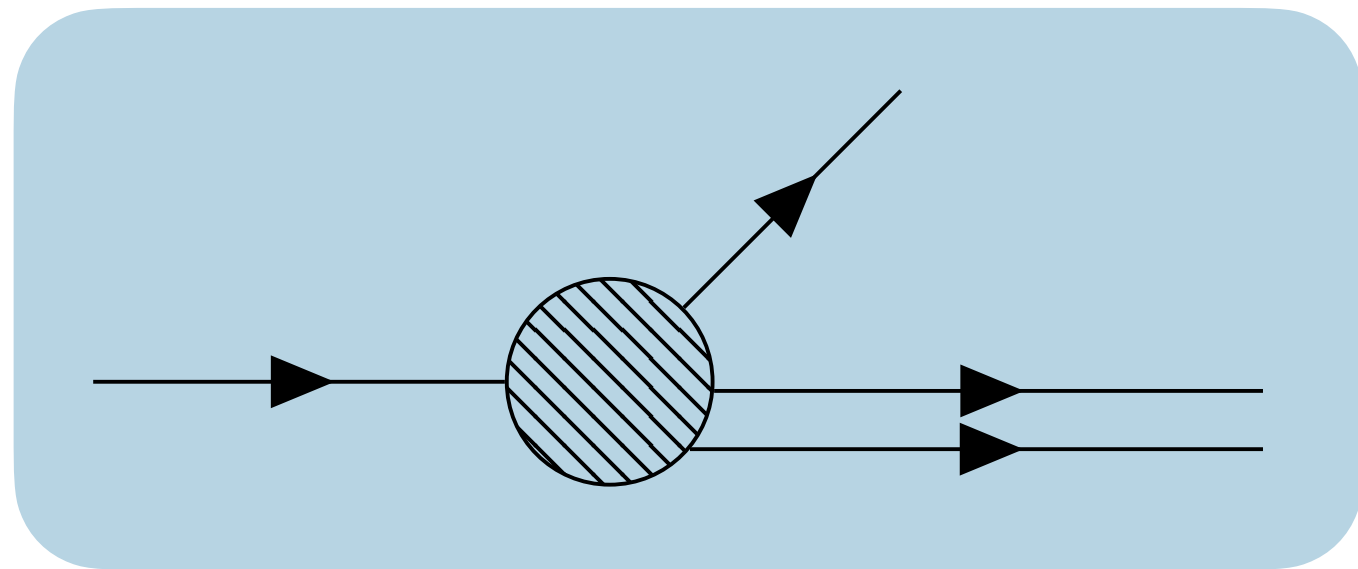
Parton Density Functions



Hard Process

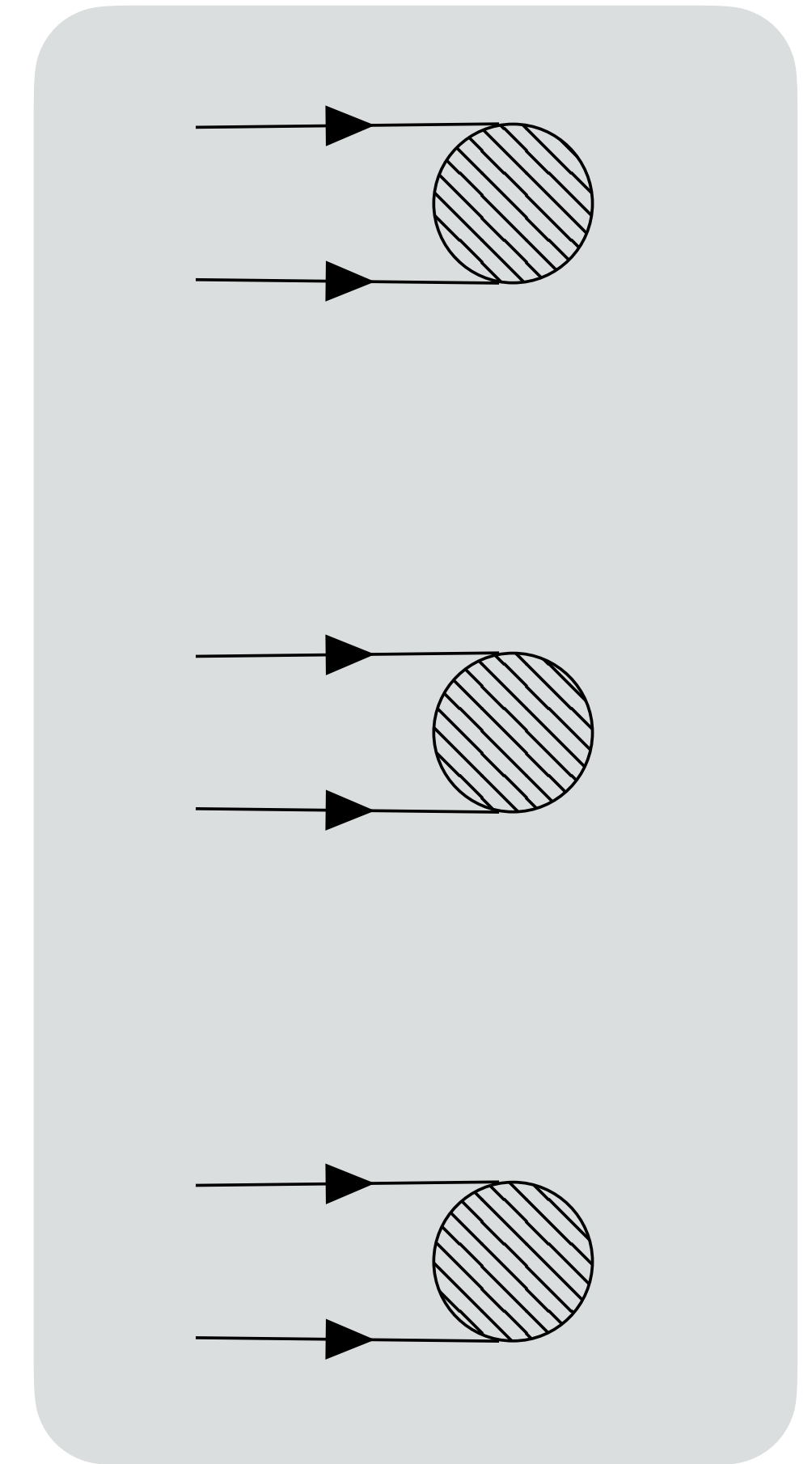


[Phys. Rev. D 103, 076020](#)



[Phys. Rev. D 103, 034027](#)

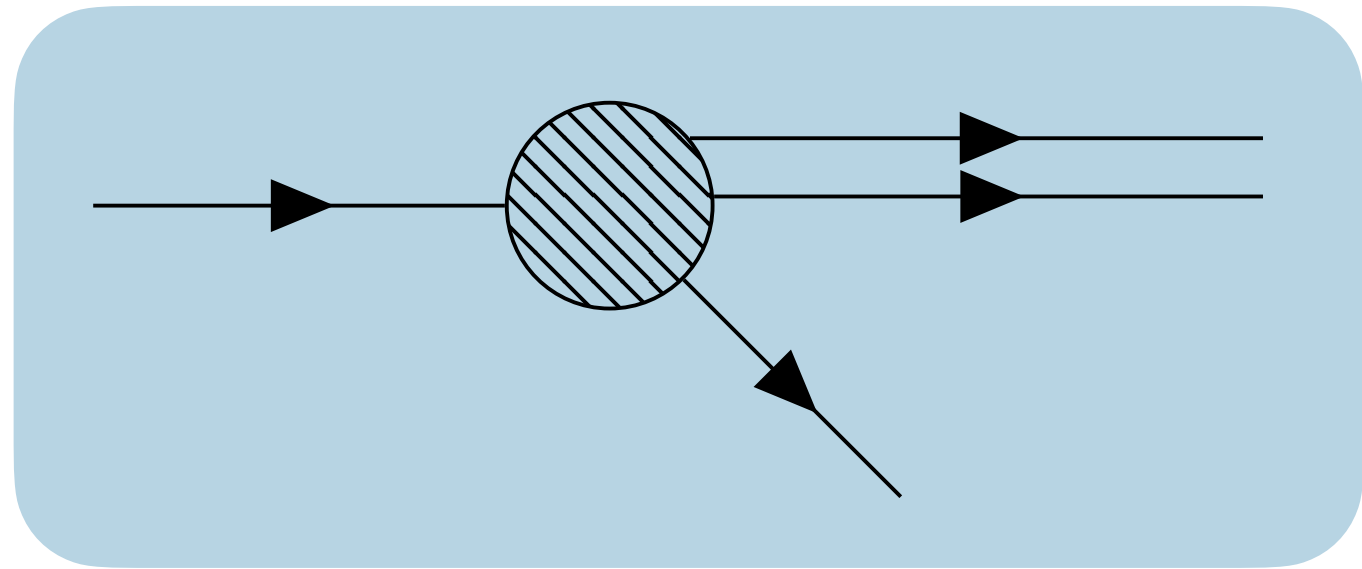
Hadronisation



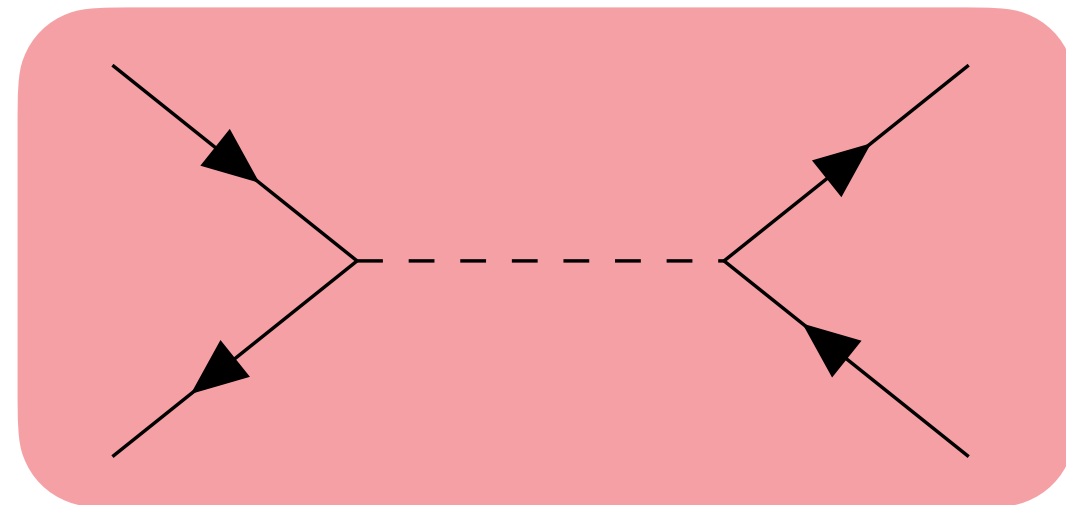
[JHEP 11 \(2022\) 035](#)

Event Generation - What's the problem?

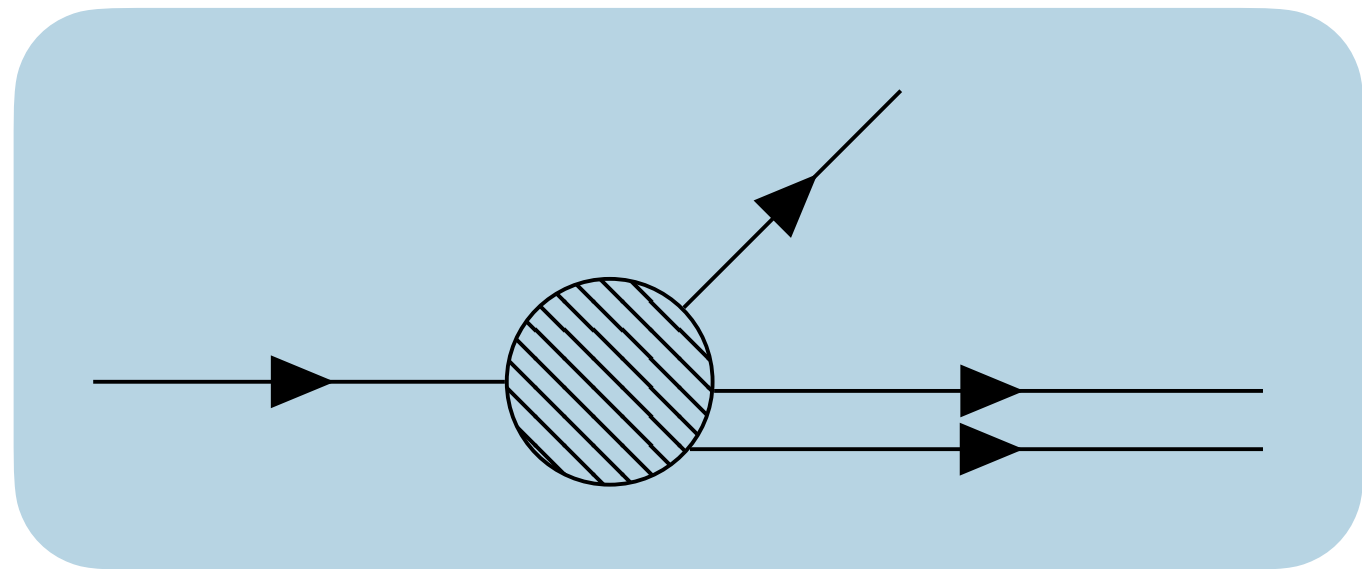
Parton Density Functions



Hard Process



[Phys. Rev. D 103, 076020](#)

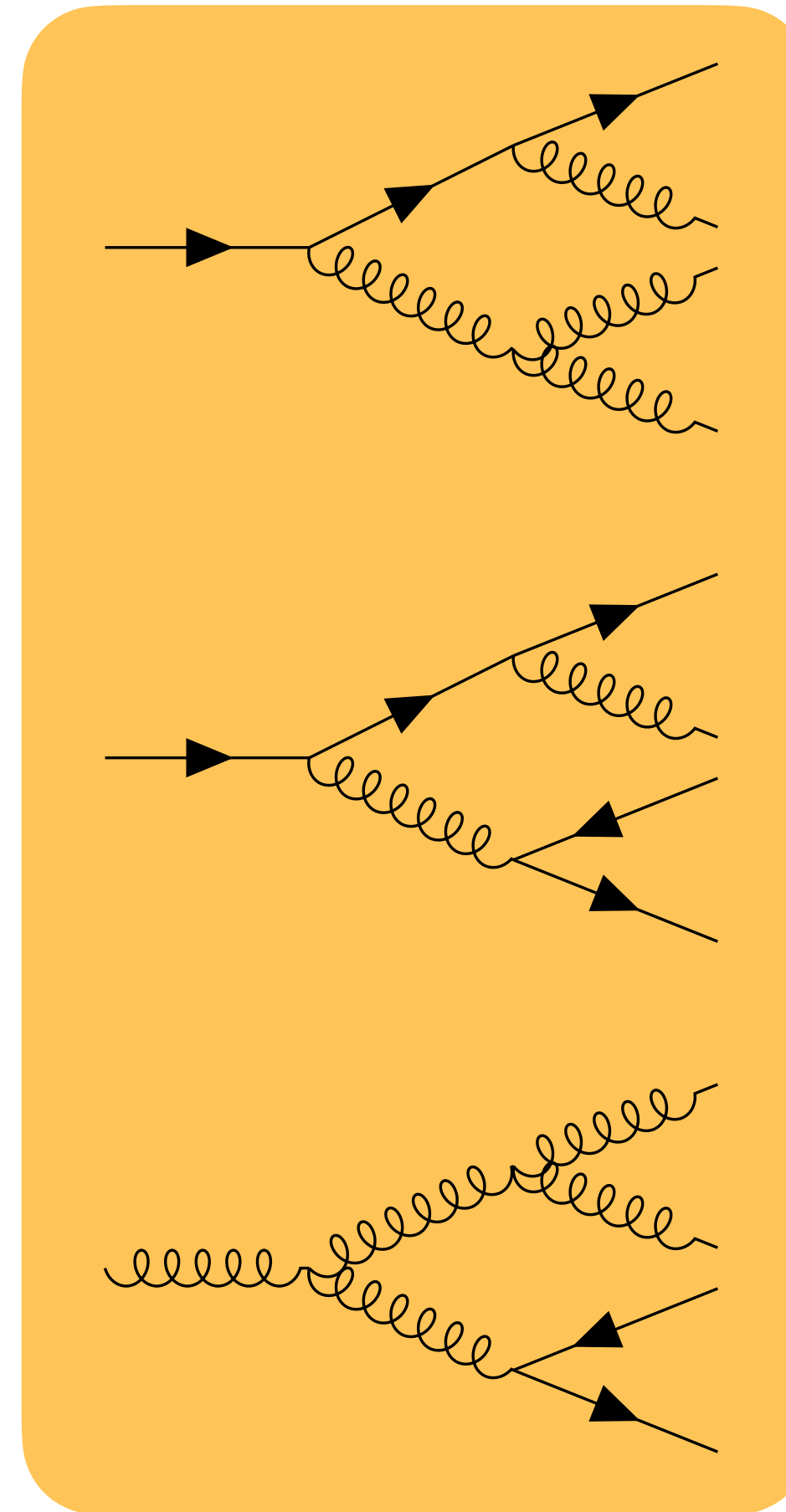


[Phys. Rev. D 106, 056002](#)

[Phys. Rev. D 103, 034027](#)

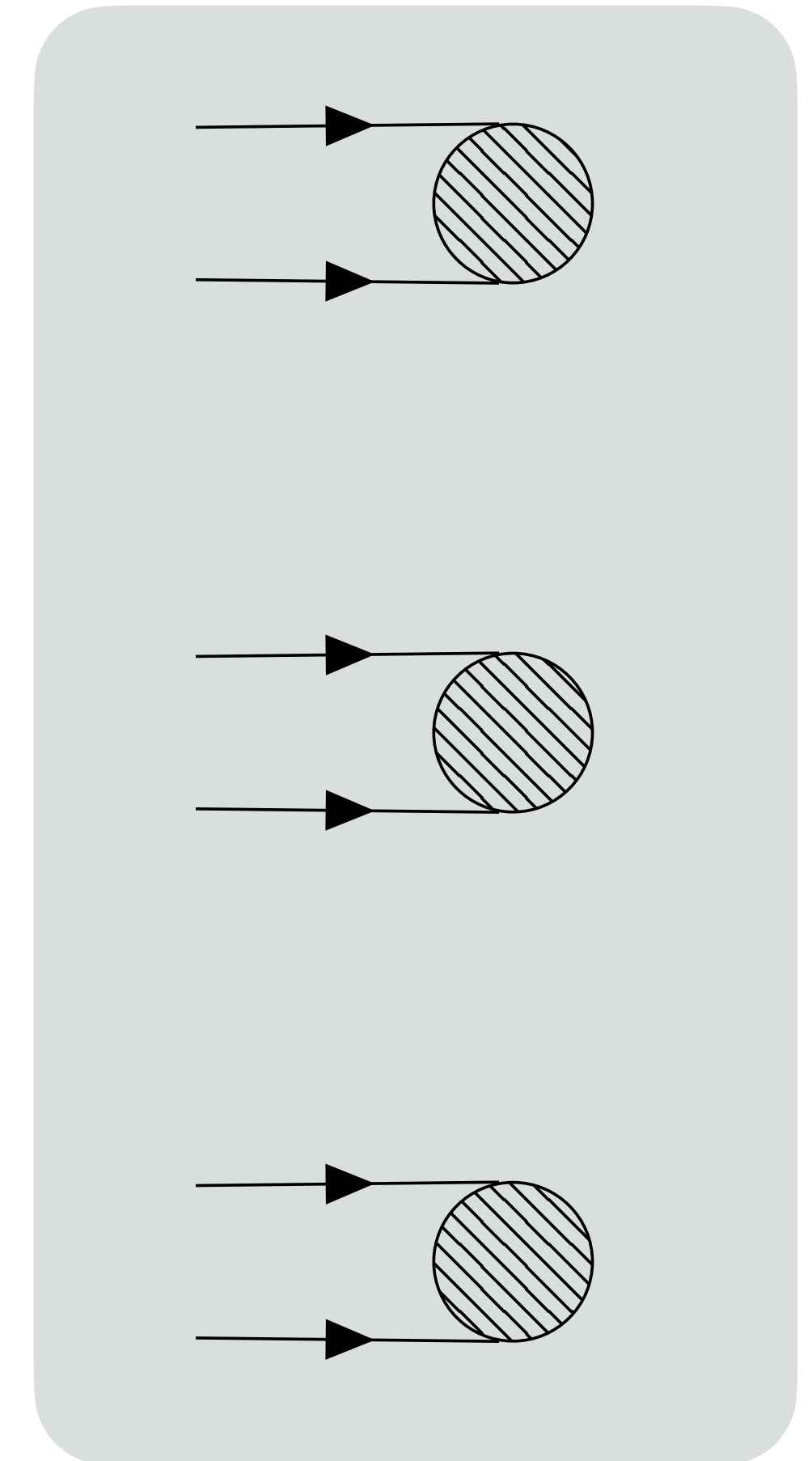
[Phys. Rev. Lett. 126, 062001](#)

Parton Shower

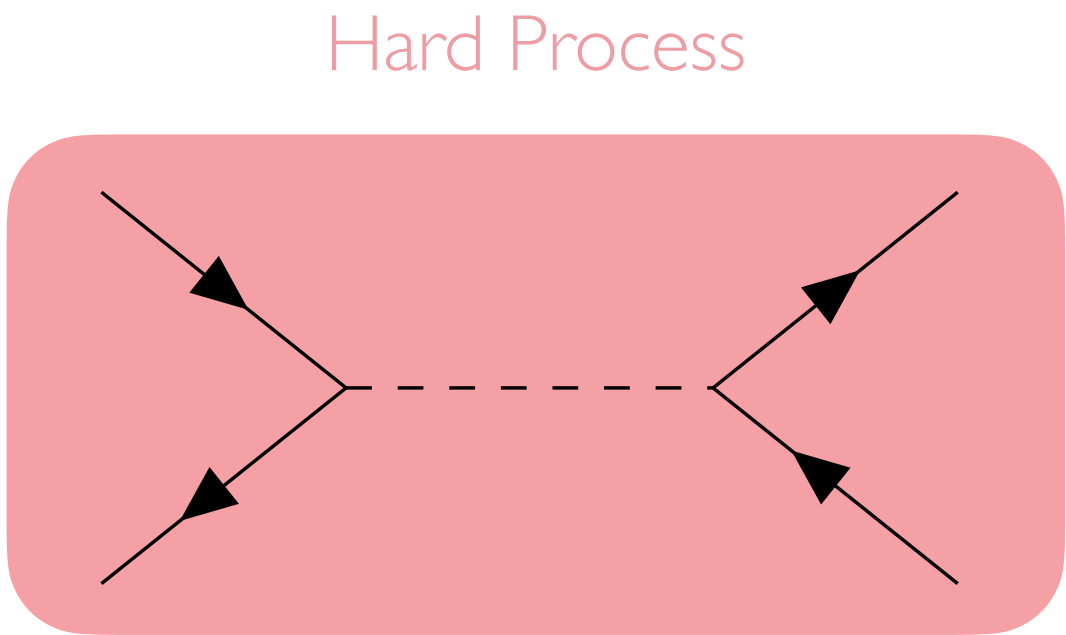


[JHEP 11 \(2022\) 035](#)

Hadronisation



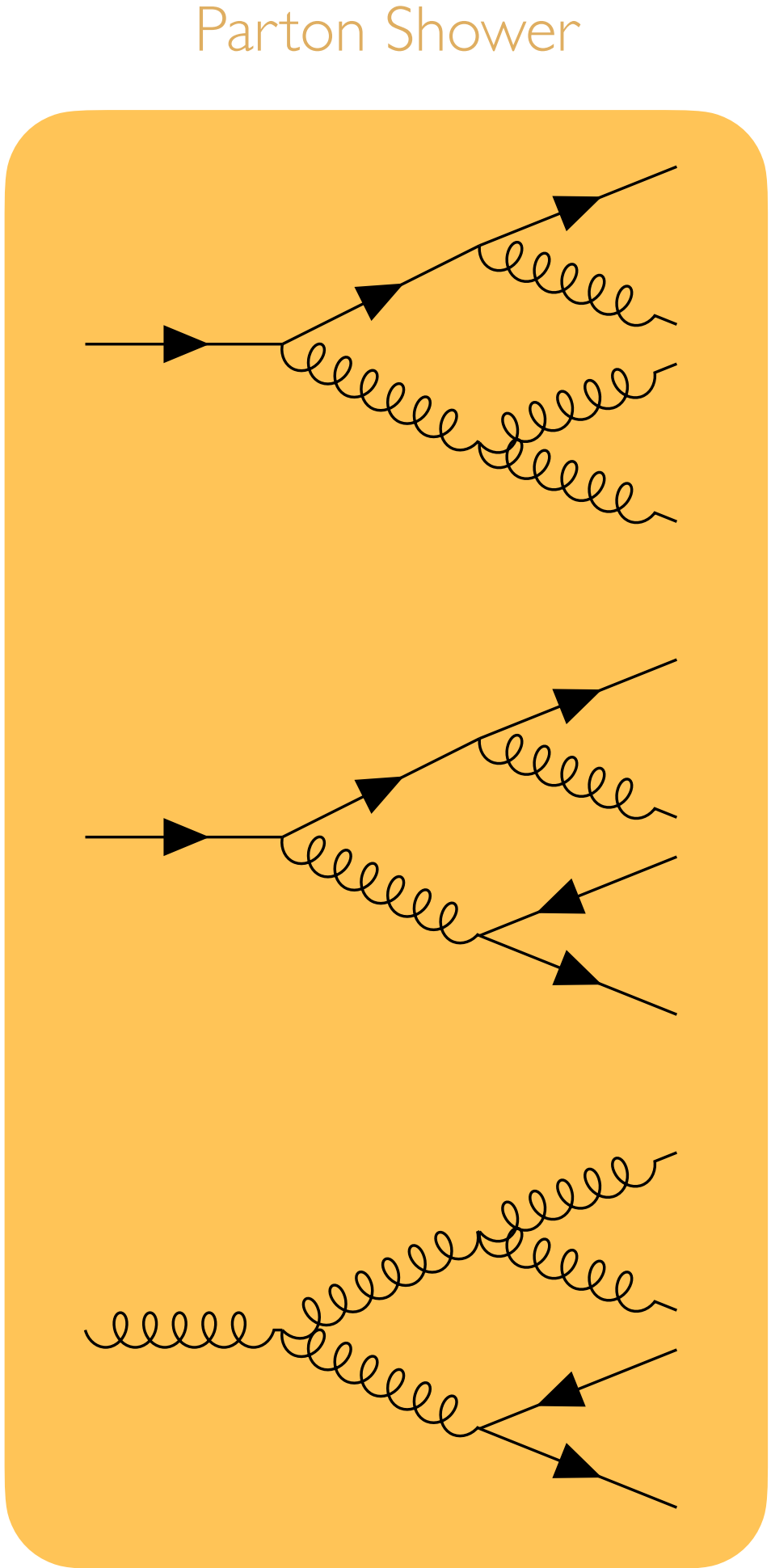
Event Generation - What's the problem?



[Phys. Rev. D 103, 076020](#)

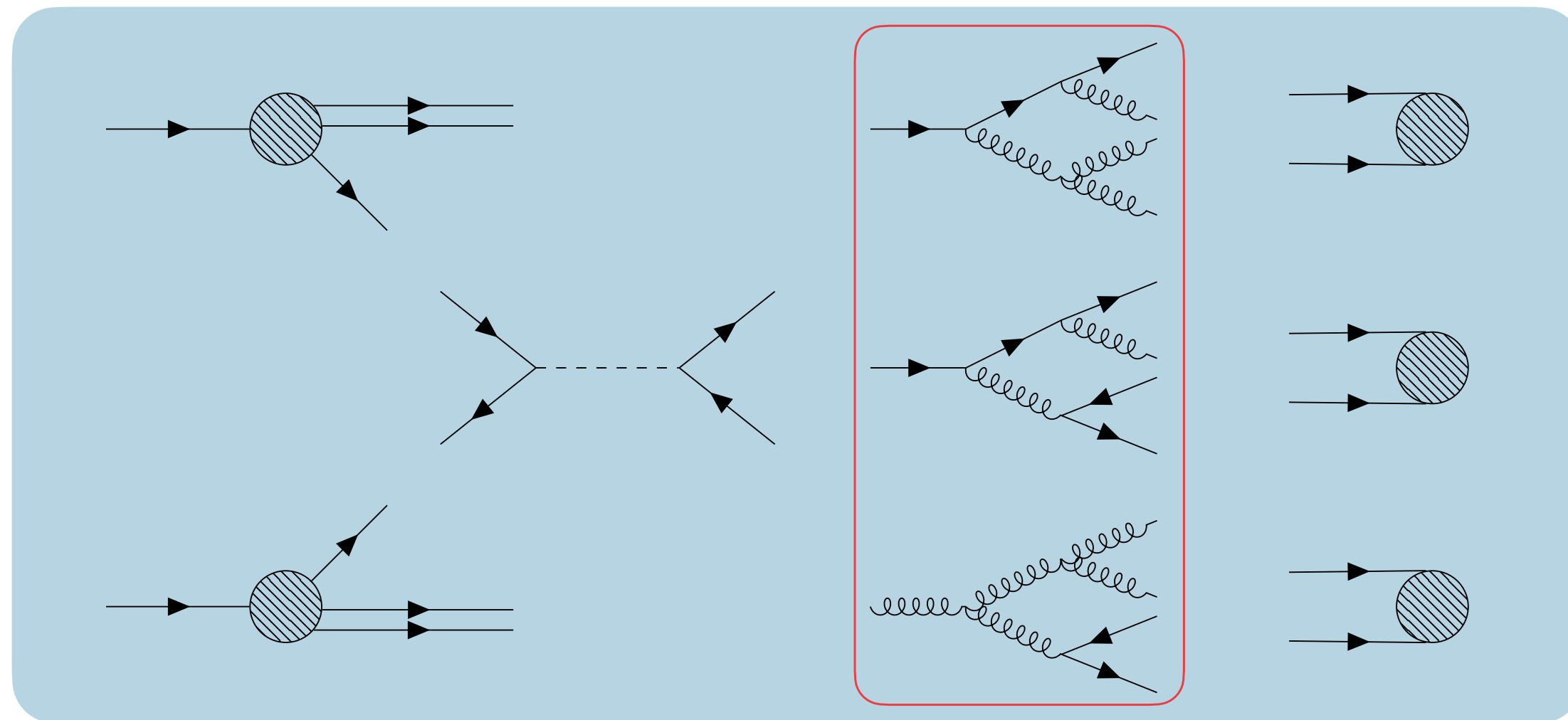
[Phys. Rev. D 106, 056002](#)

[Phys. Rev. Lett. 126, 062001](#)

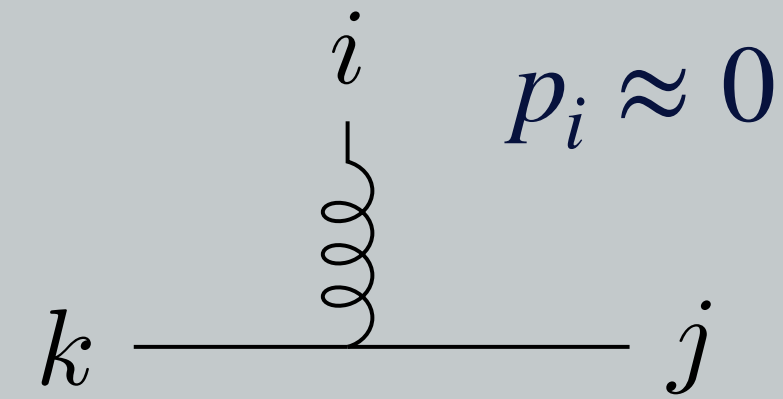


[JHEP 11 \(2022\) 035](#)

The Parton Shower



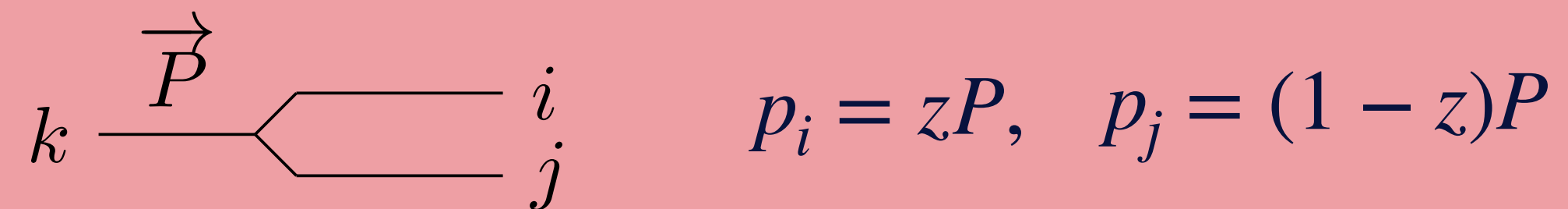
Soft mode:



Interference effects only allow for partial factorisation

Leading contributions to the decay rate in the collinear limit are included in the soft limit

Collinear mode:



Successive decay steps factorise into independent quasi-classical steps

In this limit, the decay from high energy to low energy proceeds as a **colour-dipole cascade**.

