



# Slow Extraction Techniques from Fixed Field Accelerators

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*ISIS2/LhARA common themes*

*1st June 2023*



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# Motivation

Fixed Field Accelerator = FFA

There is regular interest in building FFAs for hadron therapy facilities.



Conceptually interesting, with some benefits, but **major unaddressed challenges: particularly the extraction.**

The LhARA accelerator for the UK ITRF has interest in an FFA for Stage 2



Requirement for **flexible dose delivery systems**, including a pencil beam scanning method, such as that used clinically.

This requires **slow extraction** from an FFA: this study has not been performed before.



Typically only consider single-turn extraction, or avoiding third-order resonances during non-scaling FFA acceleration.

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702 IEEE TRANSACTIONS ON NUCLEAR SCIENCE, JUNE 1967

SLOW EXTRACTION FROM FFAG ACCELERATORS\*

C.L. Hammer, R.O. Haxby, and R. Tucker  
Institute for Atomic Research and Department of Physics,  
Iowa State University, Ames, Iowa

Summary

The results of earlier analytical calculations on the slow extraction of particles from an alternating gradient accelerator are applied to the special case of a fixed field alternating gradient accelerator. The extraction efficiency and rate of growth of the betatron oscillations are described.

Linearized Equations

In the spiral sector FFAG accelerator the magnetic field is represented by

$$B_z = B_0(1+x)^k [1 + \sum_j f_j \cos(j\theta - \frac{j}{2} \ln|1+x| - \beta_j)]$$

where  $k = (n - n_0)/n_0$ .

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Typically only consider single-turn extraction, or avoiding third-order resonances during non-scaling FFA acceleration.

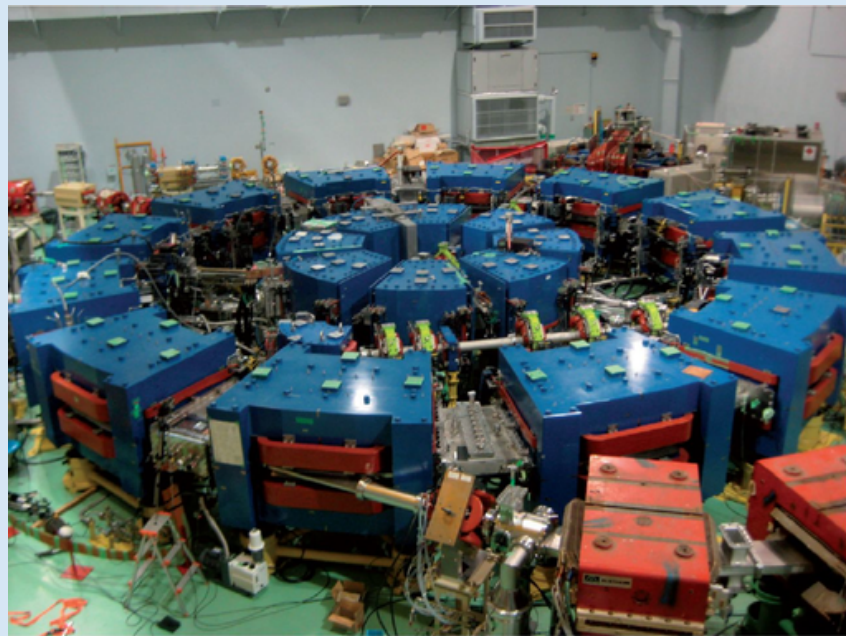
# Fixed Field Accelerators in Hadron Therapy

## KURNS

**FFA Type:** Scaling, up to 150 MeV protons, user facility

**Status:**

Operational since 2002  
Doing medical tests in 2020 (e.g. ion acoustic)

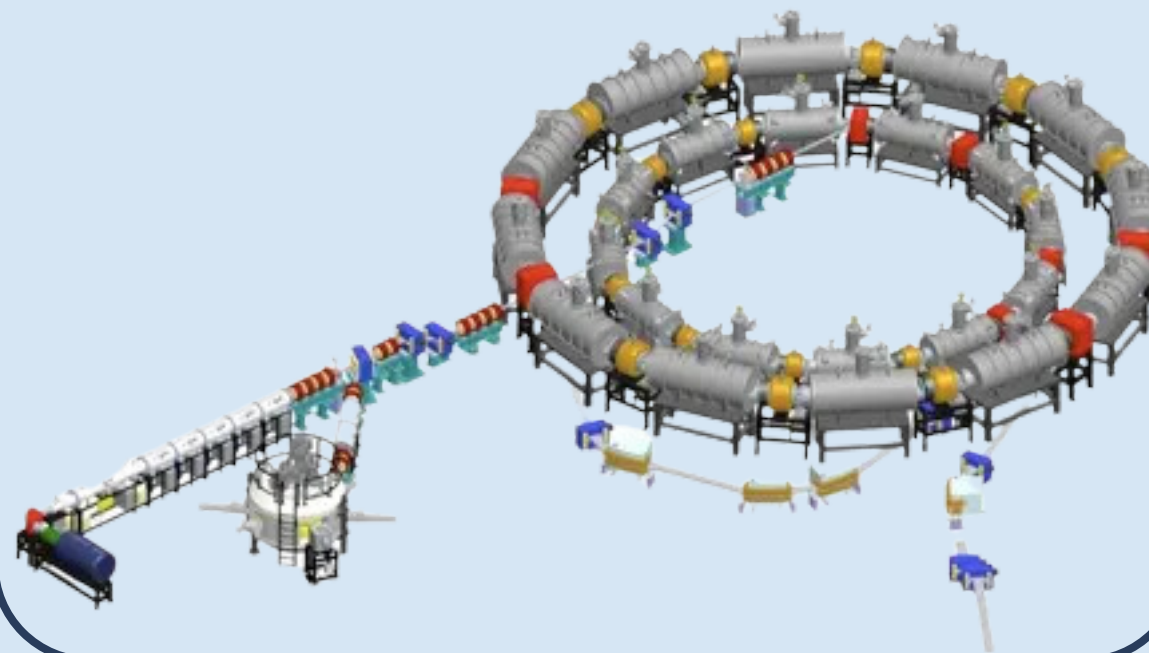


## PAMELA

**FFA Type:** Approx. Scaling, for protons (240 MeV) and carbon ions (450 MeV/u)

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Design study (2006 - 2010)

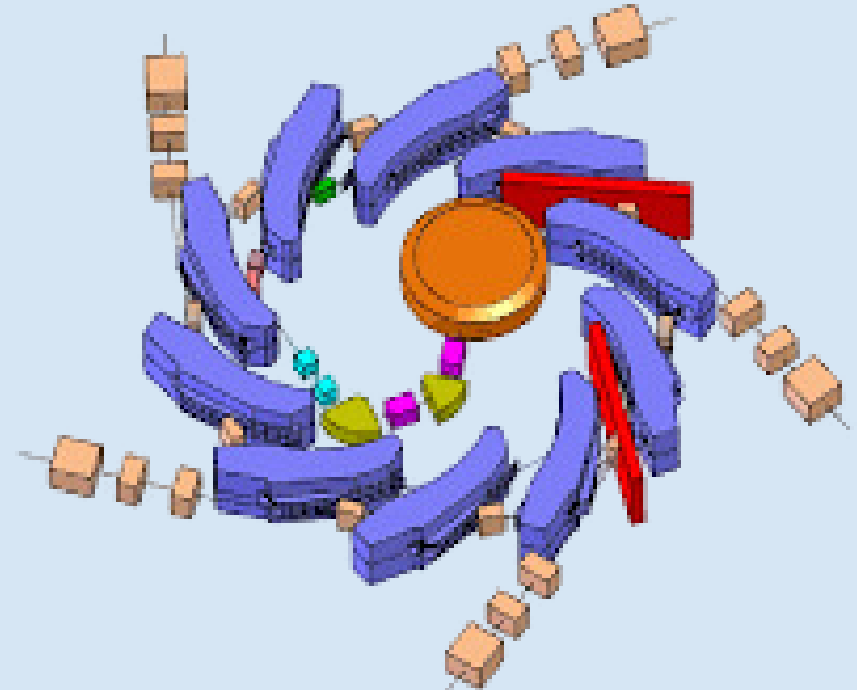


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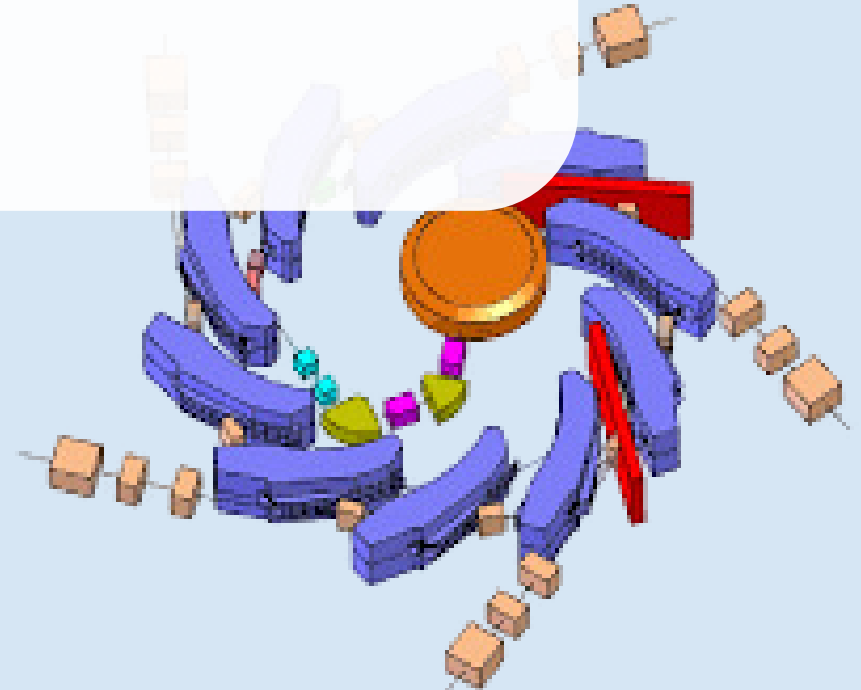
**Beam delivery with gantry?**  
**Road to clinical approval?**



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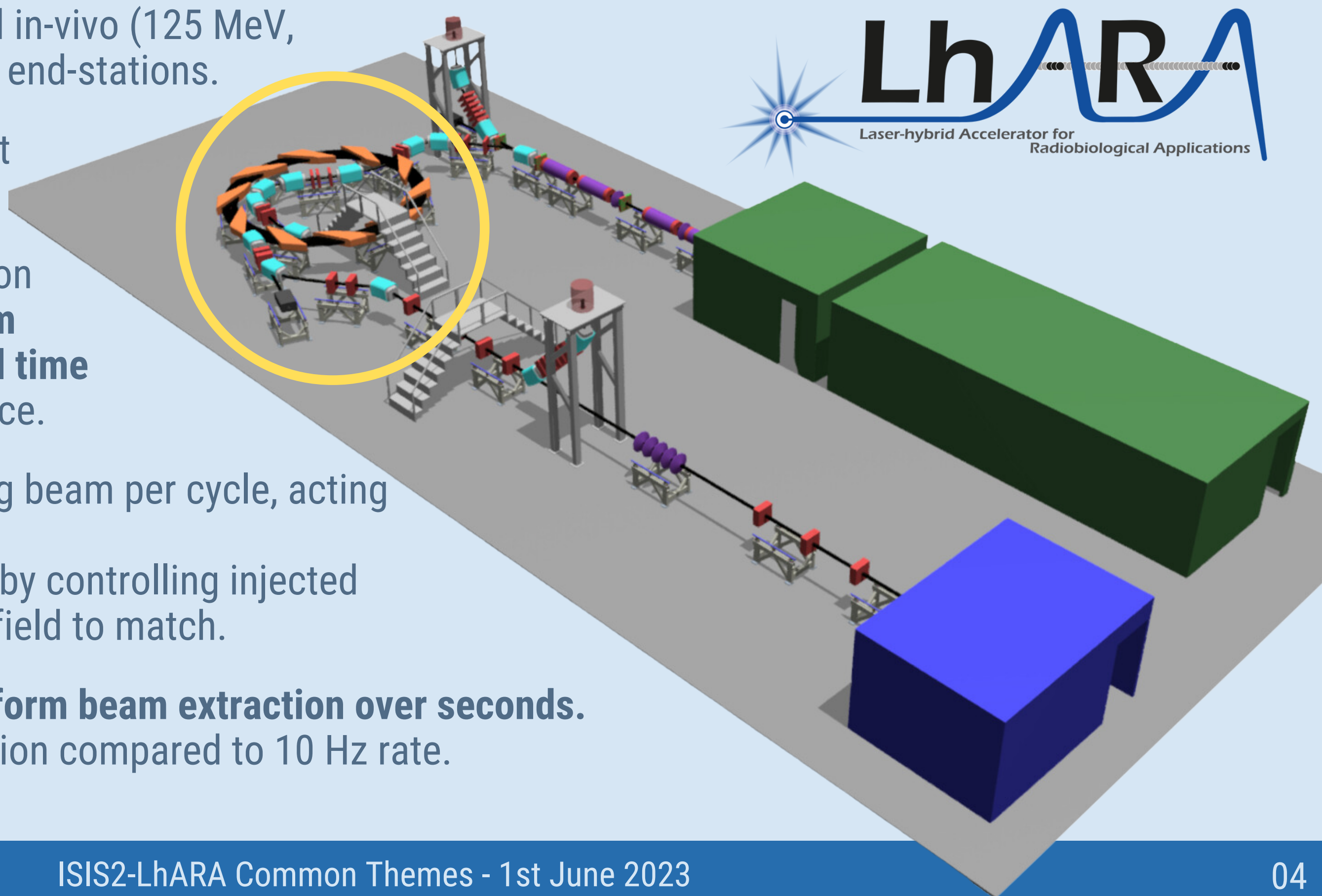
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**Considering beam delivery mechanisms?**  
Pencil beam scanning (raster scanning) vs voxel scanning vs beam shaping with leaf collimator

