Collective effects in particle accelerators

Part 6
Luminosity & Beam-beam effects

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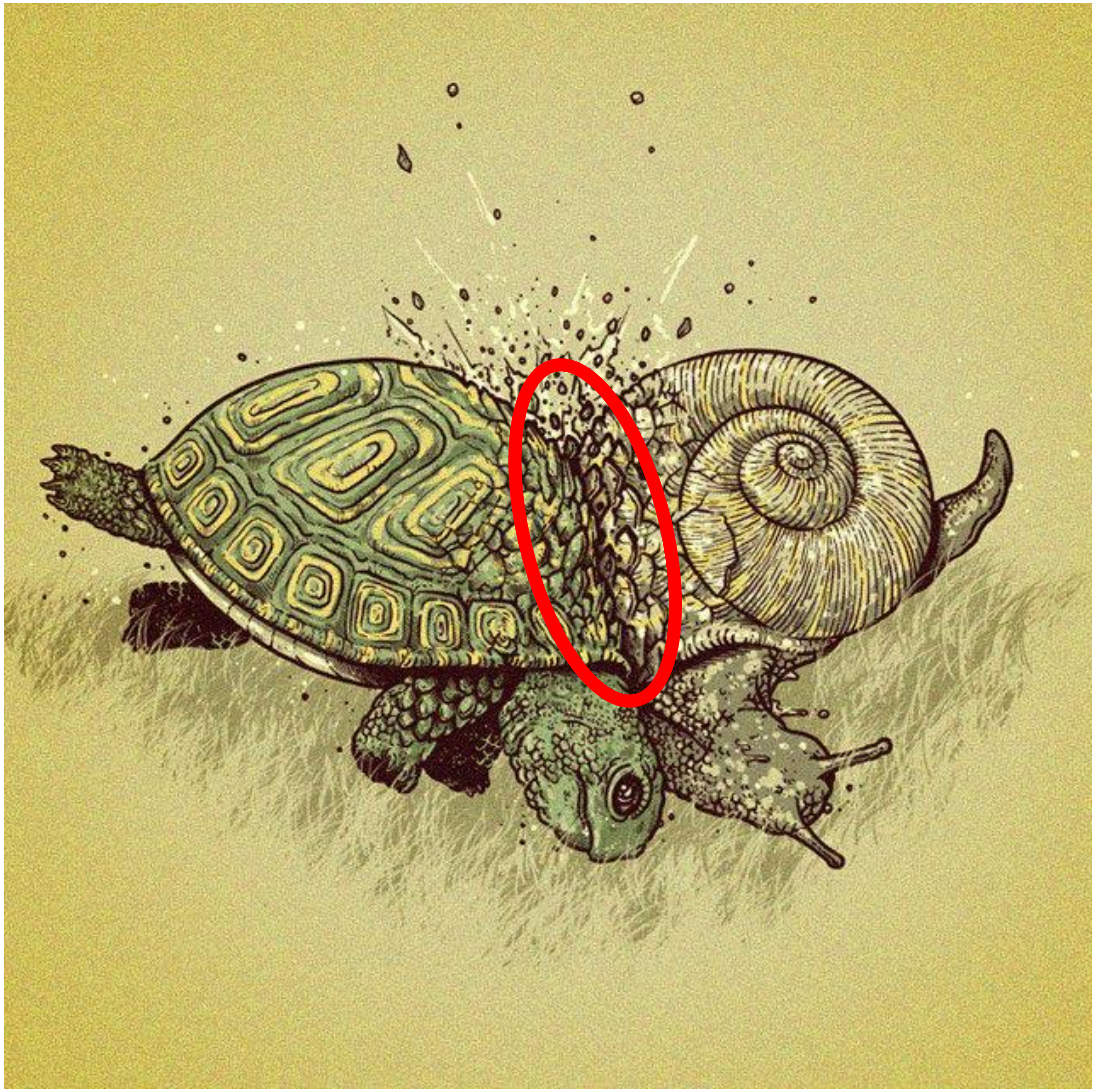
Collective effects in particle accelerators

- There are six lectures in this course on collective effects in accelerators:
  1. Space charge and scattering
  2. Wake fields and impedances
  3. Potential well distortion and the microwave instability
  4. Head-tail instability
  5. Coupled-bunch instabilities
  6. Luminosity and the beam-beam effect
- Literature: "Concept of luminosity", "Beam-beam", *CAS09*, Werner Herr, Bruno Muratori

Collisions & cross sections



- From the side & very slow …

Collisions

- From the back
- Quite fast ...
- Still not very efficient at all!

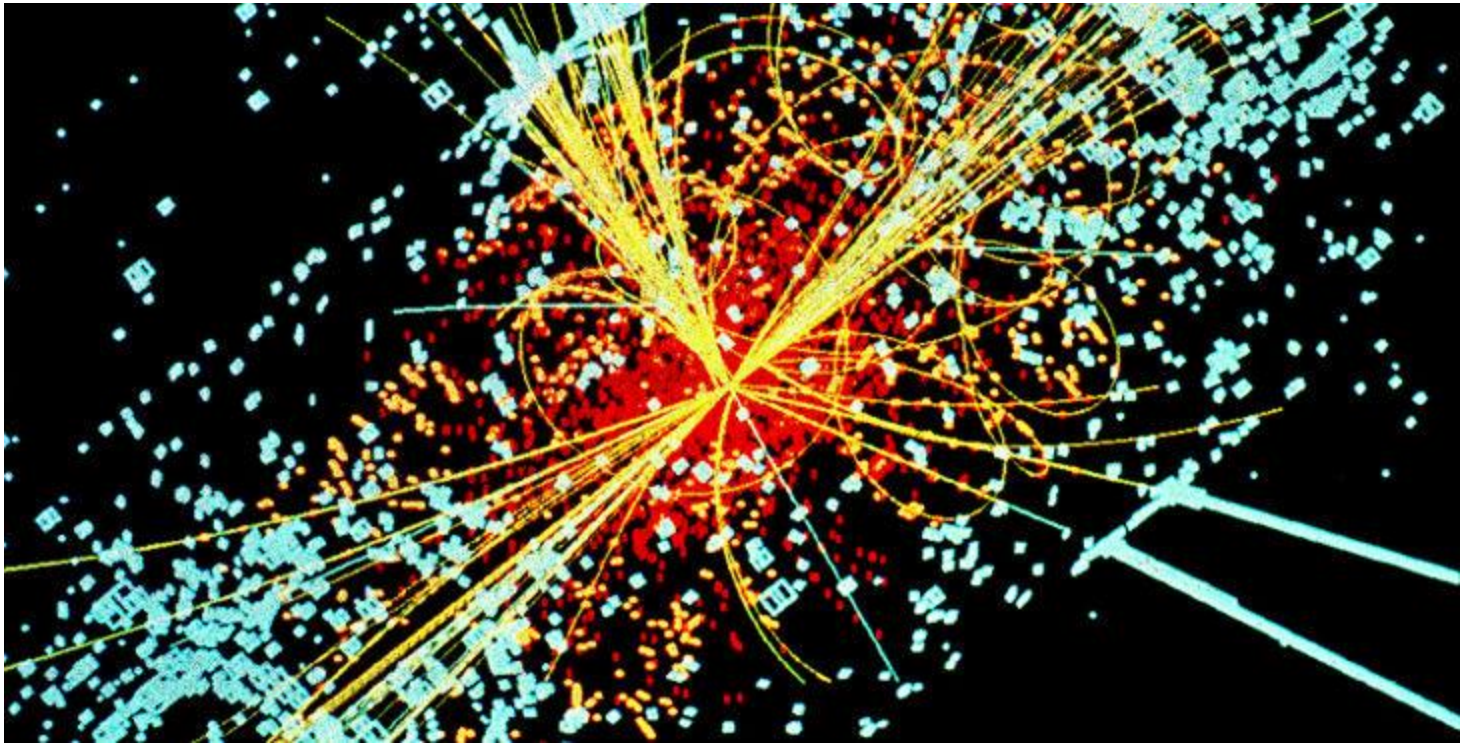
Collisions

- Head-on – most efficient

Collisions

- Fixed target ☺

Collisions



- What can we do to optimise the performance ?
- Want useful collisions (instead of any collisions)
- Avoid pile-up & background where possible
- What is best for the detectors ?

Performance Issues

- Available energy
- Useful collisions (as opposed to just collisions)
- Maximise total number of interactions
- At the same time, take into account:
  - Time spread of the interactions (when ?) or how often & how many simultaneously ?
  - Spatial spread of the interactions (where ?) or overall size of the interaction region
  - Quality of the interactions (how ?) or dead-time / pile-up / background
  - Pile-up for the LHC is around 20 & upgrade is ~40

Collective effects in particle accelerators

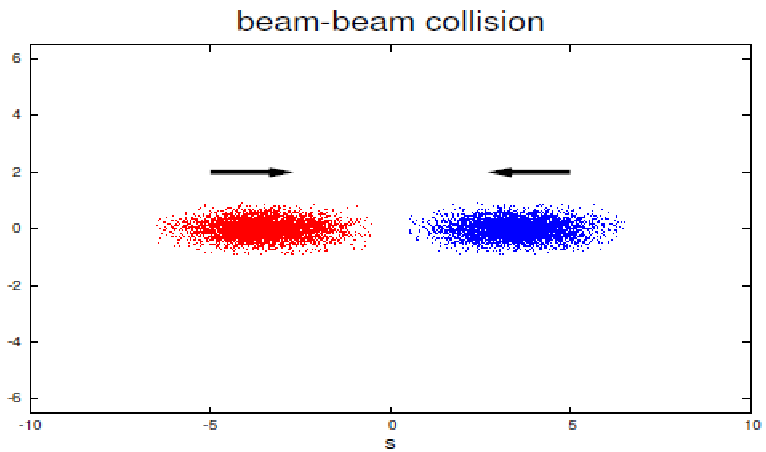
- In this lecture we shall discuss luminosity and the beam-beam instability. We will look at the implications and possible compensations for this instability
- By the end of this lecture you should know:
  - What luminosity is & how to increase it etc.
  - What the beam-beam instability is & how it works & what the beam-beam parameter is
  - How luminosity & beam-beam are related & some of the challenges they present when designing or upgrading a collider
  - Possible compensations for the beam-beam effect

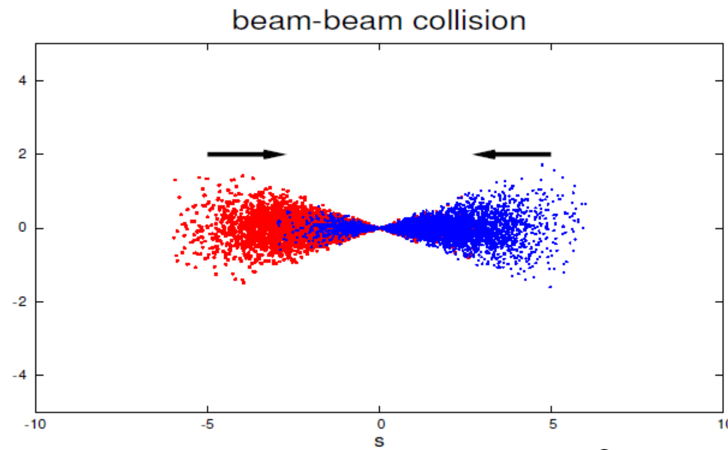
What is Pile Up?