This interpretation allows for straightforward interference patterns and momentum conservation

The Parton Shower - The Veto Algorithm

The choice of the variables ξ and t is known as the **phase space parameterisation**

Non-Emission Probability

$$\Delta(t_n, t) = \exp \left(- \int_t^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2s_{ik}(t, \xi)}{s_{ij}(t, \xi) s_{jk}(t, \xi)} \right)$$

$$\mathcal{F}_n(\Phi_n, t_n, t_c; O) = \Delta(t_n, t_c) O(\Phi_n)$$

Master Equation

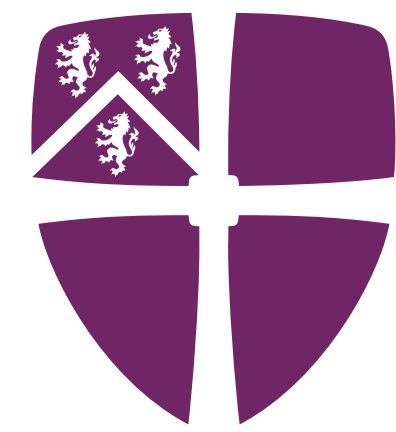
$$+ \int_{t_c}^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2s_{ik}(t, \xi)}{s_{ij}(t, \xi) s_{jk}(t, \xi)} \Delta(t_n, t) \mathcal{F}_n(\Phi_{n+1}, t, t_c; O)$$

Inclusive Decay Probability

$$d\mathcal{P}(q(p_I)\bar{q}(p_K) \rightarrow q(p_i)g(p_j)\bar{q}(p_k)) \simeq \frac{ds_{ij}}{s_{IK}} \frac{ds_{jk}}{s_{IK}} C \frac{\alpha_s}{2\pi} \frac{2s_{IK}}{s_{ij}s_{jk}}$$

Current interpretations of the veto algorithm treat the phase space variables ξ and t as **continuous**

IBM Q



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Quantum Parton Shower

G. Gustafson, S. Prestel, M. Spannowsky and S. Williams, Collider Events on a Quantum Computer, *JHEP* 11 (2022) 035, [arXiv:2207.10694](https://arxiv.org/abs/2207.10694)



LUND
UNIVERSITY

Imperial College
London

Discrete QCD - Abstracting the Parton Shower Method

1. Parameterise phase space in terms of gluon transverse momentum and rapidity:

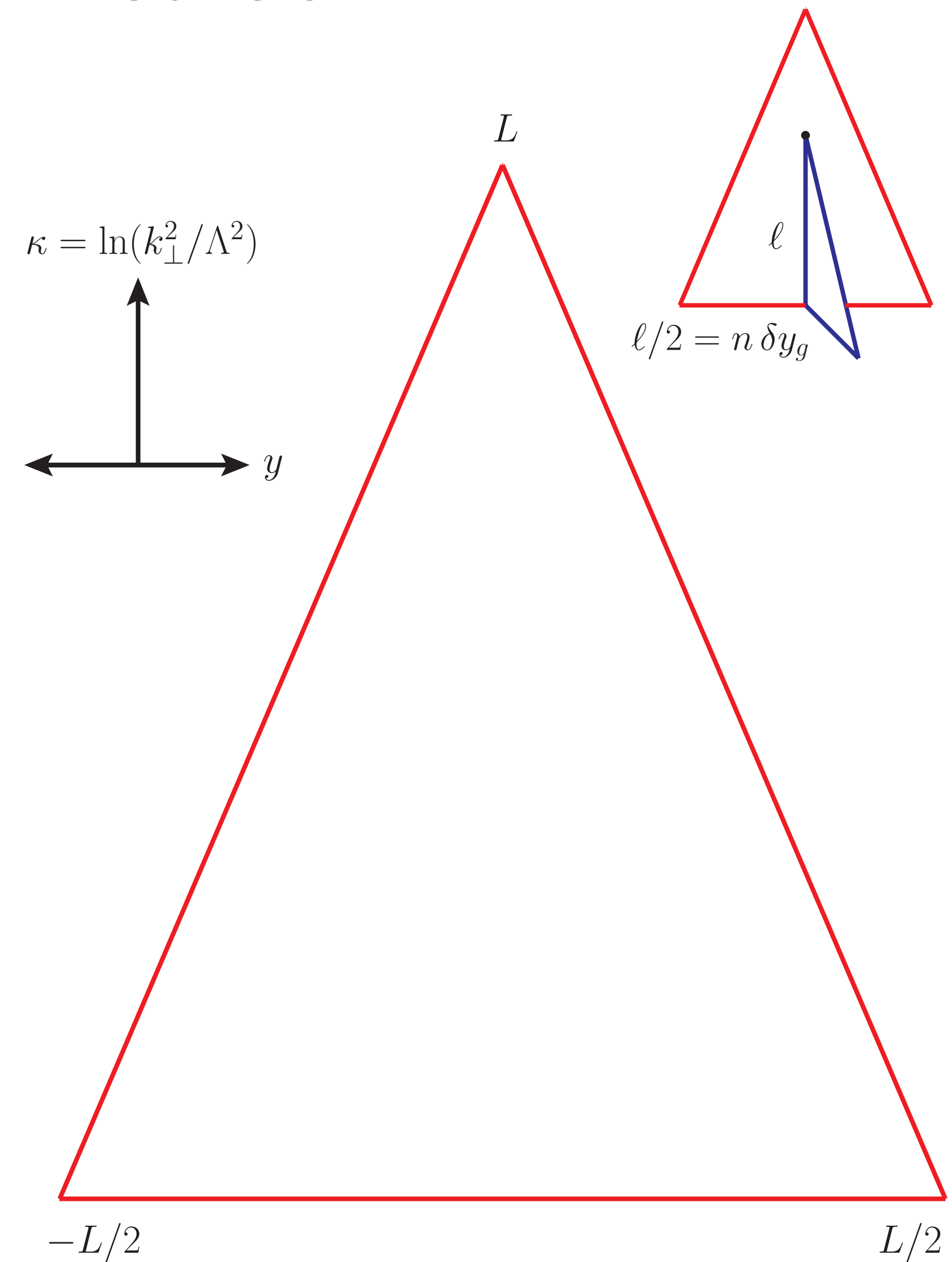
$$k_{\perp}^2 = \frac{s_{ij}s_{jk}}{s_{IK}} \quad \text{and} \quad y = \frac{1}{2} \ln \left(\frac{s_{ij}}{s_{jk}} \right)$$

which leads to the inclusive probability:

$$d\mathcal{P} (q(p_I)\bar{q}(p_K) \rightarrow q(p_i)g(p_j)\bar{q}(p_k)) \simeq \frac{C\alpha_s}{\pi} d\kappa dy$$

where $\kappa = \ln \left(\frac{k_{\perp}^2}{\Lambda^2} \right)$ and Λ is an arbitrary mass scale

Due to the colour charge of emitted gluons, the rapidity span for subsequent dipole decays is increased. This is interpreted as **“folding out”**



Discrete QCD - Abstracting the Parton Shower Method

2. Neglect $g \rightarrow q\bar{q}$ splittings and examine transverse-momentum-dependent running coupling

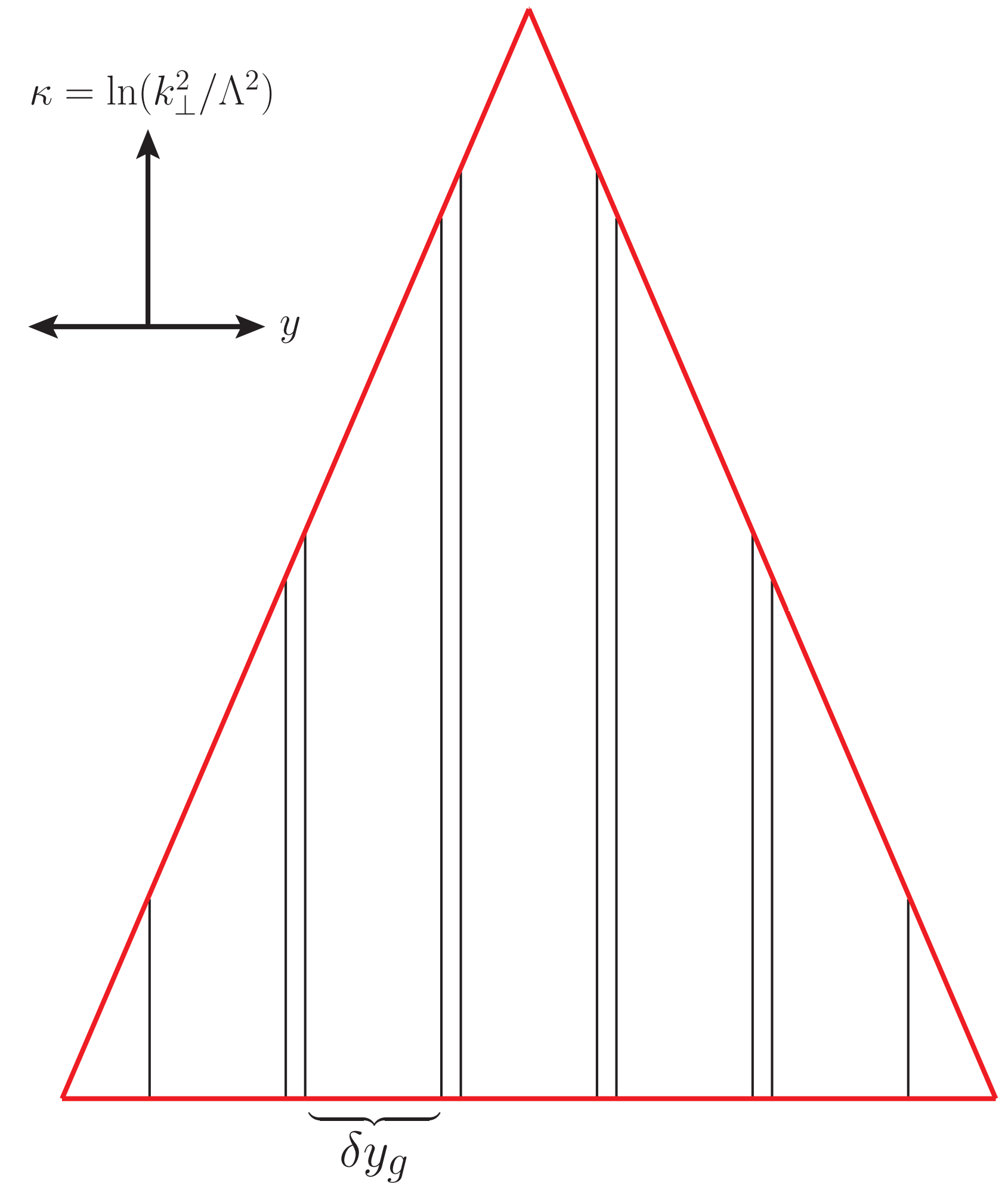
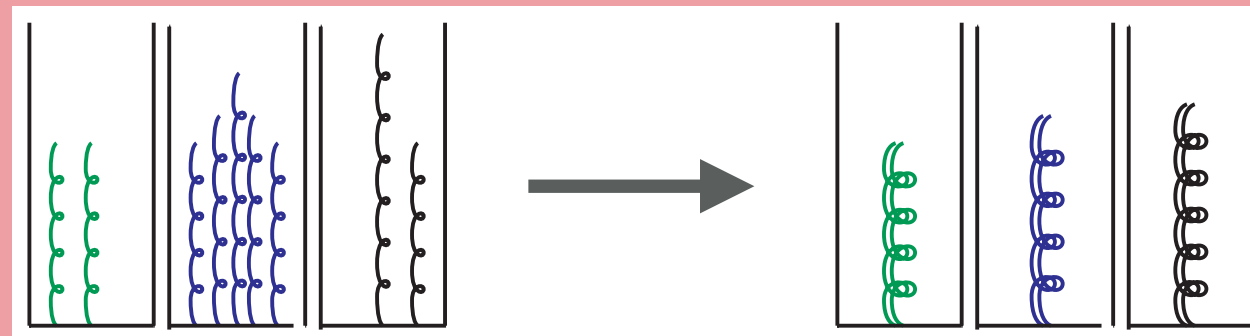
$$\alpha_s(k_{\perp}^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_{\perp}^2/\Lambda_{\text{QCD}}^2)}$$

leads to the inclusive probability

$$d\mathcal{P}(q(p_I)\bar{q}(p_K) \rightarrow q(p_i)g(p_j)\bar{q}(p_k)) \simeq \frac{d\kappa}{\kappa} \frac{dy}{\delta y_g} \quad \text{with} \quad \delta y_g = \frac{11}{6}$$

Interpreting the running coupling renormalisation group as a gain-loss equation:

Glucos within δy_g act coherently as one effective gluon



Discrete QCD - Abstracting the Parton Shower Method

2. Neglect $g \rightarrow q\bar{q}$ splittings and examine transverse-momentum-dependent running coupling

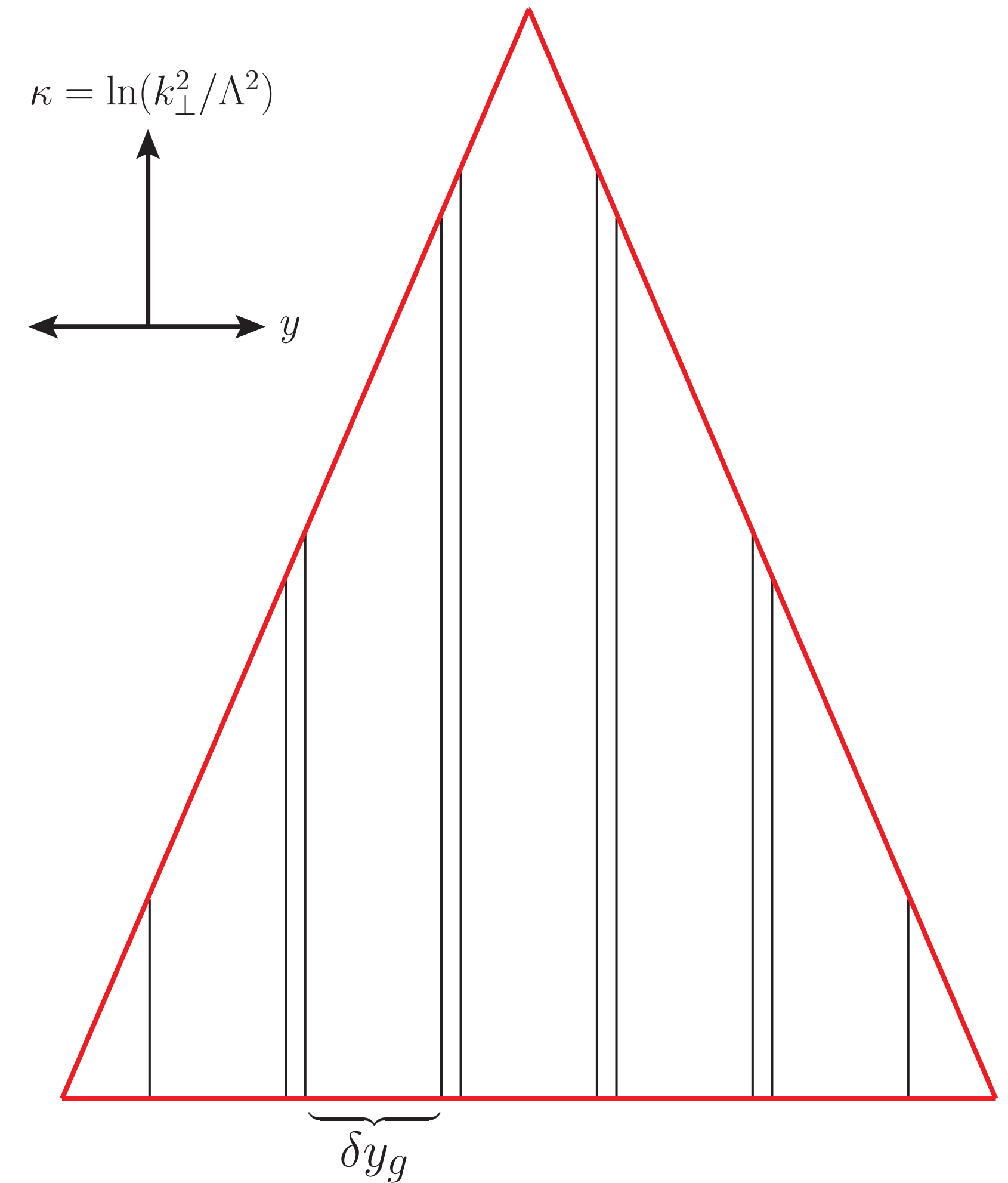
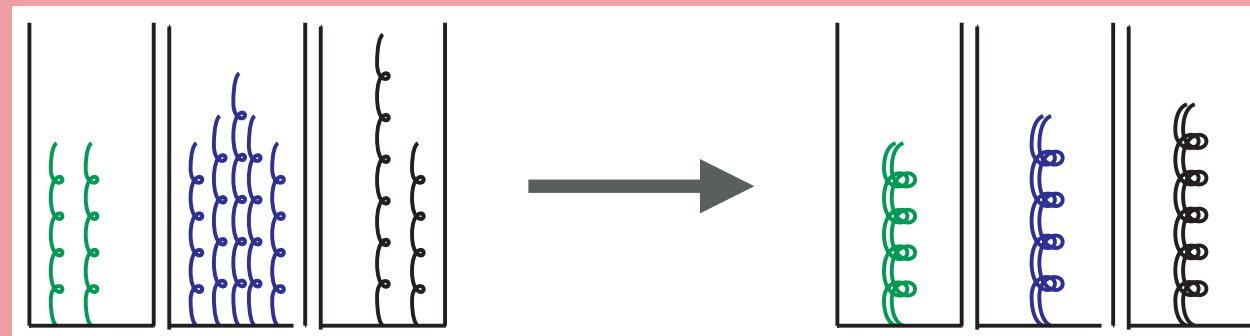
$$\alpha_s(k_{\perp}^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_{\perp}^2 / \Lambda_{\text{QCD}}^2)} = \frac{\text{const.}}{\kappa}$$

leads to the inclusive probability

$$d\mathcal{P}(q(p_I)\bar{q}(p_K) \rightarrow q(p_i)g(p_j)\bar{q}(p_k)) \simeq \frac{d\kappa}{\kappa} \frac{dy}{\delta y_g} \quad \text{with} \quad \delta y_g = \frac{11}{6}$$

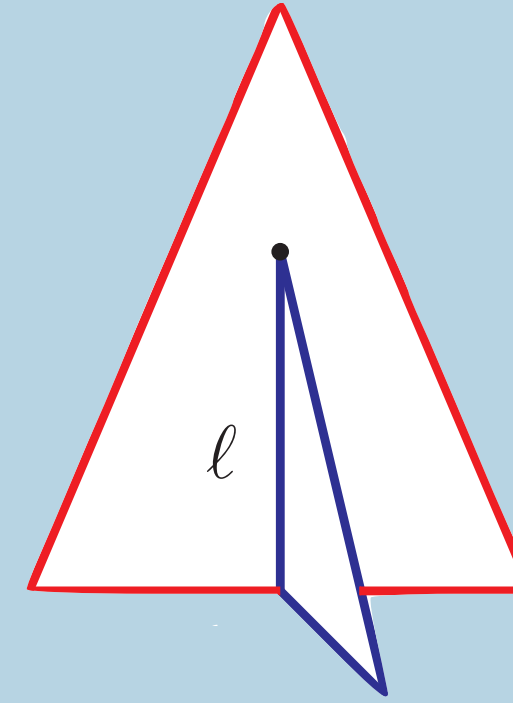
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Discrete QCD - Abstracting the Parton Shower Method

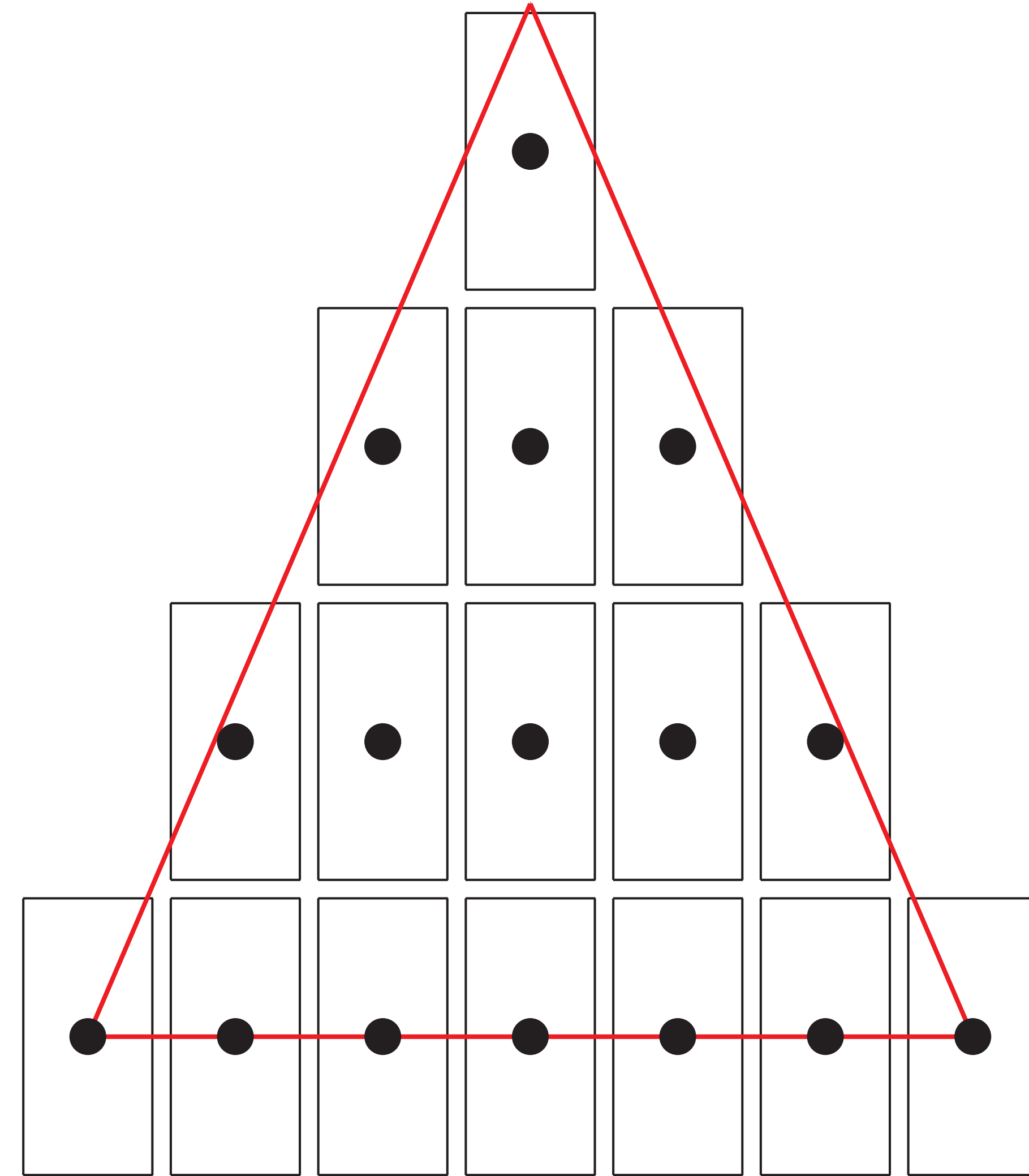
Folding out extends the baseline of the triangle to positive y by $\frac{l}{2}$, where l is the height at which to emit effective gluons



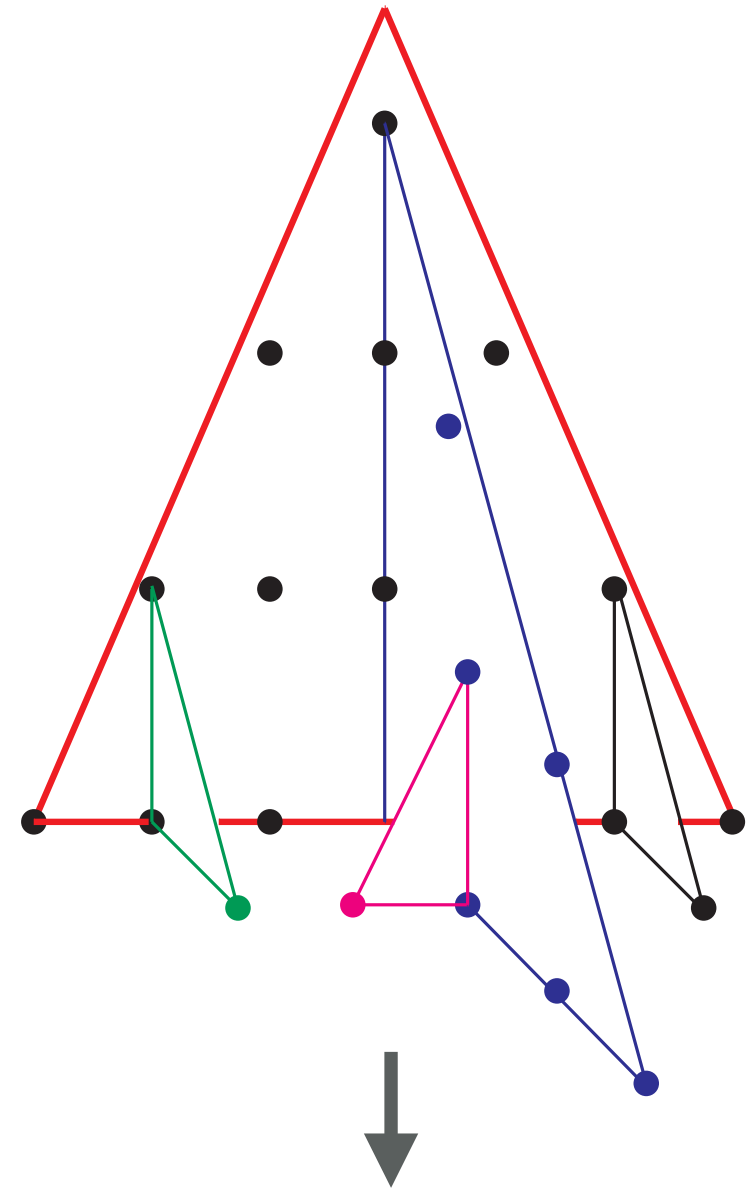
A consequence of folding is that the κ axis is quantised into multiples of $2\delta y_g$

Each rapidity slice can be treated independently of any other slice. The exclusive rate probability takes the simple form:

$$\frac{d\kappa}{\kappa} \exp\left(-\int_{\kappa}^{\kappa_{max}} \frac{d\bar{\kappa}}{\bar{\kappa}}\right) = \frac{d\kappa}{\kappa_{max}}$$

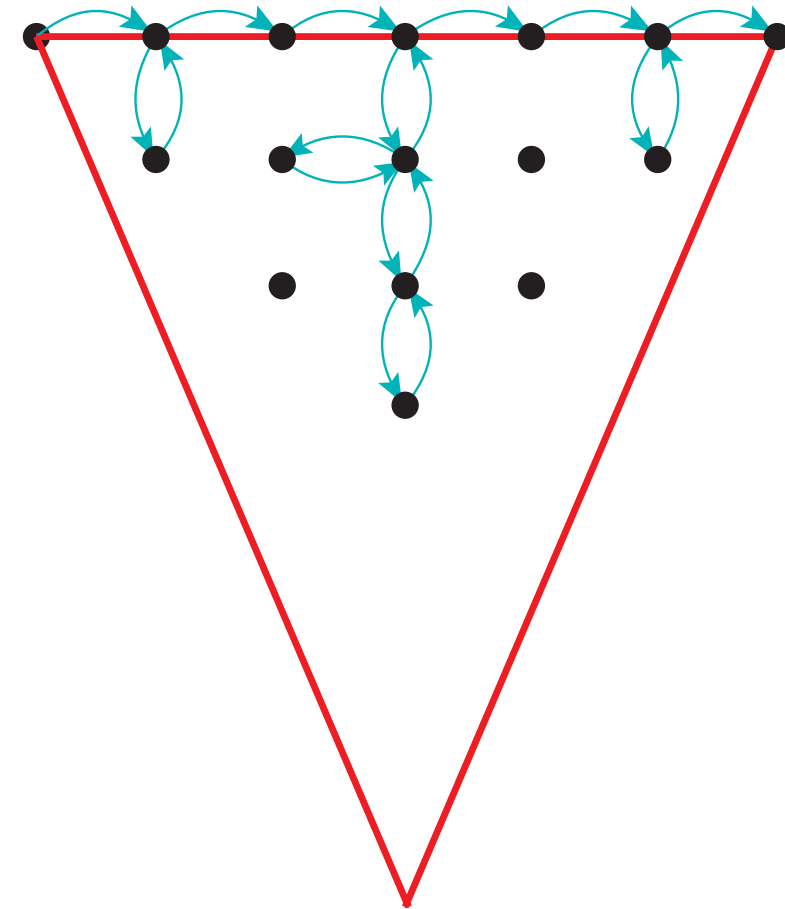
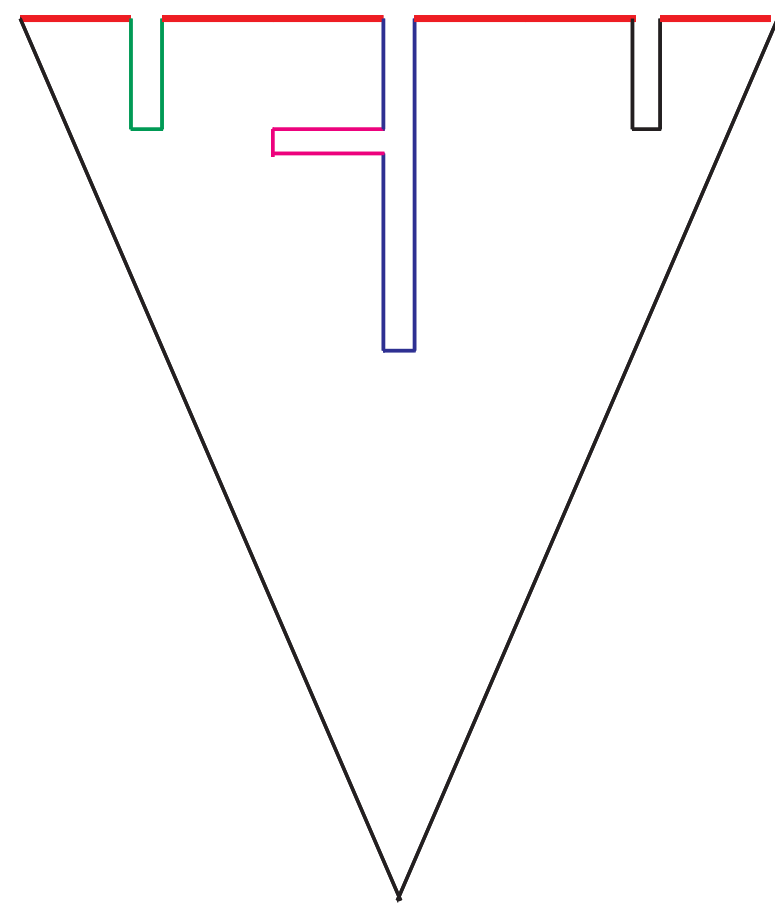