# Laser-hybrid Accelerator for Radiobiological Applications

- In-vitro (15 MeV protons) and in-vivo (125 MeV, 33 MeV/u carbon) irradiation end-stations.
- Beam generated from laser at a rate of 10 Hz.
- FFA for Stage 2 of acceleration required to handle **large beam currents** preserving the **novel time structure** from the laser source.
- Operating with one circulating beam per cycle, acting as 'energy multiplier':
  - Extraction energy varied by controlling injected beam energy & ramping field to match.
- **Considering continuous, uniform beam extraction over seconds.**
  - Different mode of operation compared to 10 Hz rate.

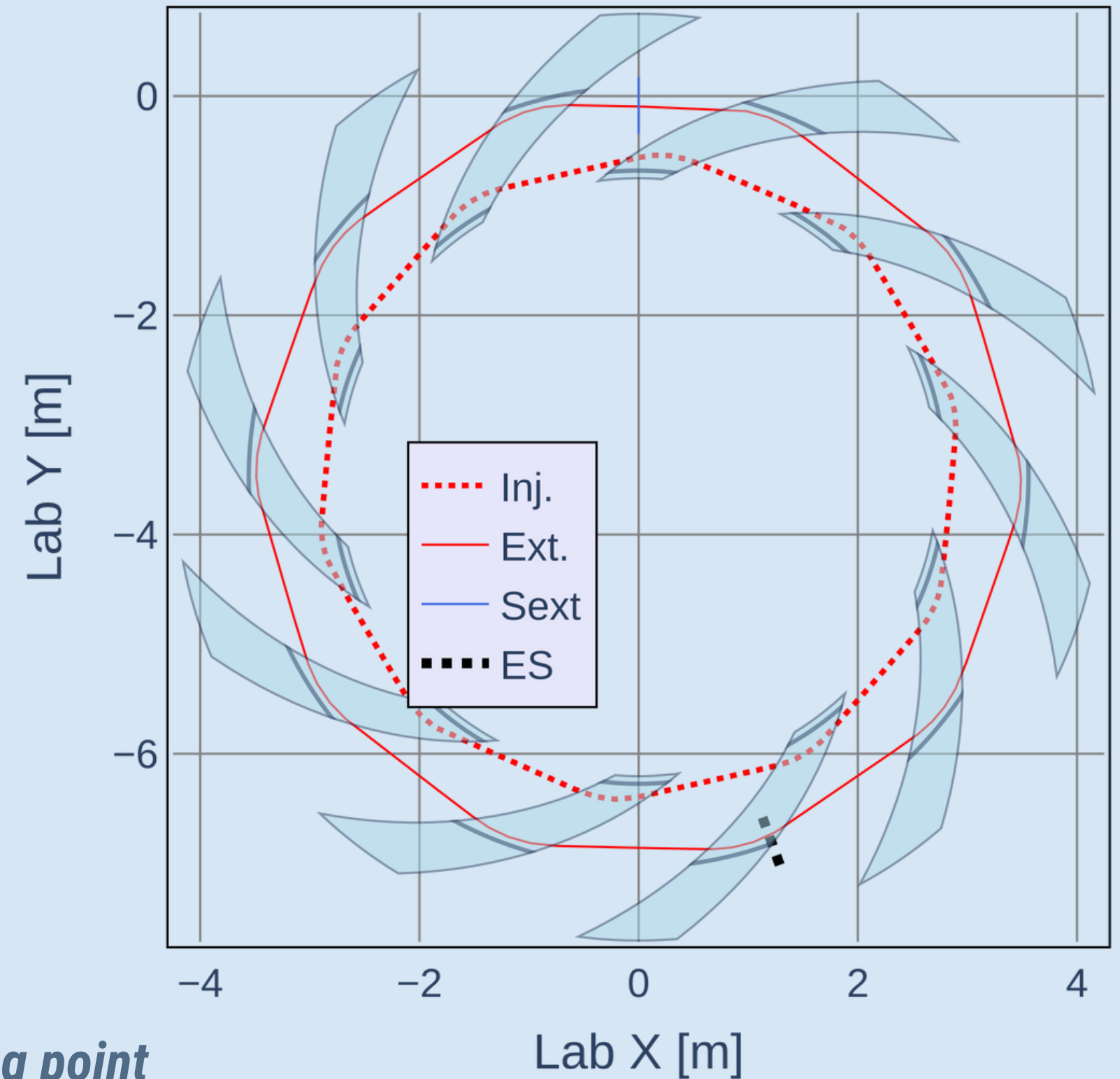
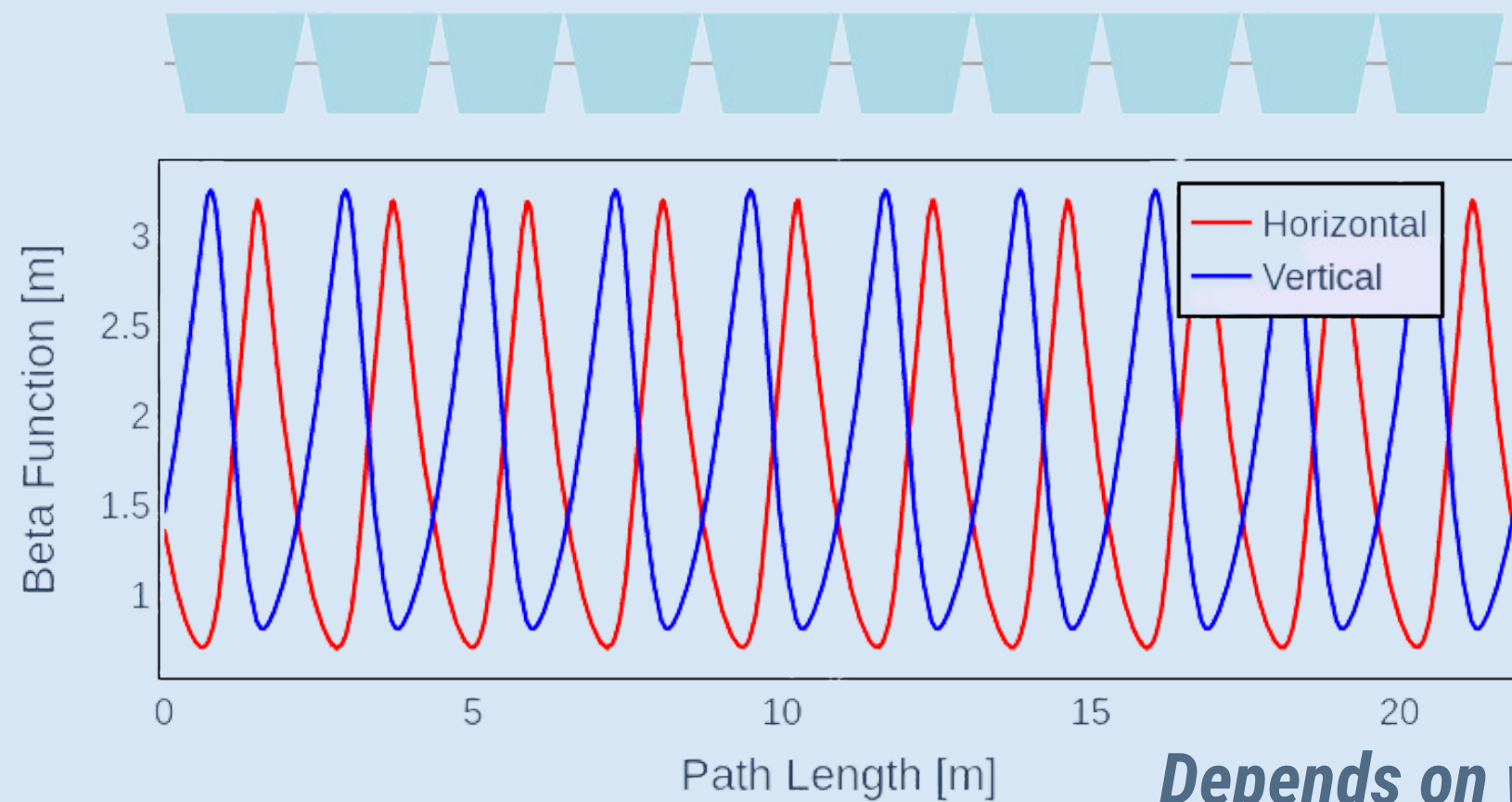




# Fixed-Field Accelerators in Hadron Therapy

## LhARA FFA - based on RACCAM

Parameter	Value	Parameter	Value
$E$ [MeV]	15 - 127	Packing Factor	0.34
$B_0$ [T]	1.4	$r_0$ [m]	3.47704
Magnet length	0.7537	$\varepsilon_x$ [ $\pi\text{mm}\cdot\text{mrad}$ ]	0.41



# Slow Extraction

- Required to deliver continuous beam spill of timescale of 1s - 30s
- Used for clinical dose delivery of hadron therapy via beam scanning across tumour volume

## Ingredients:

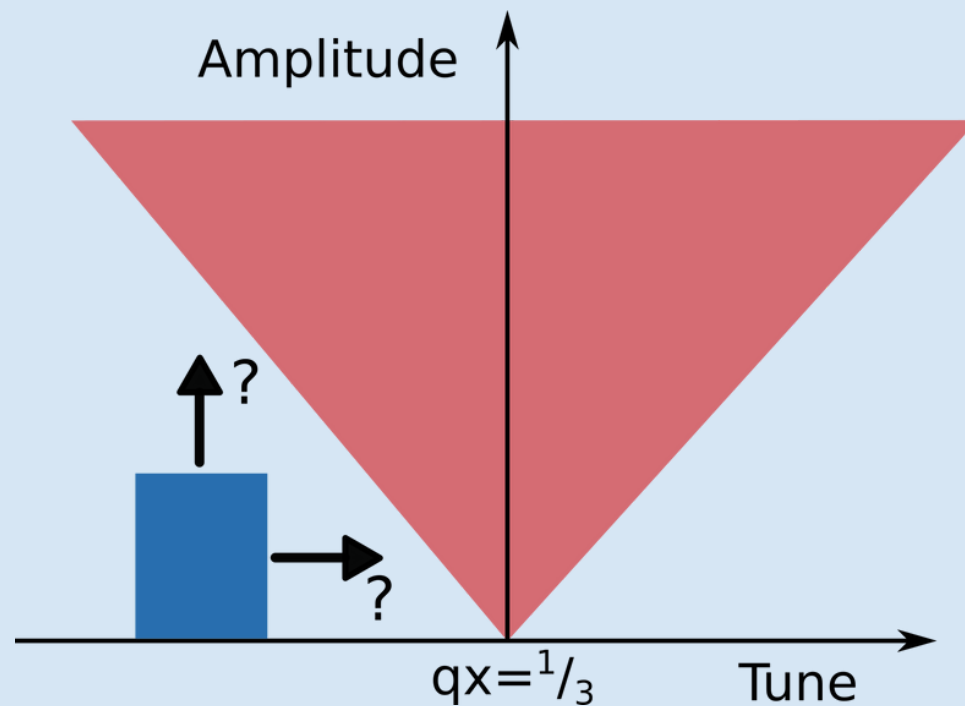
1. Set horizontal tune  $Q_x$  near  $1/3$  resonance
2. Drive resonance with strong sextupole
3. Move particles into resonance with RF-KO or with tune
4. Kick high-amplitude particles with electrostatic septa (ES)