- Unwanted collisions:
  - In time pile up: additional proton-proton collisions in the same bunch crossing
  - Out of time pile up: collisions taking place either before or after but affecting the detectors
  - Cavern background: gas of neutrons & photons inundating the cavern & causing random events
  - Beam halo: bunch scraping against upstream collimator
  - Beam gas: collisions between proton bunch & residual gas

What is beam-beam ?

- Occurs when two beam collide
- Two types of beam-beam effect:
  - High energy collision between particles (wanted)
  - Distortion of beams by electromagnetic forces (unwanted)
- Unfortunately both go together …





Courtesy of W. Herr

- Typically *0.001 %* of particles collide & rest is simply distorted … ☹

Beam-beam

- Strong-strong interaction (both beams strong)
  - Both beam affect the other in equal ways (both in simulation & reality)
  - Effects can be challenging & complicated to model
  - Examples: LEP, LHC, RHIC, …
- Weak-strong interaction (*1* beam much stronger)
  - Only the weak beam is affected by the beam-beam interaction (both in simulation & reality)
  - Examples: SPS (collider), Tevatron, …
- Weak-weak does not exist & would either be the same as strong-strong or nothing happens …

Beam-beam

- In circular colliders interactions happen at least once per turn & more for multiple IPs
- Treat beam as a collection of charges
  - Forces of beam on itself (space charge) & opposing beam (beam-beam effect)
  - This is the main limit in colliders (past, present, future)
  - Important for high density beams (high intensity / small beams or both)
- We need to introduce the concept of luminosity

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

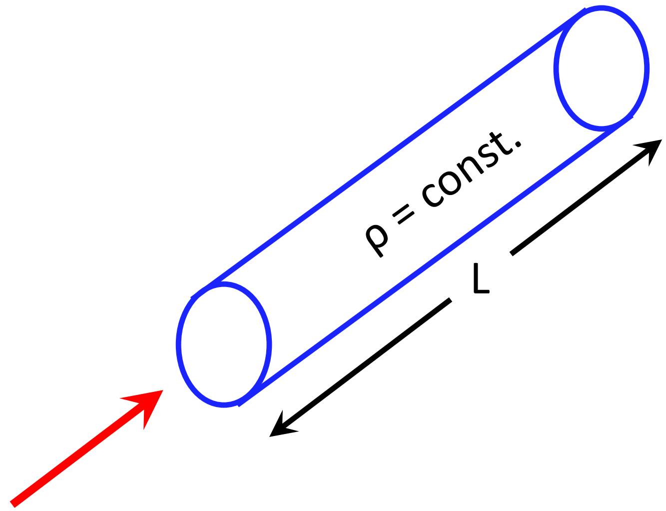
Luminosity

- Proportionality factor between the cross section σ_p at the IP and the no. of interactions / second

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$ units cm^{-2} s^{-1}

- For a fixed target:

$$\frac{dR}{dt} = \underbrace{\Phi \rho L}_{\mathcal{L}} \times \sigma_p$$

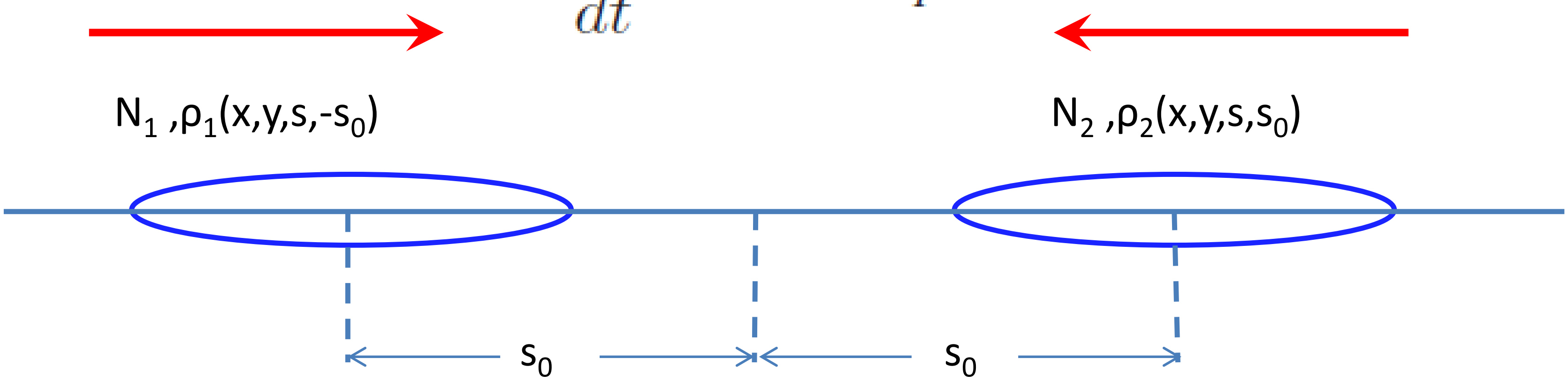


Flux $\Phi = N/s$

Luminosity

- For a collider:

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$



- N = particles / bunch, s_0 is time s_0 = ct
- ρ = density ≠ const.

$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)dxdydsds_0$$

- Kinematic factor: $K = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2/c^2}$

Luminosity

- Luminosity for Gaussian beams is:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

- N_1 & N_2 are the number of particles per bunch in beams *1* & *2* respectively
- N_b is the number of colliding bunches per beam
- σ_x & σ_y are the transverse beam dimensions
- f is the revolution frequency
- How is this derived ?

Luminosity

- Assume beams are Gaussian in all directions and independent of each other:

$$\rho^{(i)}(x,y,s,ct) = \rho_x^{(i)}(x)\rho_y^{(i)}(y)\rho_s^{(i)}(s \pm ct)$$

$$\rho_z^{(i)}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right),$$

$$\rho_s^{(i)}(s \pm ct) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s \pm ct)^2}{2\sigma_s^2}\right),$$

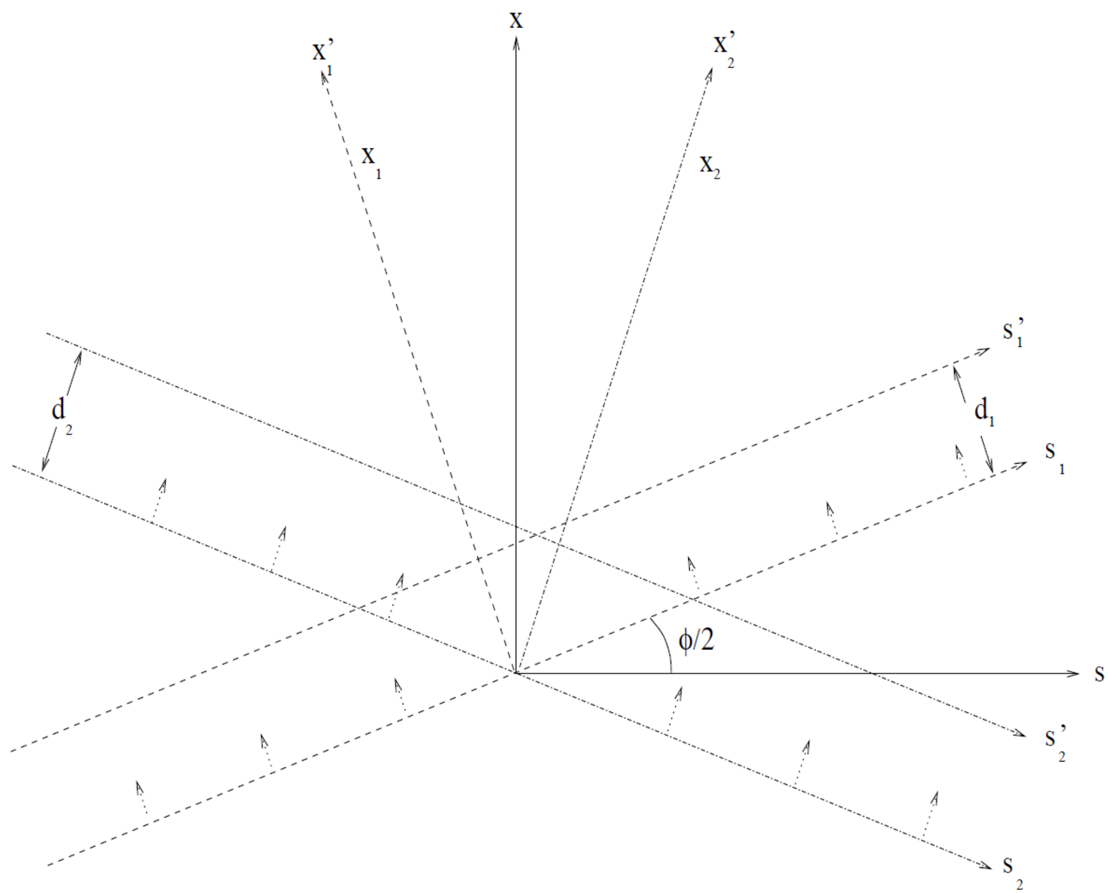
$$i = 1, 2,\ z = x, y,$$

- Introduce the most general crossing angle and offsets

Luminosity

- Introduce crossing angle and offsets

$$x_1 = d_1 + x\cos(\phi/2) - s\sin(\phi/2), \quad s_1 = s\cos(\phi/2) + x\sin(\phi/2),$$
$$x_2 = d_2 + x\cos(\phi/2) + s\sin(\phi/2), \quad s_2 = s\cos(\phi/2) - x\sin(\phi/2)$$