Discrete QCD as a Quantum Walk

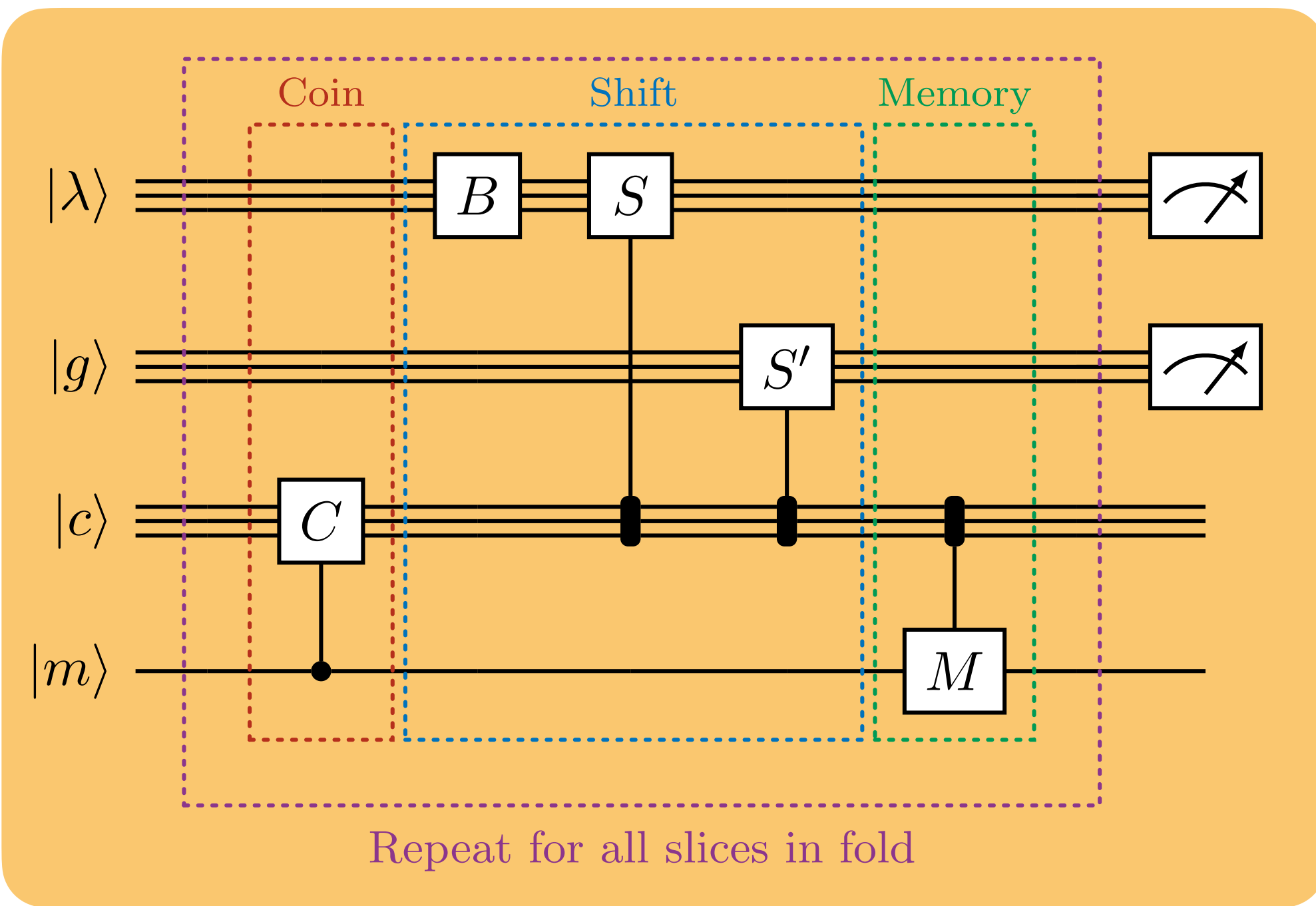


The **baseline** of the grove structure contains all kinematics information

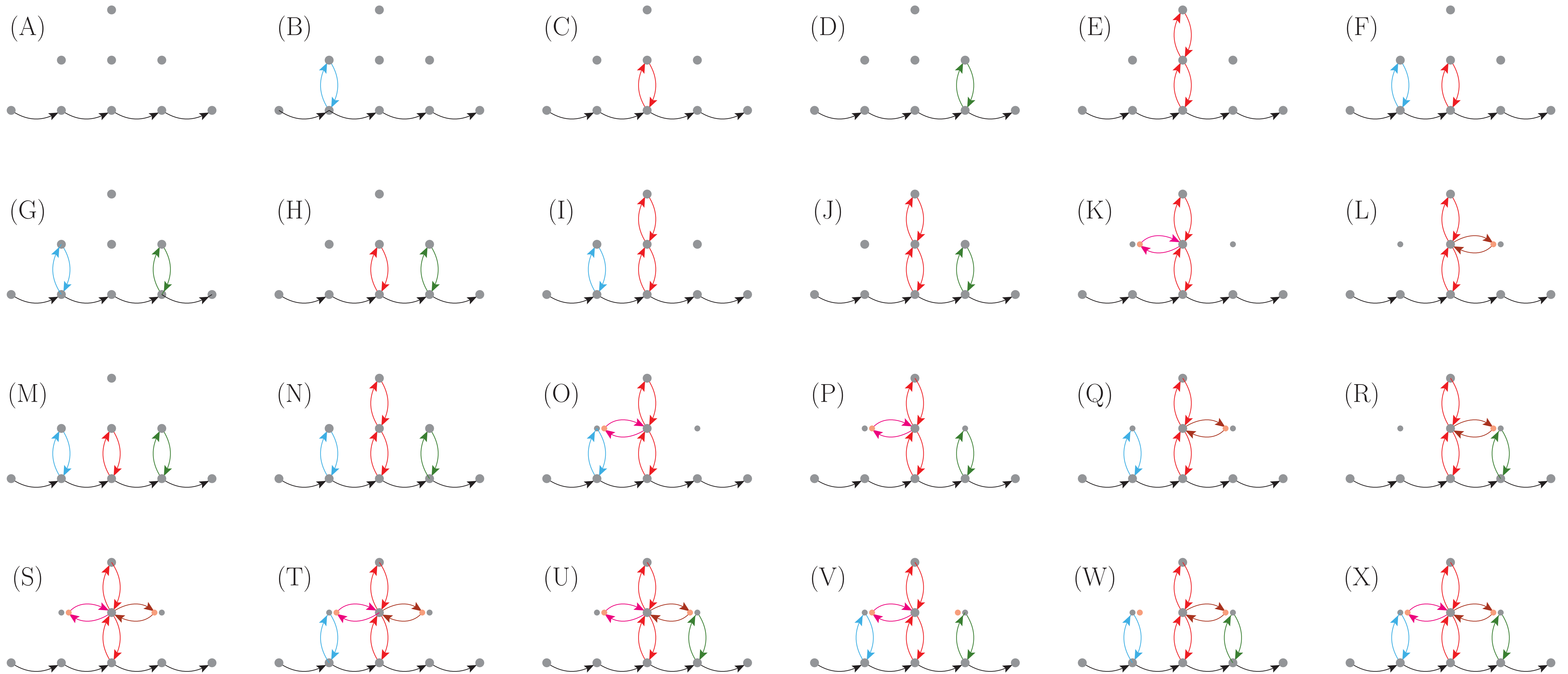
For LEP data there are **24 unique grove structures** for $\Lambda_{\text{QCD}} \in [0.1, 1]$ GeV



The Discrete-QCD dipole cascade can therefore be implemented as a simple **Quantum Walk**



Discrete QCD - Grove Structures



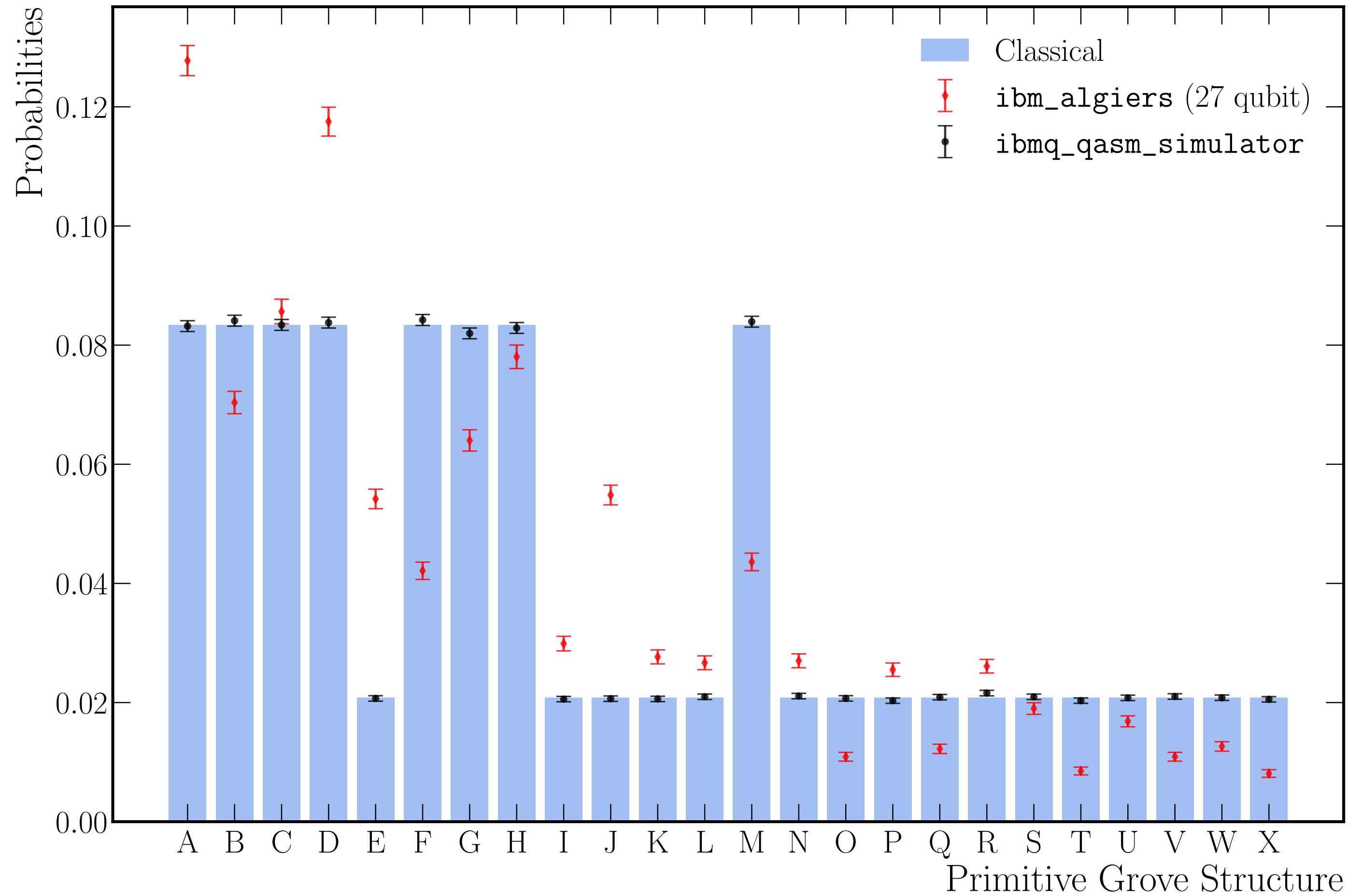
Generating Scattering Events from Groves

Once the grove structure has been selected, event data can be synthesised in the following steps using the baseline:

1. Create the highest κ effective gluons first (i.e. go from top to bottom in phase space)
2. For each effective gluon j that has been emitted from a dipole IK , read off the values s_{ij} , s_{jk} and s_{IK} from the grove
3. Generate a uniformly distributed azimuthal decay angle ϕ , and then employ momentum mapping (here we have used [Phys. Rev. D 85, 014013 \(2012\), 1108.6172](#)) to produce post-branching momenta

The algorithm has been run on both the `ibm_qasm_simulator` and the `ibm_algiers 27` qubit device. A like-for-like classical implementation has been used as a comparison.

Discrete QCD as a Quantum Walk - Raw Grove Simulation



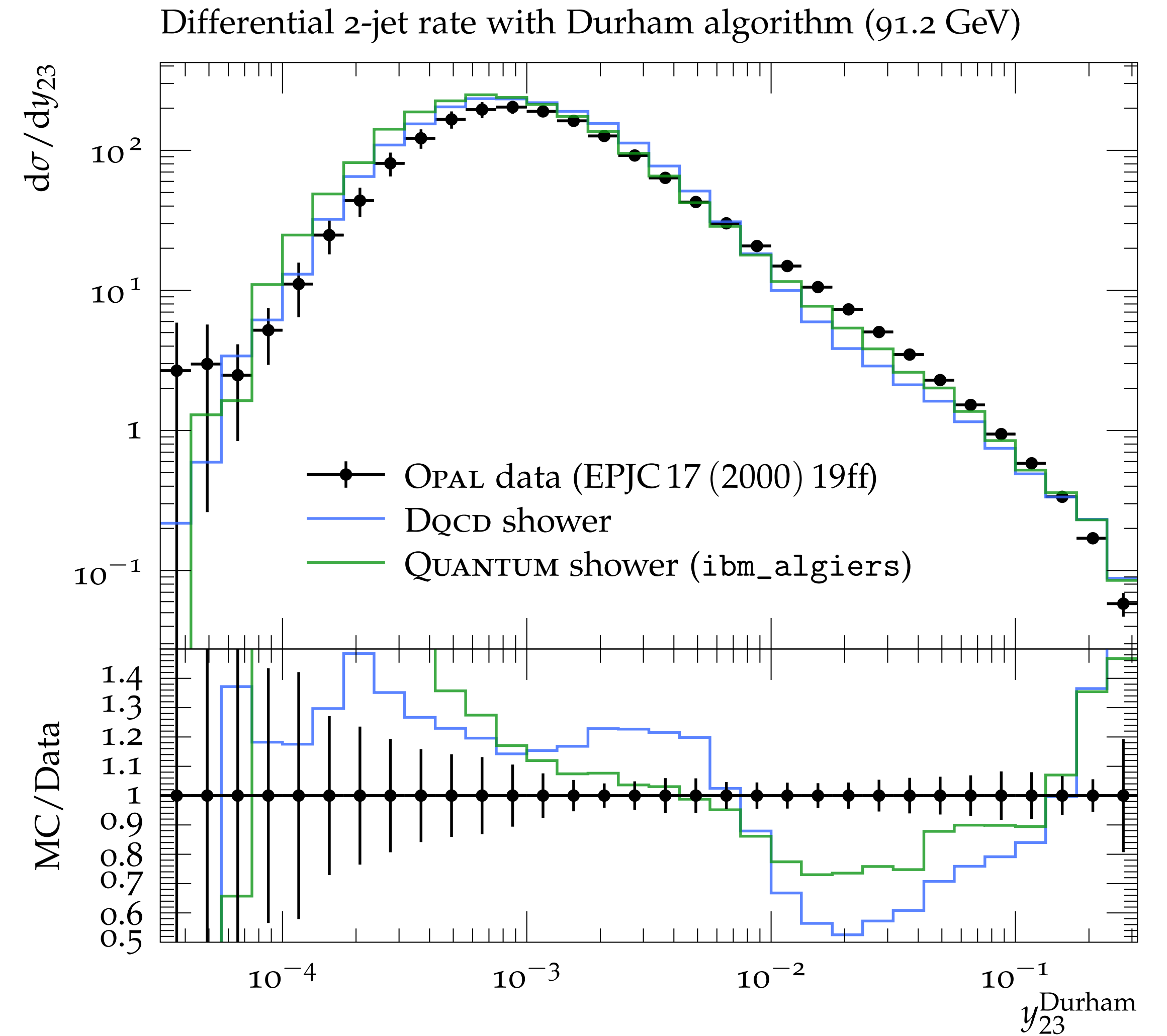
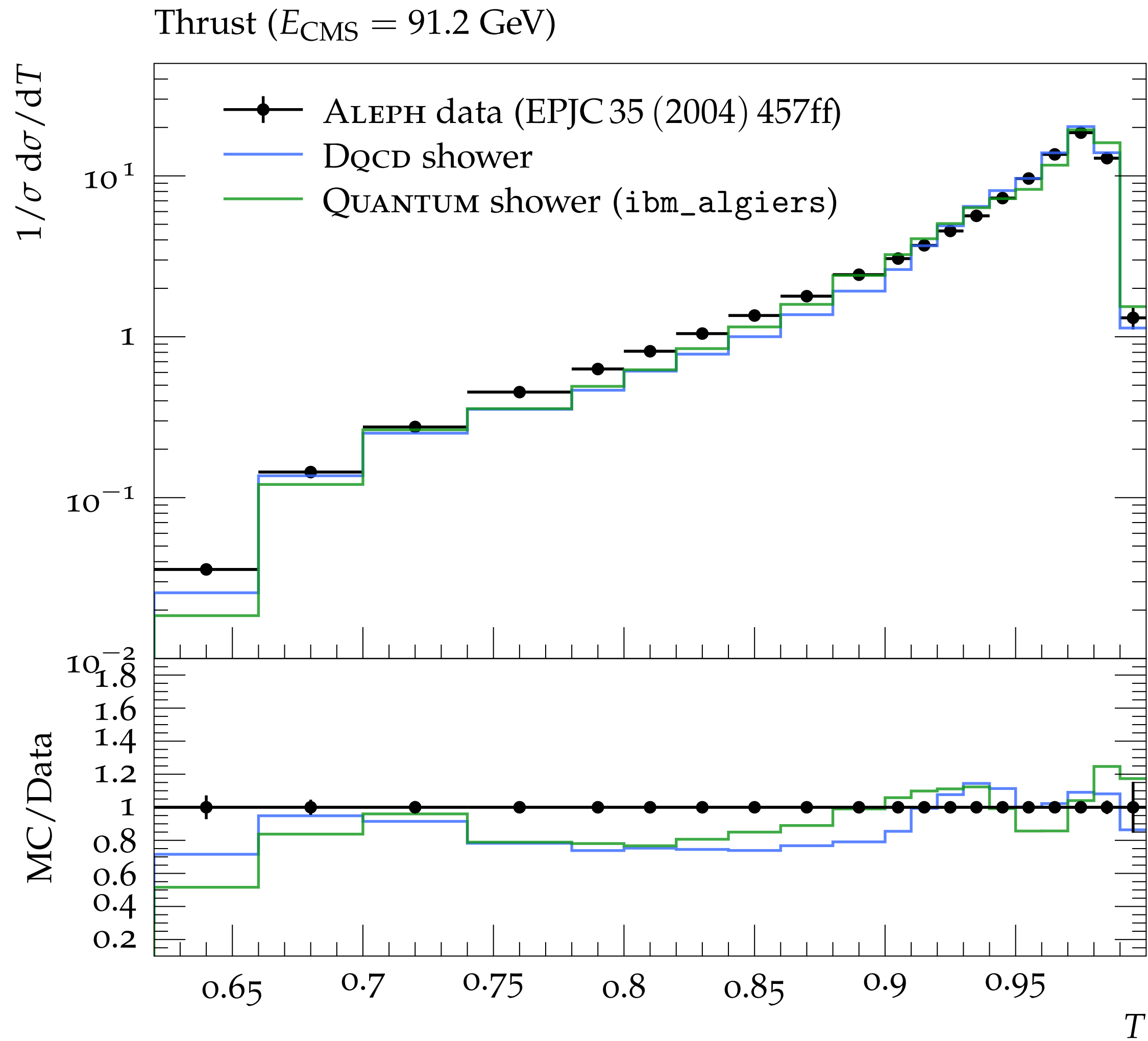
The algorithm has been run on the **IBM Falcon 5.1 Ir chip**

The figure shows the uncorrected performance of the **ibmq_algiers** device compared to a simulator

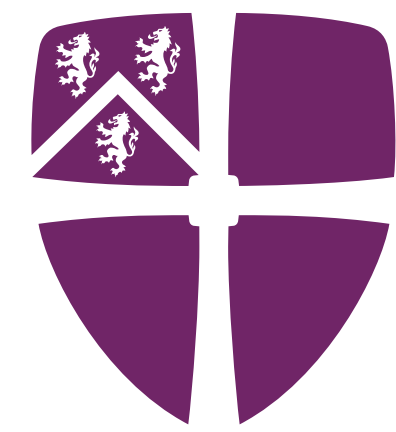
The 24 grove structures are generated for a $E_{CM} = 91.2$ GeV, corresponding to typical collisions at LEP.

Main source of error from CNOT errors from large amount of SWAPs

Collider Events on a Quantum Computer



IBM Q



Durham
University

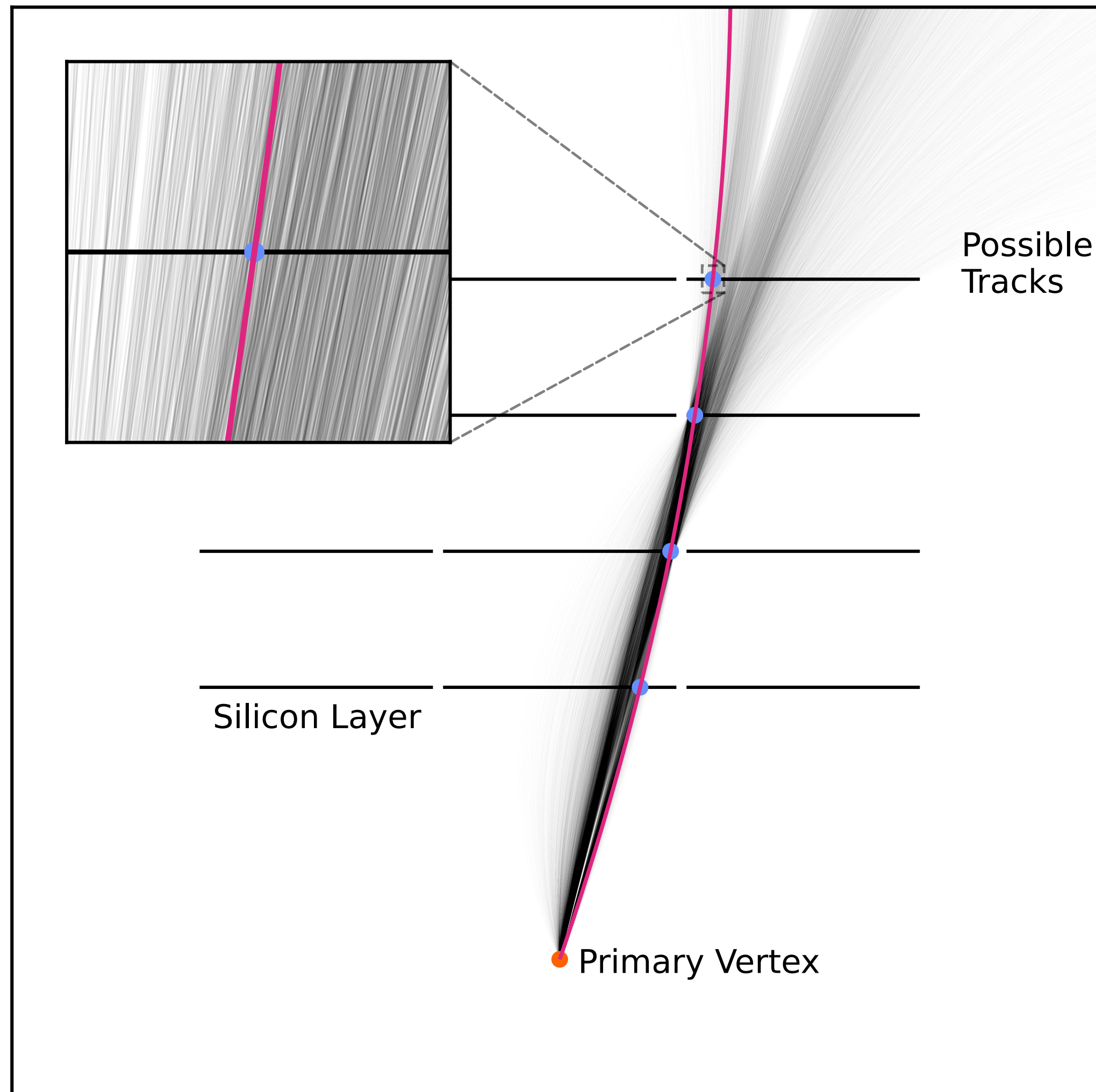


Quantum Charged Track Finding

Quantum Pathways for Charged Track Finding in High-Energy Collisions,
C. Brown, M. Spannowsky, A. Tapper, SW and I. Xiotidis, [arXiv:2311.00766](https://arxiv.org/abs/2311.00766)

Imperial College
London

Track Finding via Associative Memory

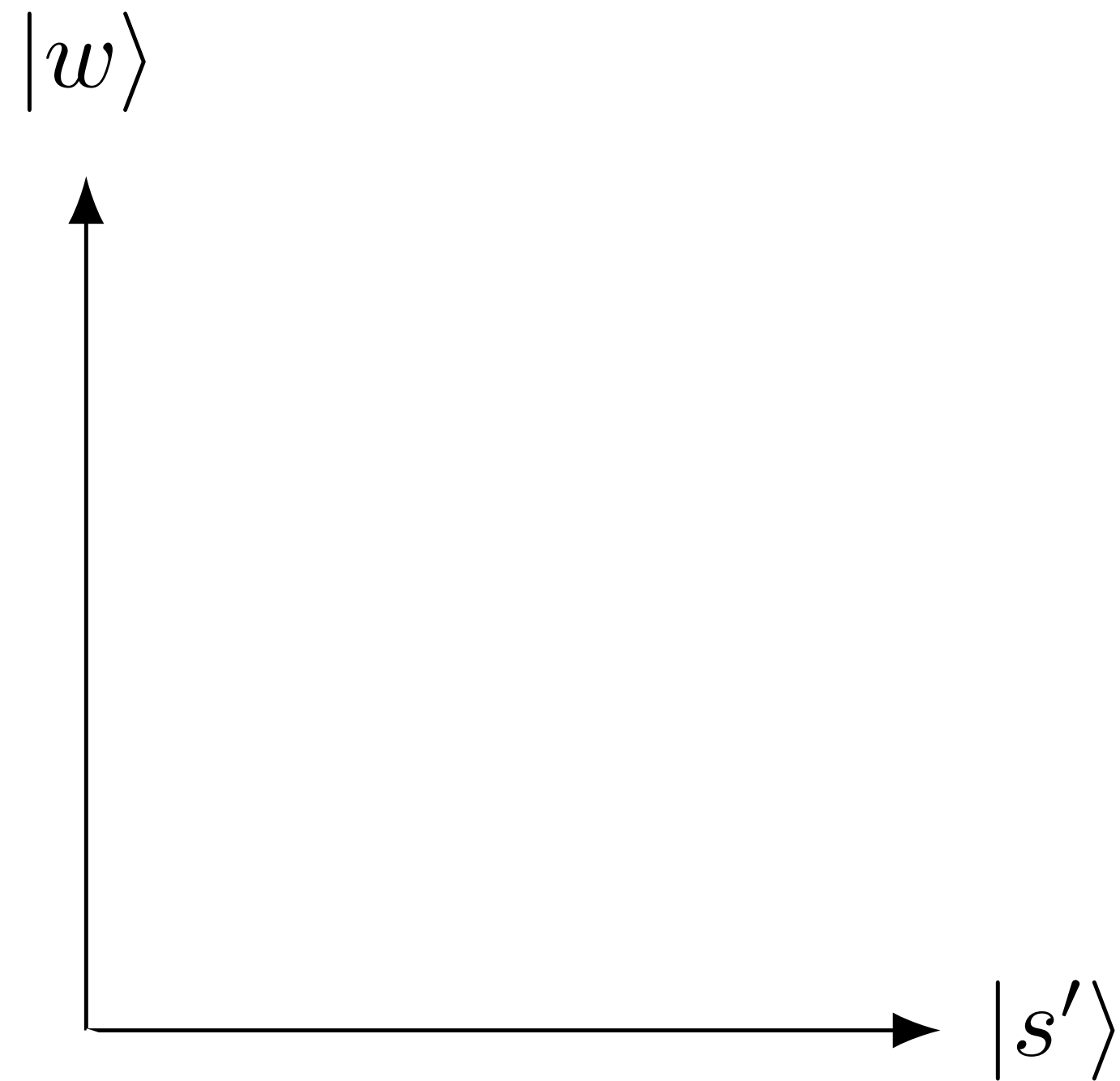


A critical stage of event reconstruction and classification in modern colliders is the identification of **charged particle trajectories**

Highly **granular** detectors are used to efficiently measure the **position** of **charged particles** as they move through the detector

Classical techniques like **Associative Memory** have been shown to be **highly effective**, but **new approaches** are required as collider **energy and luminosity increase** to handle the growing number of **tracks and combinatorics**

Quantum Amplitude Amplification



The aim is to **identify** interesting states in a database $X = \{x_0, x_1, \dots, x_N\}$ with **interesting states** m_i encoded on a quantum device as $|s\rangle = \mathcal{A} |0\rangle^{\otimes n}$

Marking interesting states, $|m\rangle$ using the **oracle**

$$f(x) = \begin{cases} 1 & \text{if } x = m, \\ 0 & \text{otherwise.} \end{cases} \quad \longrightarrow \quad S_f |x\rangle = (-1)^{f(x)} |x\rangle$$

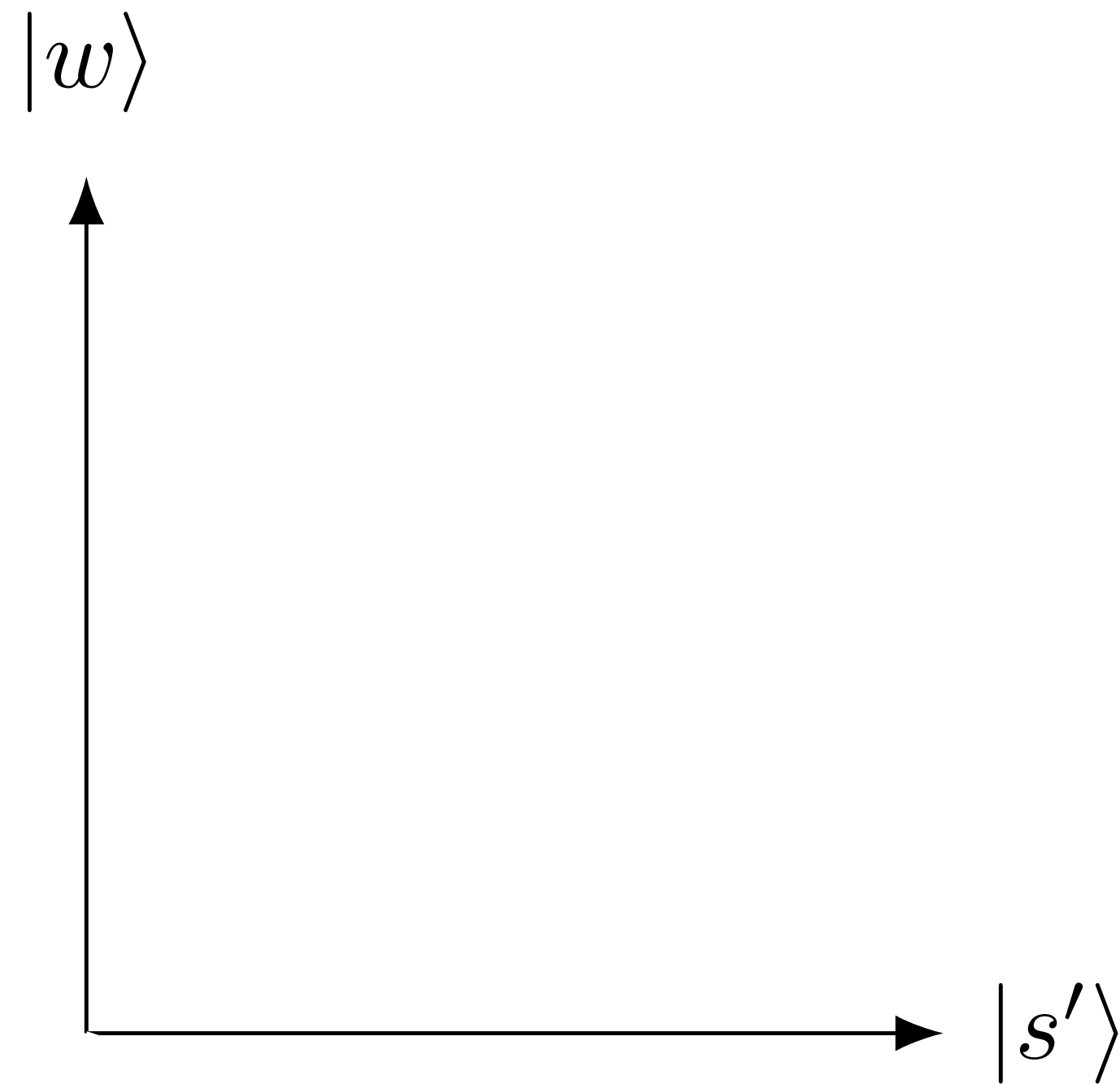
Amplify marked states using the diffusion operation:

$$D = \mathcal{A}^\dagger S_0 \mathcal{A}$$

Therefore, can iteratively apply the **Grover Iterator**:

$$Q = \mathcal{A}^\dagger S_0 \mathcal{A} S_f$$

Quantum Amplitude Amplification



$$|s'\rangle = \frac{1}{\sqrt{N-1}} \sum_{n=1}^{N-1} |n-1\rangle$$

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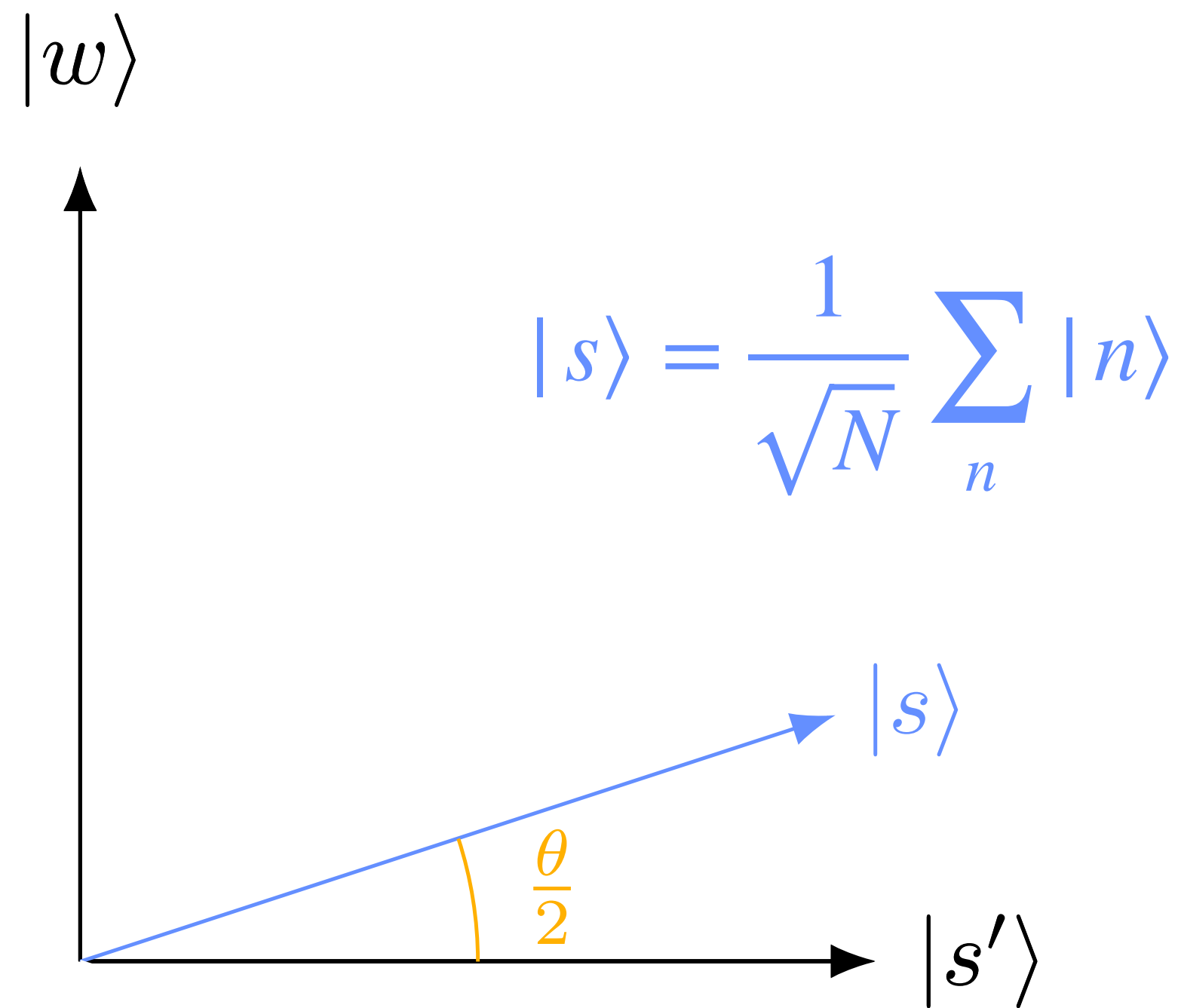
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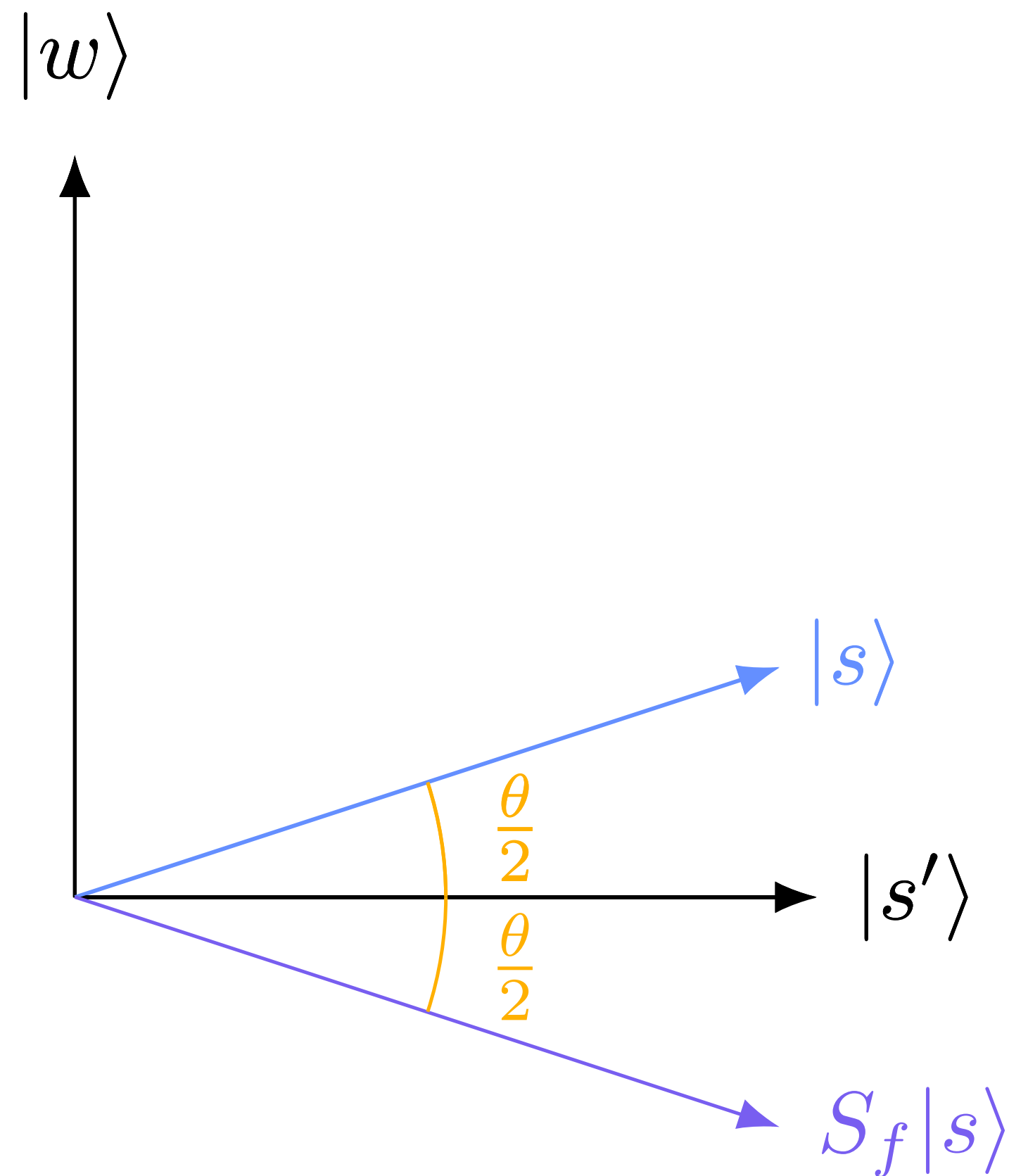
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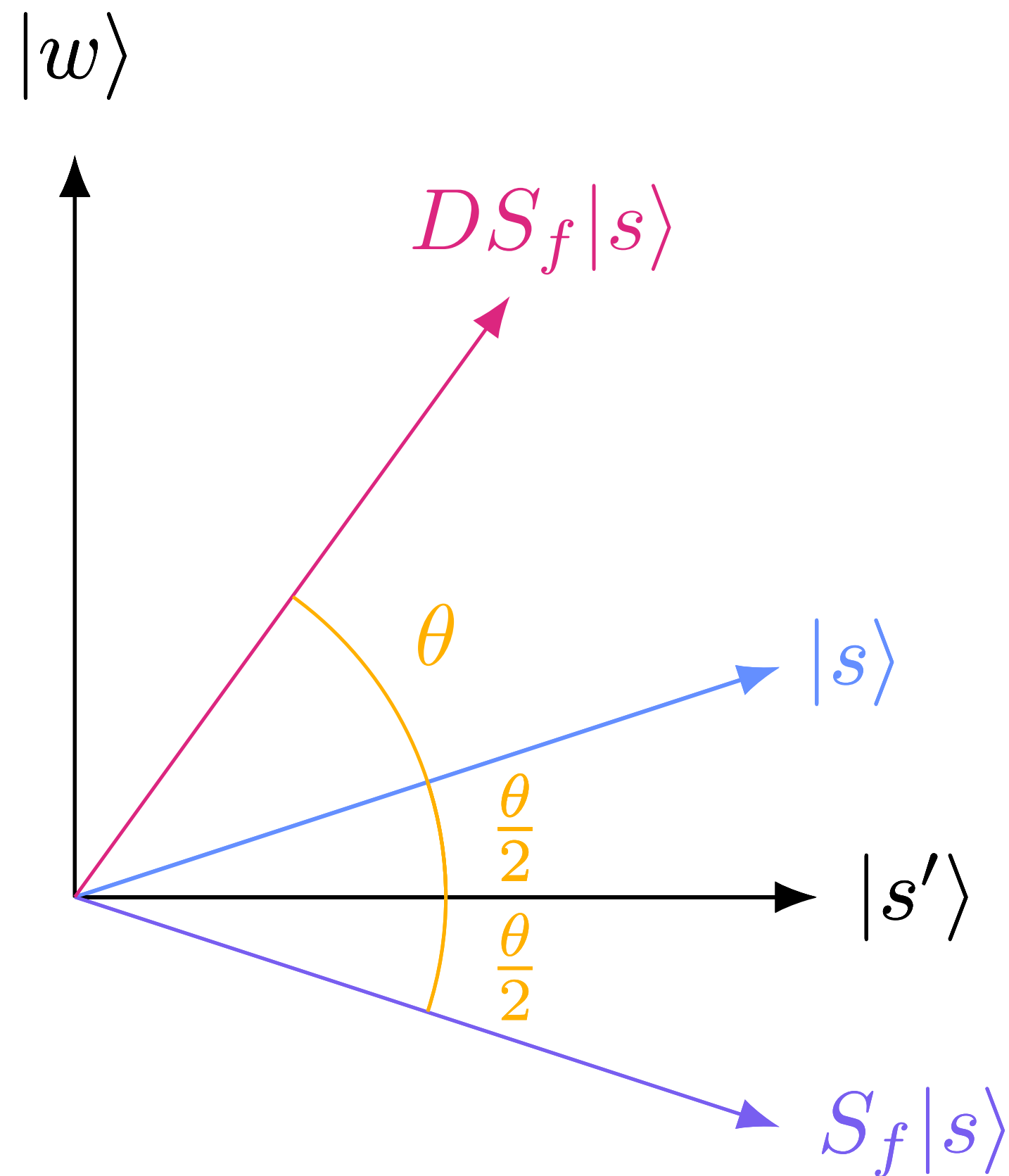
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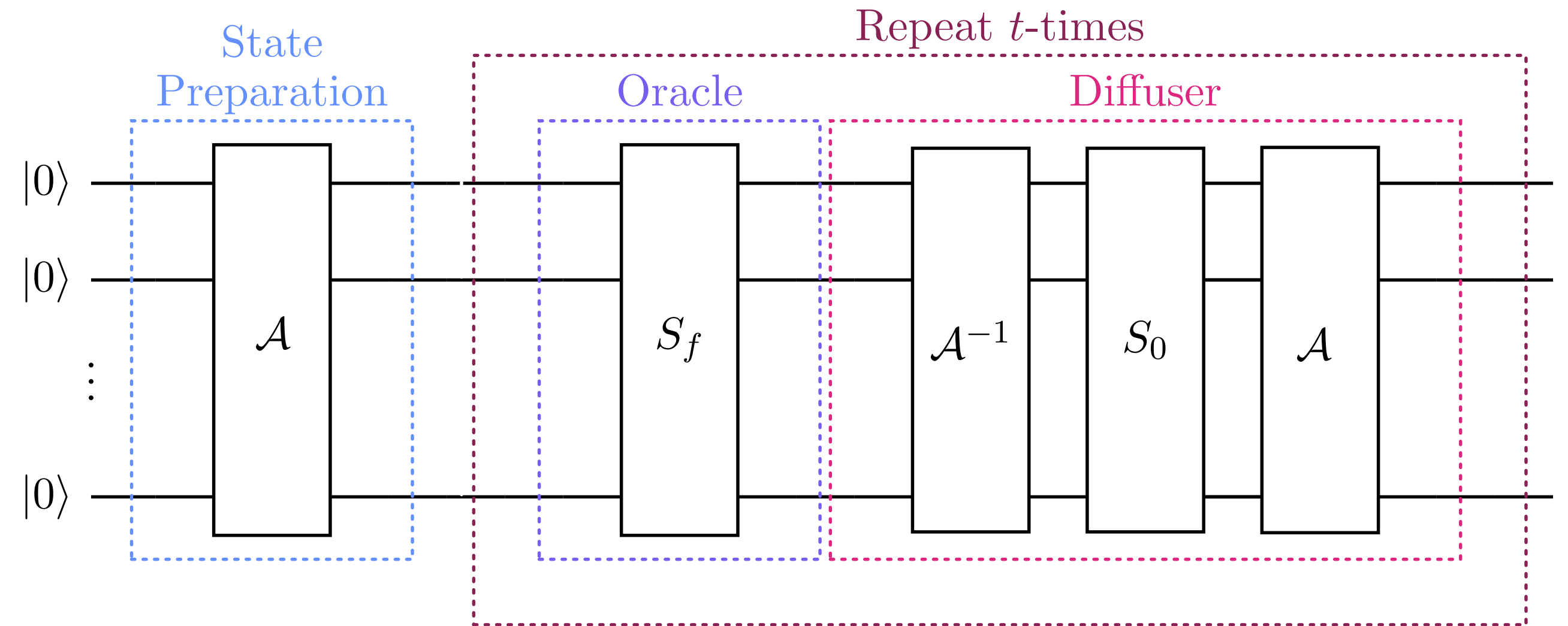
Quantum Amplitude Amplification

The optimal number of iterations of the QAA routine \mathcal{Q} is given by

$$t = \left\lceil \frac{\pi}{4} \sqrt{\frac{N}{m}} \right\rceil$$

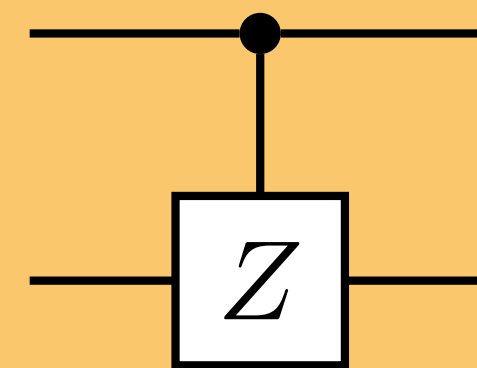
After t iterations of \mathcal{Q} , measurement will return a marked state with high probability

QAA therefore scales as $\mathcal{O}(\sqrt{N})$, thus achieving a **polynomial speedup** over classical search algorithms, which scale as $\mathcal{O}(N)$



Oracle Construction

Consider a two qubit example where $|11\rangle$ is the marked state



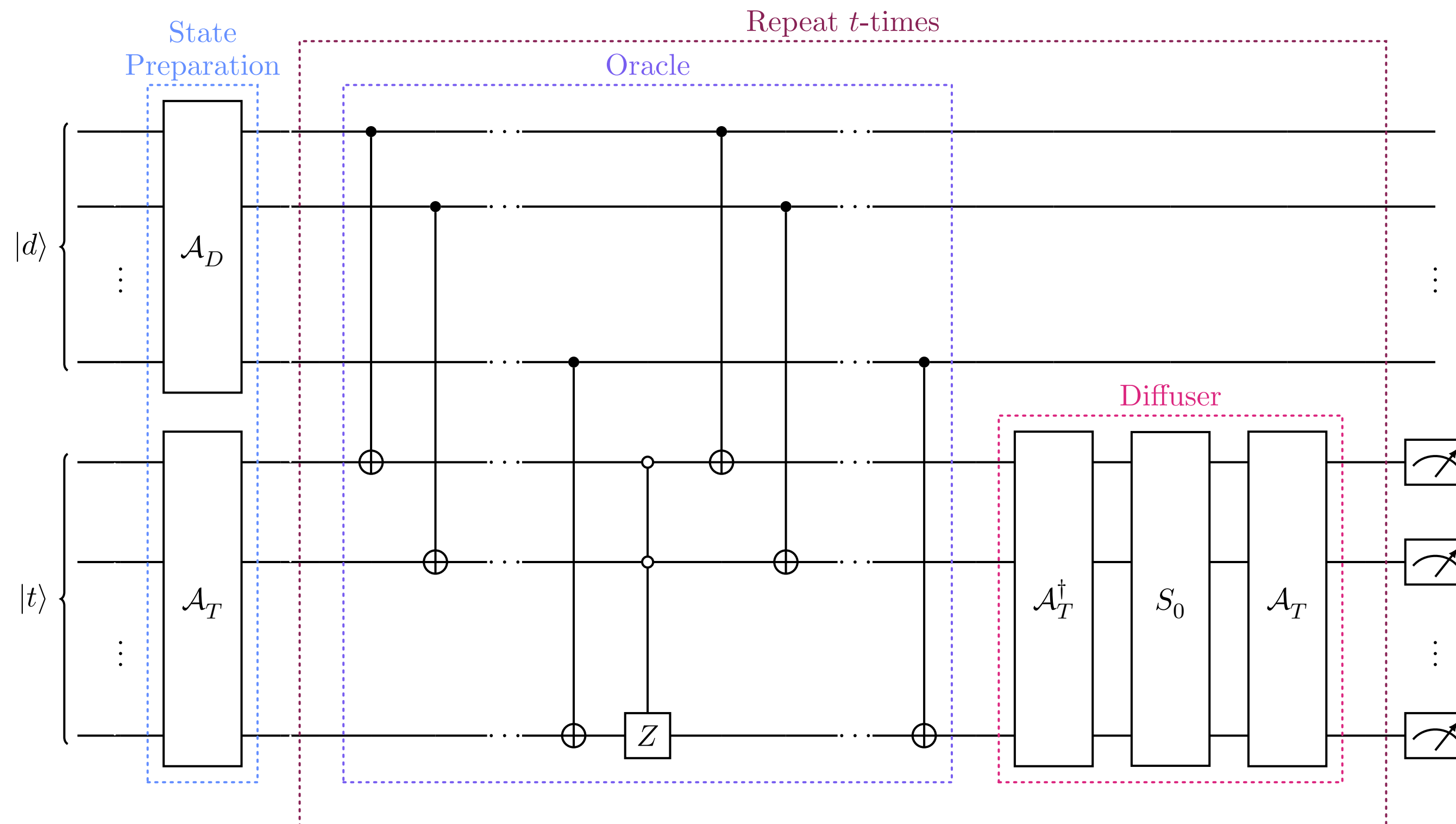
$$S_f : I \otimes |0\rangle\langle 0| + Z \otimes |1\rangle\langle 1|$$

Quantum Template Matching

The perform template matching, we must **abstract** the QAA routine by constructing a new **oracle**

Introducing a new **data register** and acting the oracle across **two registers** allows for **data** to be **parsed directly** to the algorithm

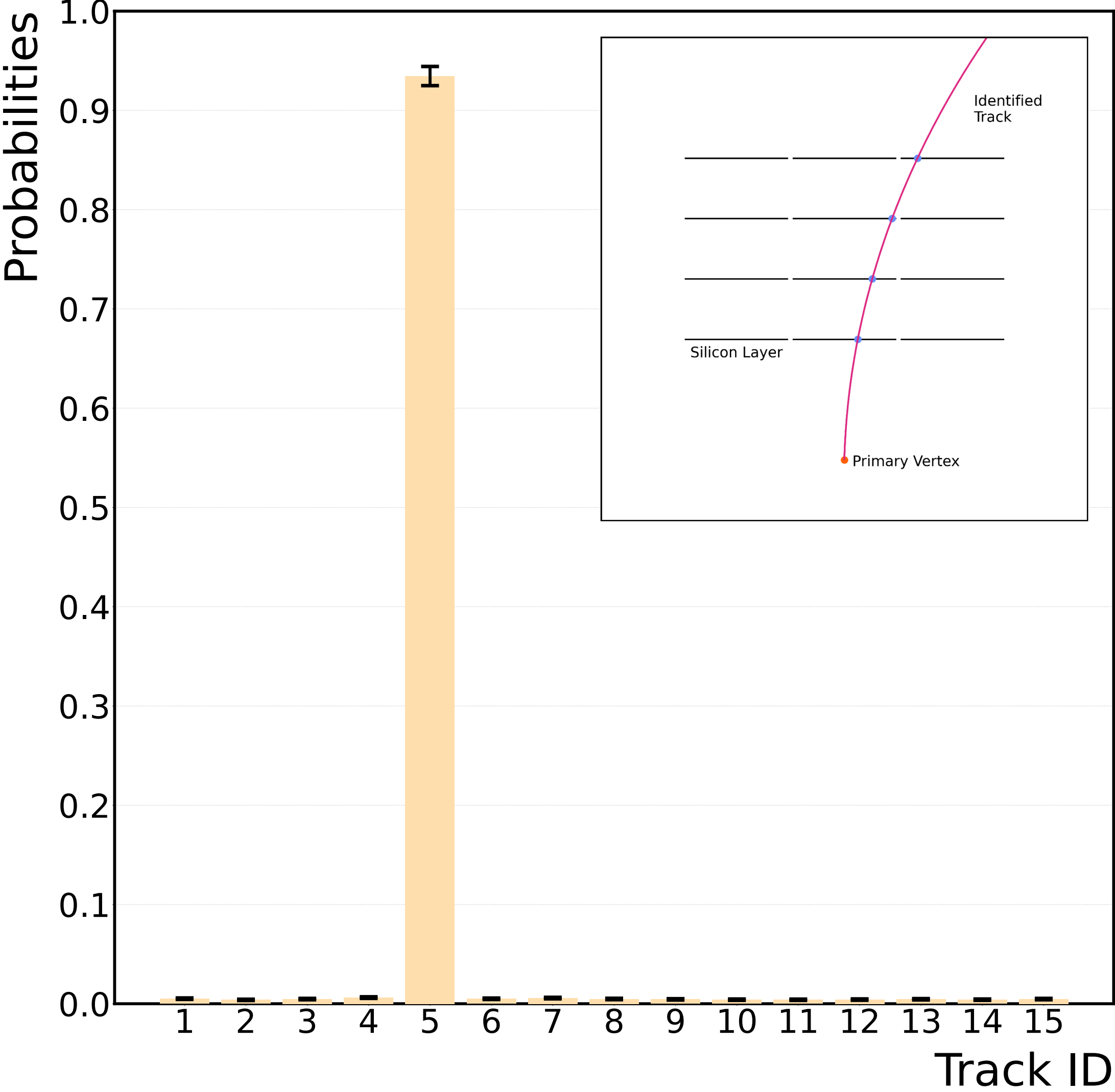
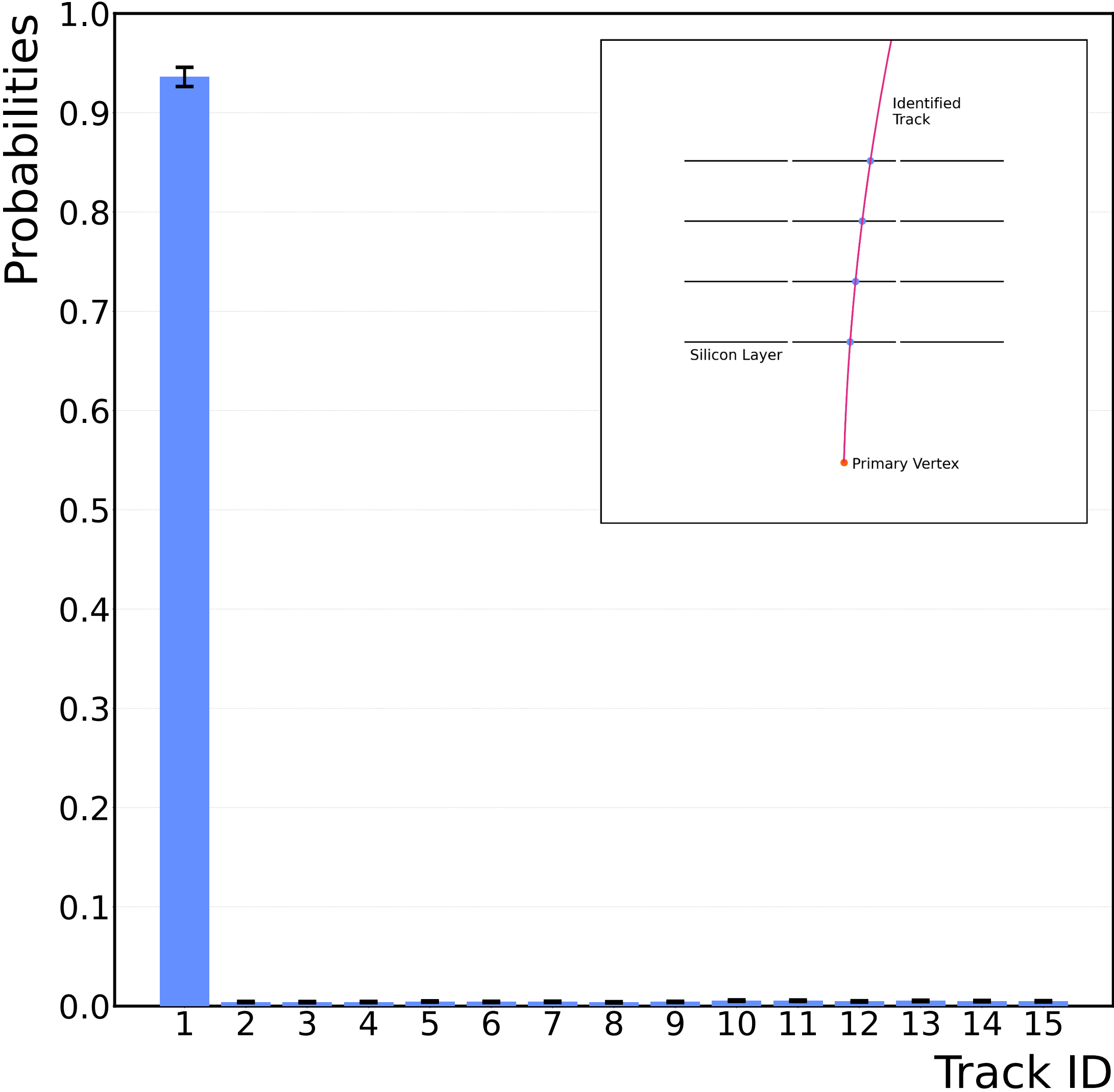
The oracle is constructed from a series of **CNOT** gates and a phase inversion about the zero state on the **template register**



The **diffusion operation** then has the same form as the regular QAA routine

$$Q = A^\dagger S_0 A S'_f$$

Quantum Template Matching for Track Finding

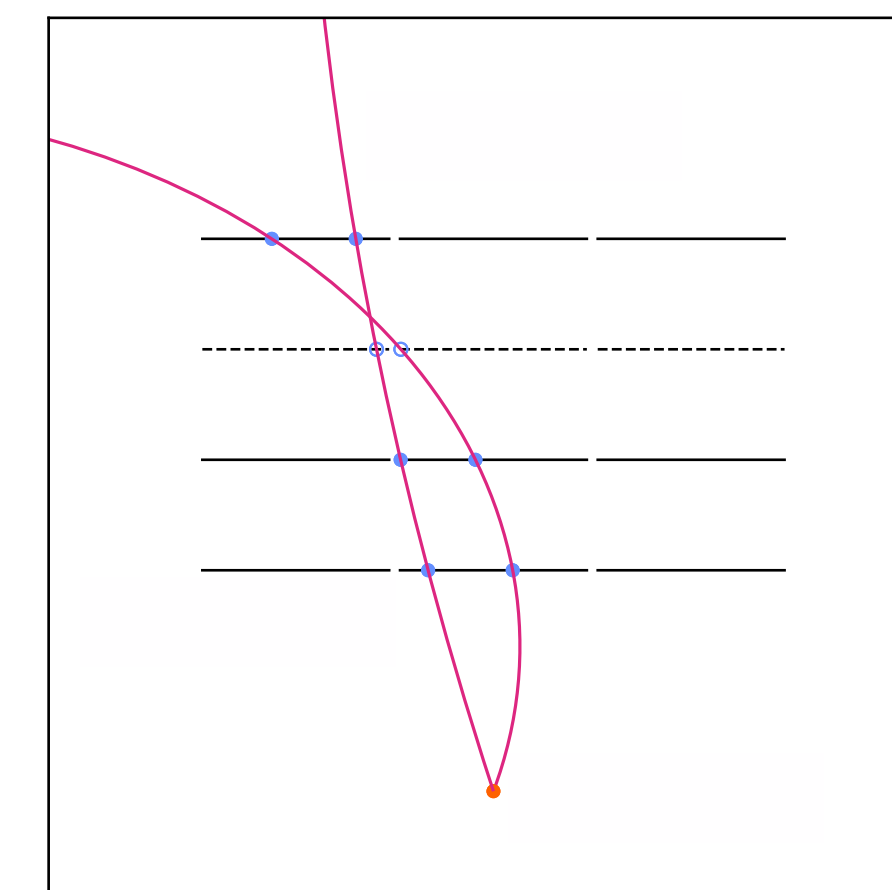
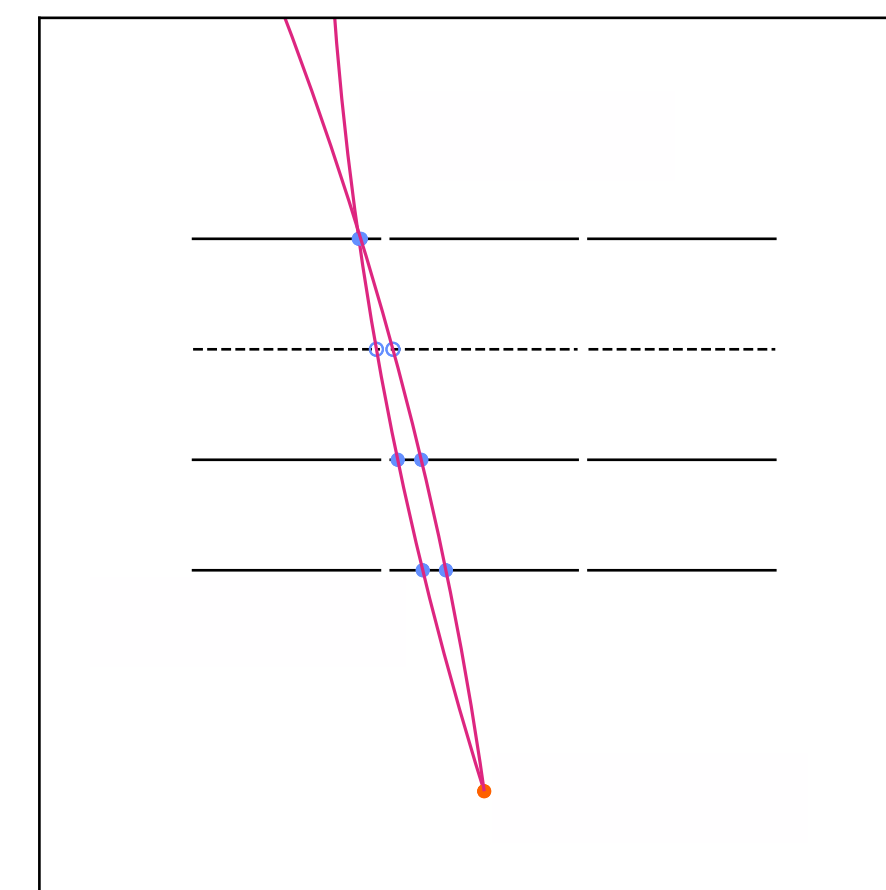
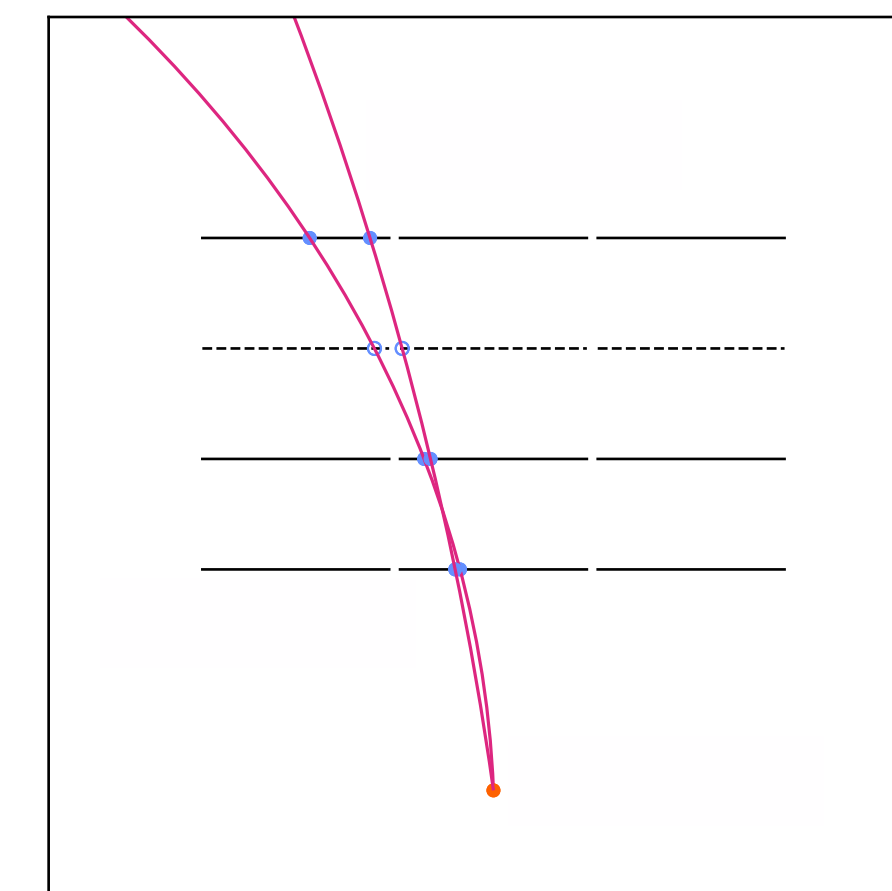
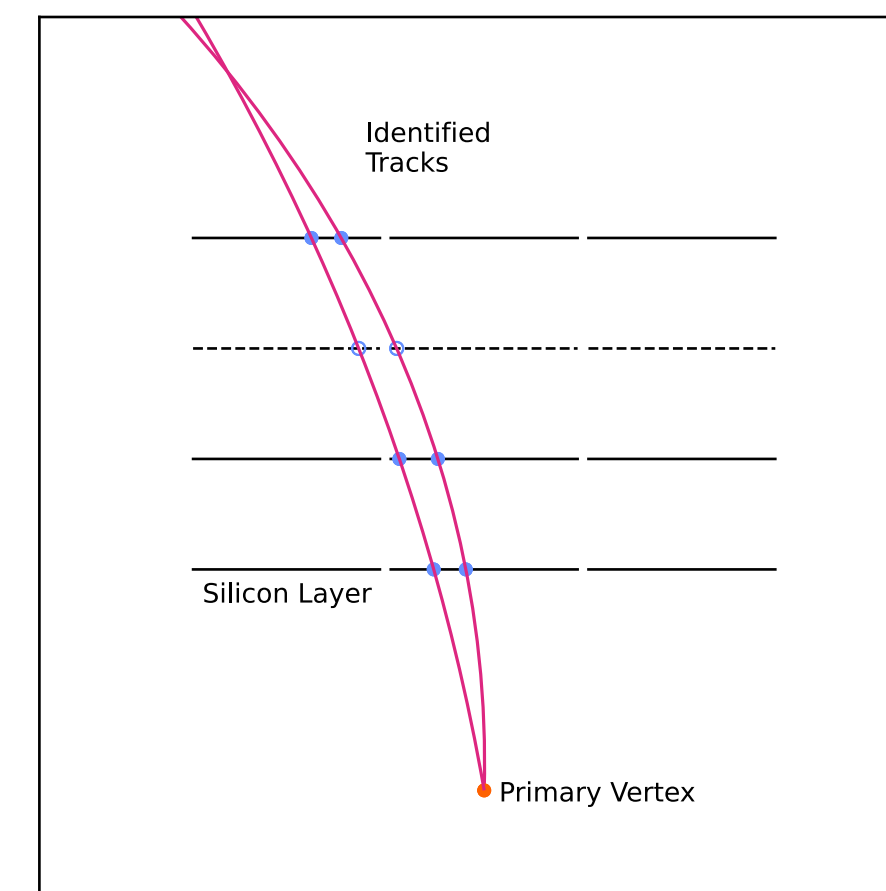