# Slow Extraction

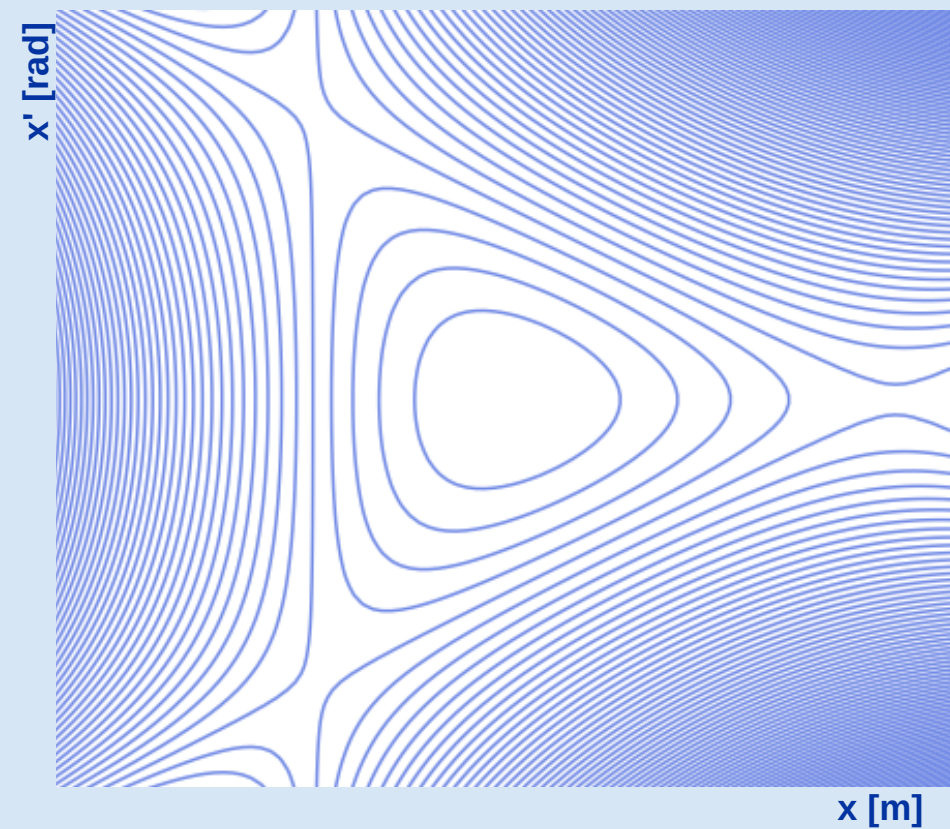
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Need a mechanism to control rate at which particles enter the resonance.

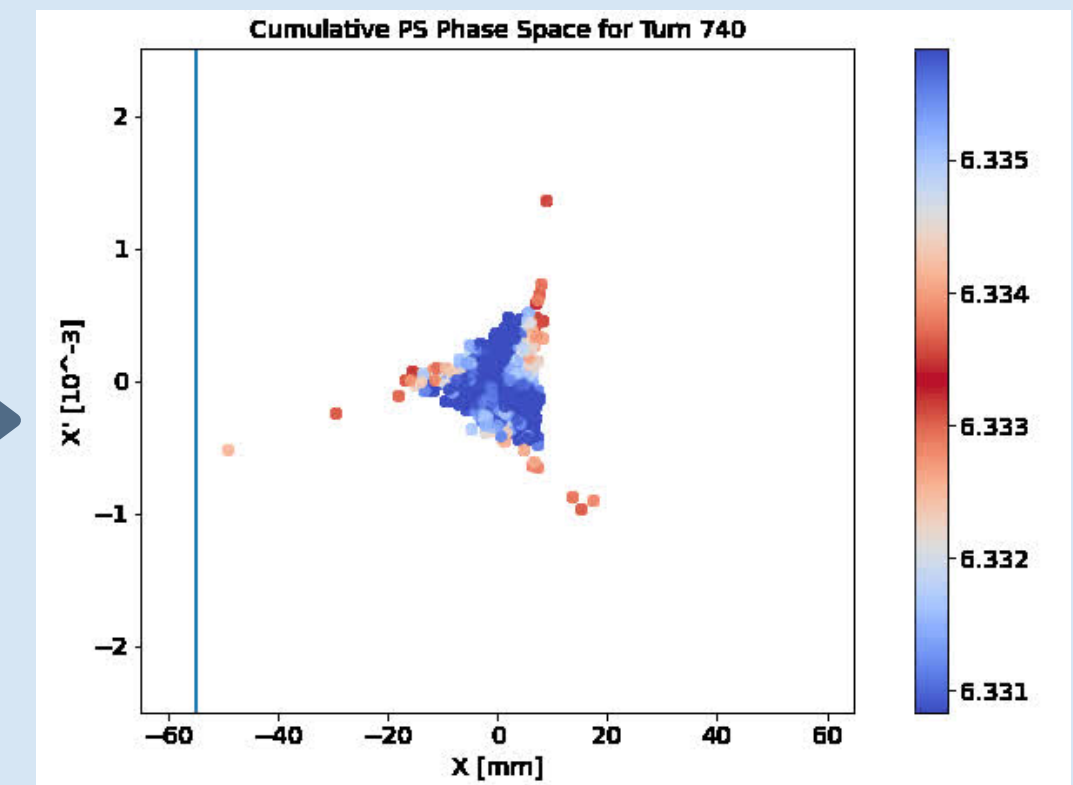


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*Hamiltonian near 1.3 resonance, showing stable triangular phase-space and 3 unstable separatrices.*

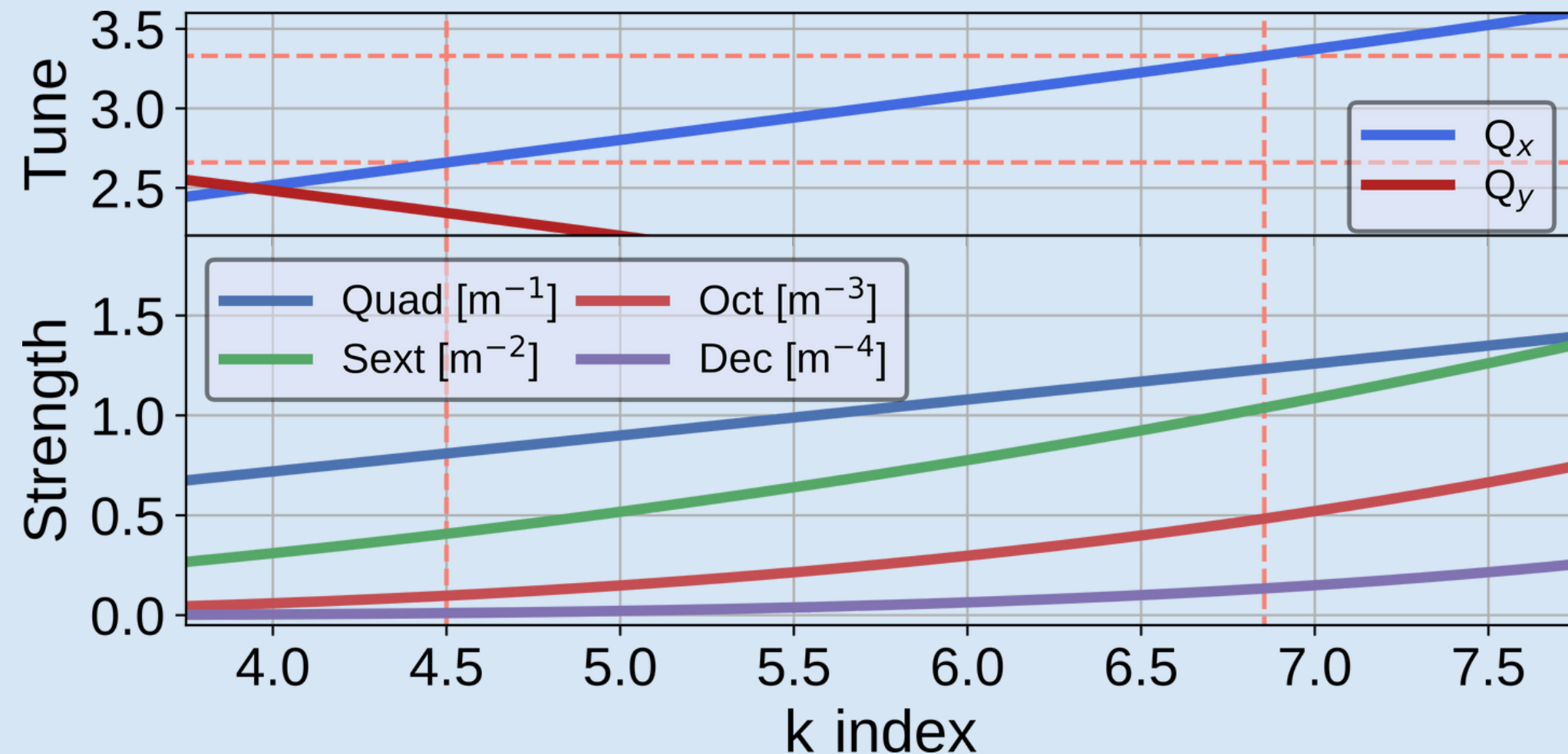


*Extraction from the PS synchrotron via quadrupole showing three separatrices.*

# Slow Extraction & Scaling FFAs

- **Scaling FFAs** have fixed tune at arbitrary energy, but this requires a nonlinear magnetic field
- FFA scaling law depends on magnetic field strength  $B_0$ , closed orbit  $r$ , spiral angle  $\zeta$  and  $k$ -index.
- A Taylor expansion to higher orders determines **quadrupole**, **sextupole** terms etc.

$$B = B_0 \left( \frac{r}{r_0} \right)^k \mathcal{F} \left( \theta - \tan(\zeta) \ln \frac{r}{r_0} \right)$$