Luminosity

- Beam size is much smaller than the bunch length and the crossing angle ϕ is small (~ *300 μrad*) so

$$s_1 = s_2 = s\cos(\phi/2) \qquad (\sigma_z << \sigma_s)$$

- Calculating all the overlap integrals to get the luminosity:

$$\mathcal{L} = 2cN_1N_2fN_b\cos^2\frac{\phi}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho_x^{(1)}(x)\rho_y^{(1)}(y)\rho_s^{(1)}(s-ct)$$

$$\times\rho_x^{(2)}(x)\rho_y^{(2)}(y)\rho_s^{(2)}(s+ct)dxdydsdt$$

- With repeated applications of:

$$\int e^{-(ax^2+2bx)}dx = e^{b^2/a}\frac{1}{2}\sqrt{\frac{\pi}{a}}\mathrm{erf}\left[\frac{b+ax}{\sqrt{a}}\right] + \mathrm{const.}$$

Luminosity

- Noting: *erf*(-*x*) = - *erf*(*x*), *erf*(*0*) = *0*, *erf*(∞) = *1*
- We obtain:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi^{\frac{3}{2}}\sigma_s} \cos\frac{\phi}{2} \int_{-\infty}^{+\infty} W \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds.$$

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}, \quad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2},$$

$$W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}.$$

- *W*, σ_x, σ_y are still inside the integral as they may still depend on "*s*", otherwise we would have:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} W e^{\frac{B^2}{A}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}.$$

Luminosity

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} W e^{\frac{B^2}{A}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}.$$

- This shows luminosity is independent of offsets provided $d_1 = d_2$, which makes sense from the crossing angle, however, the interaction could now lie *outside* the detector …
- Also written as: $\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} W e^{\frac{B^2}{A}} S,$
- *S* is the luminosity reduction factor $S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}},$
- Where we assumed: $\tan(\phi/2) \approx \phi/2$ valid for a small crossing angle
- *W* is due to the offset & the rest involves both

Luminosity

- Early LHC parameters were as follows: $N_1 = N_2 = 1.1 \times 10^{11}$, with *2808* bunches per beam & $f = 11.2455$ *kHz*, $\gamma = 7461$, $\phi = 300\ \mu rad$, $\beta^* = 0.5\ m$, $\sigma_s = 7.7\ cm$ and $\varepsilon_n = 3.75\ \mu m$, therefore, the luminosity can be calculated as (exercise):

$$\mathcal{L} = 1.21 \times 10^{34} \times 0.809\ \mathrm{cm}^{-2}\mathrm{s}^{-1} = 9.79 \times 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- First number = nominal luminosity & second = S
- For illustration, if we have offsets $d_1 = 10\ \mu m$, $d_2 = 0$, then (exercise):

$$W = 0.906, \quad e^{\frac{B^2}{A}} = 1.035, \quad S = 0.809$$

$$\mathcal{L} = 1.21 \times 10^{34} \times 0.758\ \mathrm{cm}^{-2}\mathrm{s}^{-1} = 9.17 \times 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Luminosity

- How does this compare to other colliders ?

	Energy (GeV)	$\mathcal{L}_{max}$ $cm^{-2}s^{-1}$	rate s^{-1}	σ_x/σ_y $\mu m/\mu m$	Particles per bunch
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$	$4\ 10^{5}$	60/30	$\approx 10\ 10^{10}$
Tevatron ($p\bar{p}$)	1000x1000	$100\ 10^{30}$	$7\ 10^{6}$	30/30	$\approx 30/8\ 10^{10}$
HERA (e^+p)	30x920	$40\ 10^{30}$	40	250/50	$\approx 3/7\ 10^{10}$
LHC (pp)	7000x7000	$10000\ 10^{30}$	10^{9}	17/17	$\approx 11\ 10^{10}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$	≤ 1	200/2	$\approx 50\ 10^{10}$
PEP (e^+e^-)	9x3	$8000\ 10^{30}$	NA	150/5	$\approx 2/6\ 10^{10}$

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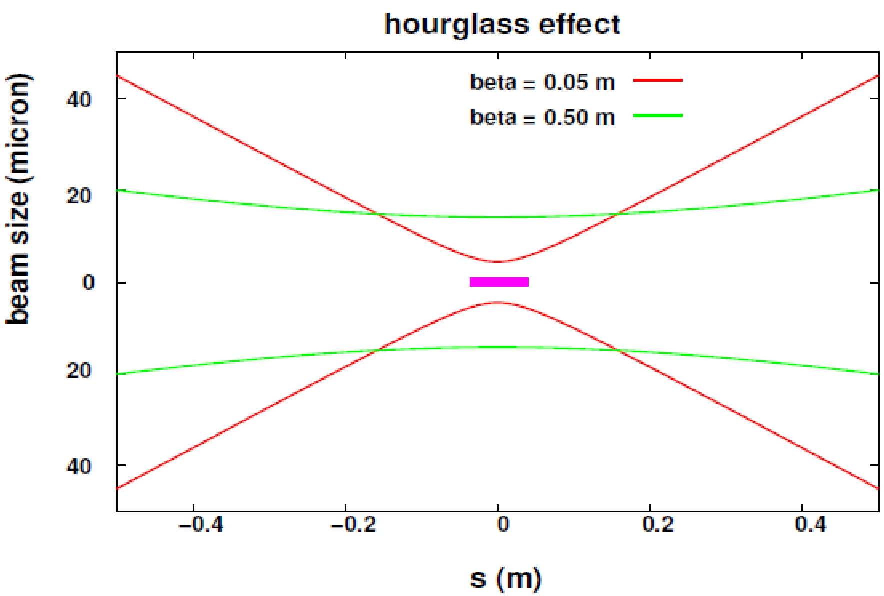
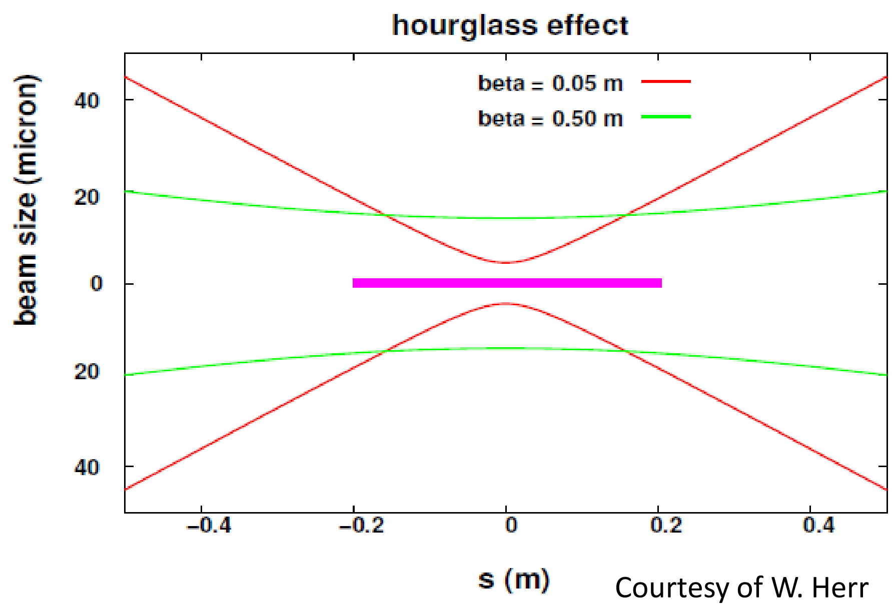
Luminosity (Hourglass effect)

Luminosity

- What if the beam is squeezed at the IP ?





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- Hourglass effect leads to a further reduction factor if the bunch length is long enough
- β function either side of the IP behaves as:

$$\beta(s) \approx \beta^* (1 + \left(\frac{s}{\beta^*}\right)^2)$$

Luminosity

- So the beam size either side of the IP behaves as:

$$\sigma_z = \sigma_z^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2},$$

- For the parameters we had earlier this means:

$$\mathcal{L}_{HG} = \left(\frac{N_1 N_2 f N_b}{4\pi\sigma_x^*\sigma_y^*}\right) \frac{\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} W \frac{e^{-(As^2+2Bs)}}{1+(\frac{s}{\beta^*})^2} ds,$$

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} = \frac{\sigma_s^2 \sin^2\frac{\phi}{2} + (\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\cos^2\frac{\phi}{2}}{(\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\sigma_s^2}.$$

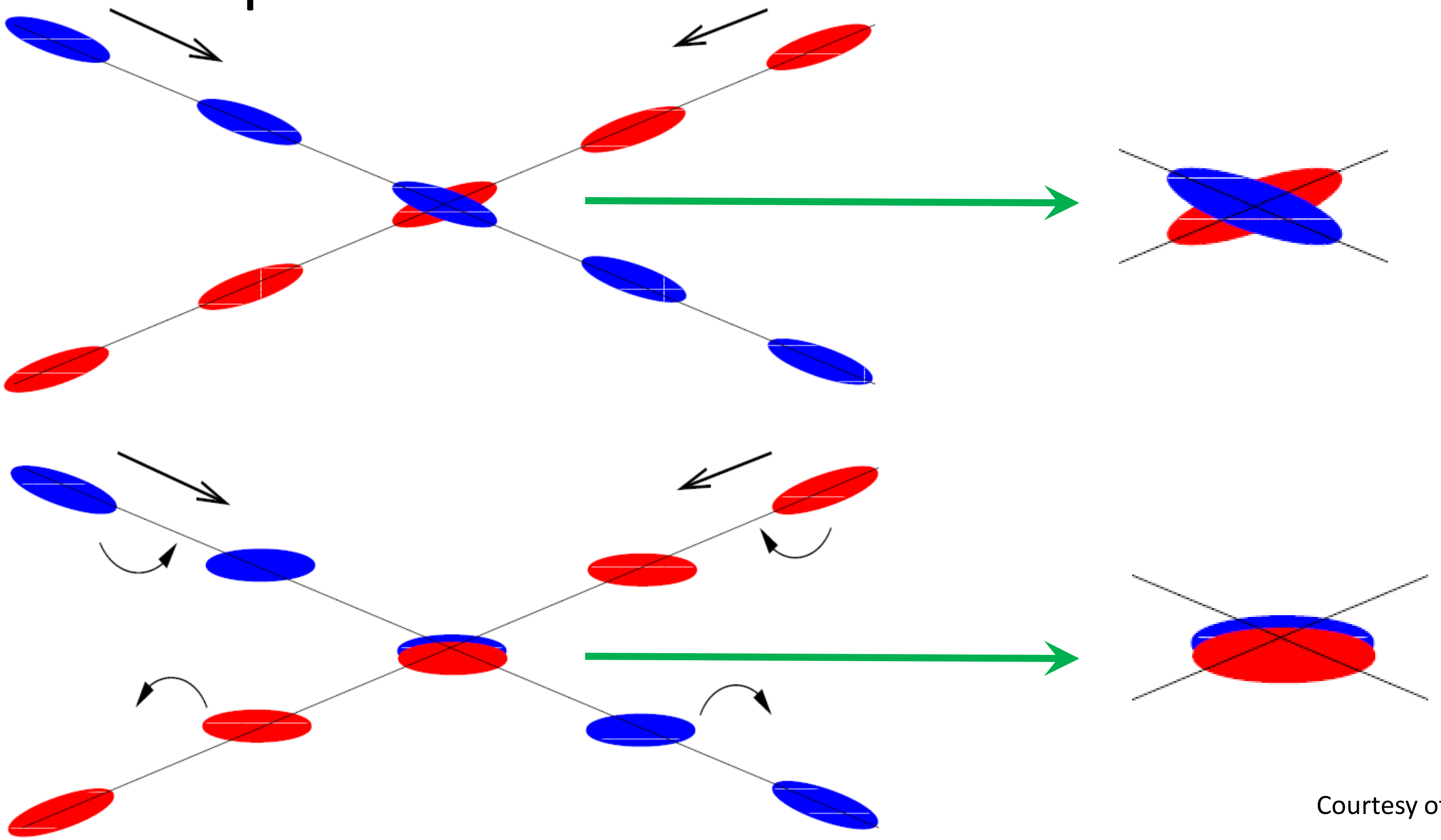
- So, evaluating the integral above numerically:

$$\mathcal{L}_{HG} = 1.21 \times 10^{34} \times 0.755 \text{ cm}^{-2}\text{s}^{-1} = 9.14 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$$

Luminosity (Crab crossing)

Luminosity

- Crab crossing done with crab cavities to give a twist to the colliding bunches to ensure a total overlap at the IP



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Integrated luminosity

- This can be defined straightforwardly, together with the average luminosity as:

$$\mathcal{L}_{int} = \int_0^T \mathcal{L}(t)dt \qquad <\mathcal{L}> = \frac{\int_0^{t_r} \mathcal{L}(t)dt}{t_r + t_p} = \mathcal{L}_0 \times \tau \times \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$$

- Figure of merit: $\mathcal{L}_{int} \times \sigma_p$ = number of events
- Luminosity decays due to decays in intensity and emittance through collisions or other
- Exponential decay is assumed which is realistic:
- E.g.

$$\mathcal{L}(t) \rightarrow \mathcal{L}_0 \exp\left(\frac{t}{\tau}\right)$$

Integrated luminosity

- If we know how much preparation time is required then we can optimise $\mathcal{L}_{int}$ easily:

Integrated luminosity

- Typical run times for LEP:
- $t_r \approx 8 - 10$ hours
- For the LHC a long preparation time t_p is usual
- Therefore it is possible to optimise t_r & t_p so as to have the maximum integrated luminosity
- t_r can usually be treated as a free parameter which can be chosen in this optimisation & so we can find a theoretical maximum for t_r:

$$t_r \approx \tau \times \ln\left(1 + \sqrt{2t_p/\tau} + t_p/\tau\right)$$

- For the LHC: $t_p \approx 10$ hr, $\tau \approx 15$ hr, → $t_r \approx 15$ hr

Luminosity

- How can the best luminosity be achieved ?
- Increase the intensity
- Decrease the beam sizes (small ε_n & β^*)
- Get as many bunches as possible
- Have as small a crossing angle as possible or compensate for it by having crab cavities
- Try to achieve as exact head-on collisions as possible, minimising separation etc.
- Get bunches to be as short as possible
- At the same time – try to minimise beam-beam !

Beam-beam

- Recall the maximum luminosity is defined as:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

- To find the fields, we transform to the rest frame where we only have $\vec{E}$ and $\vec{B} = 0$ & the densities:

$$\rho_u(u) = \frac{1}{\sigma_u\sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \text{ where } u = x, y$$

- We can write the potential $U(x, y, \sigma_x, \sigma_y)$ so:

$$U(x, y, \sigma_x, \sigma_y) = \frac{ne}{4\pi\epsilon_0} \int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2 + q)(2\sigma_y^2 + q)}} dq$$

Beam-beam

- The potential:

$$U(x,y,\sigma_x,\sigma_y) = \frac{ne}{4\pi\epsilon_0}\int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}}dq$$

- Satisfies: $\vec{E} = -\nabla U(x,y,\sigma_x,\sigma_y)$

- For elliptical beams with $\sigma_x > \sigma_y$ we can write:

$$E_x = \frac{ne}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\mathrm{Im}\left[\mathrm{erf}\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - e^{\left(-\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)}\mathrm{erf}\left(\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right]$$

$$E_y = \frac{ne}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\mathrm{Re}\left[\mathrm{erf}\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - e^{\left(-\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)}\mathrm{erf}\left(\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right]$$

$$\mathrm{erf(t)} = e^{-t^2}\left[1+\frac{2i}{\sqrt{\pi}}\int_0^t e^{z^2}\,dz\right], B_y = -\beta_r E_x/c, B_x = \beta_r E_y/c$$

Beam-beam

- So the potential

$$U(x,y,\sigma_x,\sigma_y) = \frac{ne}{4\pi\epsilon_0}\int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}}dq$$

- Can be used to calculate the beam-beam force in conjunction with the Lorentz force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Similarly, for round beams $\vec{F} = q(E_r + \beta c B_\Phi) \times \vec{r}$

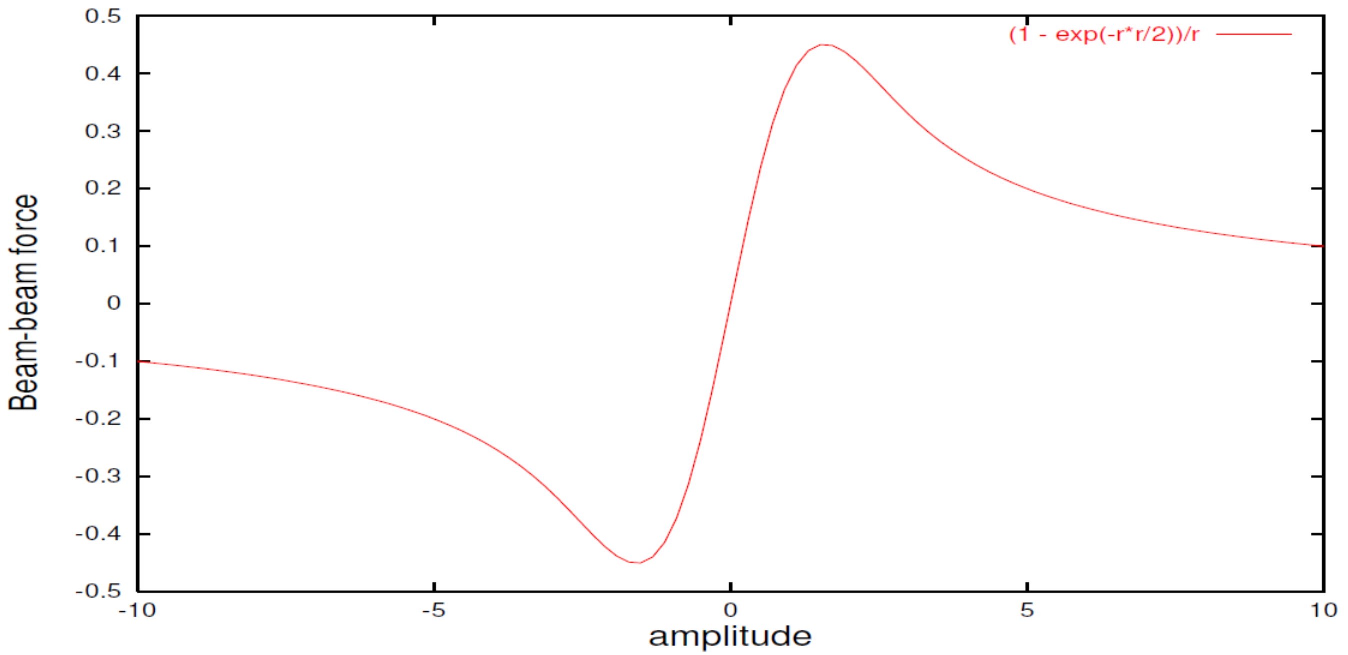
$$E_r = -\frac{ne}{4\pi\epsilon_0} \cdot \frac{\delta}{\delta r}\int_0^\infty \frac{\exp(-\frac{r^2}{(2\sigma^2+q)})}{(2\sigma^2+q)}dq$$

$$r^2 = x^2 + y^2 \qquad B_\Phi = -\frac{ne\beta c\mu_0}{4\pi} \cdot \frac{\delta}{\delta r}\int_0^\infty \frac{\exp(-\frac{r^2}{(2\sigma^2+q)})}{(2\sigma^2+q)}dq$$

Beam-beam

- So the radial force can be expressed as:

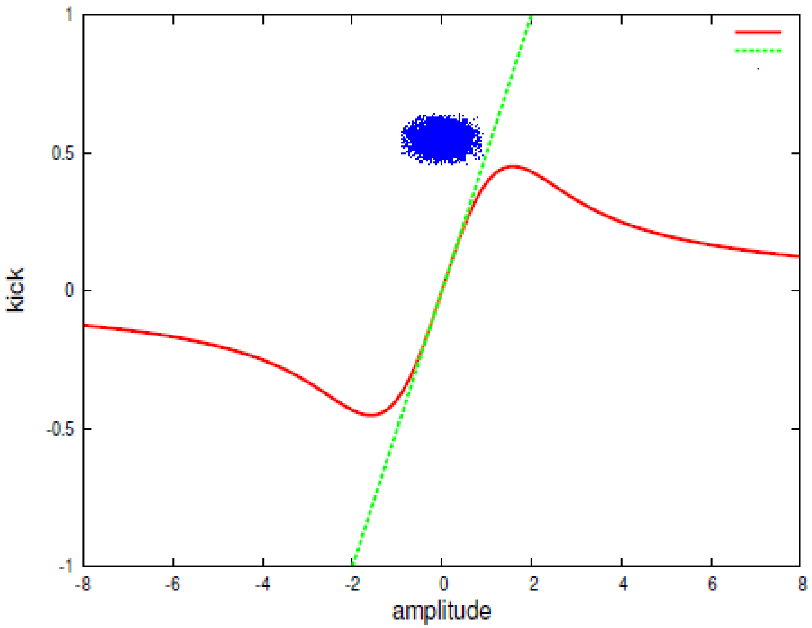
$$F_r(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0}\cdot\frac{1}{r}\cdot\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]$$