Quantum Track Finding with Missing Hits

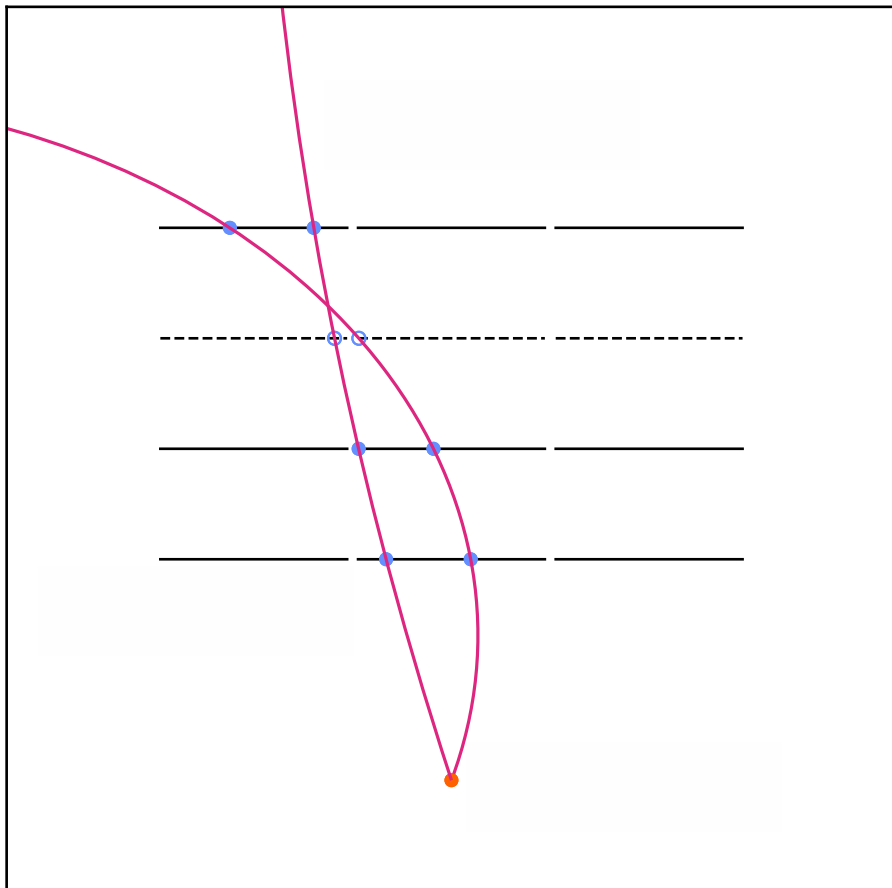
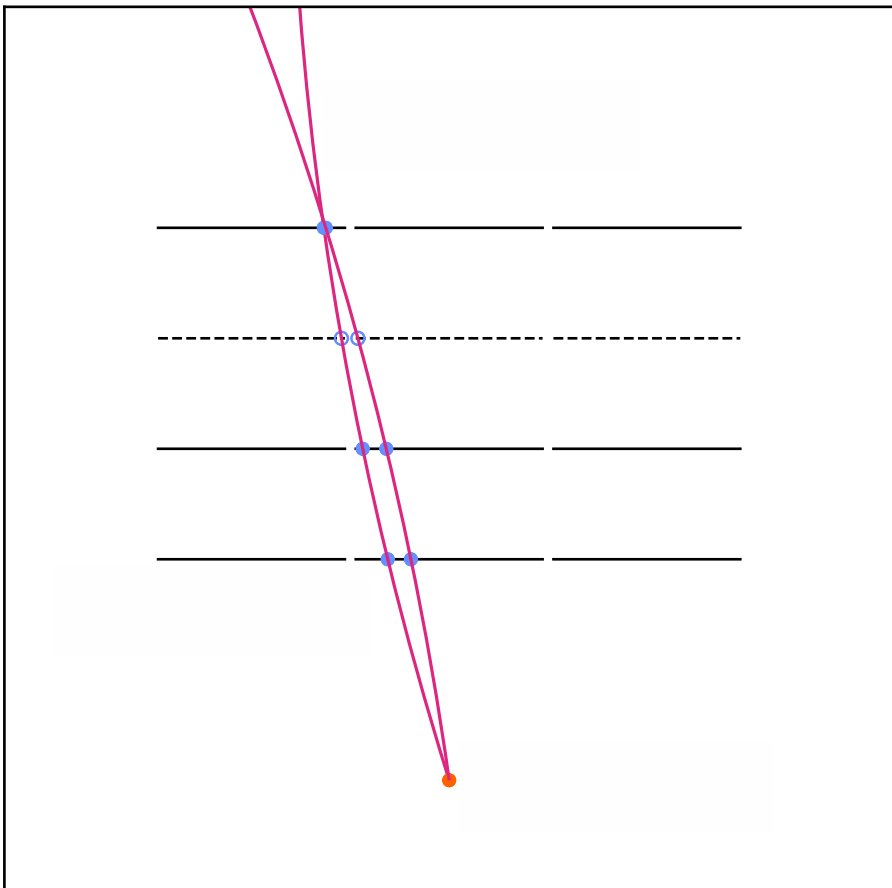
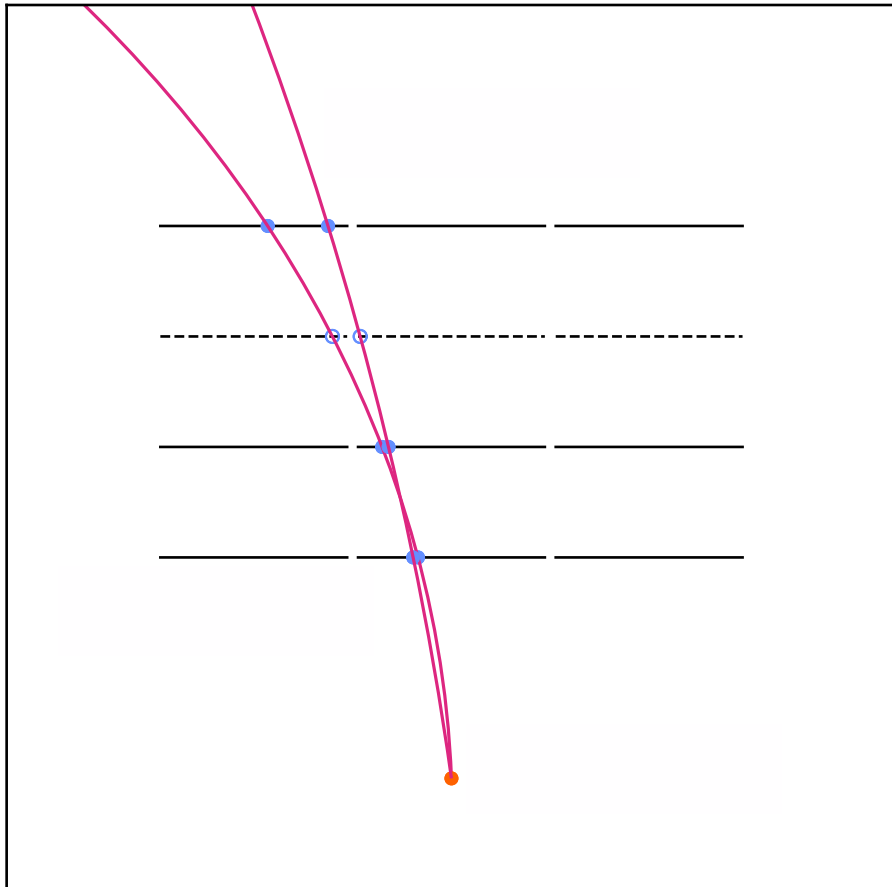
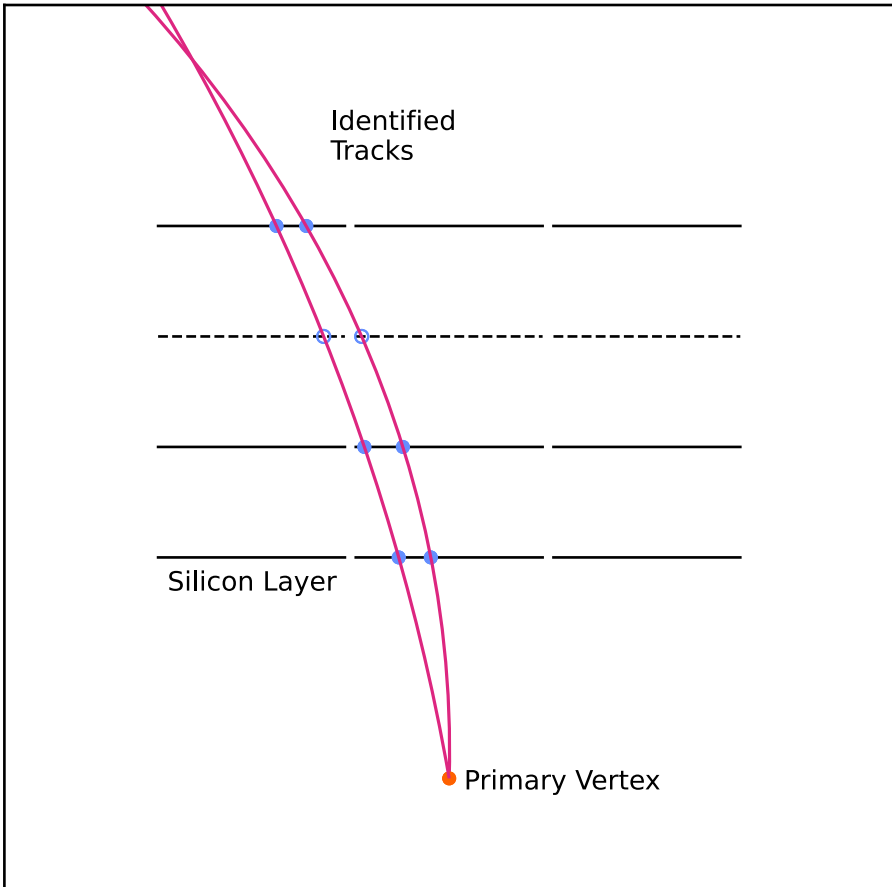
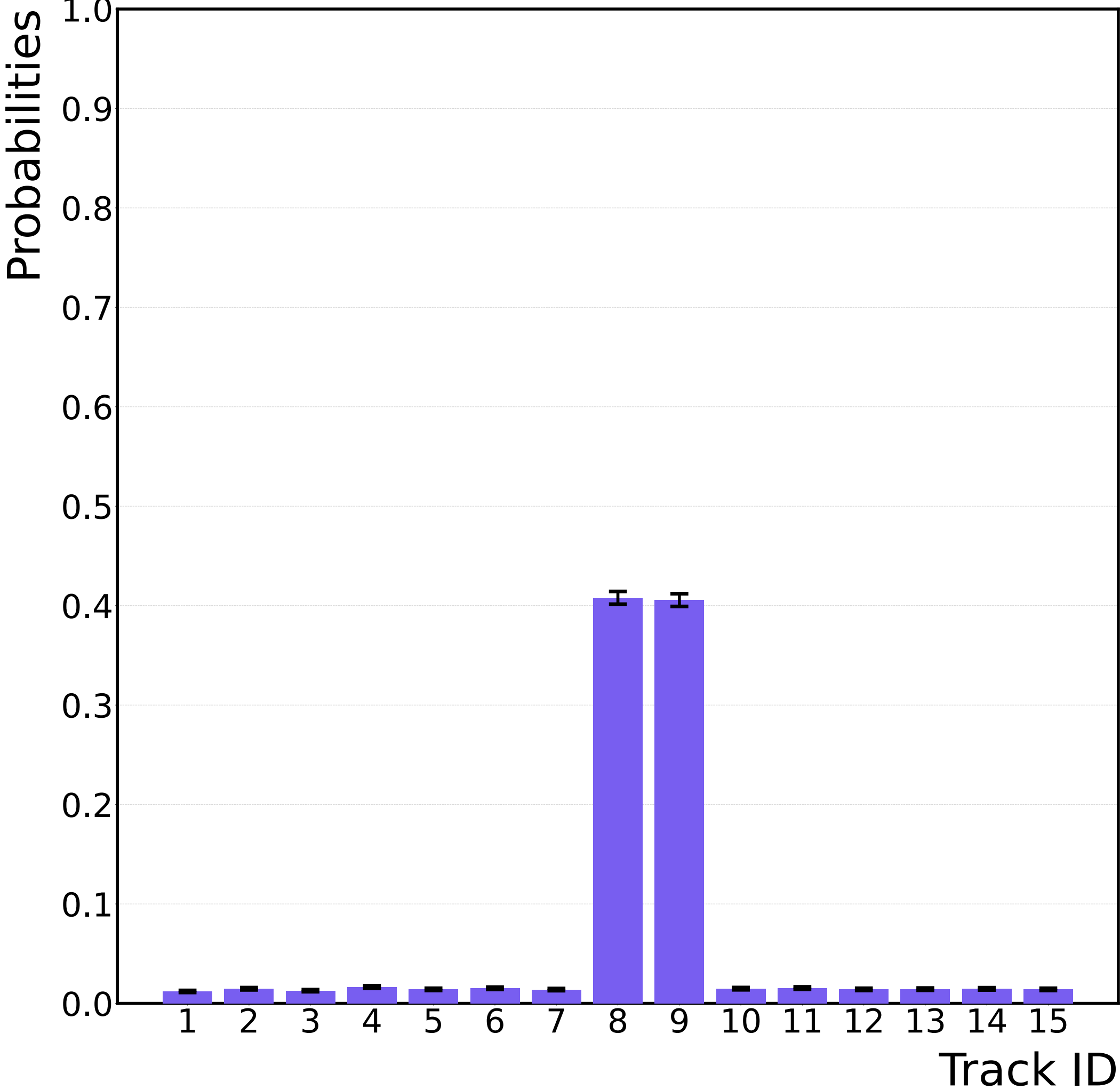
A primary challenge for track finding algorithms is when a particle traverses a detector without registering a hit in one or more detector module

An Associative Memory approach to track finding cannot manage **missing hit data**

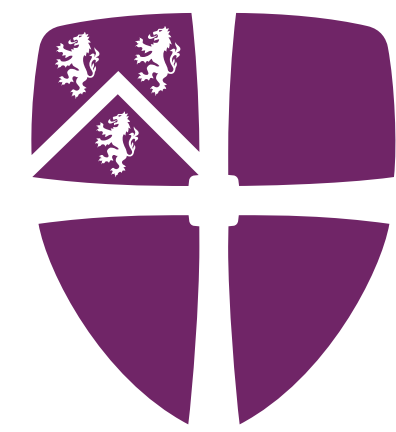
Modifying the oracle allows for the quantum template algorithm to efficiently search on missing hit data, **without an increase in resources** and retaining the **high accuracy** and **speedup**



Quantum Track Finding with Missing Hits



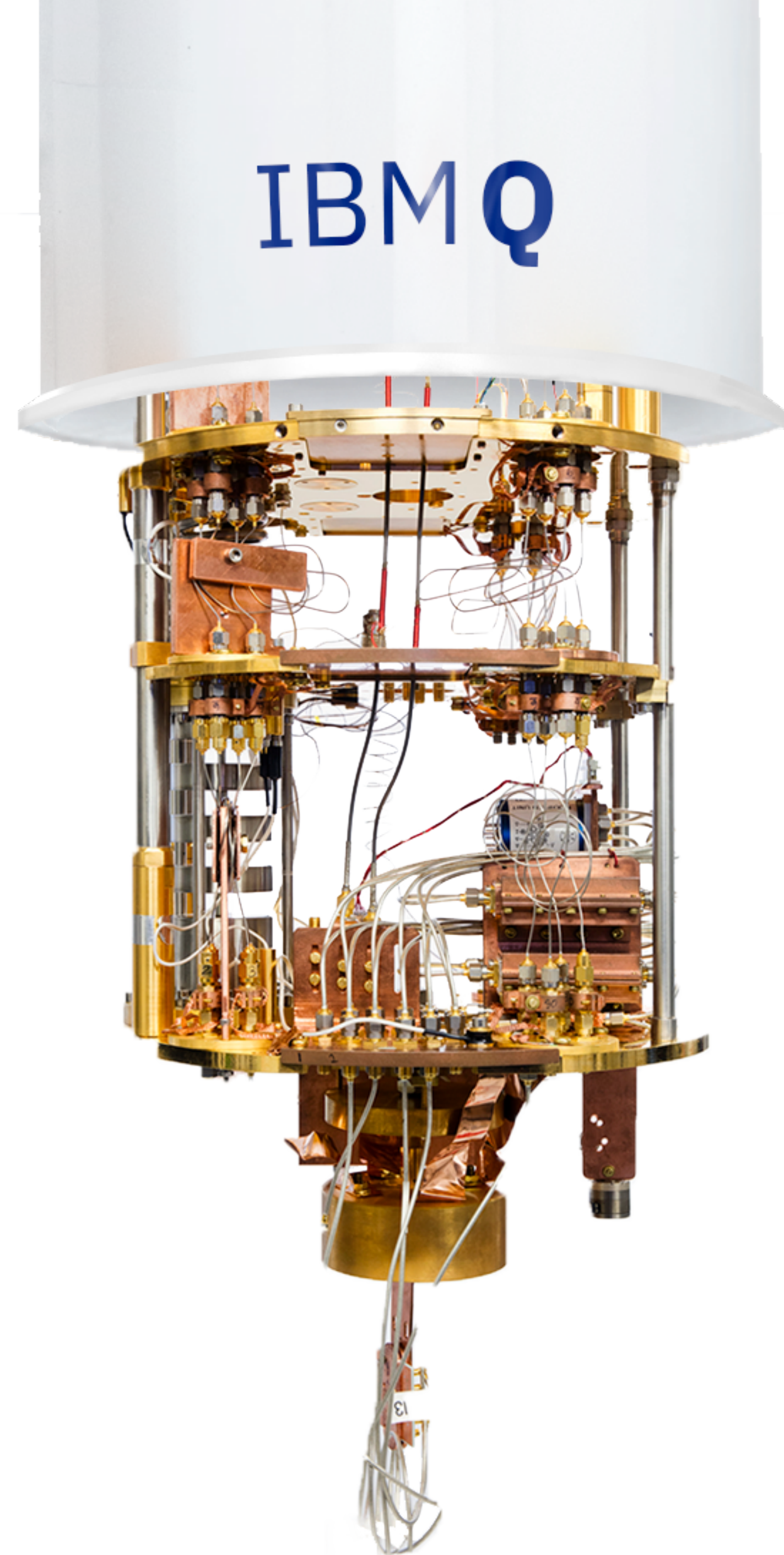
IBMQ



Durham
University

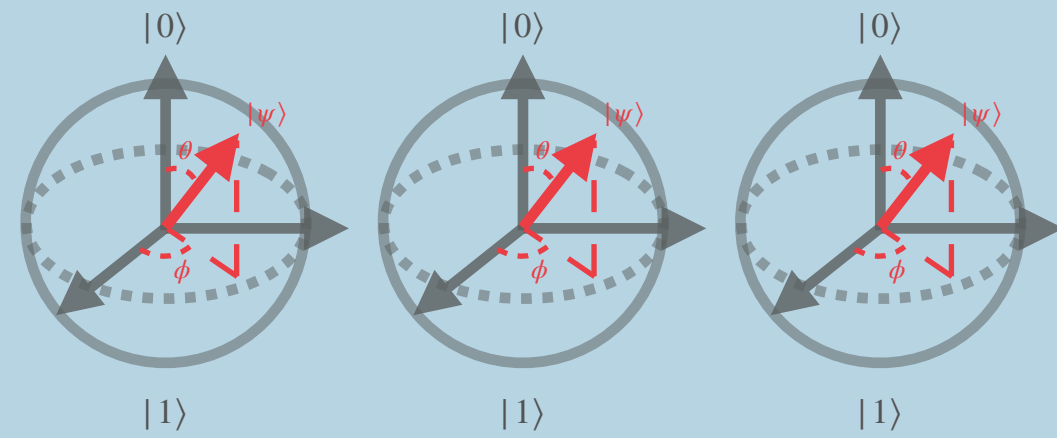


What next for Quantum Computing in Particle Physics?



The Future of Quantum Computing

More qubits?



A lot of emphasis on more qubits, but without fault tolerance, large qubit devices become

impractical

Be better architects?

Realistic algorithms are already being created for NISQ devices. Efficient architectures allow for **practical algorithms** on NISQ devices.

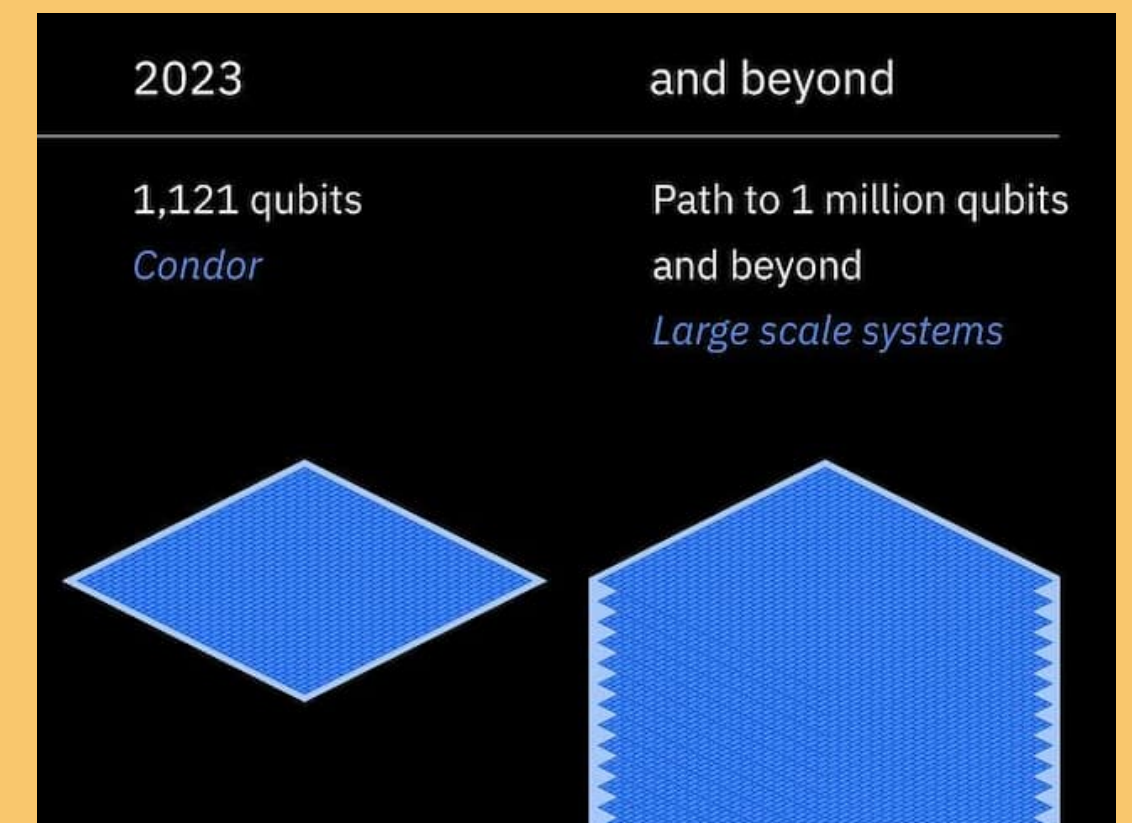
Better technology?

New technology could be the answer - will new qubit hardware be more **fault tolerant?**

IBM Roadmap

On track to deliver

1000 qubits in 2023





IBM Q

Summary

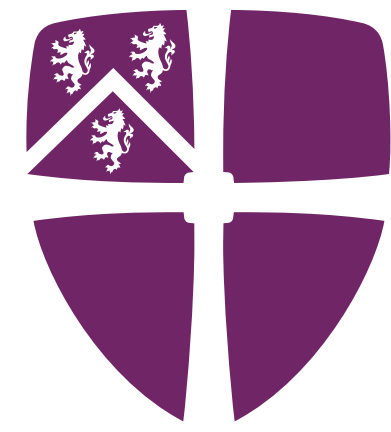
High Energy Physics is on the edge of a **computational frontier**, the High Luminosity Large Hadron Collider and FCC will provide **unprecedented amounts of data**

Quantum Computing offers an impressive and powerful tool to **combat computational bottlenecks**, both for theoretical and experimental purposes

The **first realistic simulation** of a **high energy collision** has been presented using a compact **quantum walk** implementation, allowing for the algorithm to be run on a **NISQ device**

We present an **efficient** approach to track finding using quantum computers by exploiting the **QAA** routine and employing a **novel oracle** paving the way for **practical quantum track finding**

IBMQ



Durham
University



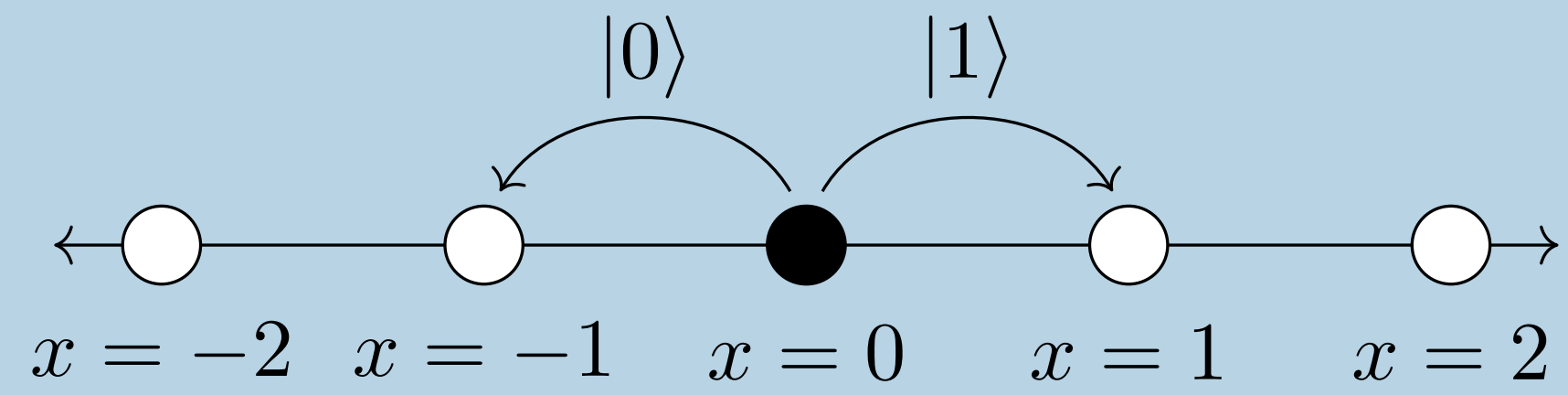
Backup Slides

Simon Williams

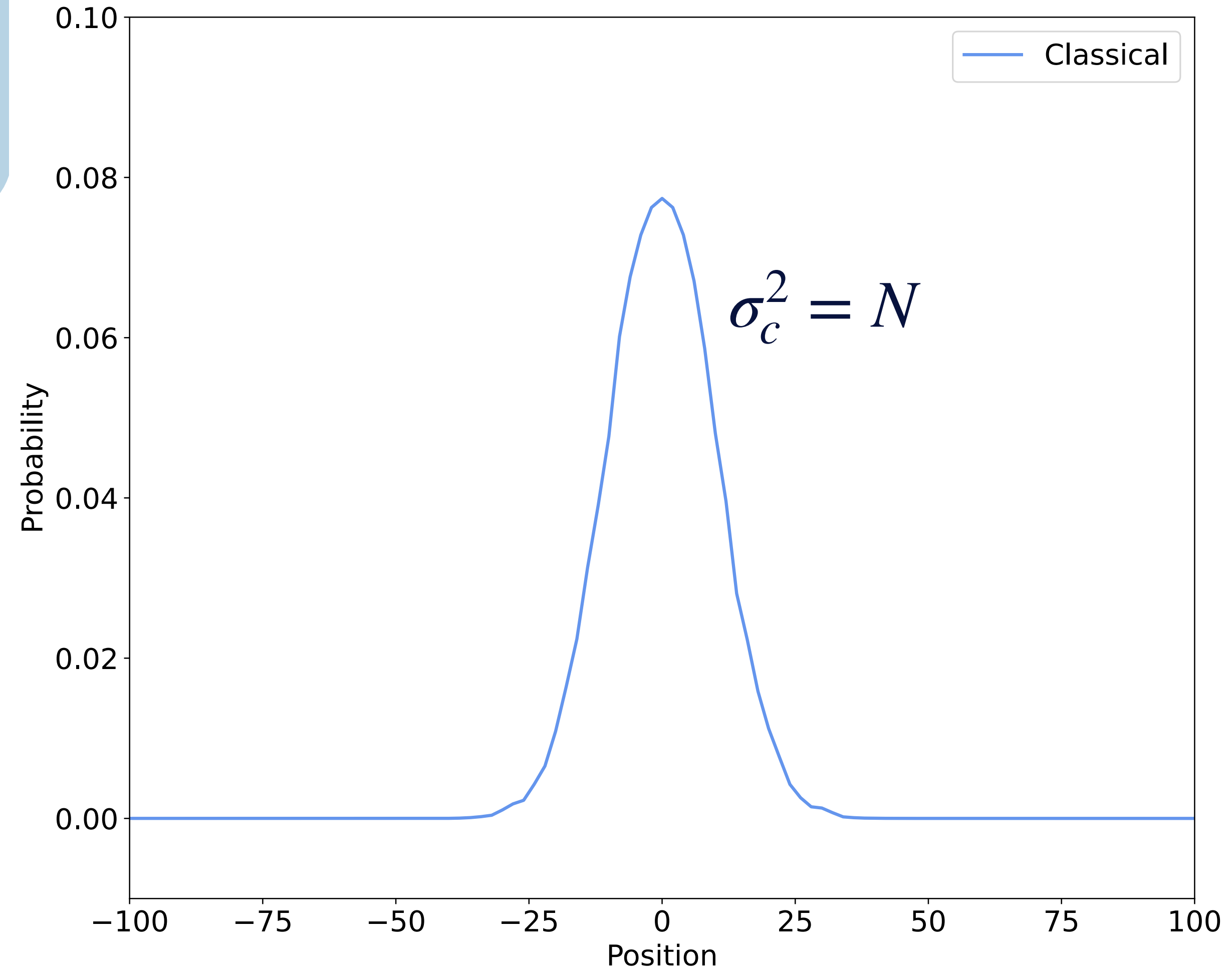
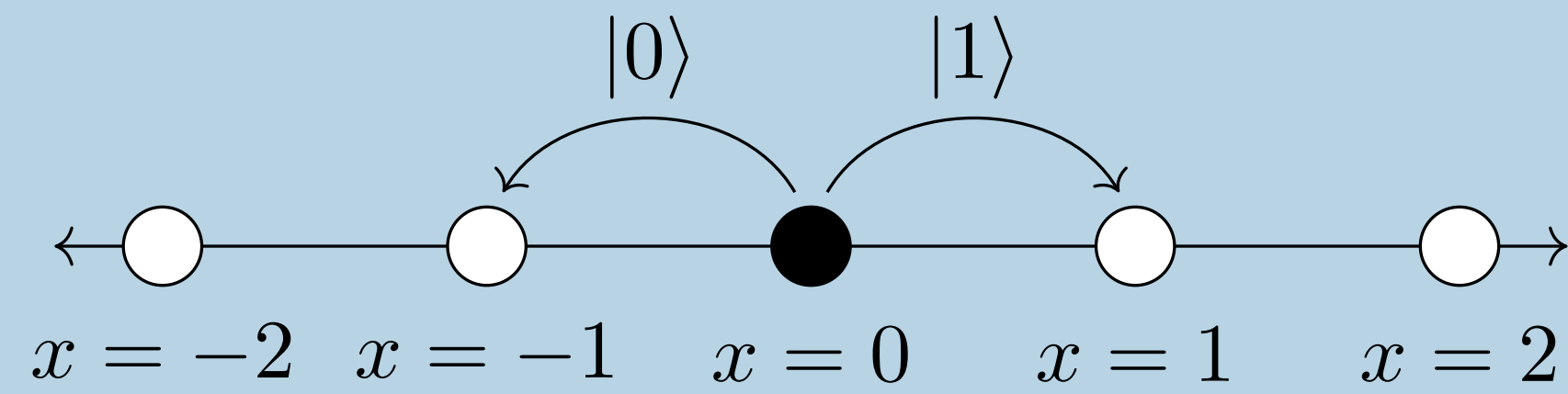
Rutherford Appleton Laboratory,
7th February 2024