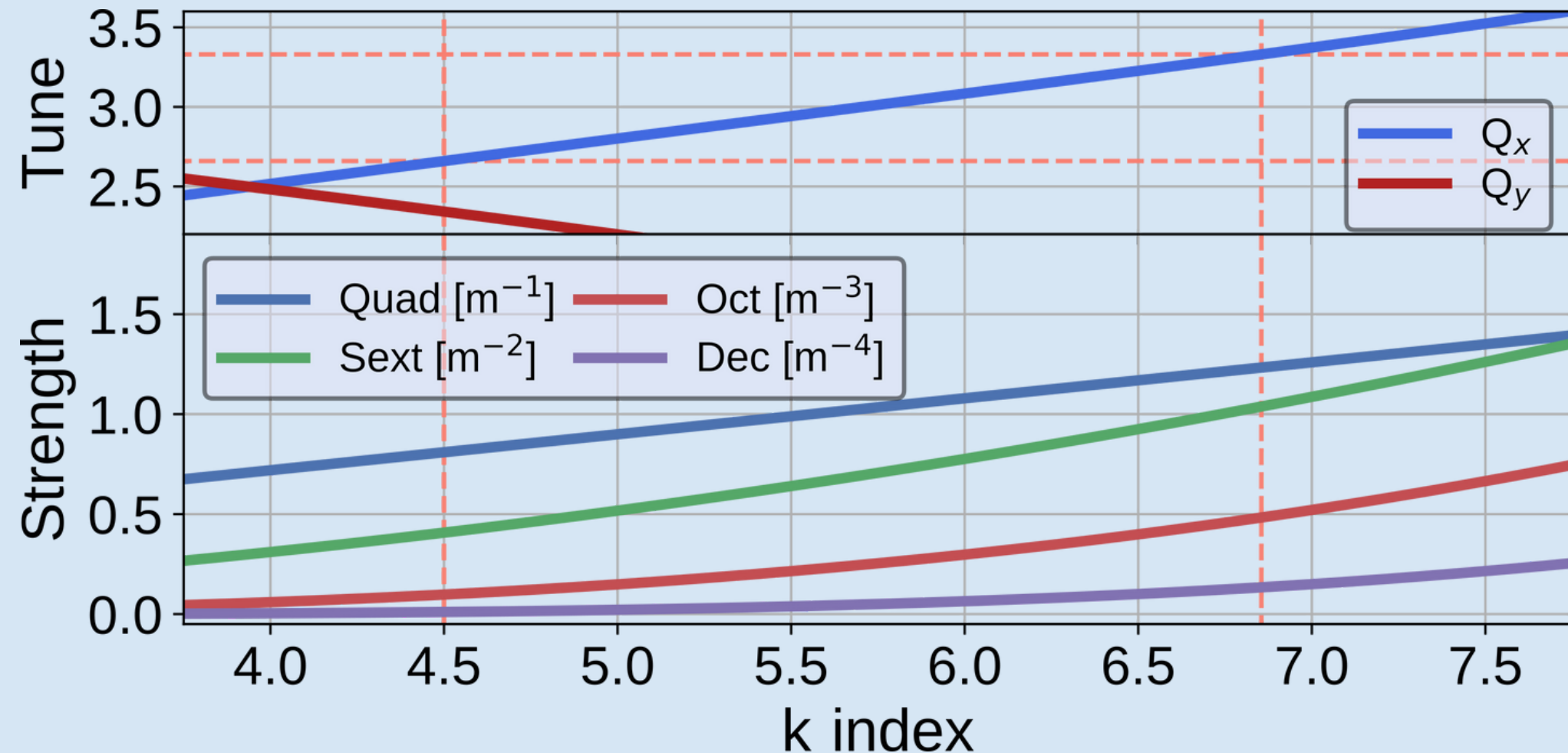


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- Choose the **k-index** value for a particular third-order resonance ( $q_x = 1/3$ ).
  - Relative multipole strength is fixed. Otherwise scaling law is broken.
  - Larger integer tune = larger k-index = larger sextupole  $k_2$  component.
- Then select spiral angle  $\zeta$  to ensure stability in vertical plane.





# Slow Extraction & Scaling FFAs

**Tune is constant** throughout acceleration.



Challenging when tune must be varied to control extraction rate.

Strong, fixed **non-linear magnets** at constant field.



Challenging when sextupole strength must be fine-tuned.

**High packing factor** keeps accelerator footprint small.



Limited geometric options for additional extraction hardware.

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Need to decide:

- Are the intrinsic FFA non-linearities sufficient to drive the resonance in slow extraction?
- If not, how and where should an extra sextupole be added?
- How should the beam be excited into the resonance without ramping the tune?

# Simulation Methodology

- Fast MADX simulations used to get intuitive idea of machine & parameters.
- Dedicated Zgoubi simulations used for complete model of FFA and particle tracking.

## MAD-X

Fast Simulations/Modelling

Widely Used

Doesn't Handle Large Excursions

Misses Effects of Large Edge Angles

## Zgoubi

Relatively Slow Ray Tracing

User Base Small

Deals with All Geometric Effects

Nonlinear Fields are No Obstacle



# Using Inherent Sextupole Strength

# Using Additional Sextupole Strength



# Using Inherent Sextupole Strength

**The ideal case: No additional sextupoles.**

Zero-dispersive regions impossible, so sextupoles would introduce chromaticity and break the tune-energy relation of the FFA.

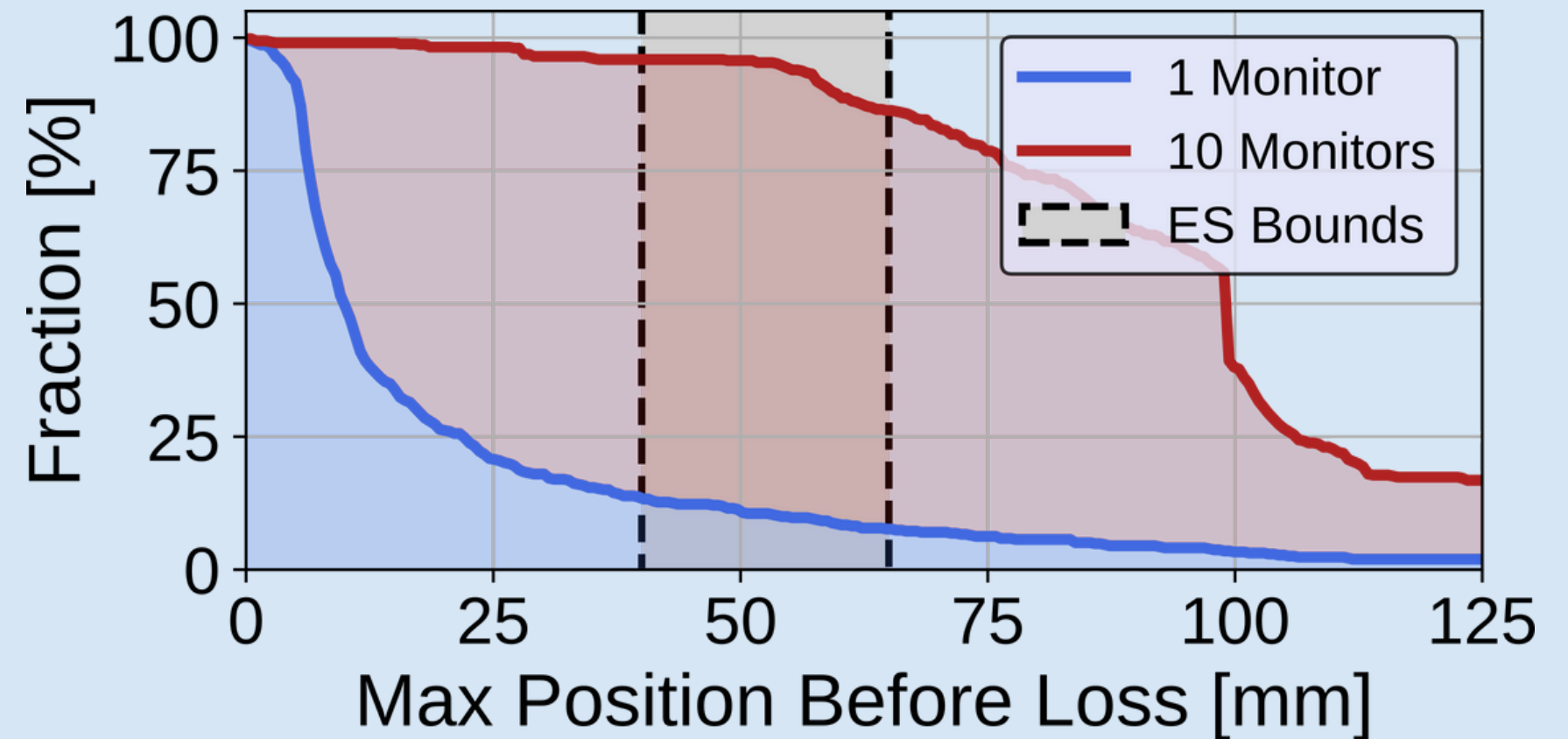
Number of cells: **10**

Machine Tune: **3.333**

Tune per cell: **1/3**

Resonance in this case is too strong:

- Causes a spiral step increase every 3 magnets, rather than every 3 turns.
- **Particle lost before reaching electrostatic septa.**



*Only get 10% extraction efficiency, unless ES septa placed at all 10 cells.*

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## Using Additional Sextupole Strength

Machine Tune: **2.666**

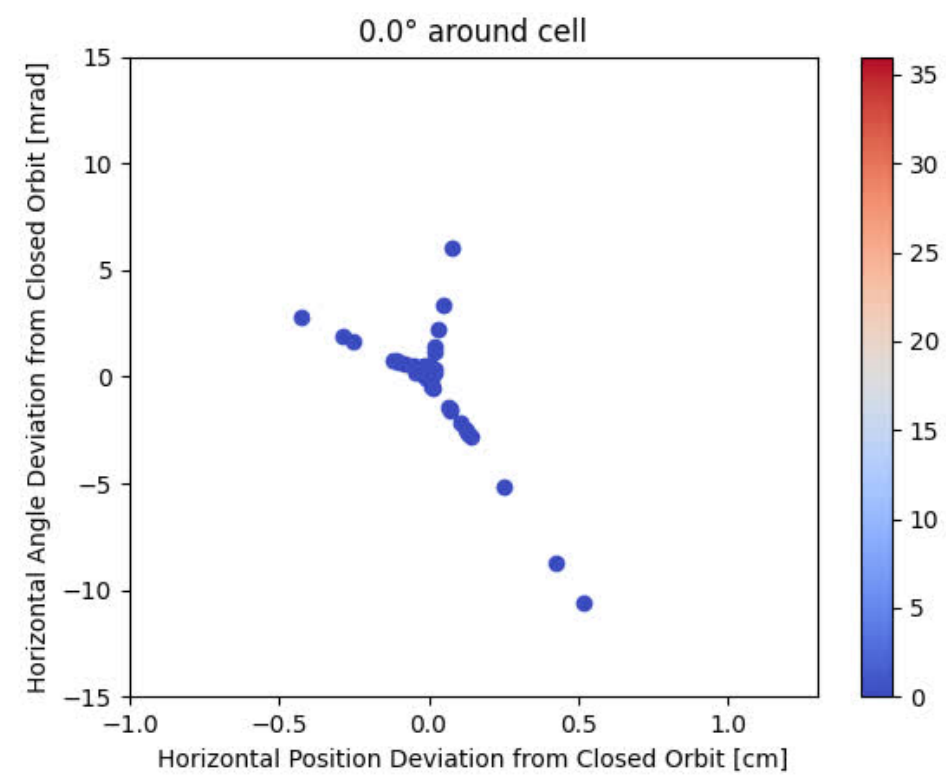
Strength insufficient for slow extraction, without addition of external sextupole

- **Novel C-shaped sextupole** or customised extra windings needed to only affect extraction orbit particles.
- Sextupole would be **turned off during acceleration** to keep tune constant.
- Could explore multiple-sextupole option, to counter chromatic effect but keep resonance.