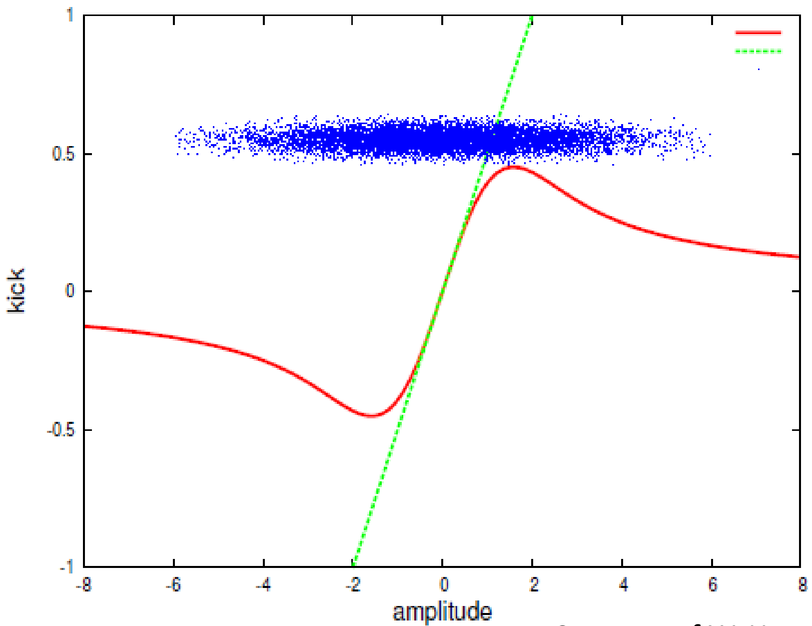


- This is extremely nonlinear and has potentially very negative effects on the colliding beams

Beam-beam

- What does the beam-beam force do ?
- For small amplitudes, beam-beam kick ≡ quadrupole → simple tune shift
- For large amplitudes → amplitude dependent tune shift





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Beam-beam

- Start with a 2 dimensional force & assume it is spread over a longitudinal distribution which depends on both *s*, *t* & has a Gaussian shape σ_s:

$$F_r(r,s,t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0\sigma_s}\cdot\frac{1}{r}\cdot\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]\cdot\left[\exp(-\frac{(s+vt)^2}{2\sigma_s^2})\right]$$

- We can use Newton's law & integrate to get the total deflection (*N* = total number of particles)

$$\Delta r' = \frac{1}{mc\beta\gamma}\int_{-\infty}^{\infty} F_r(r,s,t)dt$$

$$\Delta r' \;=\; -\frac{2Nr_0}{\gamma}\cdot\frac{1}{r}\cdot\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

Beam-beam

- So the beam-beam kick is:

$$\Delta r' \;=\; -\frac{2Nr_0}{\gamma}\cdot\frac{1}{r}\cdot\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

- *N* is the total number of particles
- In the two transverse planes we have:

$$\Delta x' = -\frac{2Nr_0}{\gamma}\cdot\frac{x}{r^2}\cdot\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta y' = -\frac{2Nr_0}{\gamma}\cdot\frac{y}{r^2}\cdot\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]$$

- r_0 is the classical particle radius: $r_0 = e^2/4\pi\epsilon_0 mc^2$

Beam-beam

- We can take the limit of the beam-beam kick for small *r*:

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

- To obtain:

$$\Delta r'|_{r \to 0} = -\frac{Nr_0 r}{\gamma\sigma^2}$$

- For small amplitudes the linear beam-beam force is like a quadrupole with focal length *f*:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{Nr_0}{\gamma\sigma^2} = \left[\frac{\xi \cdot 4\pi}{\beta^*}\right]$$

Beam-beam

- Small amplitude beam-beam ≡ quadrupole with focal length *f*:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \left[\frac{\xi \cdot 4\pi}{\beta^*}\right]$$

- With ξ the linear beam-beam parameter defined as:

$$\xi = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

- For non-round beams this becomes:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Beam-beam

- Examples of beam-beam parameters:

	LEP (e^+e^-)	LHC (pp)
Beam sizes	160 - 200μm · 2 - 4μm	16.6μm · 16.6μm
Intensity N	4.0 · 10^{11}/bunch	1.15 · 10^{11}/bunch
Energy	100 GeV	7000 GeV
$\epsilon_x \cdot \epsilon_y$	(≈) 20 nm · 0.2 nm	0.5 nm · 0.5 nm
$\beta_x^* \cdot \beta_y^*$	(≈) 1.25 m · 0.05 m	0.55 m · 0.55 m
Crossing angle	0.0	285 μrad
Beam-beam parameter(ξ)	0.0700	0.0037

Beam-beam

- The Beam-beam parameter is often used to quantify the strength of the beam-beam interaction but it only takes the linear part of the force into account
- Compare the beam-beam parameter to the nominal luminosity:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \qquad \mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

- We find them almost directly proportional so higher luminosity → higher beam-beam …

Beam-beam

- What is the linear tune shift resulting ?

$$\begin{pmatrix} \cos(2\pi(Q+\Delta Q)) & \beta^* \sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*}\sin(2\pi(Q+\Delta Q)) & \cos(2\pi(Q+\Delta Q)) \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \begin{pmatrix} \cos(2\pi Q) & \beta_0^* \sin(2\pi Q) \\ -\frac{1}{\beta_0^*}\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

- Which can be easily solved (exercise) to give:

$$cos(2\pi(Q+\Delta Q)) = cos(2\pi Q) - \frac{\beta_0^*}{2f} sin(2\pi Q)$$

$$\frac{\beta^*}{\beta_0^*} = sin(2\pi Q)/sin(2\pi(Q+\Delta Q))$$

- So tune is Q changed by ΔQ and β is changed as well (β - beating)

Beam-beam

- Beam-beam tune shift is given by:

$$\Delta Q \approx \xi$$

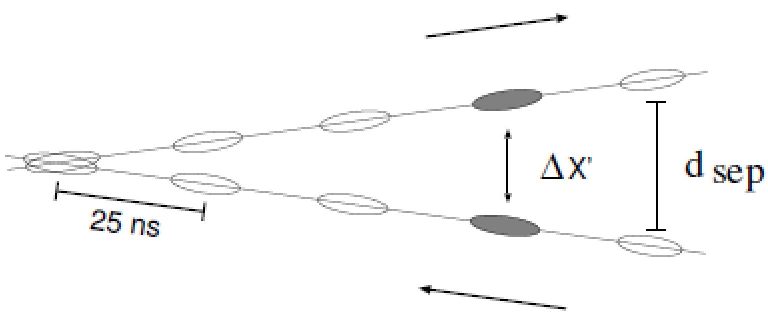
- β function can become bigger or smaller at the interaction point (IP) (dynamic β)

$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q+\Delta Q))} = \frac{\beta_0}{\sqrt{1+4\pi\xi cot(2\pi Q) - 4\pi^2\xi^2}}$$

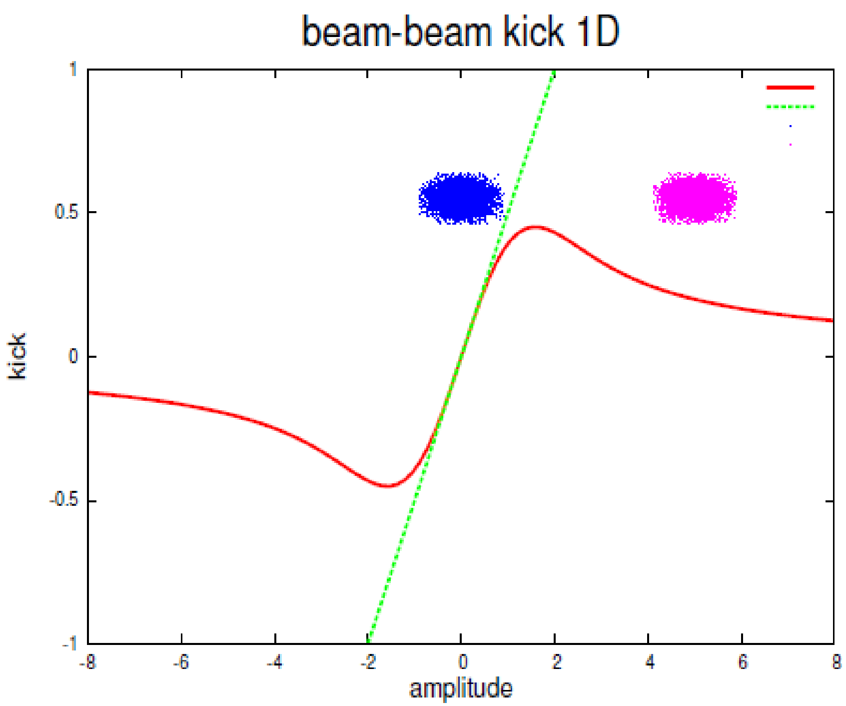
- But this is only true for small amplitude particles and different amplitudes have different kicks & the slope has the opposite sign for a large enough separation so that it focuses & defocuses at the same time !