Classical Random Walk

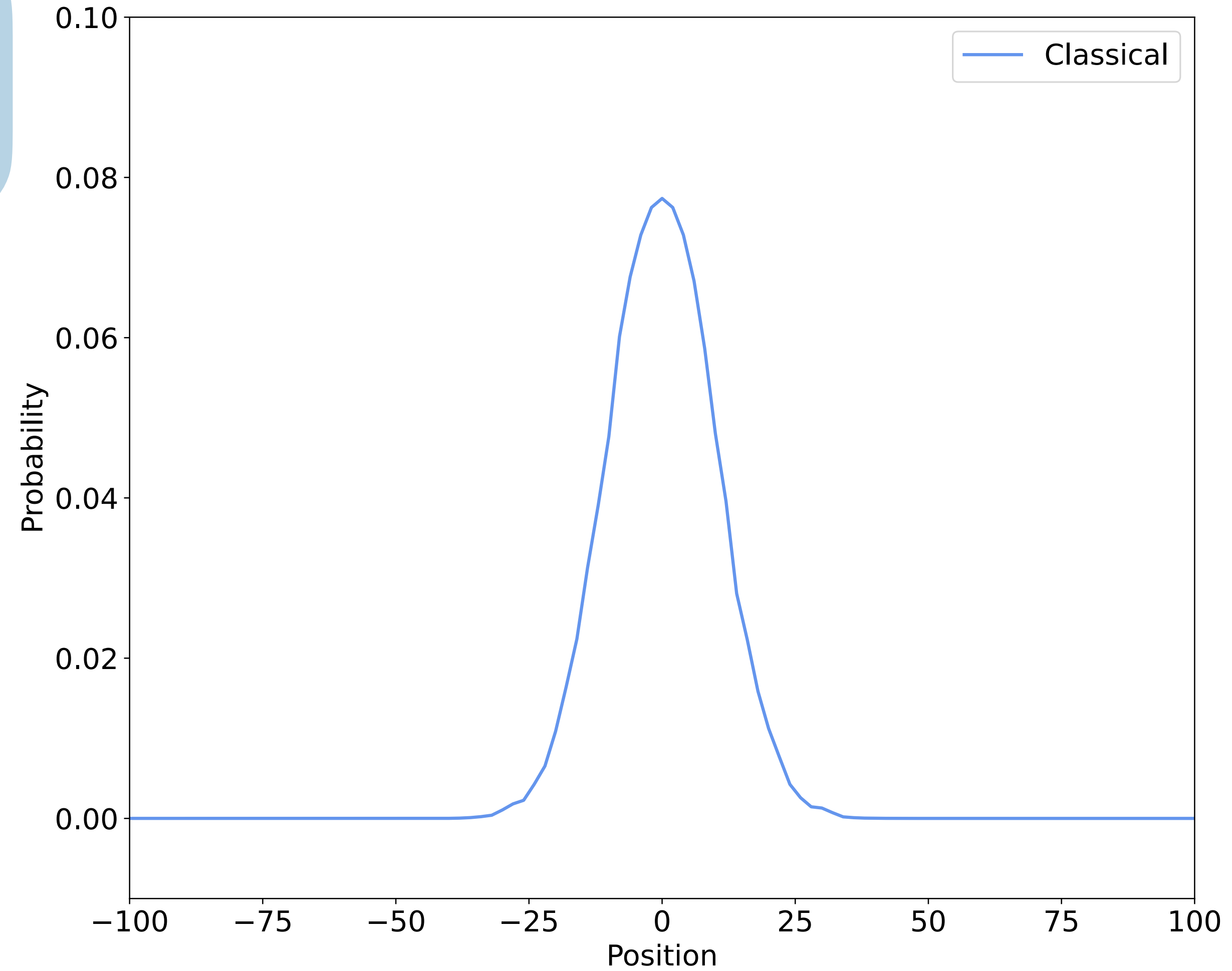
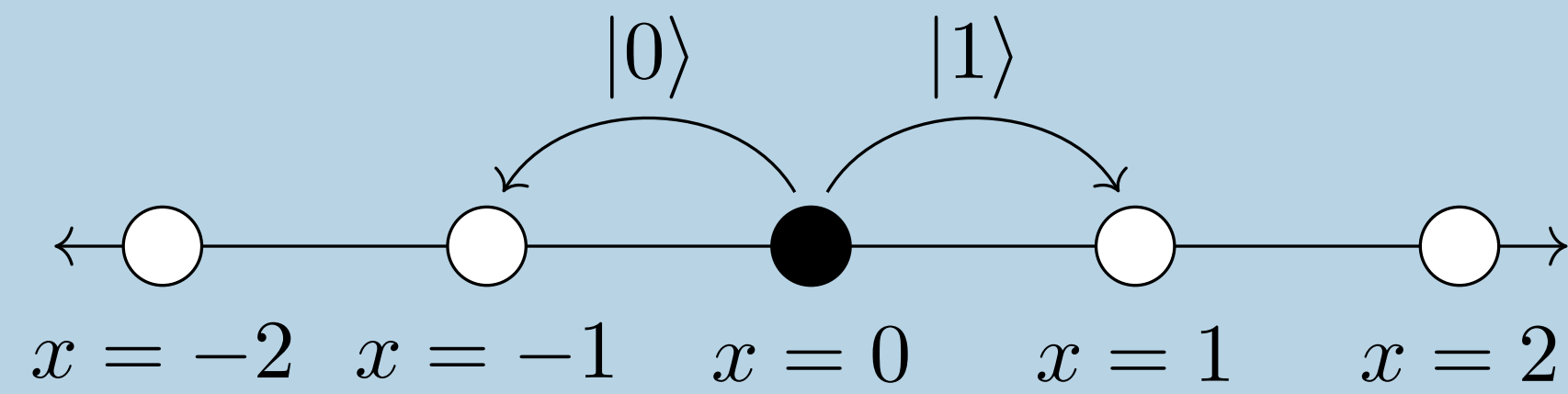
Classical Random Walk



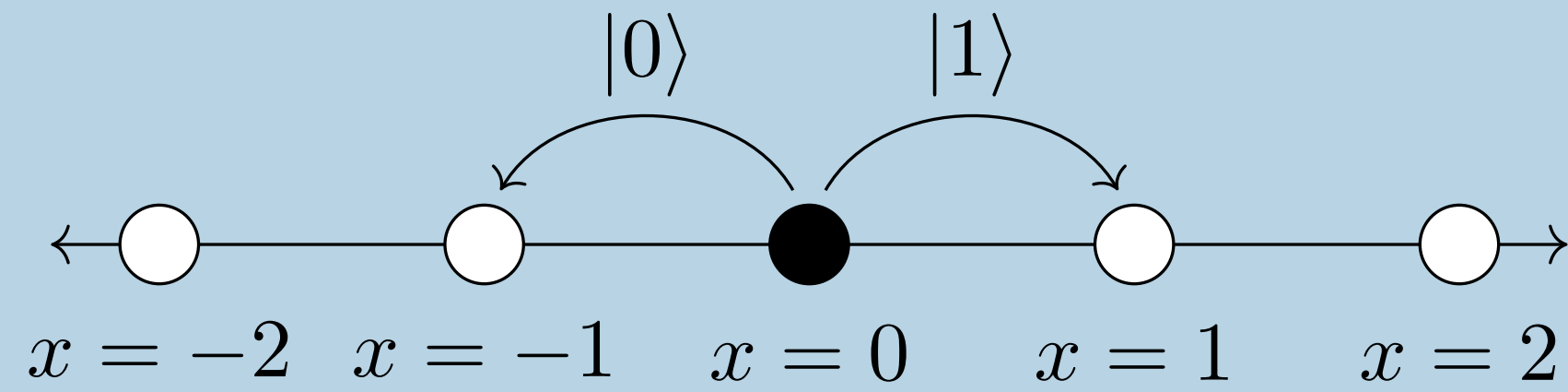
Classical Random Walk



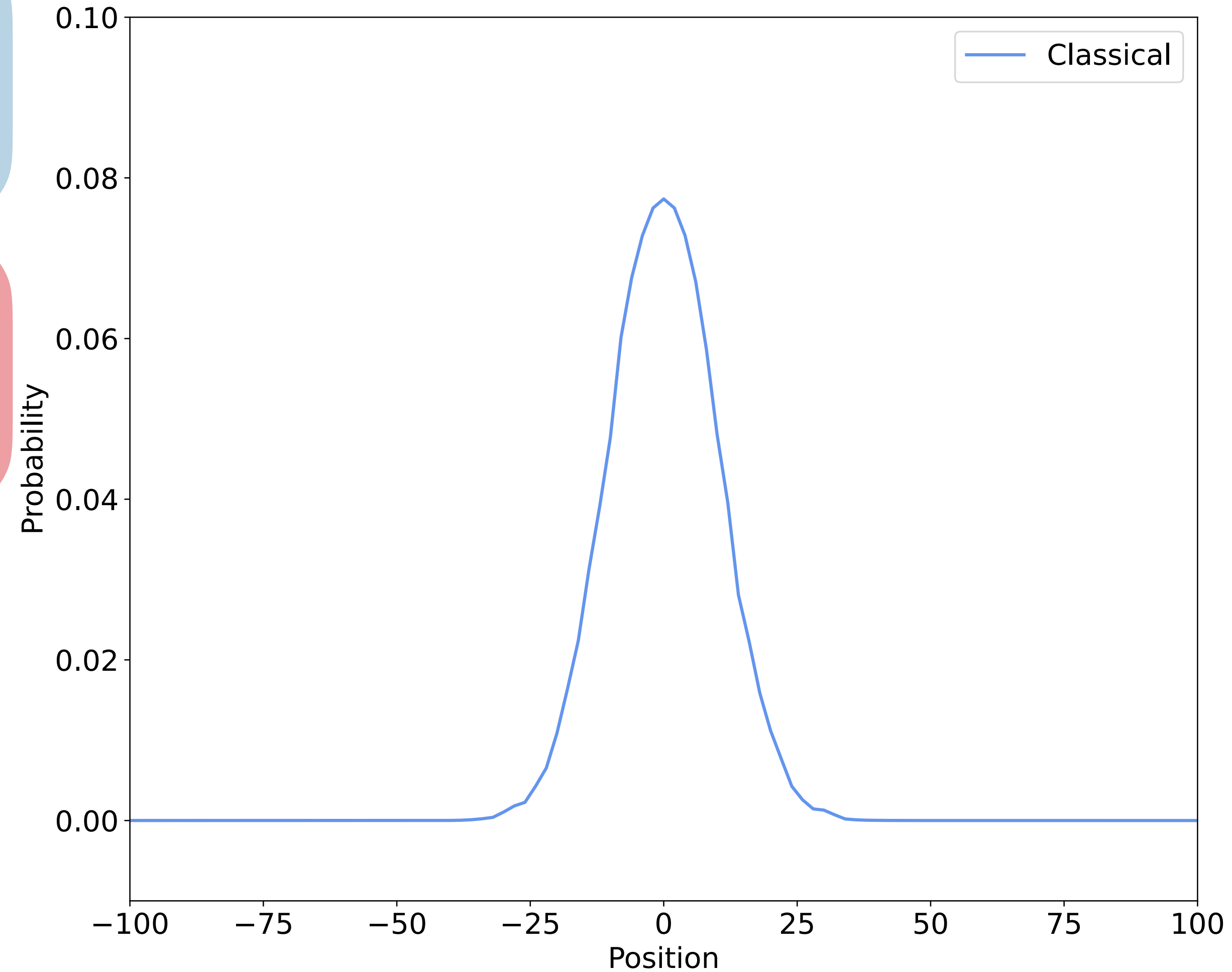
The Quantum Walk



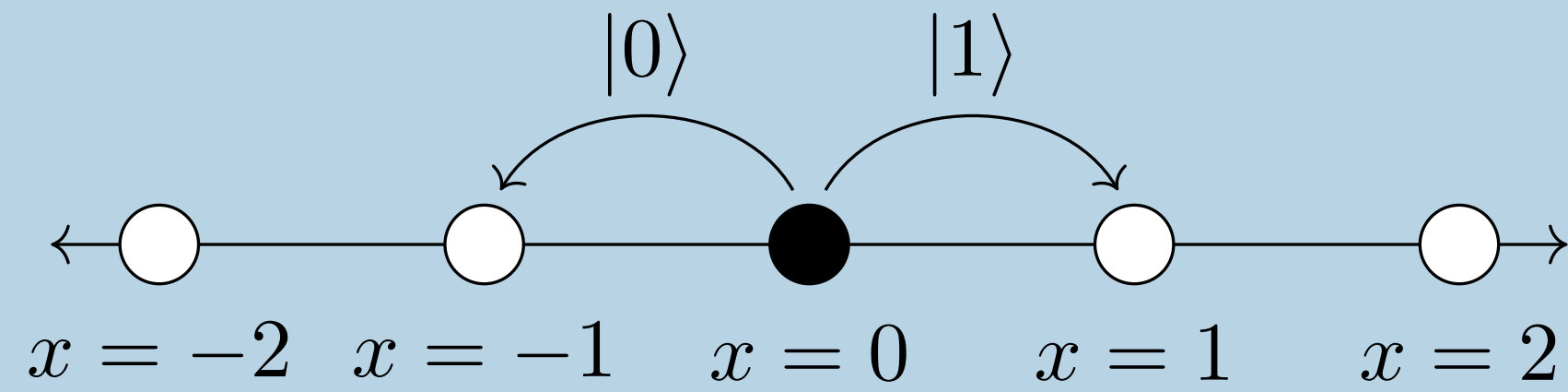
The Quantum Walk



$$\left. \begin{aligned} \mathcal{H}_P &= \{ |i\rangle : i \in \mathbb{Z} \} \\ \mathcal{H}_C &= \{ |0\rangle, |1\rangle \} \end{aligned} \right\} \mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_P$$



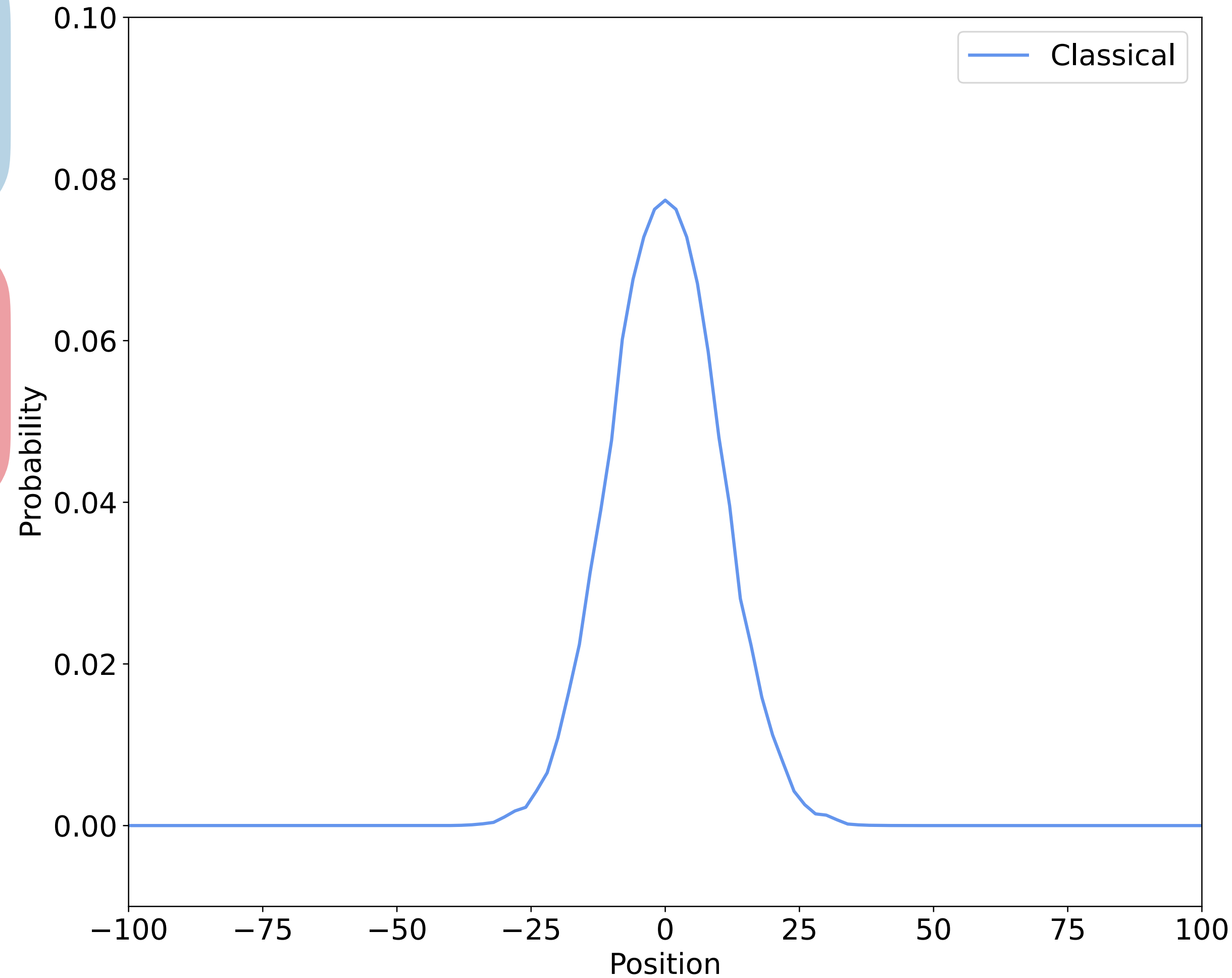
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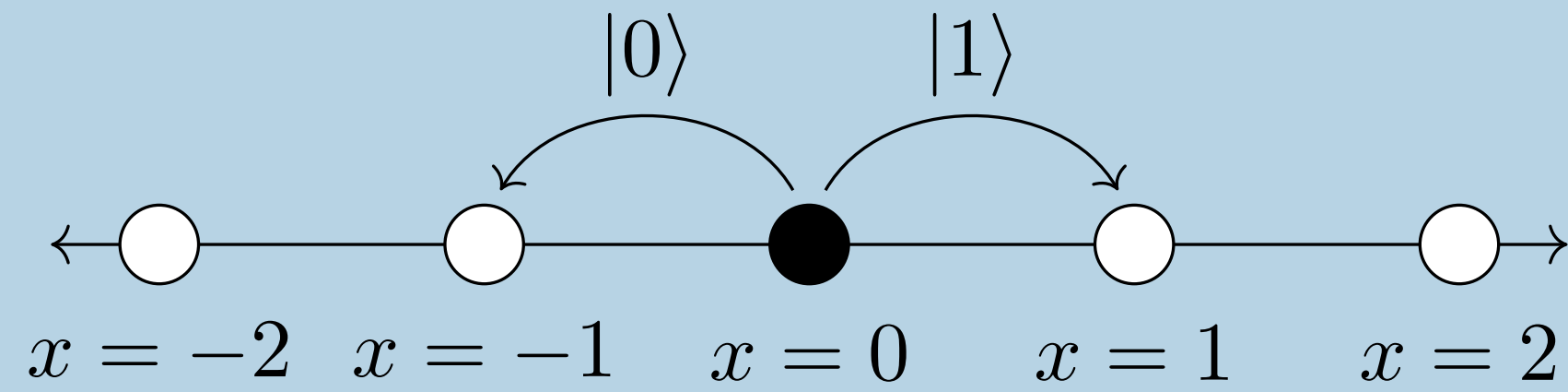
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Unitary Transformation:

$$U = S \cdot (C \otimes I)$$



The Quantum Walk



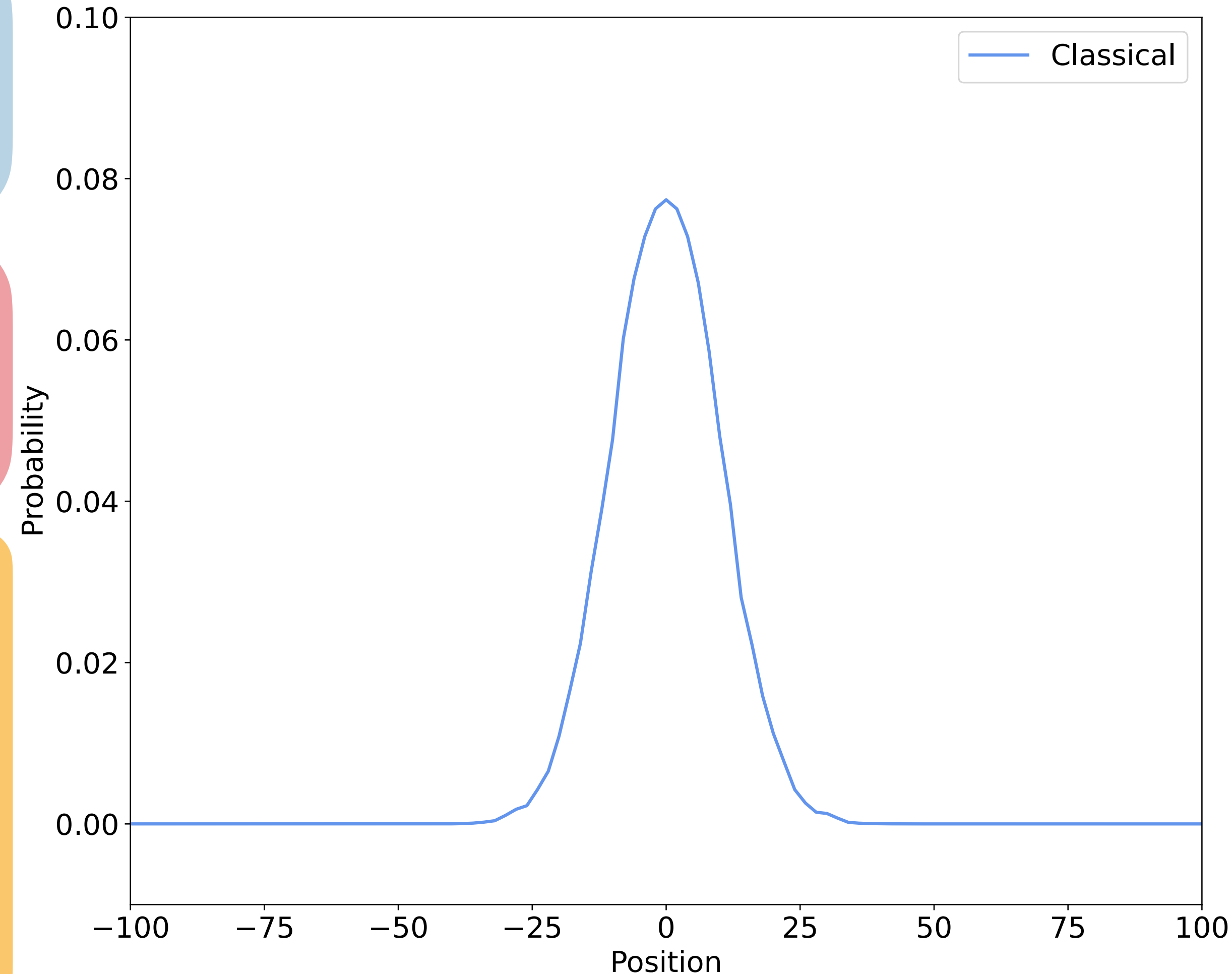
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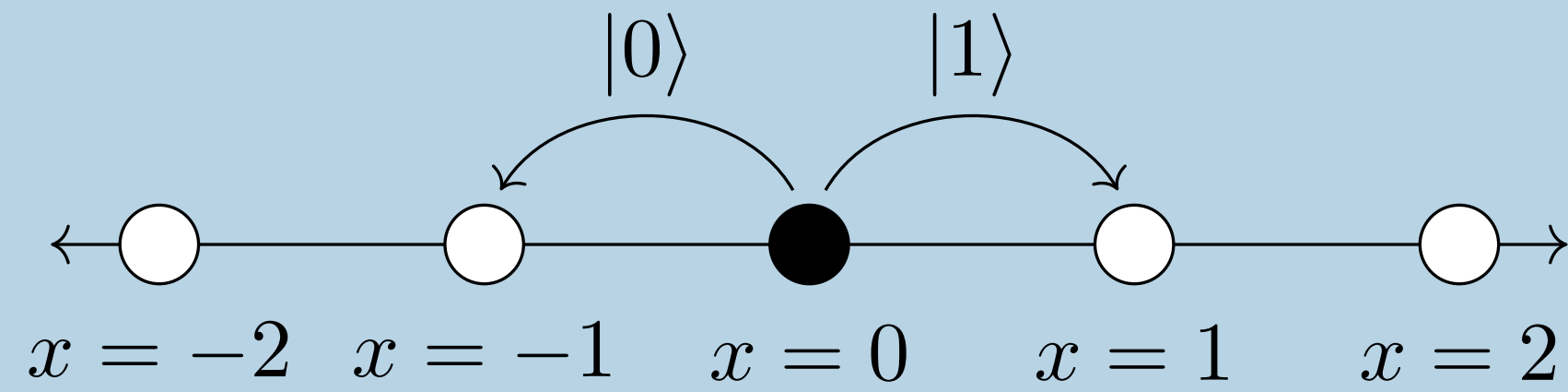
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Coin Operation:

$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$



The Quantum Walk



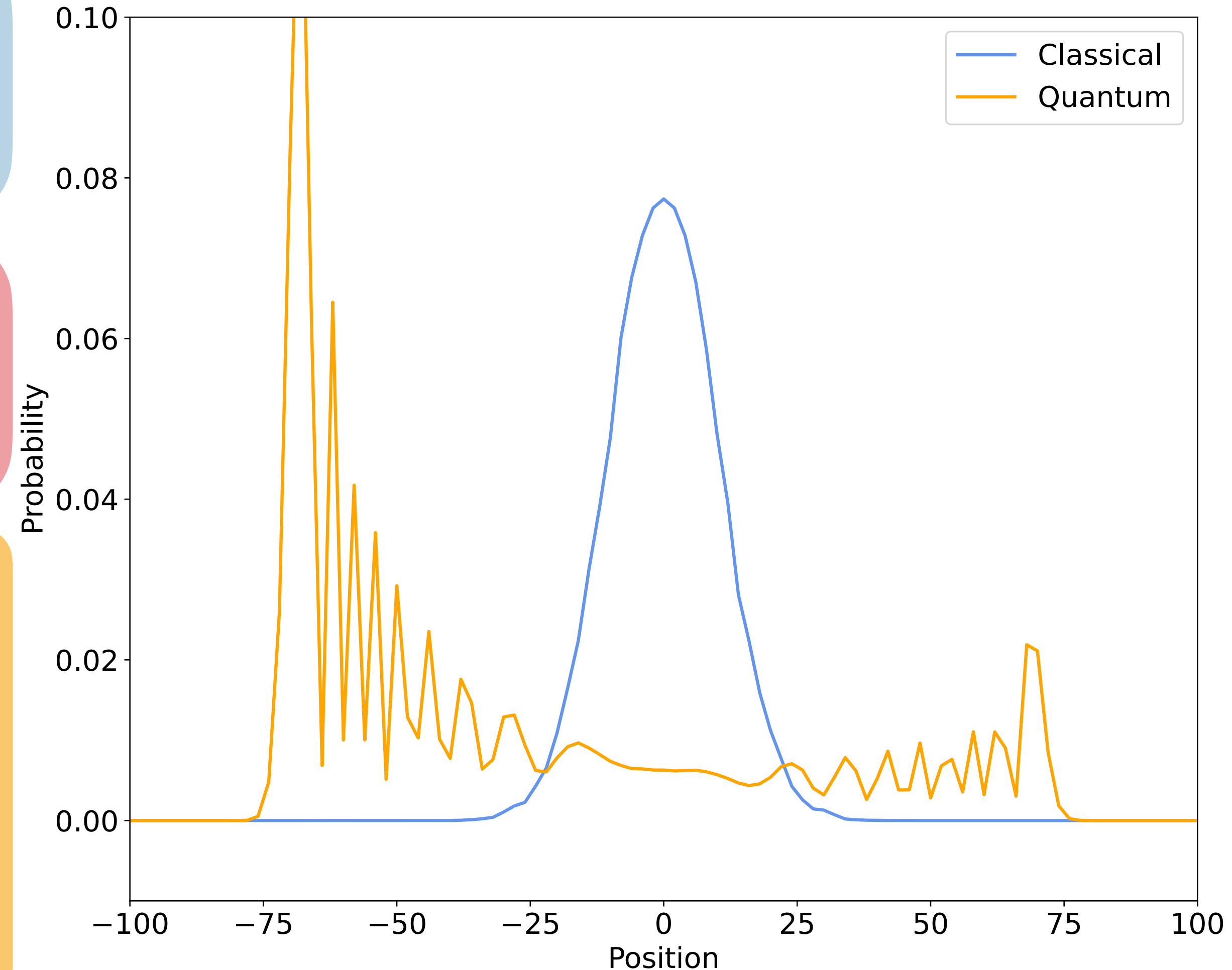
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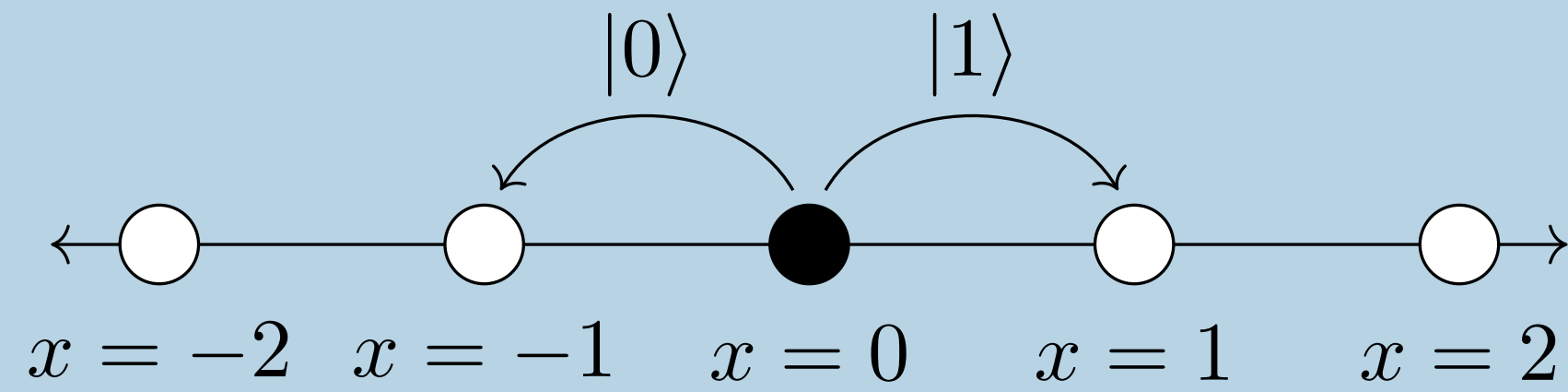
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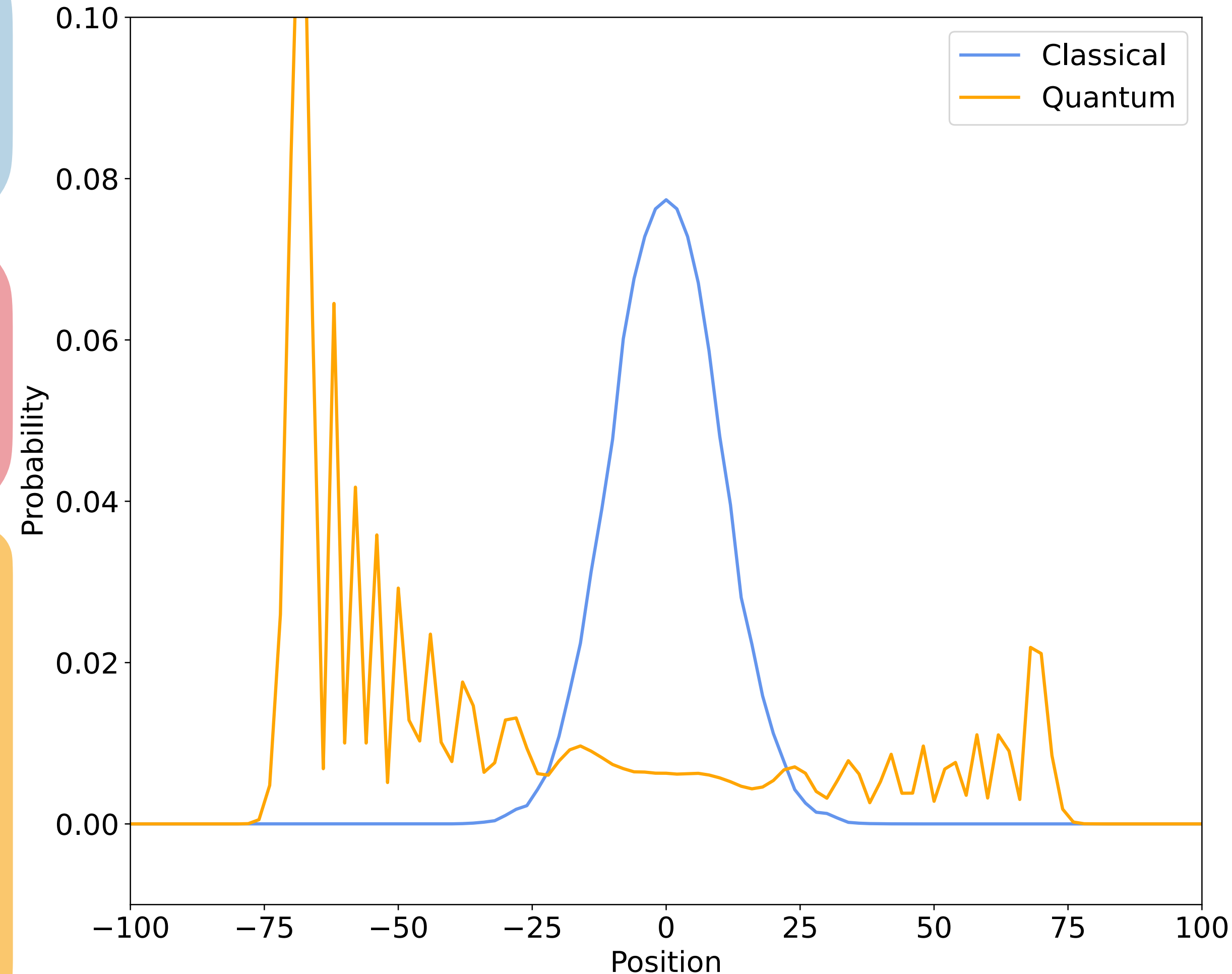
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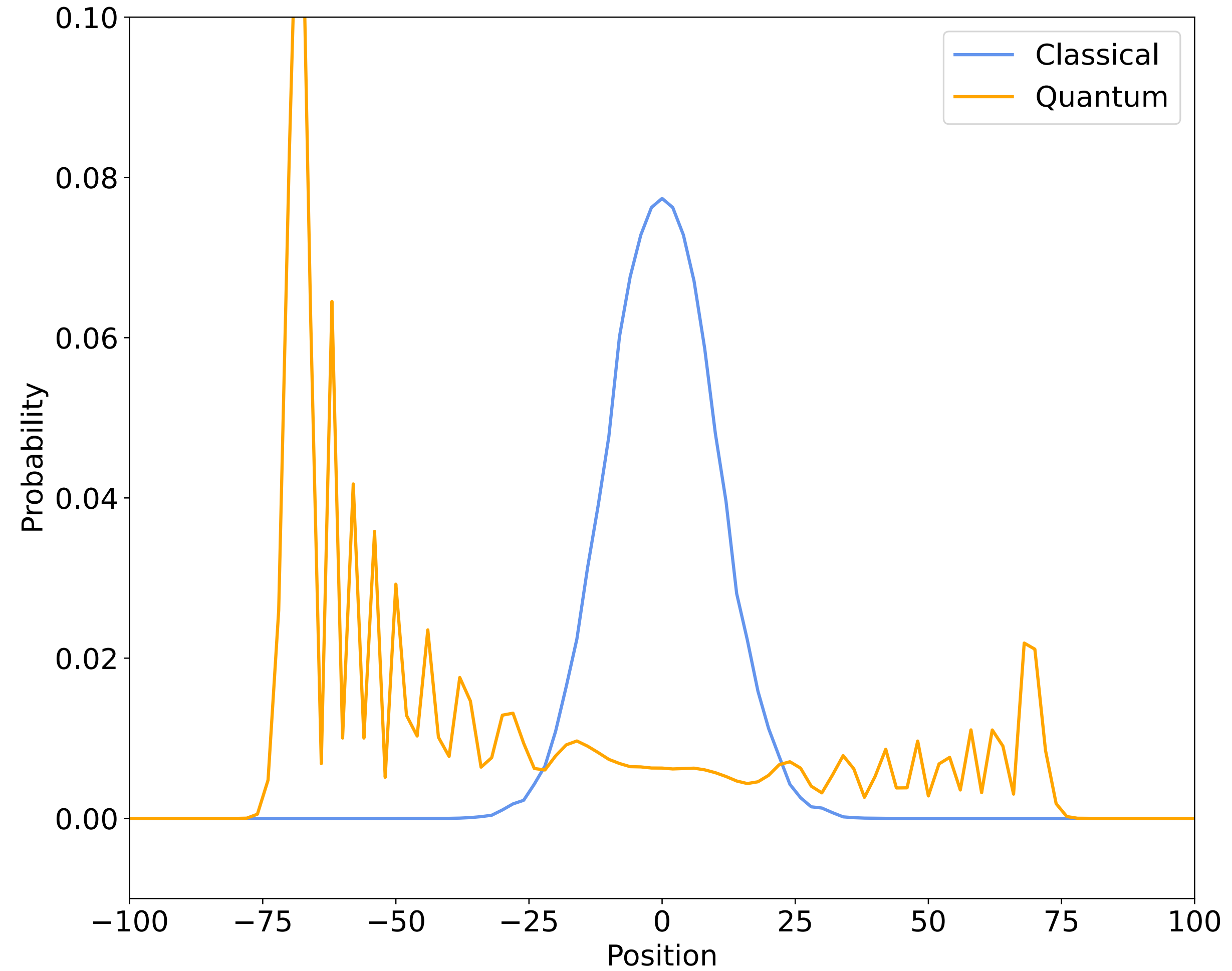
$$U = S \cdot (C \otimes I)$$