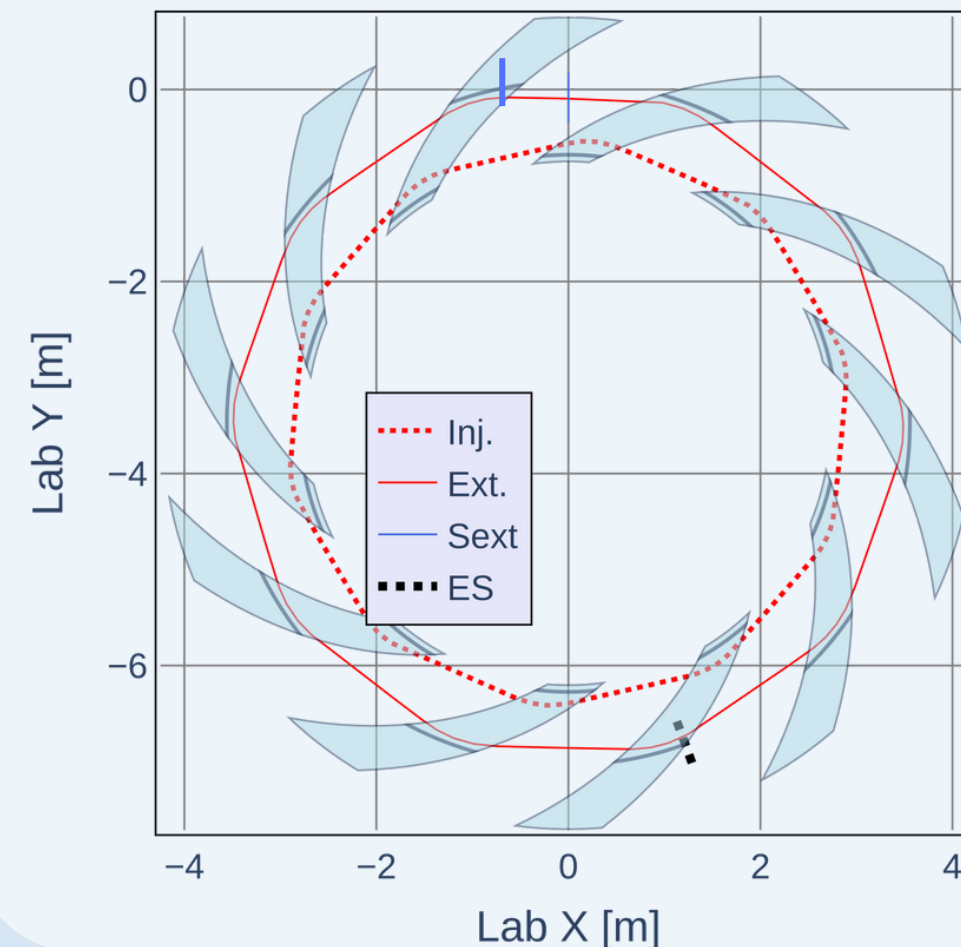
# Electrostatic Septum Geometry

**Phase-advance  $\varphi$**  between sextupole and ES is important for separatrix orientation ( $\sim 225^\circ$ ).

Difficult to get correct with short drifts, and large rotation within the magnet cells.



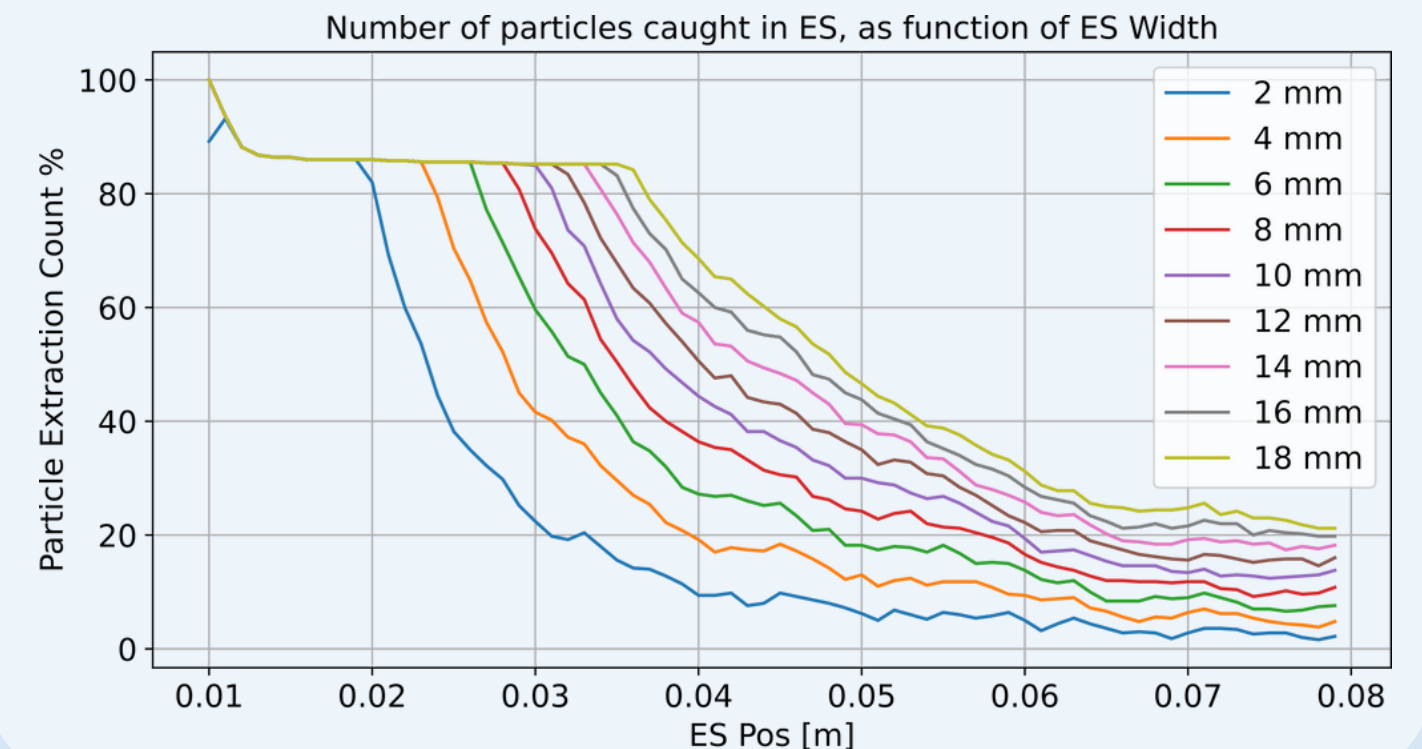
Treated the ES as a **point monitor**, due to limited spacing between FFA magnets. In reality, beam may rotate throughout ES length. Current solution has **ES within a magnet**.



Spiral step affects size of extracted beam and depends on sextupole strength  $S$ , septa offset  $X_{ES}$  and  $\varphi$ .

$$\Delta R = \frac{3}{4} \frac{S}{\cos \phi} X_{ES}^2$$

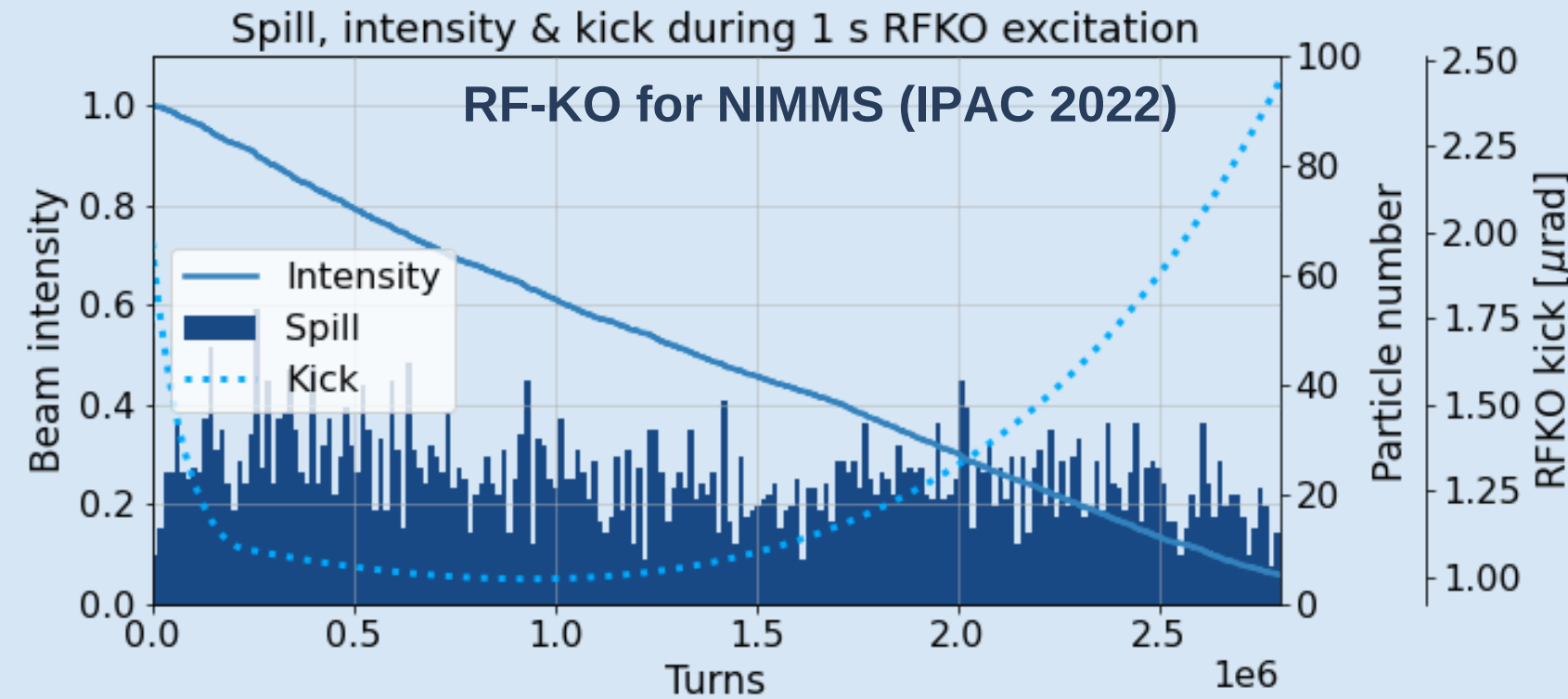
For a given sextupole strength, can observe ideal septum position and width for any desired beam size.