Beam-beam

- The interactions can therefore be split into two: long & short range, for the LHC, this can be represented as:



Courtesy of W. Herr



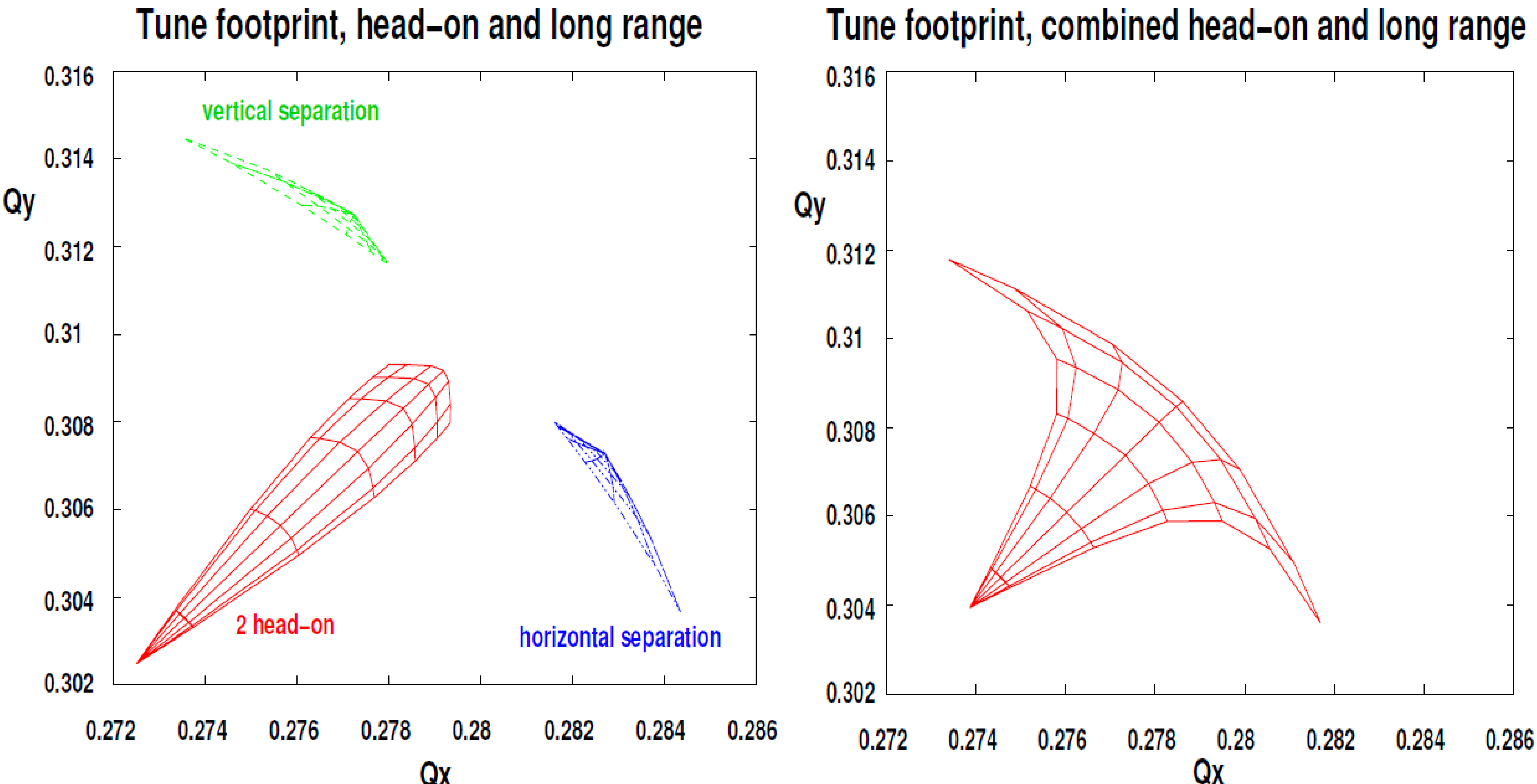
Blue bunch ≡ short range

Pink bunch ≡ long range

Courtesy of W. Herr

Beam-beam

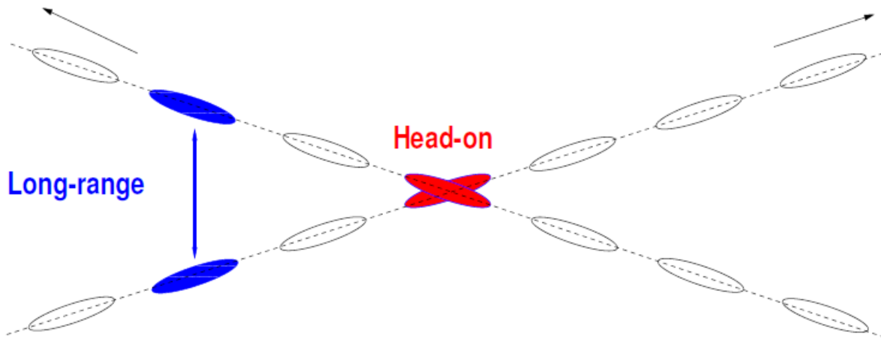
- Both types of interactions have their respective tune shifts:



Courtesy of W. Herr

Beam-beam

- If we look at the increasing separation of the two colliding beams due to the IP crossing angle:



Courtesy of W. Herr

$$\Delta x' = -\frac{2Nr_0}{\gamma}\frac{(x+d)}{r^2}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \qquad r^2 = (x+d)^2 + y^2$$

- If we expand this, we see:

$$\Delta x' \propto \frac{1}{d}\left[1-\frac{x}{d}+O\left(\frac{x^2}{d^2}\right)+\ldots\right]$$

- There is an amplitude independent contribution

Beam-beam

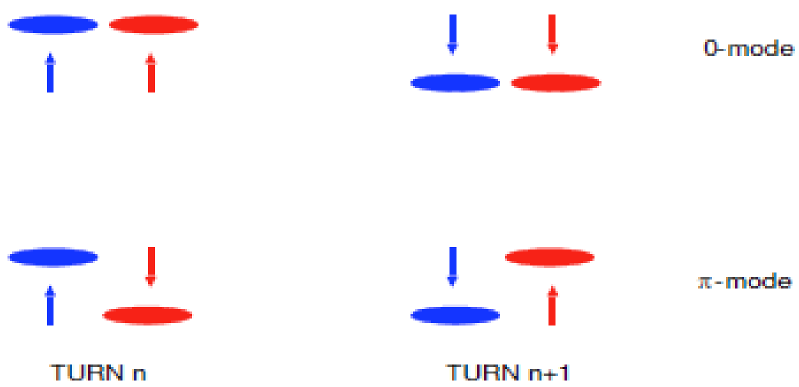
- Amplitude independent contribution:

$$\Delta x' \propto \frac{1}{d}\left[1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \ldots\right]$$

- So long range beam-beam leads to an orbit kick !
- Effect can become important & needs to be mitigated if possible & understood
- There can be both coherent and incoherent motion or oscillations of particles within the beam
- Can also see from expansion beam-beam excites all orders of multipoles

Beam-beam

- The main two modes of oscillation of the colliding bunches are

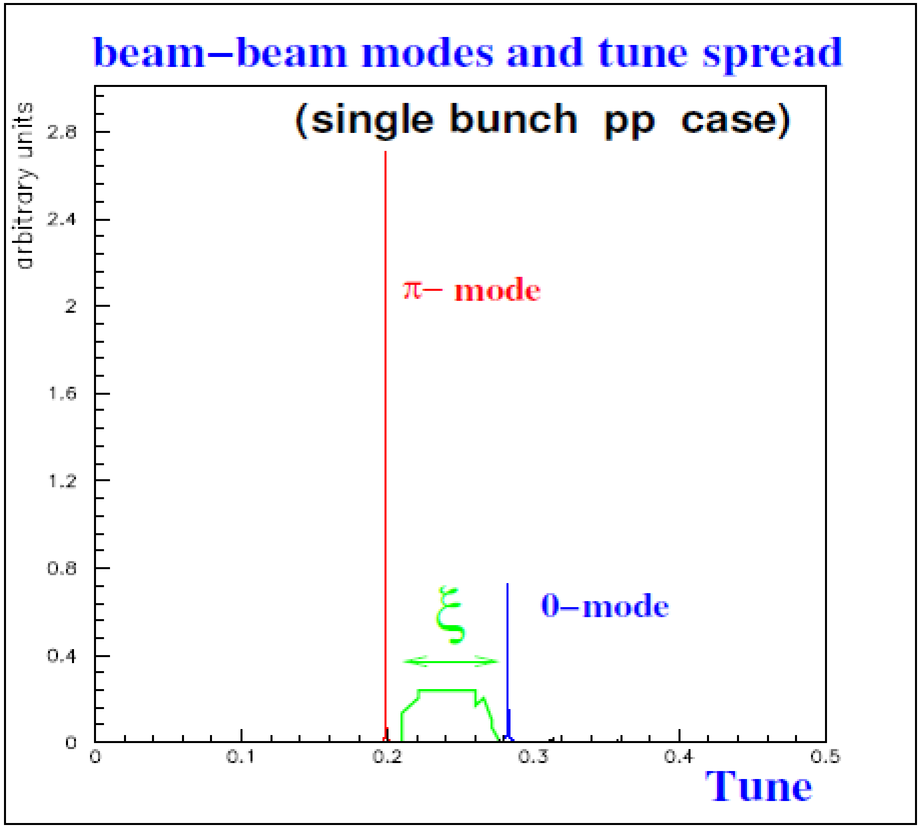


Courtesy of W. Herr

- So the two bunches are “locked” in coherent oscillation with each other
  - *0* mode is stable & with no tune shift
  - π mode – can become unstable & has a maximum tune shift

Beam-beam

- All particles in the beam are disturbed by the other when colliding & an FFT of this gives
- *0* mode unperturbed
- π mode perturbed & shifted by *1-1.3* × ξ
- Incoherent spectrum between [*0.0,1.0*] × ξ
- Strong-strong case: π mode is shifted outside the usual tune spread