Hadamard Coin:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



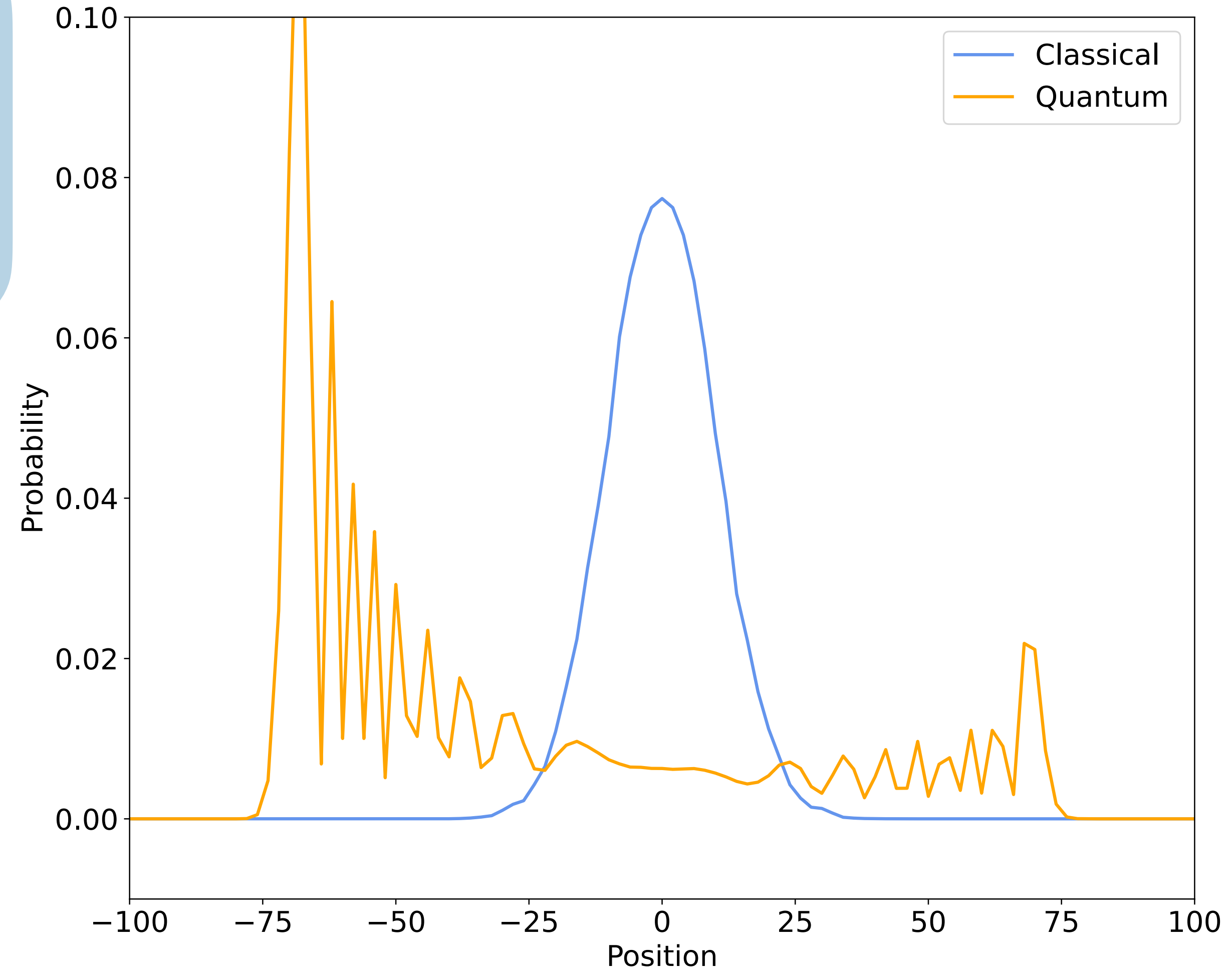
The Quantum Walk - Coin initialisation



The Quantum Walk - Coin initialisation

Initialising the coin in the $-|1\rangle$ state

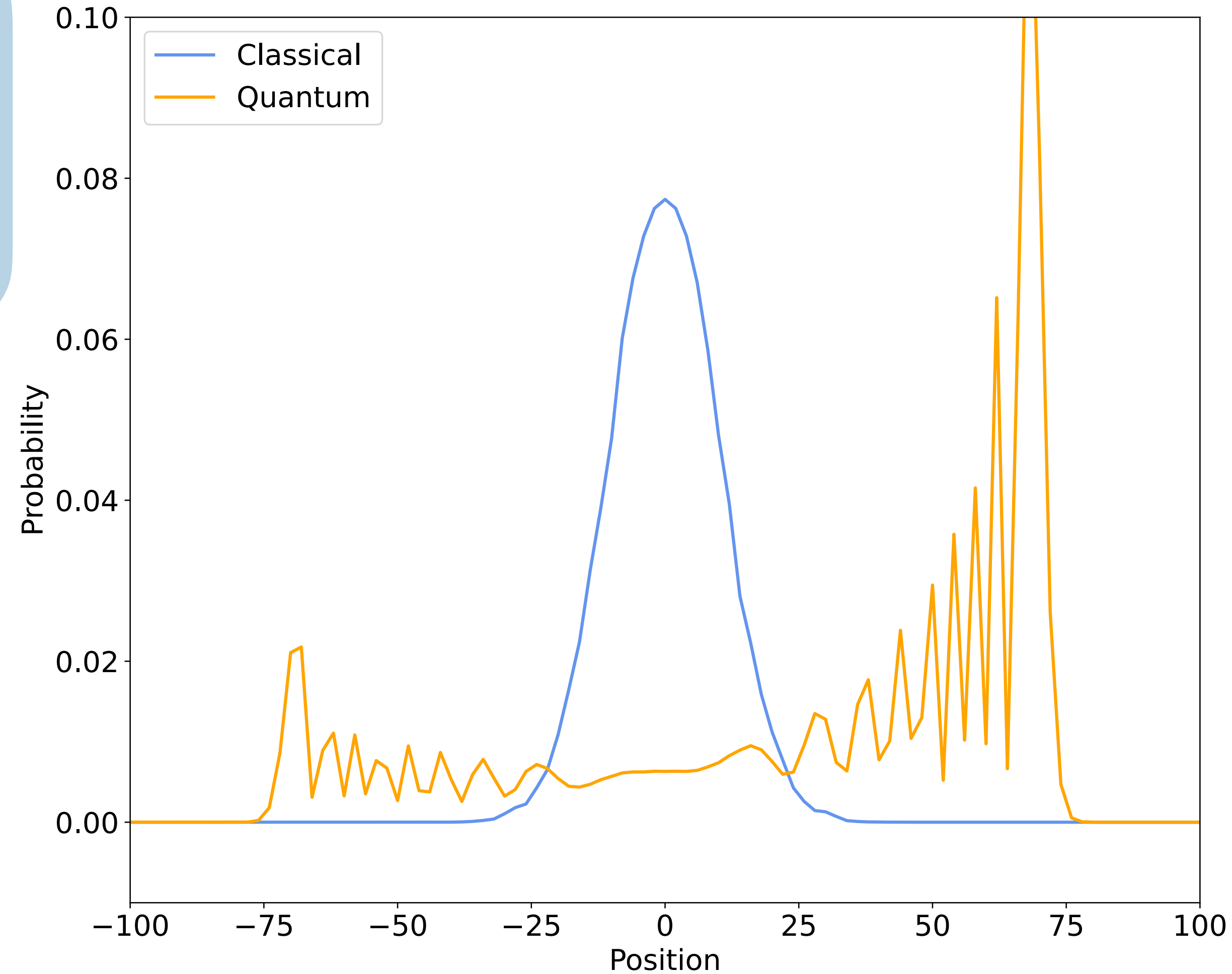
$$H(-|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



The Quantum Walk - Coin initialisation

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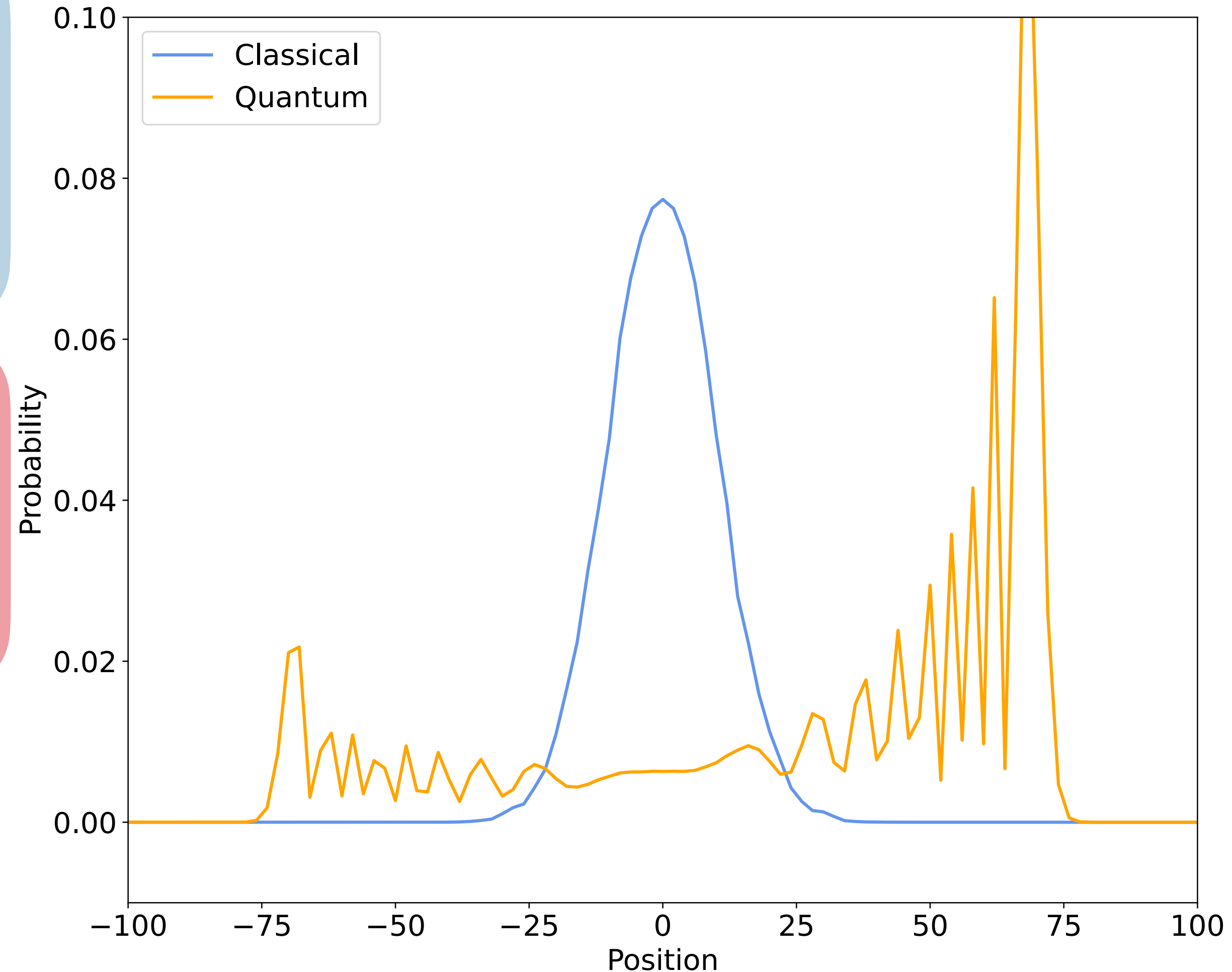
The Quantum Walk - Coin initialisation

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$$H(-|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Removing the asymmetry:

$$|c\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$



The Quantum Walk - Coin initialisation

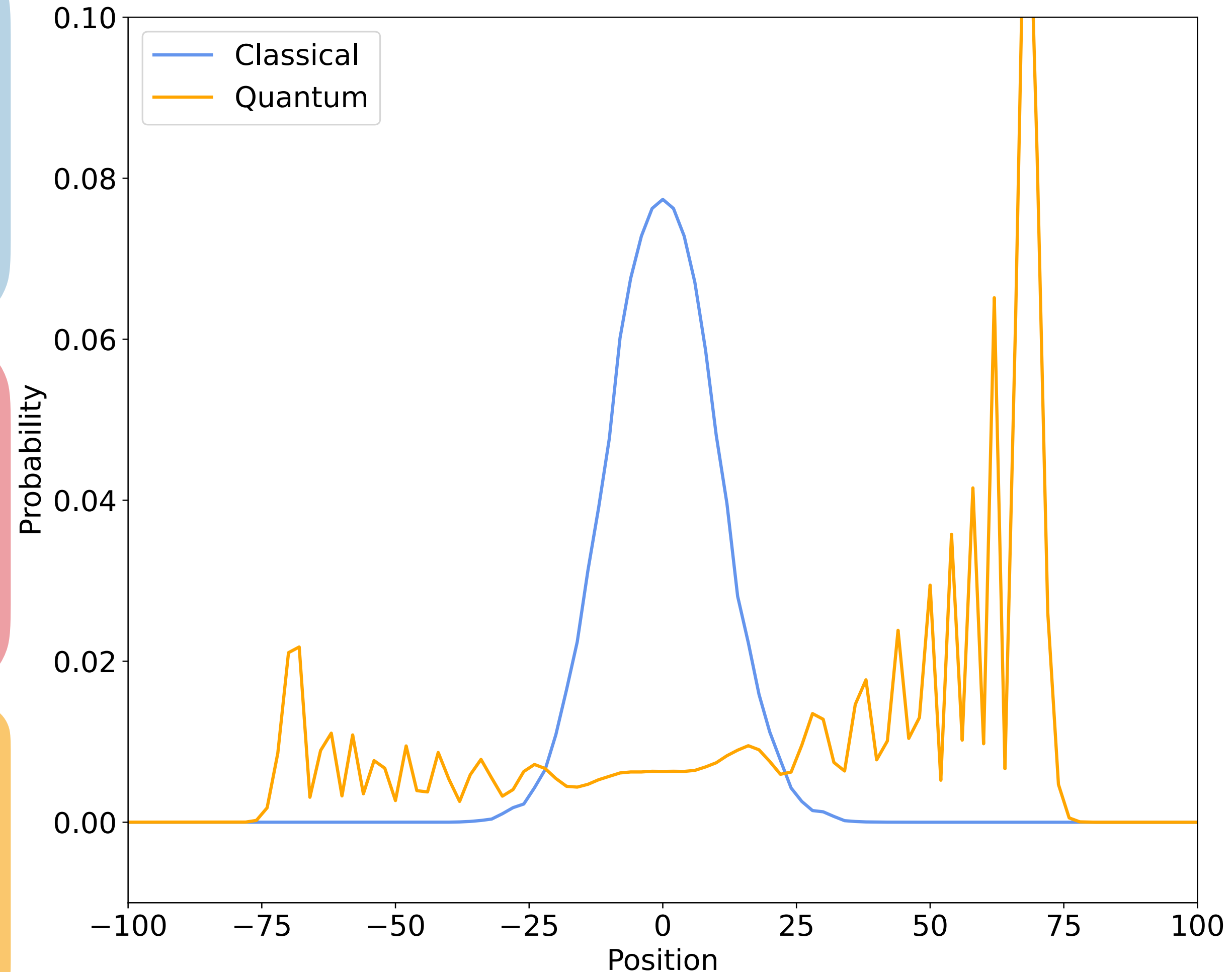
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$$H(-|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Removing the asymmetry:

$$|c\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

Left moving part ($|c\rangle = |0\rangle$) propagates in **real amplitudes**. **Right moving part** ($|c\rangle = |1\rangle$) propagates in **imaginary amplitudes**.



The Quantum Walk - Coin initialisation

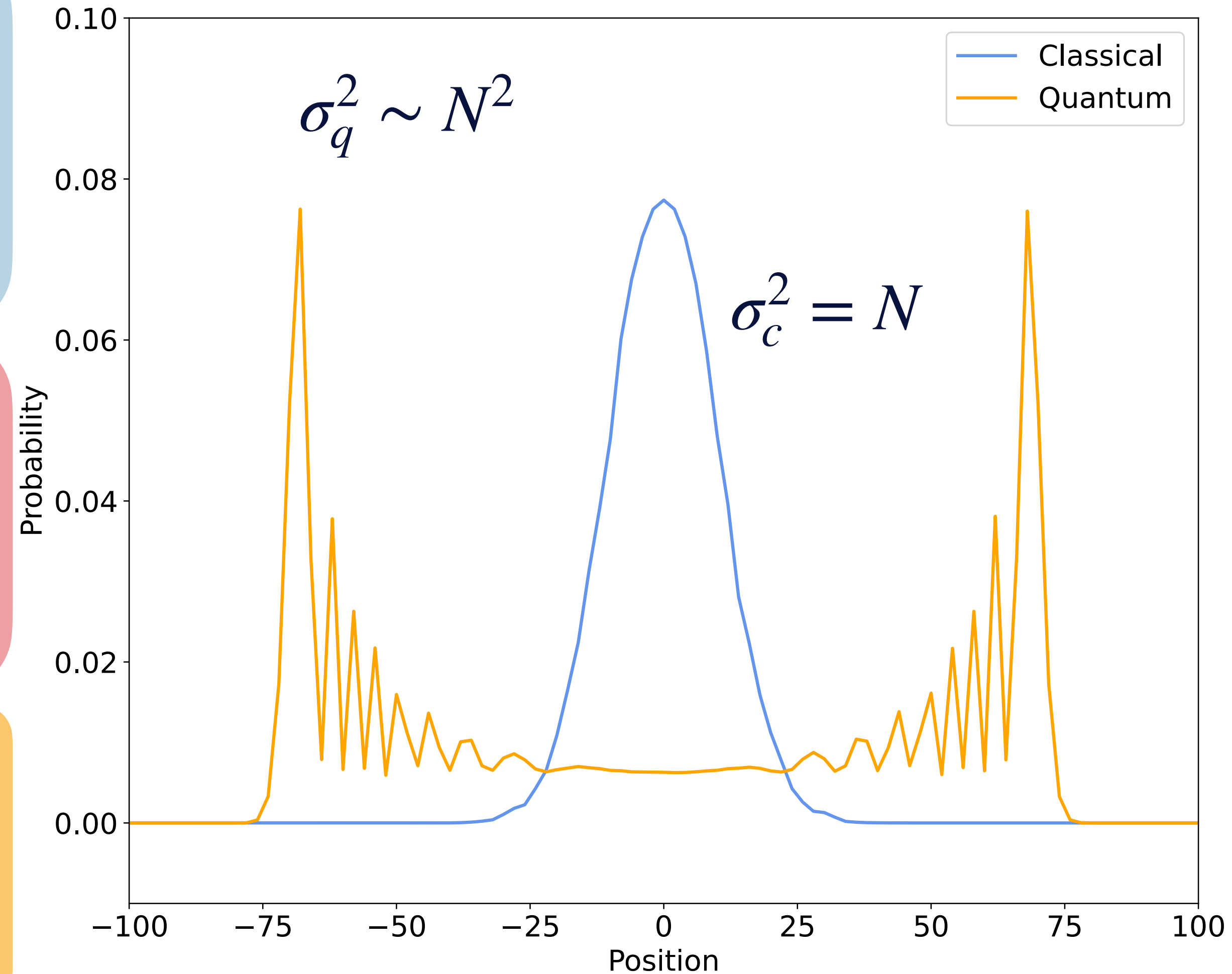
Initialising the coin in the $-|1\rangle$ state

$$H(-|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

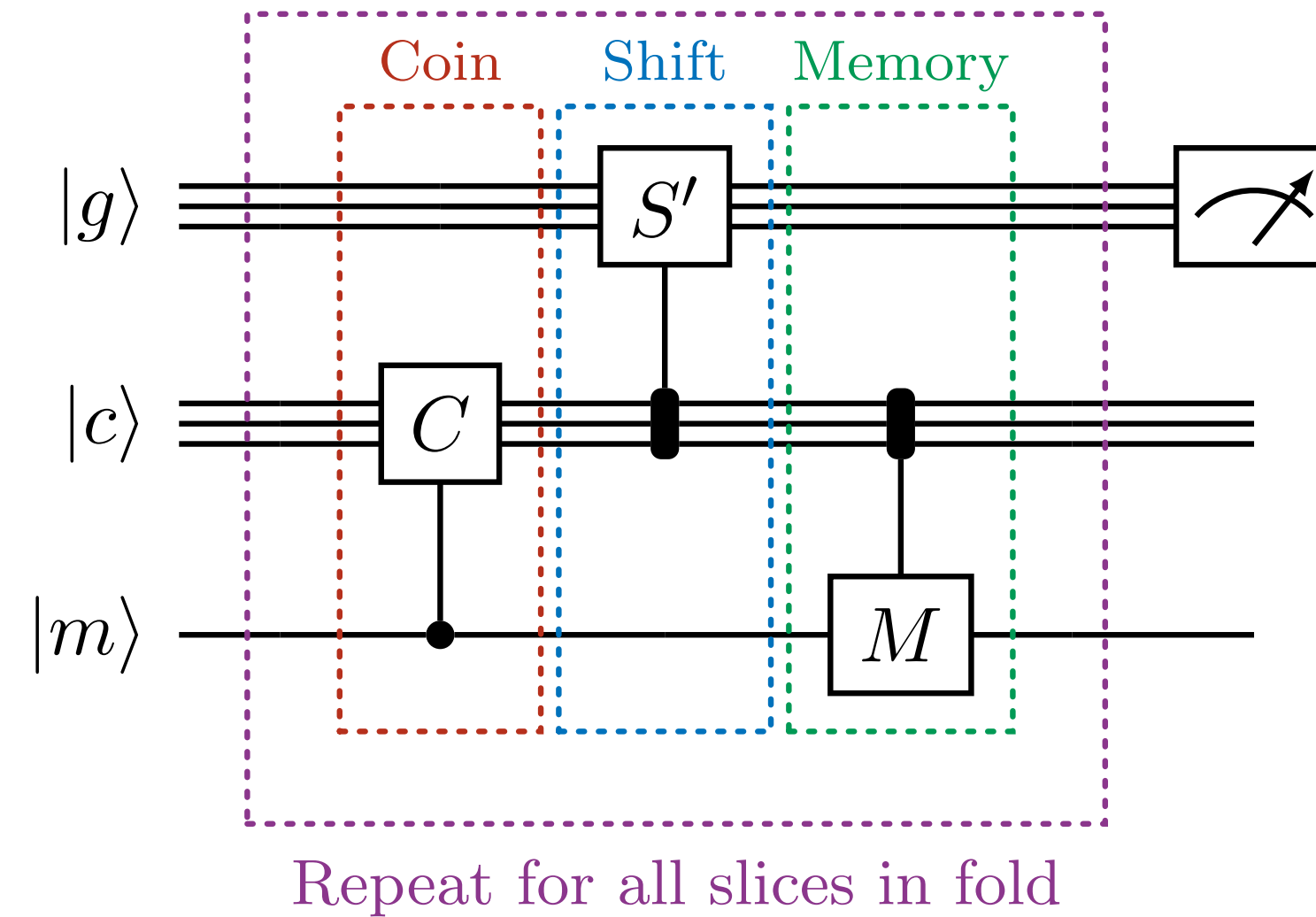
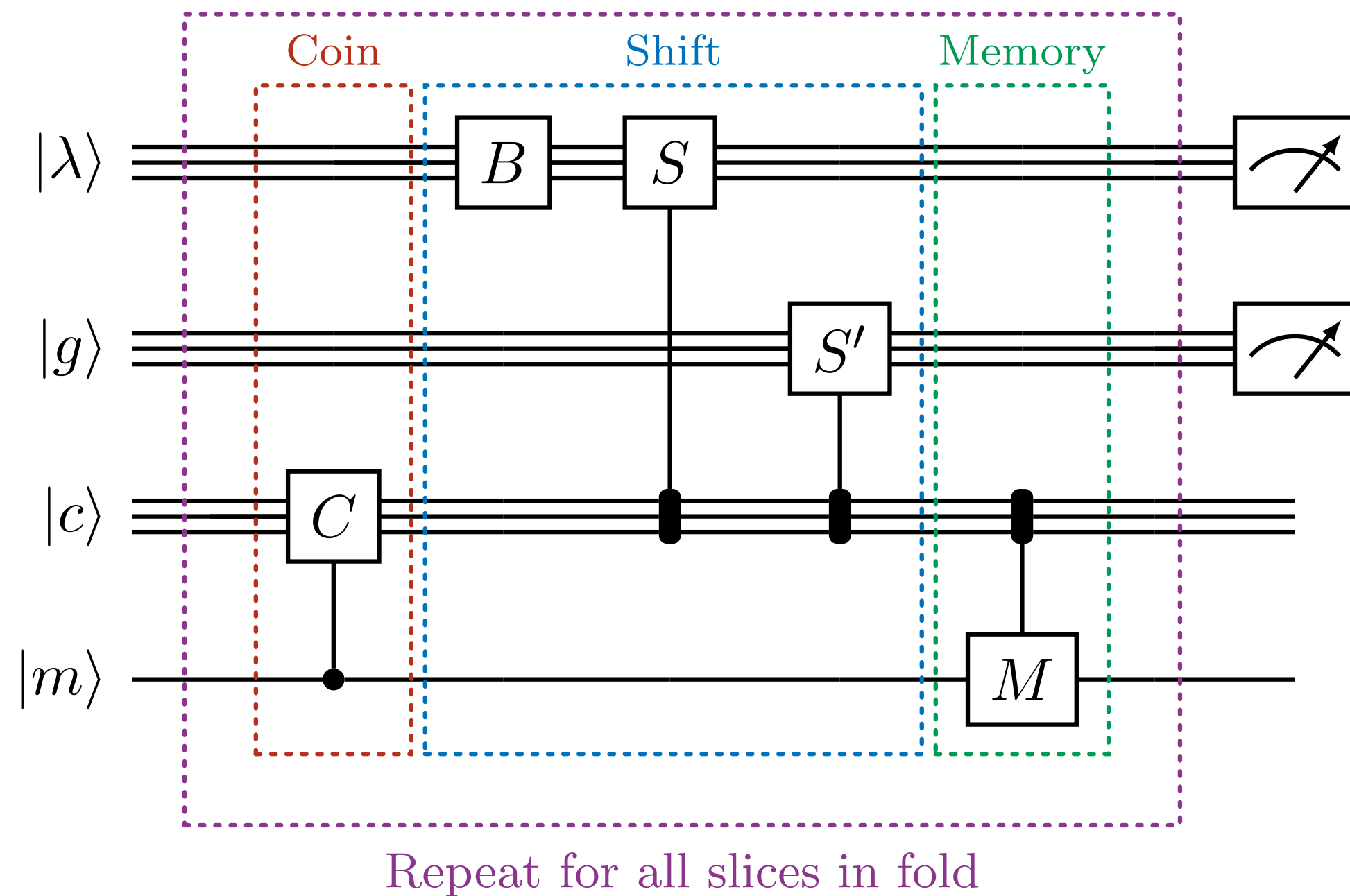
Removing the asymmetry:

$$|c\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

Left moving part ($|c\rangle = |0\rangle$) propagates in **real amplitudes**. **Right moving part** ($|c\rangle = |1\rangle$) propagates in **imaginary amplitudes**.