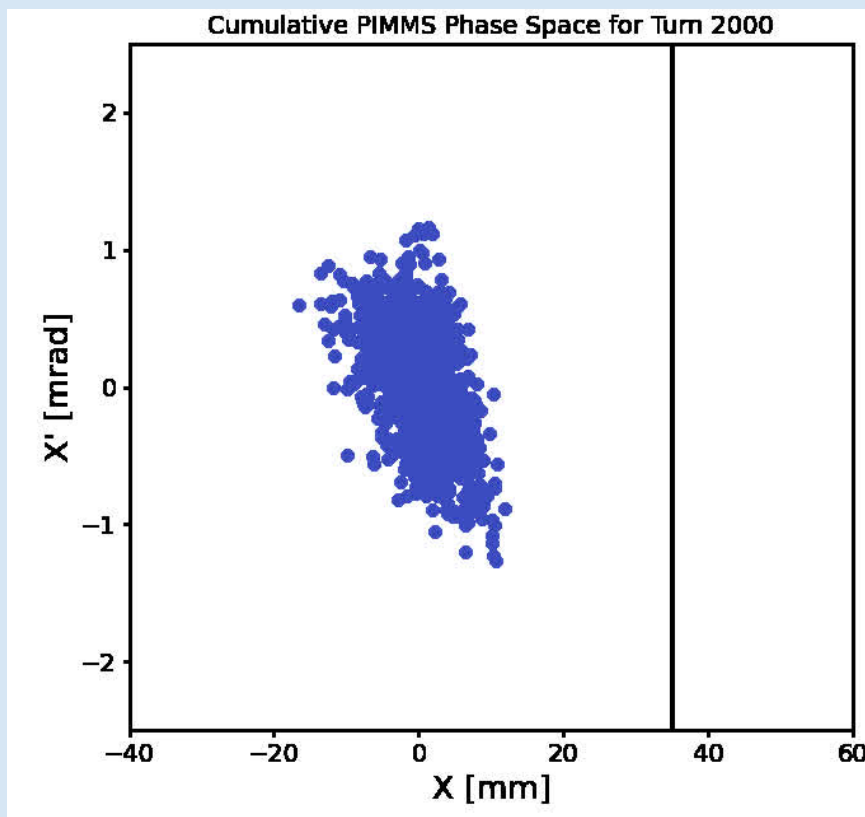


# Radiofrequency Knock-Out Extraction - NIMMS

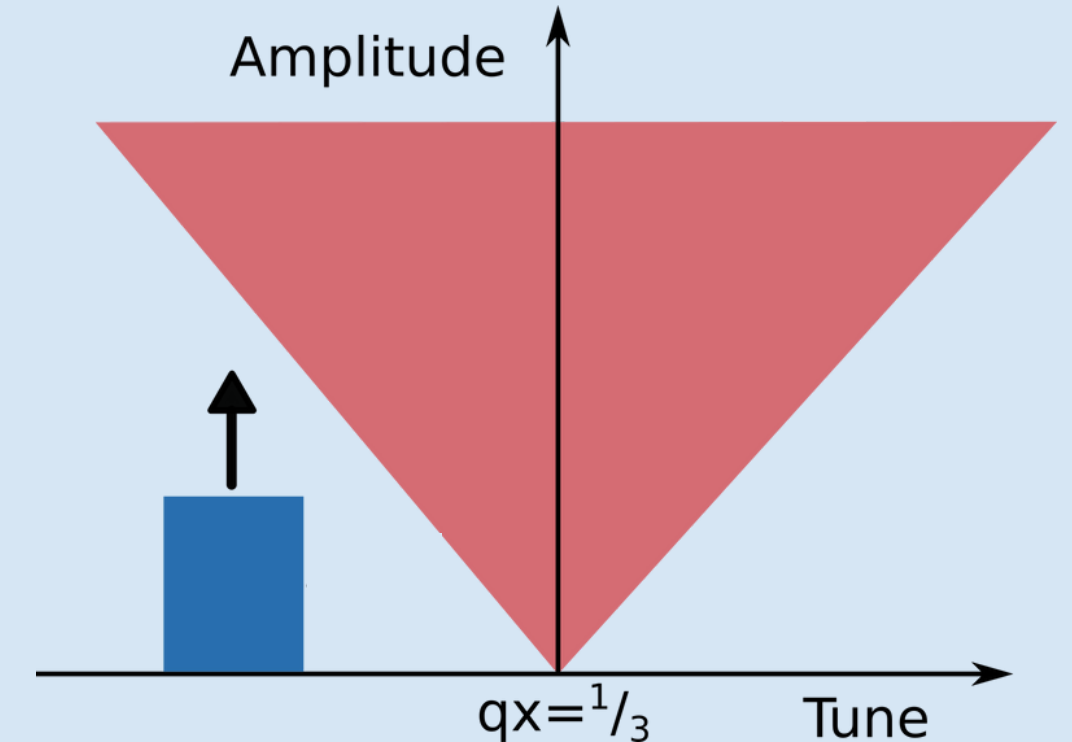
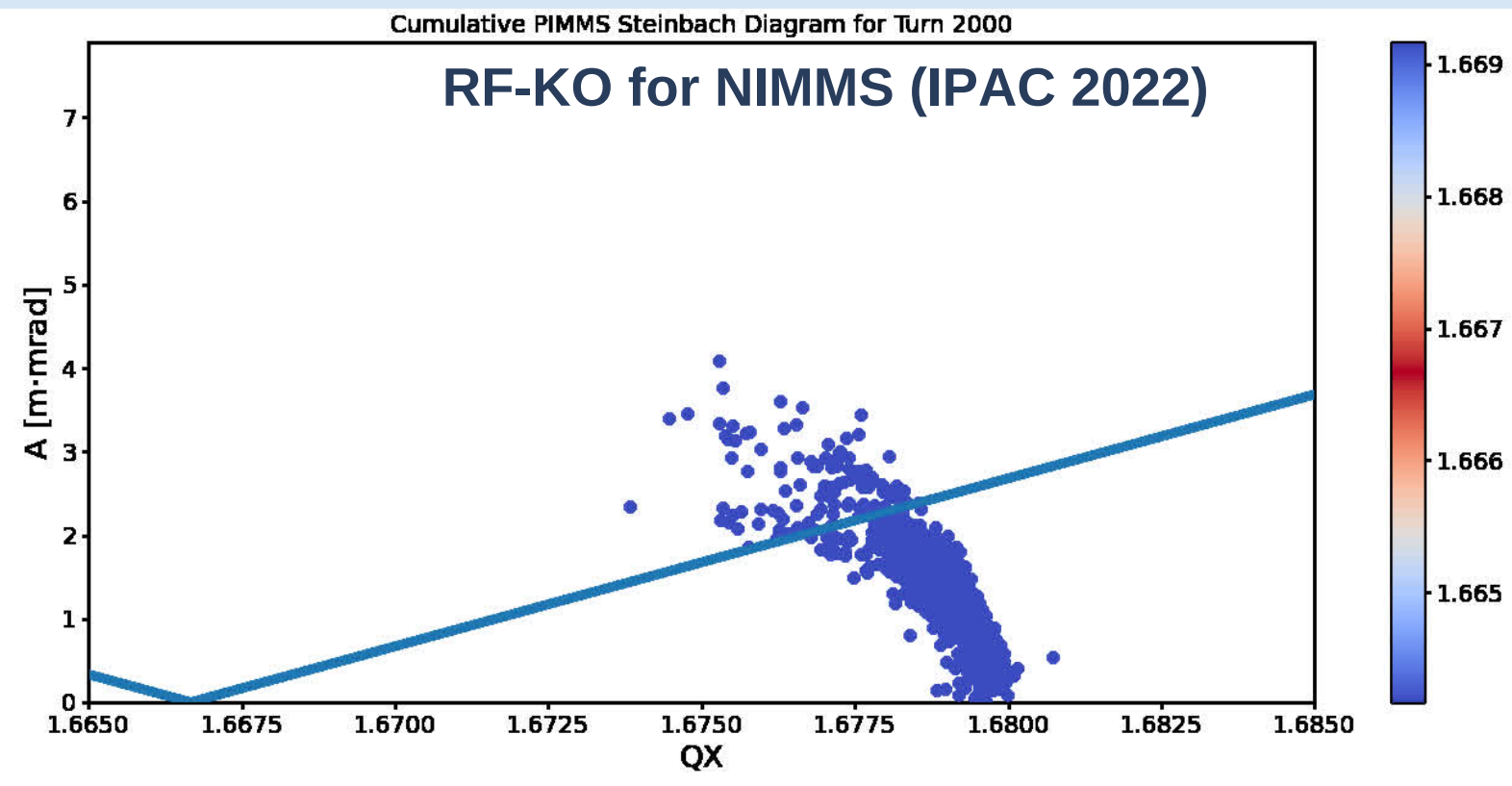
- Extraction method to **transversely excite** particle amplitude when close to the 3rd order resonance.
- Particle at stable tune near resonance.
  - Requires sinusoidal excitation at beam frequency.
- Excitation amplitude corresponding to tune density.



*Phase space*

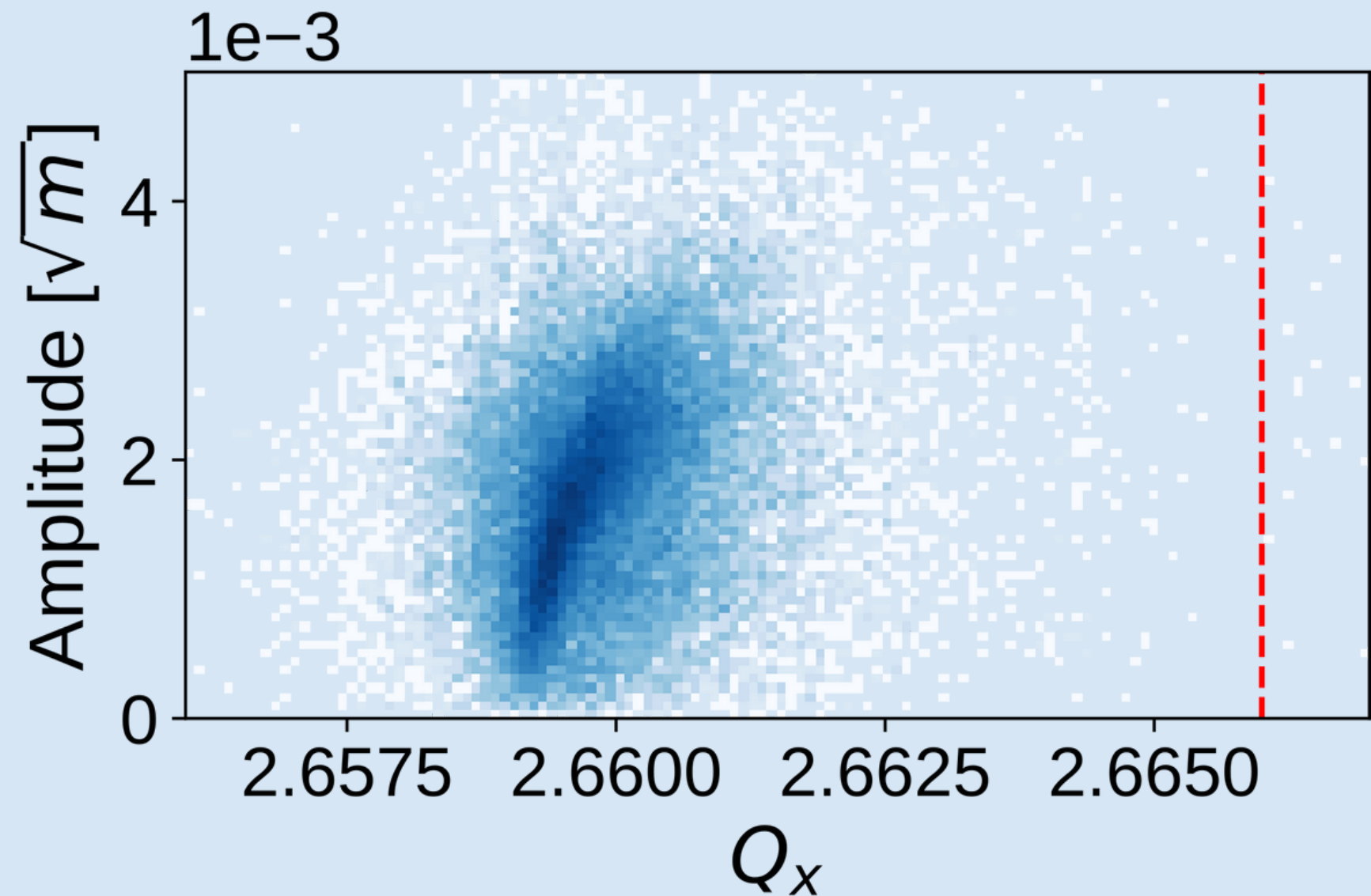
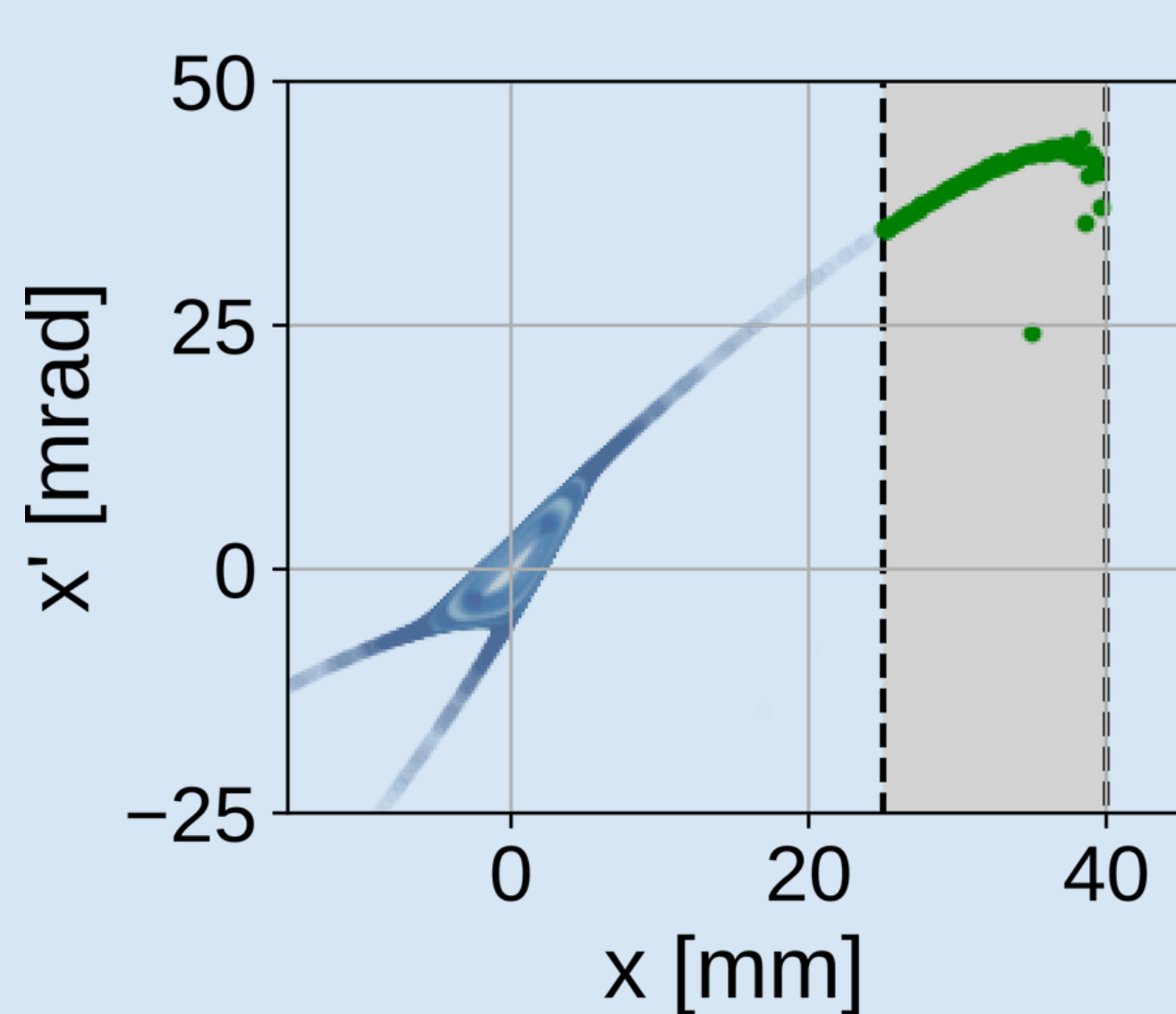