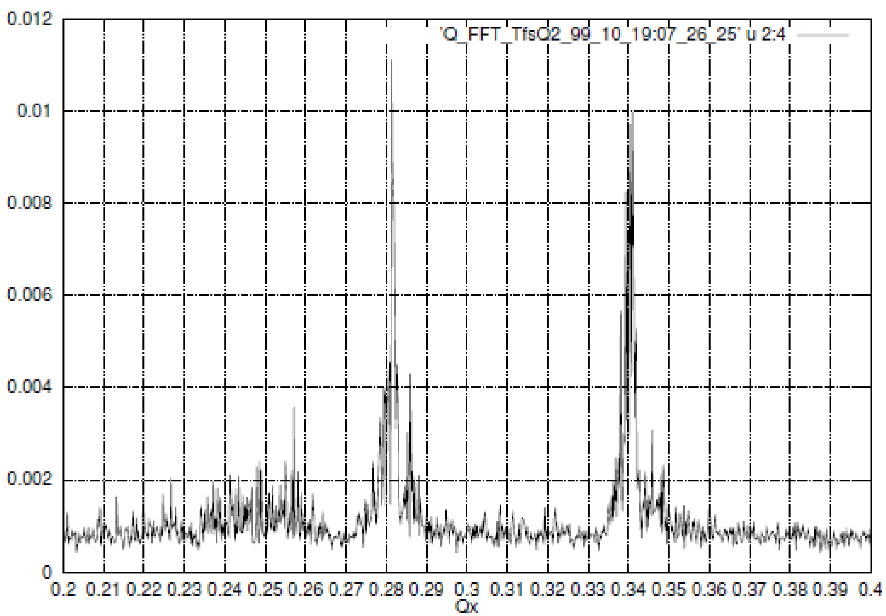


Courtesy of W. Herr

Beam-beam

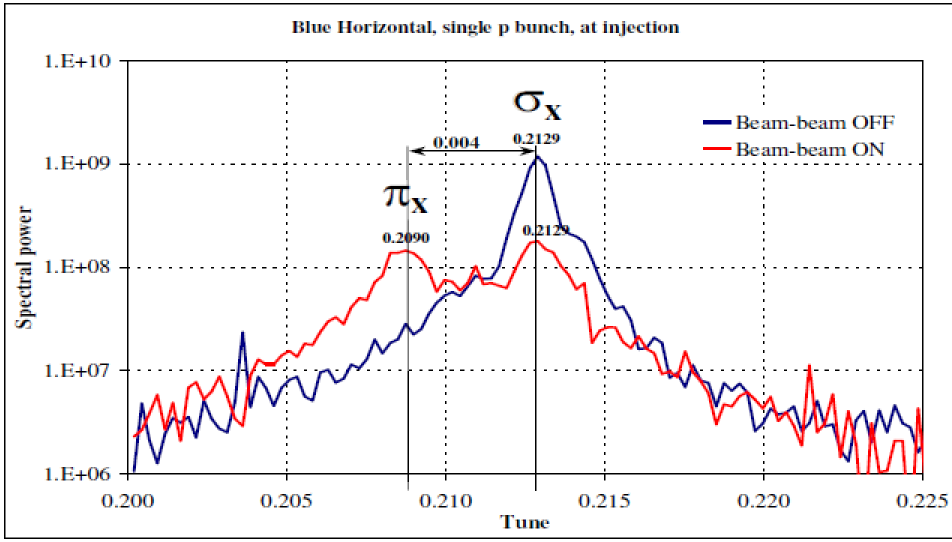
- Landau damping says that, if the π mode would be inside the incoherent spectrum, it would automatically be damped – however, it is not !
- This has been measured experimentally:

LEP:



Courtesy of G. Morpurgo

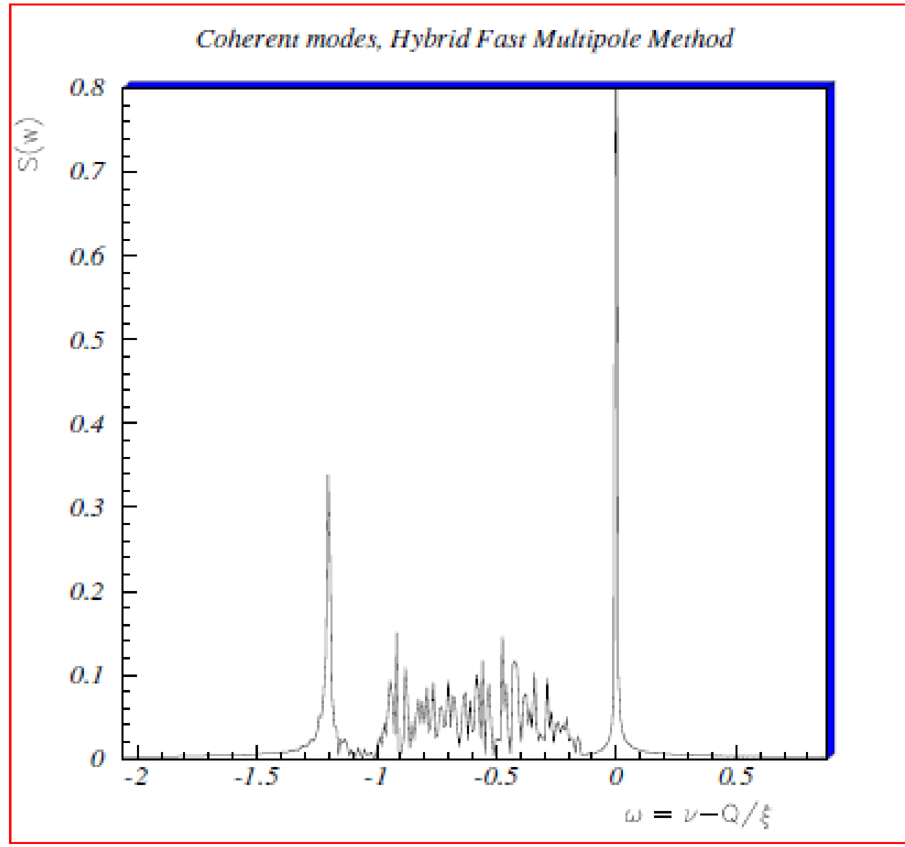
RHIC:



Courtesy W. Fischer (BNL)

Beam-beam

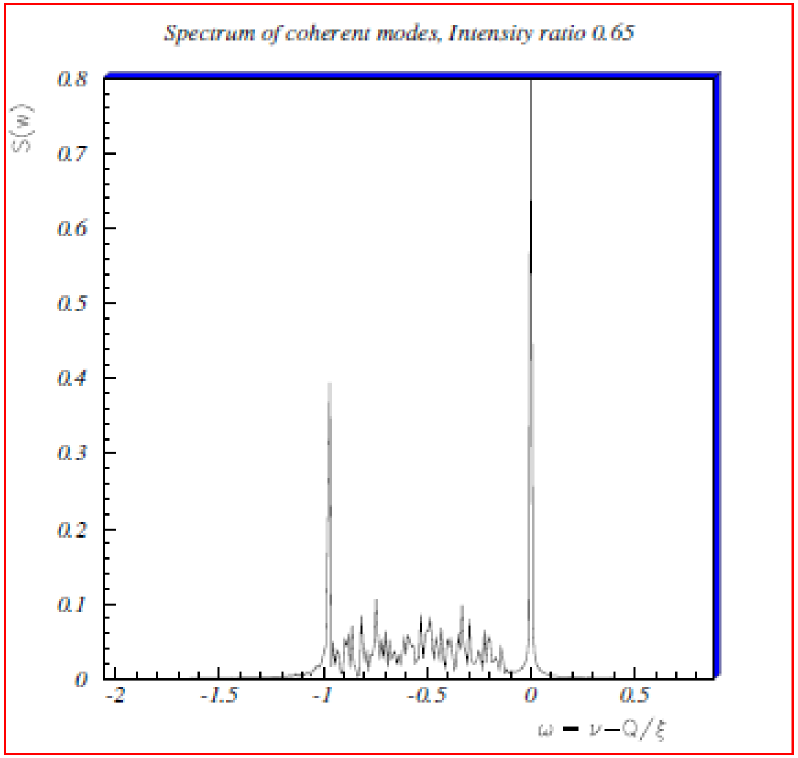
- Simulation of coherent spectra can rapidly become very complex:
- Need full simulations of both beams
- Must take into account changing fields
- Use up to *10^9* particles
- Can be very time consuming

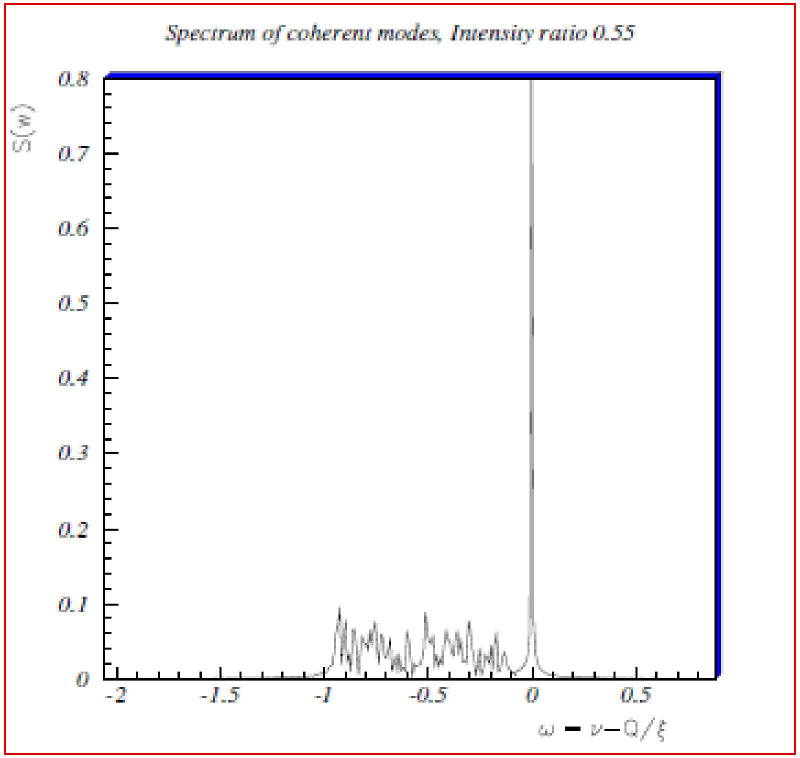


Courtesy of W. Herr

Beam-beam

- Methods of restoring Landau damping 1):
- Different intensities for colliding beams