Running on a NISQ Quantum Device - Streamlined Circuit



15 qubits

116 gate operations

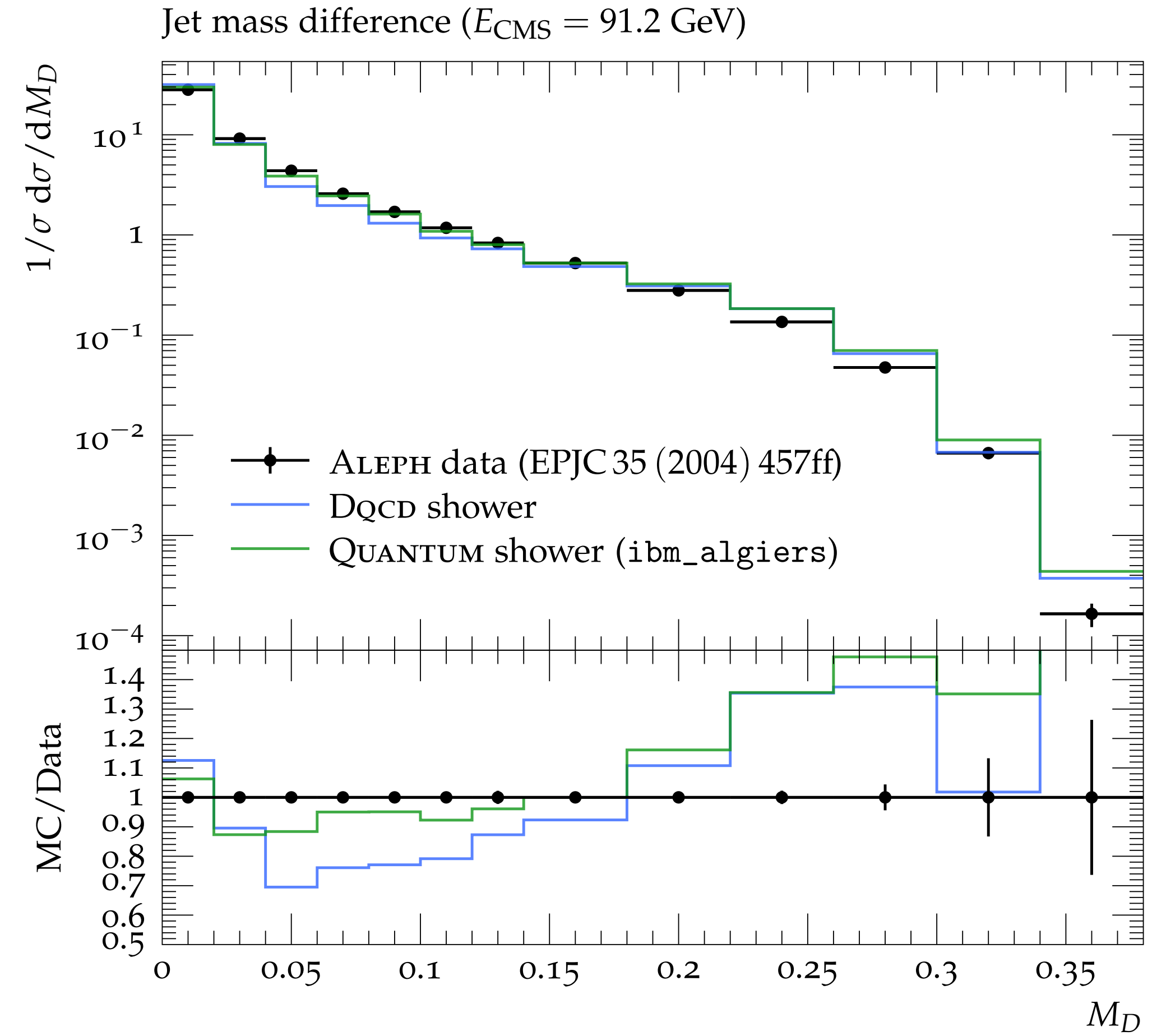
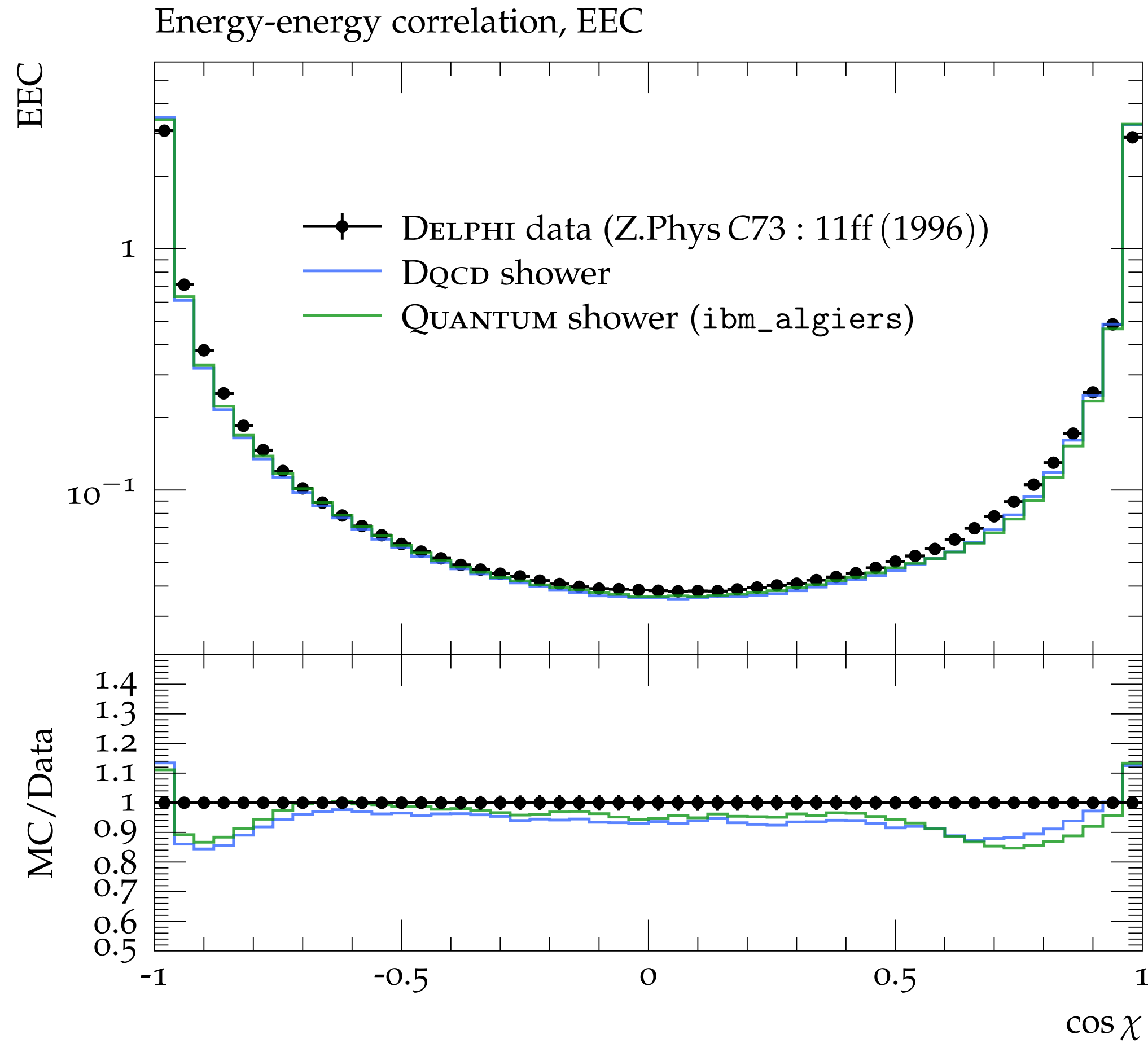
(102 multi-qubit, 14 single qubit)

10 qubits

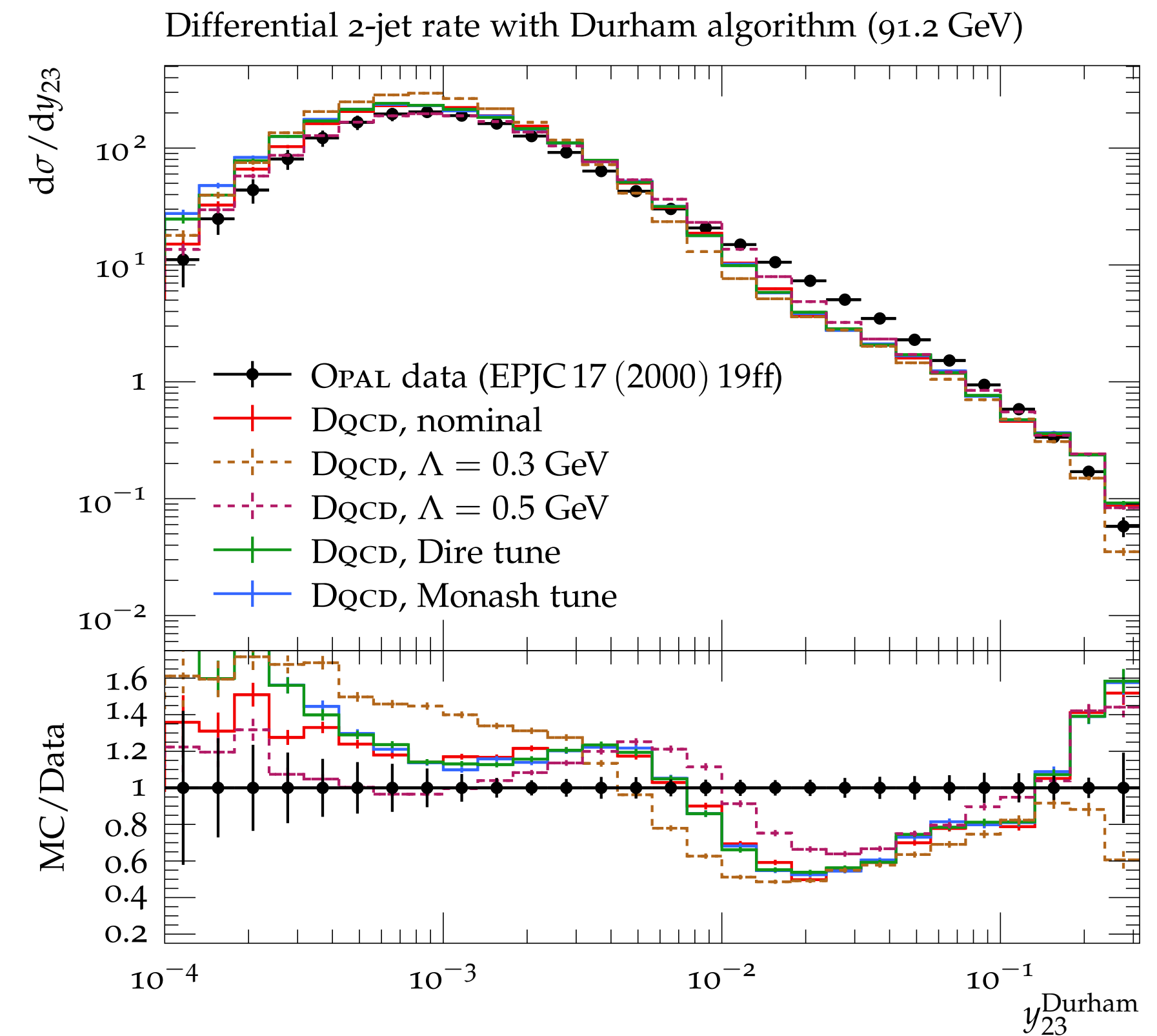
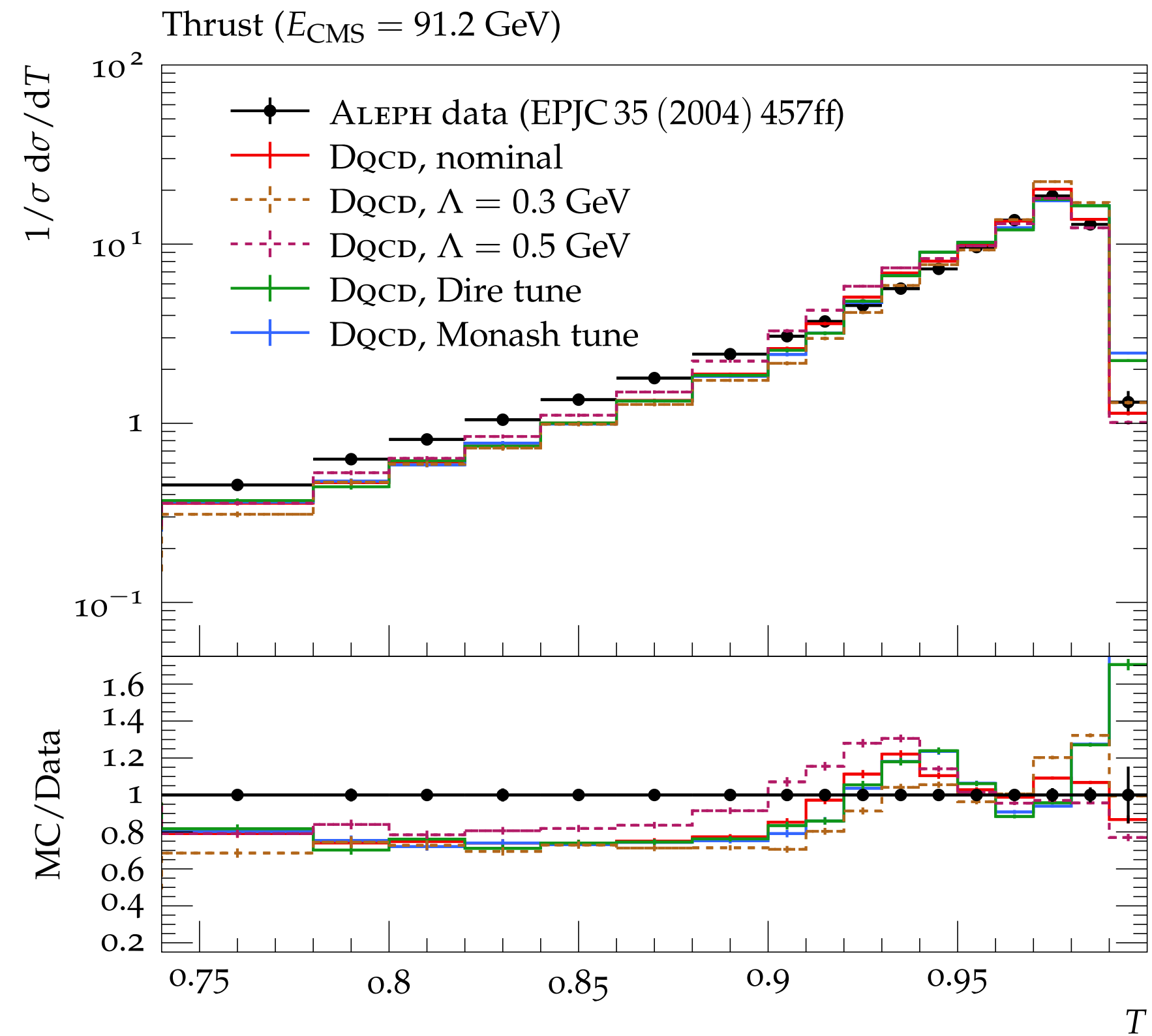
21 gate operations

(12 multi-qubit, 9 single qubit)

Collider Events on a Quantum Computer

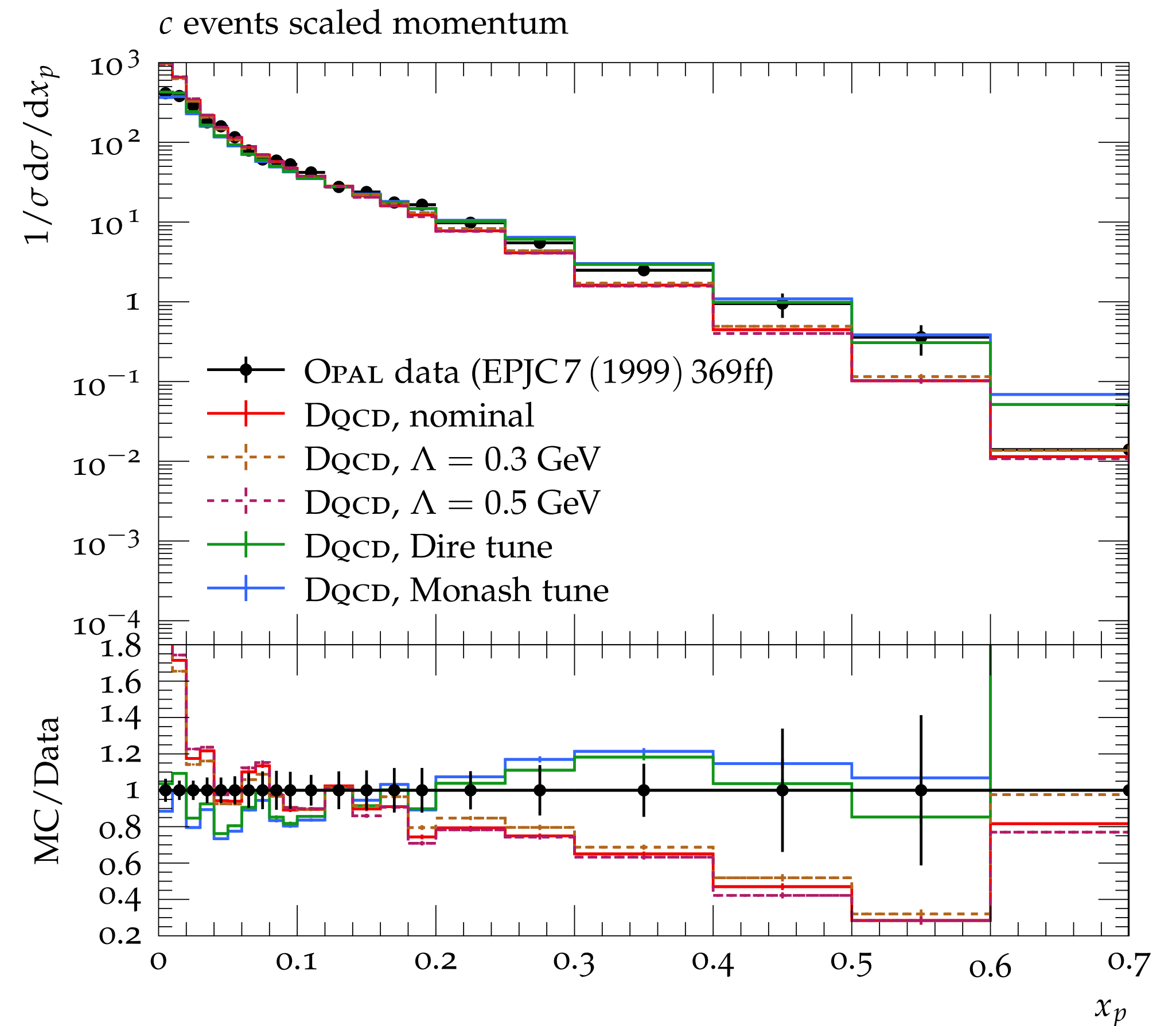
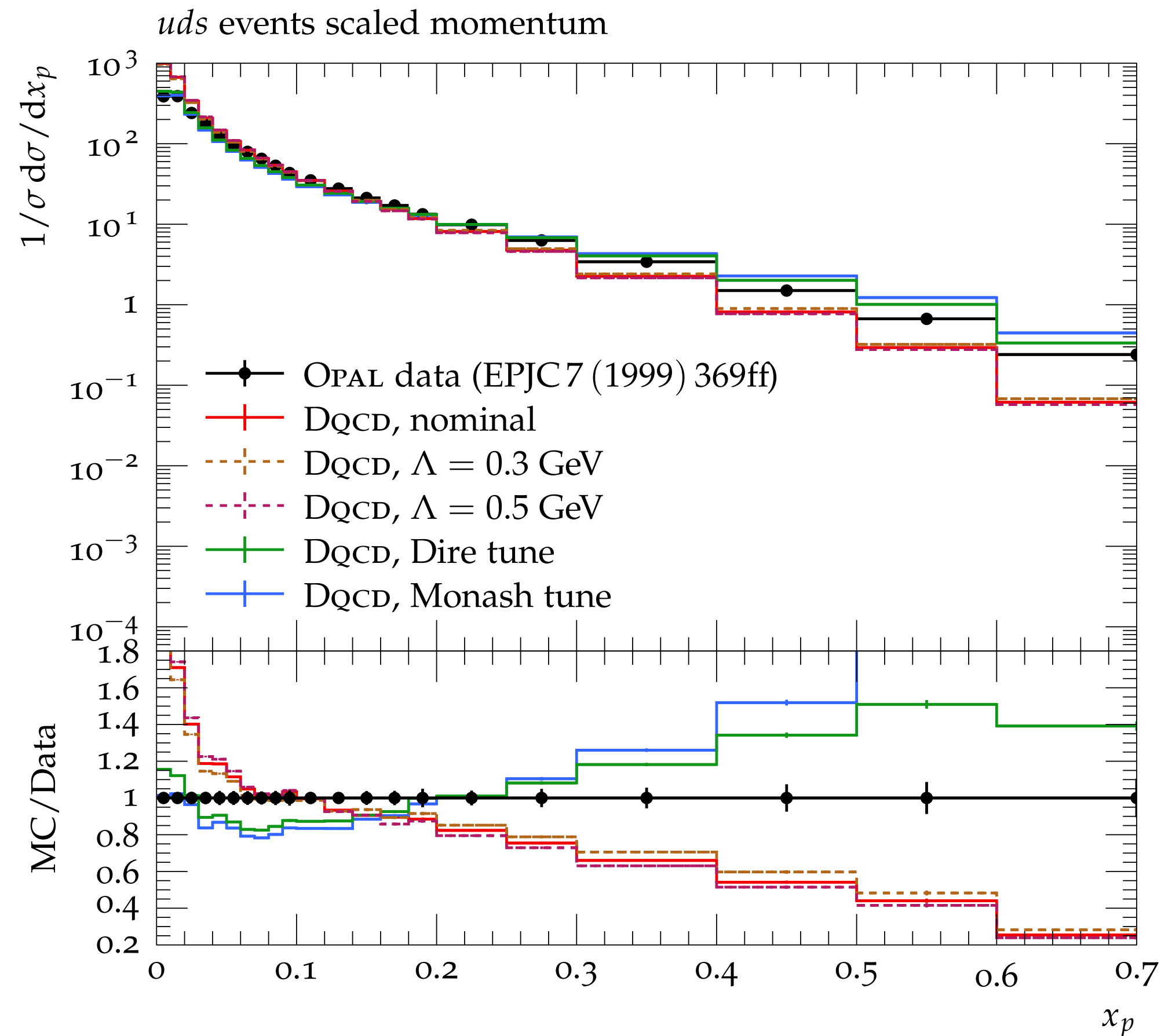


Collider Events on a Quantum Computer - Varying Λ



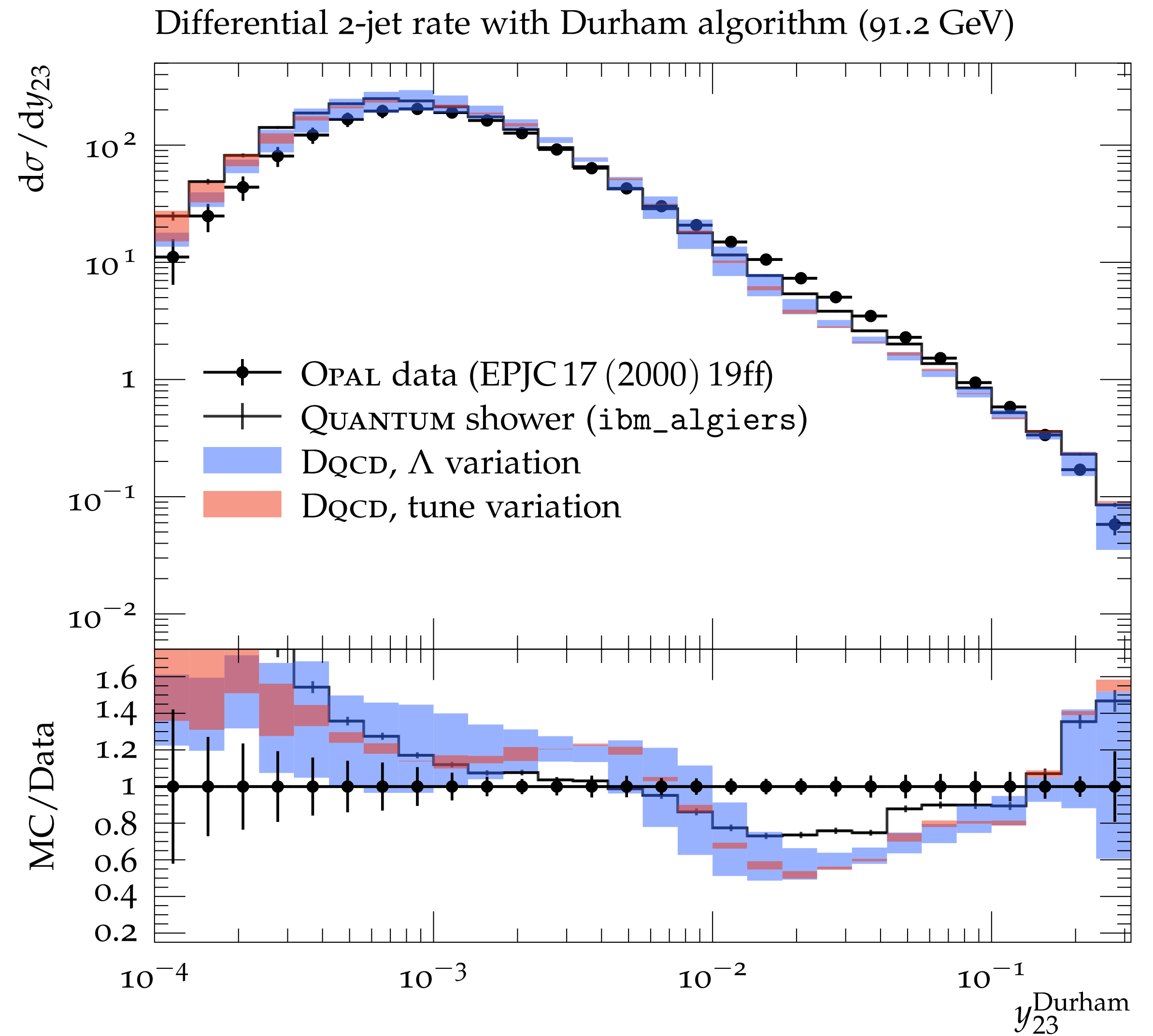
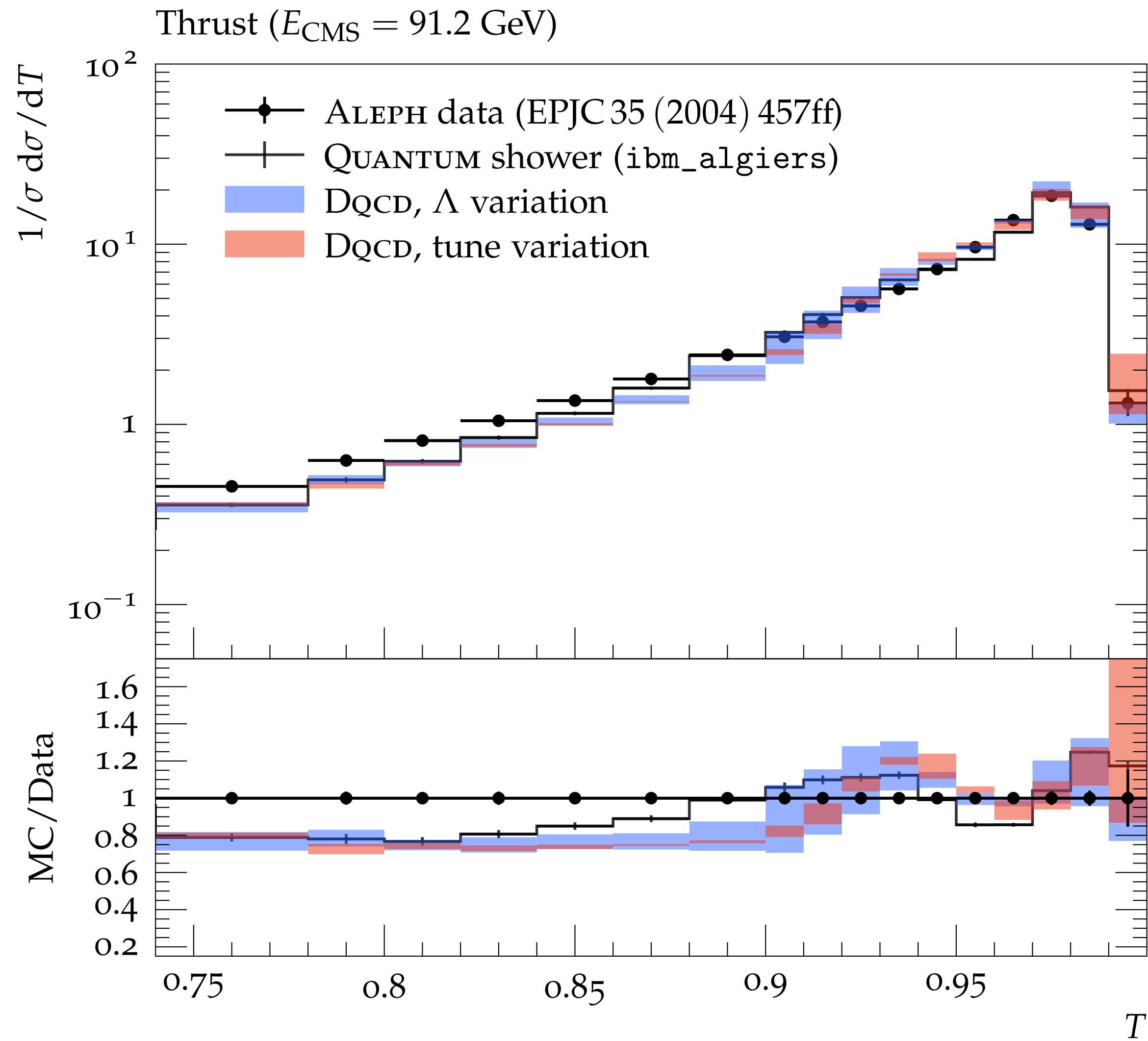
Varying values for the mass scale Λ . This leads to non-negligible uncertainties, however this is expected from a leading logarithm model.

Collider Events on a Quantum Computer - Varying Λ

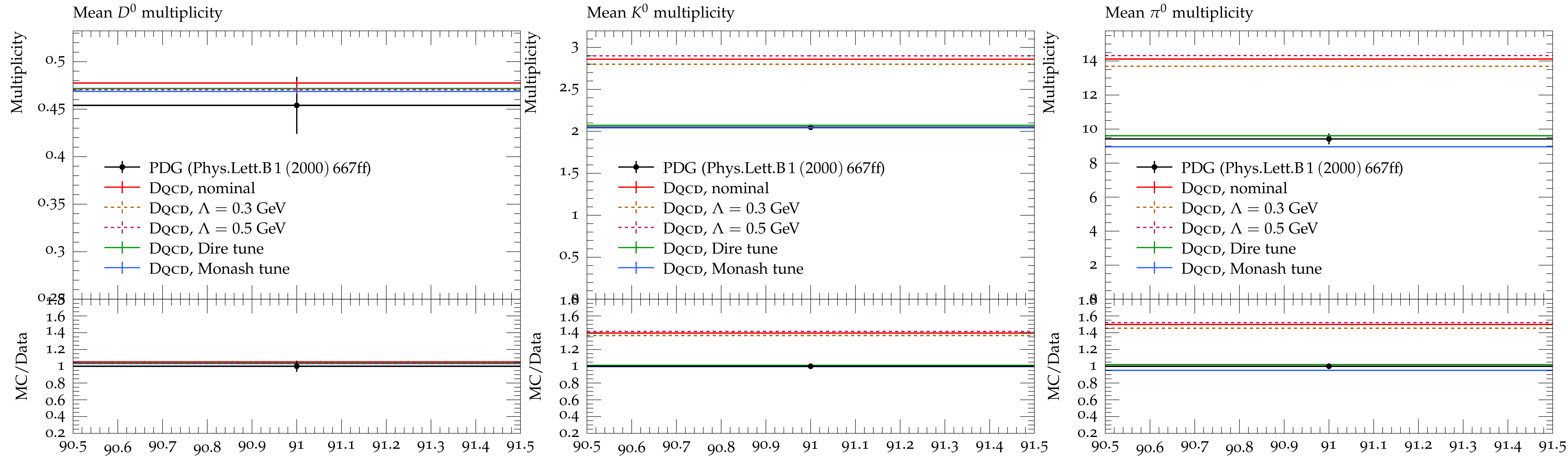


Varying values for the mass scale Λ . This leads to non-negligible uncertainties, however this is expected from a leading logarithm model.

Collider Events on a Quantum Computer



Collider Events on a Quantum Computer - Changing tune



Observables dominated by non-perturbative dynamics show mild dependence on the mass scale Λ , but are highly sensitive to changes in the tune.

Collider Events on a Quantum Computer