*Tune-Amplitude space*



# Radiofrequency Knock-Out Extraction - FFA

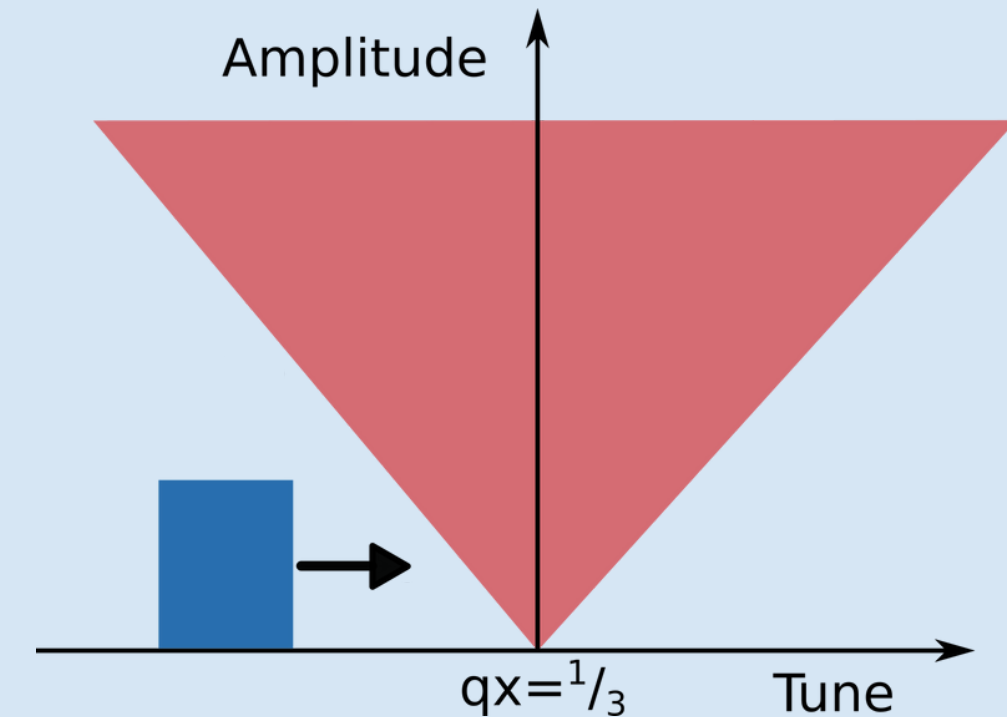
- Beam tune near resonance at  $Q_x = 2.666$
- ES centered at 32.5 mm, with 15 mm width
- Applied **sine excitation** of amplitude  $2.5E-5$  and frequency equal to beam tune, with small bandwidth.
- Observed **256 particles** over **30,000 turns**
- Plotted stable beam & separatrices (blue), extracted beam (green) and Steinbach diagram towards resonance (red).



# Alternative extractions: Tune Ramping

- Conventional quadrupole-driven extraction in synchrotrons uniformly ramp the beam into the resonance.

Can we do the same for k-index ramping or tune ramping in FFAs?



Do we need to?

- Is RF-KO stable enough near resonance that we can have constant FFA tune?
- Worth the more complicated magnet design?

Can k-index change **during** the cycle, or only in-between cycles?

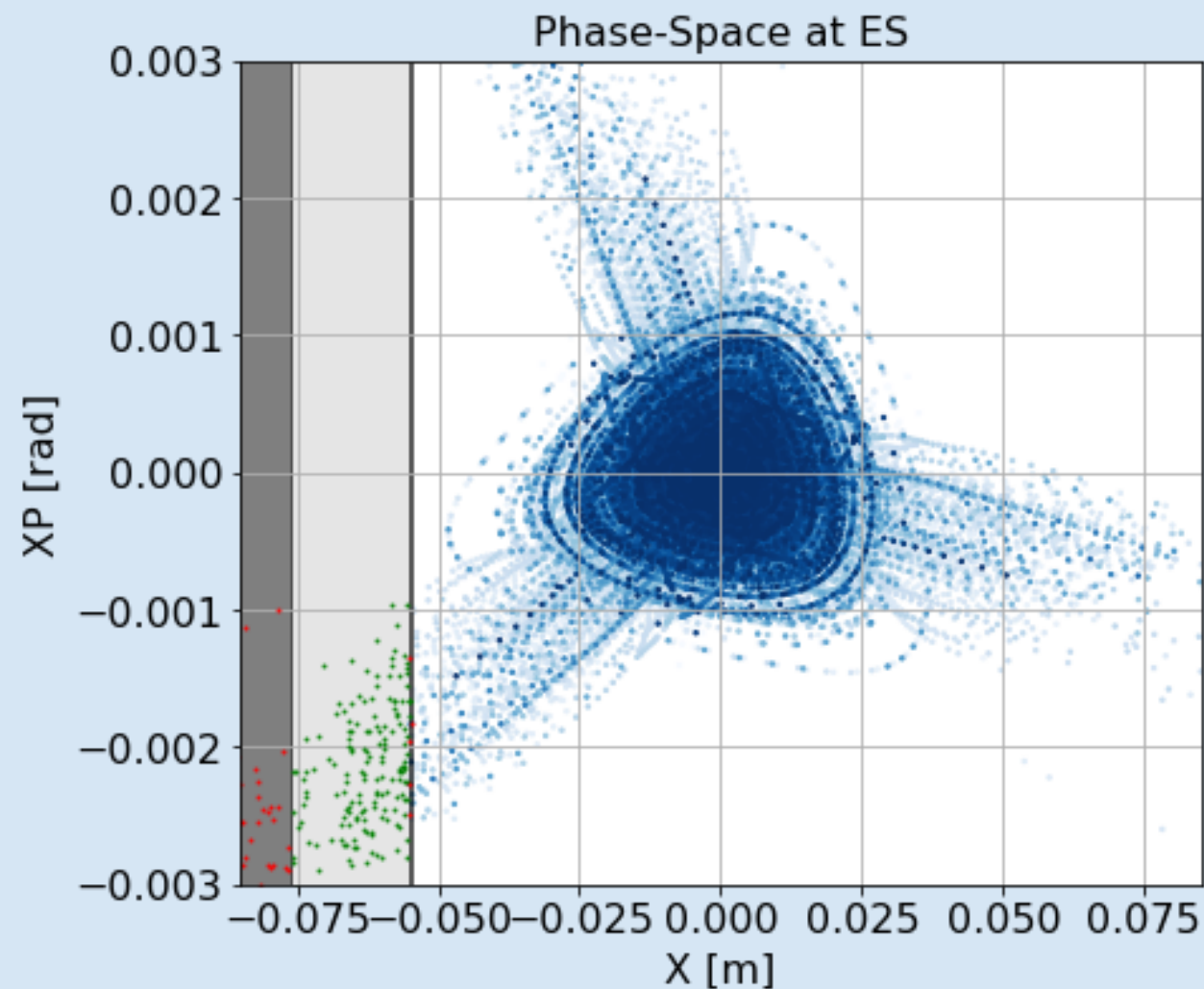
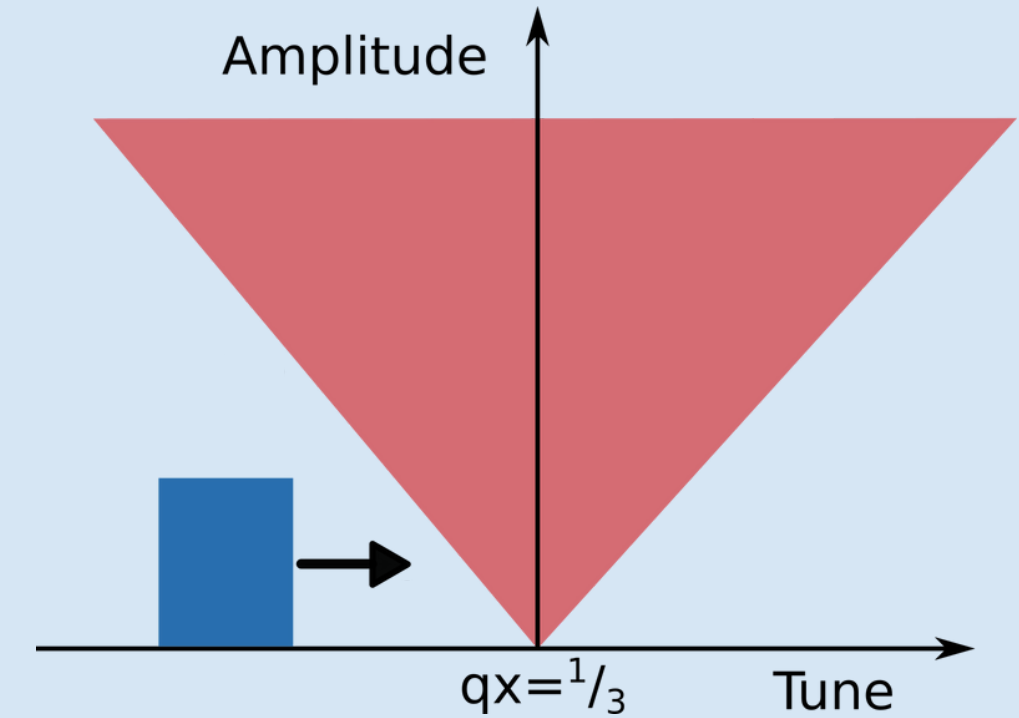
- Need to ensure **smooth uniform ramp**, with no large frequency ripples (good spill duty factor).
- Need constant optics for thin separatrices (especially with dispersion & large momentum spread)
- Change B-field same time to keep orbit constant?



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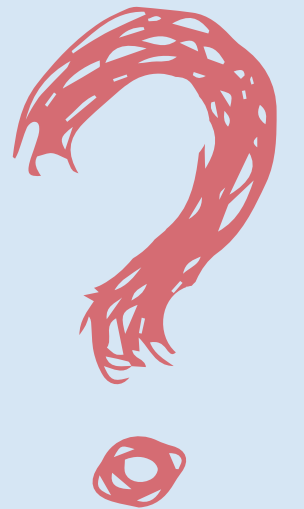


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# Alternative extractions: Getting Creative

- Attempting slow extraction from FFAs is forcing them to behave like synchrotrons:

**How can we benefit from the natural uniqueness of FFAs to get a continuous, uniform beam over seconds?**



E.g. Introduce chromaticity such that tune approaches resonance as beam approaches extraction orbit?

Consider septum-less extraction and some form of gradual orbit push. (May require interesting multipoles and amplitude detuning effects - think Multi-Turn Extraction)

# Raised Questions:

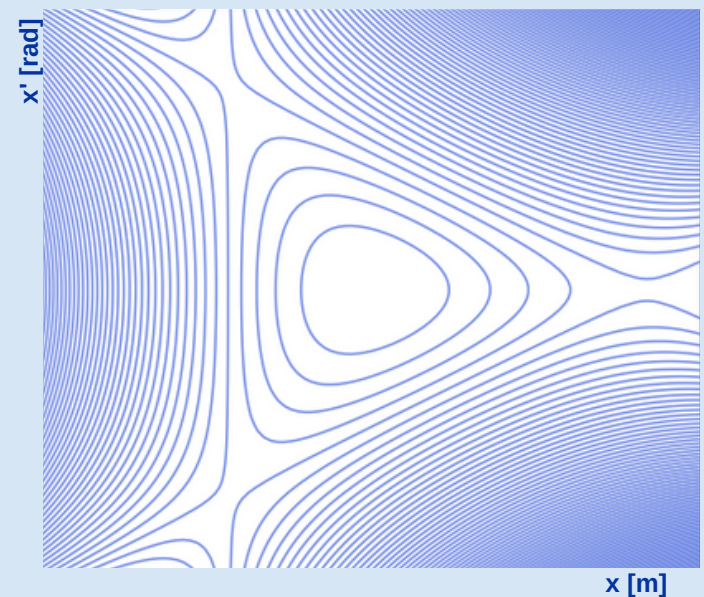
## Geometric Difficulties

- **Finite length of septa** may introduce spread in extracted beam due to large phase-advance.
- Particles from ES to MS will go through **distorting fringe fields** at large amplitudes.
- Additional magnets may need to curve with the magnet spiral, giving cylindrical/angled effects.
- May need to consider racetrack design to fit all extraction components in.
  - Racetrack design would break tune-energy relation.



## Separatrices & gradients

- May find asymmetric spiral step for large field gradients with high curvatures.
- Not observed for this machine, with low beam rigidity.
- **How does the Hamiltonian of the FFA compare to synchrotrons?**



# Conclusions

It is **feasible** to to perform slow extraction from **scaling** Fixed-Field Accelerators.

RF-KO extraction allows for control of extraction rate with **constant machine tune**. Could also consider ramping tune methods.

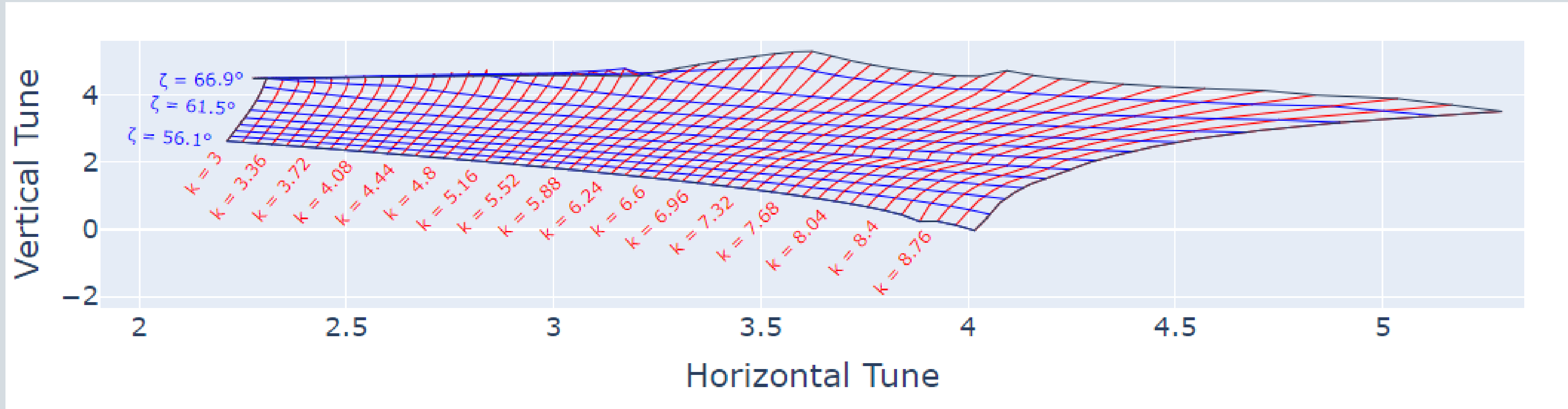
Compactness of LhARA FFA accelerator results in **geometric difficulties** to fit all components.

Difficulties of FFAs and slow extraction means it **must** be considered concurrently alongside the LhARA optical design.

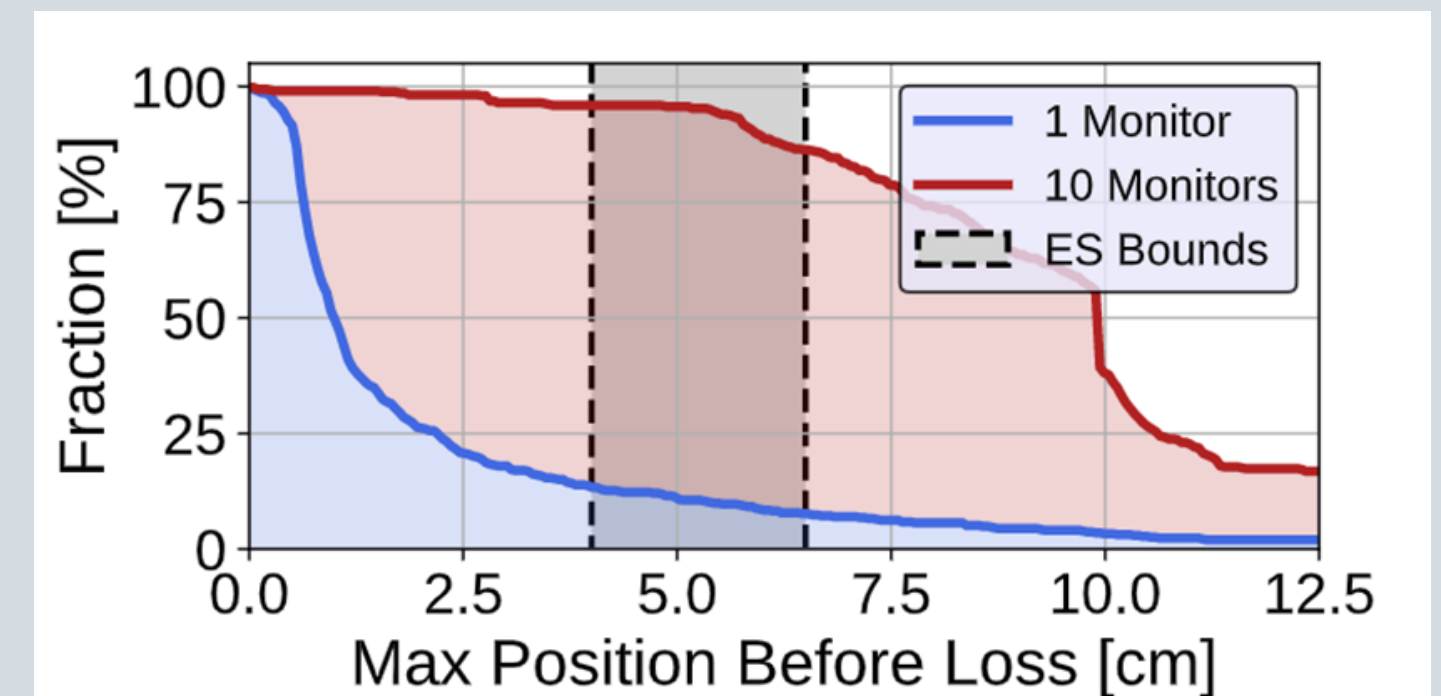
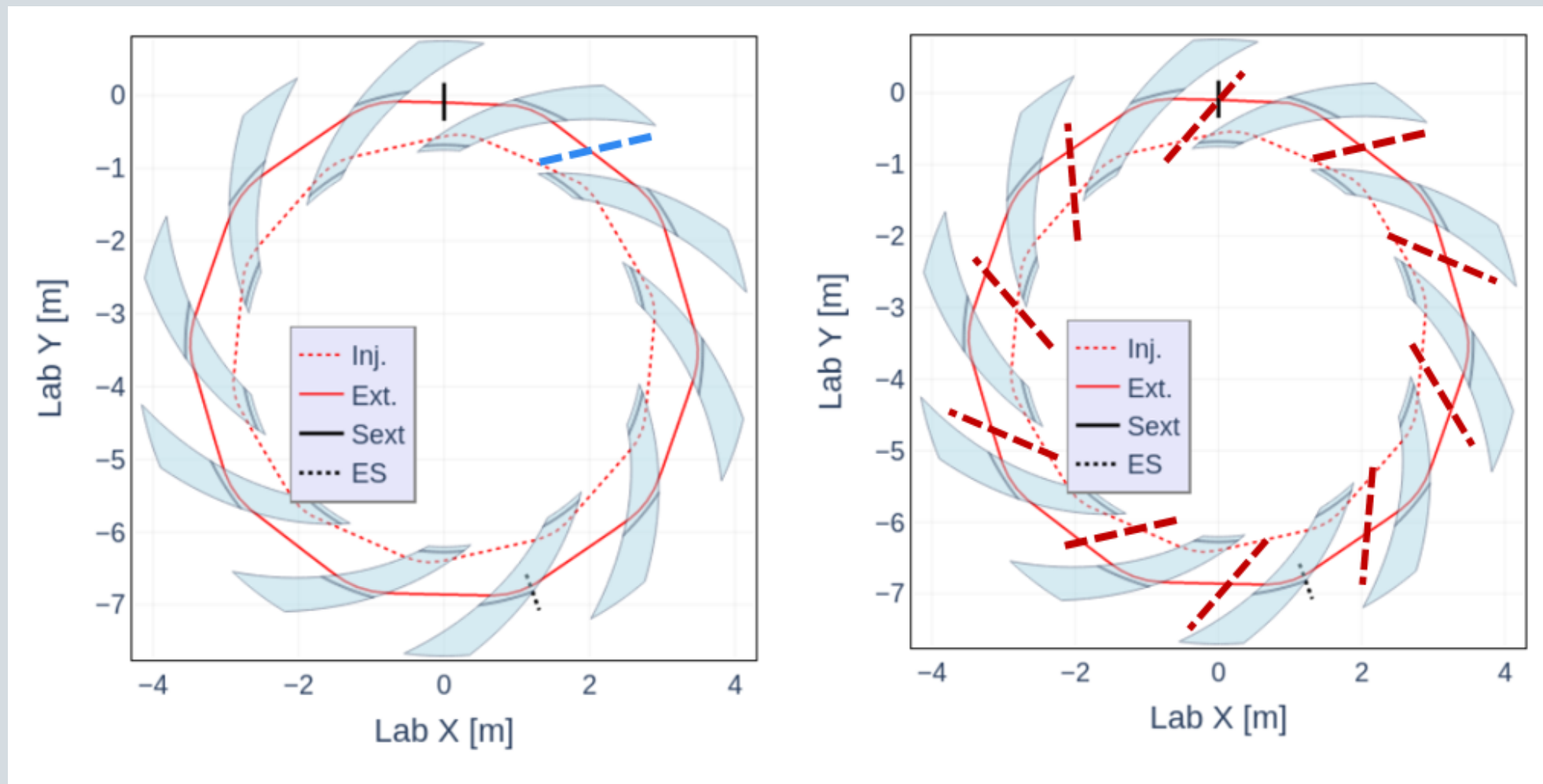
## Thank you, questions are welcome



# Extra Slides - Design/Tune Space



# One Monitor vs 10 Monitors: Visualisation



# ABP + LhARA for the FFA Extraction

- Compatibility with NIMMS: Compact ring accelerators, requirements of advanced slow extraction options.
- Requires design of four components:
  - Resonant sextupole (Difficult to extract with FFA k2 str.)
  - Electrostatic septum & magnetic septum
  - Transverse exciter for RF-KO extraction (constant tune)
- Restrictions on **phase-advance** between these components.
  - Optics gives **limited geometry**: had to treat components as thin lens, and in undesirable locations (i.e. within FFA magnet)
- Prelim study performed (IPAC23), but **optics update needed** and dedicated student to continue study.
- Question benefit of **spiral sector vs straight sector** w.r.t compactness and space between cells.
- Decision to be made on race-track design or not