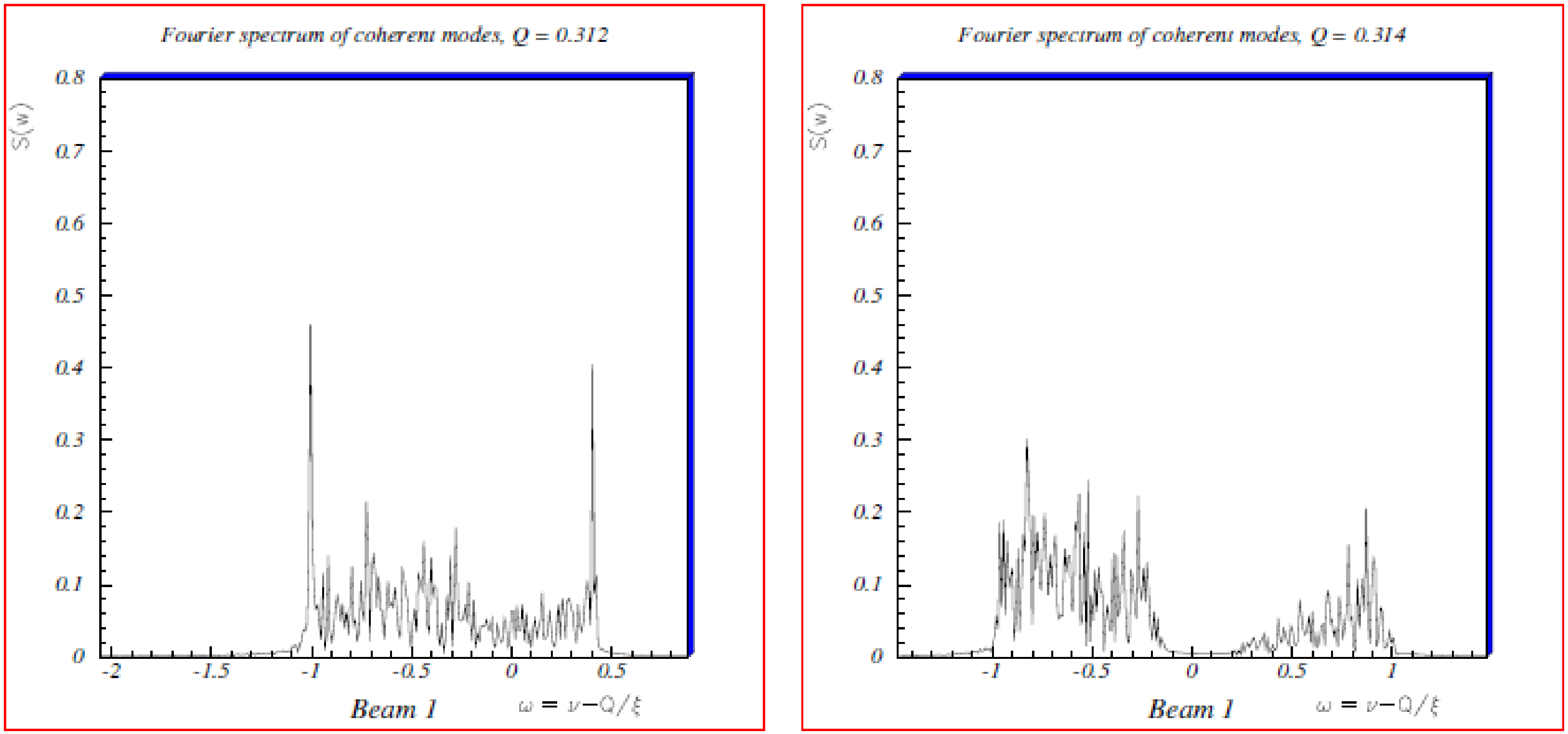


Courtesy of W. Herr

- Damping restored (*0.65* & *0.55* ratios)

Beam-beam

- Methods of restoring Landau damping 2):
- Different tunes for the two colliding beams



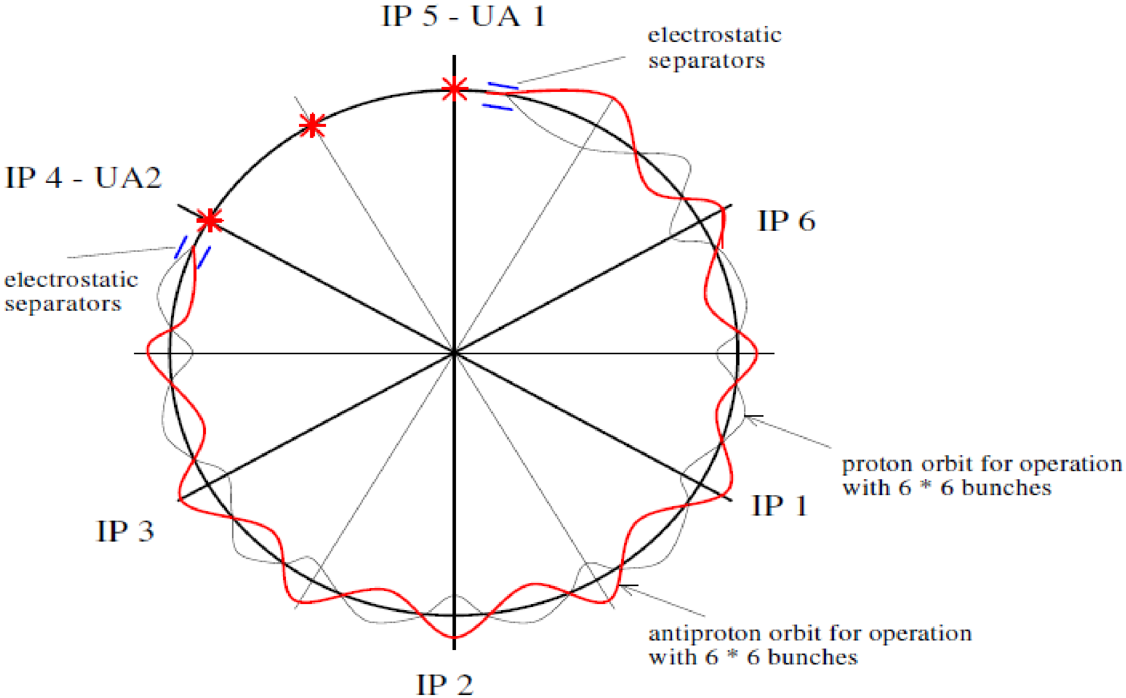
Courtesy of W. Herr

- Damping restored (*0.002* & *0.004* tune difference)

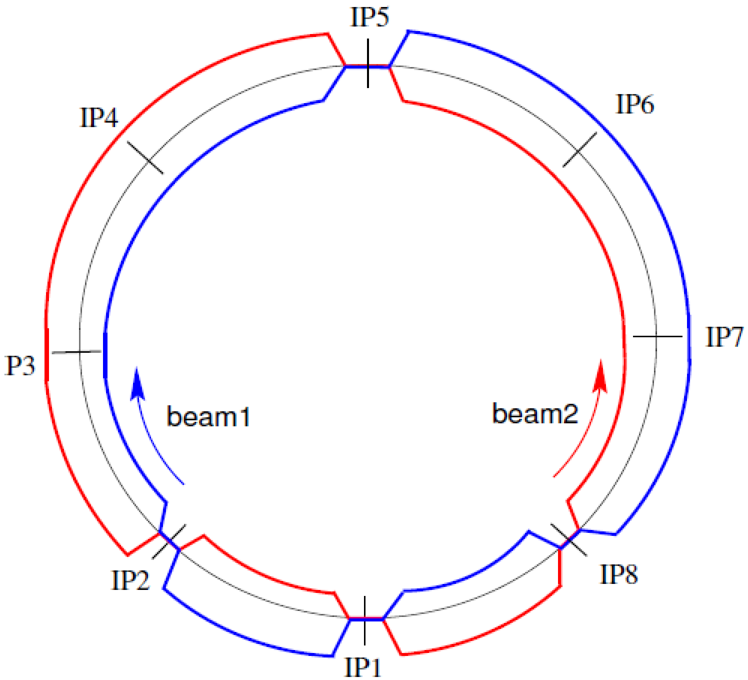
Beam-beam

- So far only two bunches & a single IP were considered – what happens when we increase this ?

SPS



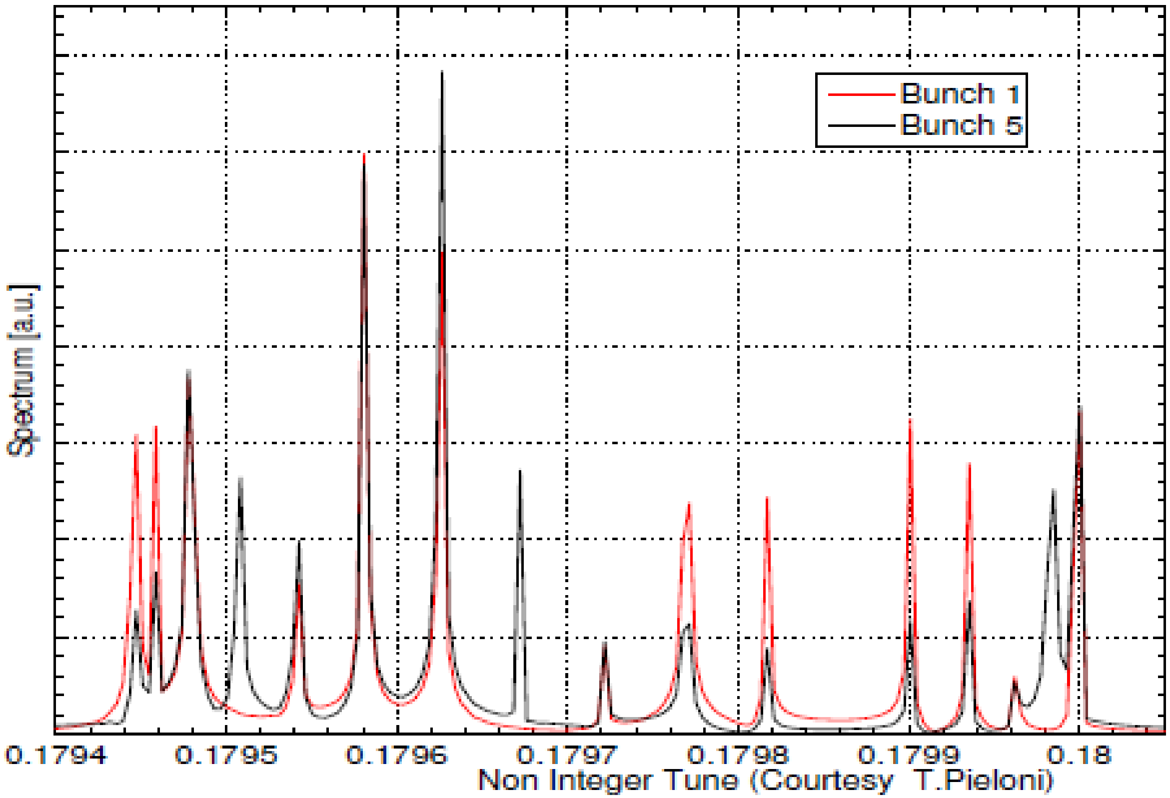
LHC



Courtesy of W. Herr

Beam-beam

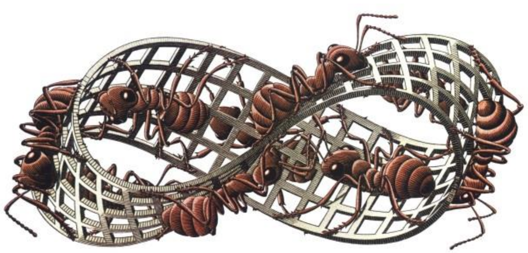
- What happens with multiple bunches & multiple interaction points ?



- Multiple *0* & π modes making things worse …

Things not covered

- Luminosity (not complete list):
  - How luminosity is measured
  - Kinematic factor derivation
  - Luminous region / luminosity levelling
- Beam-beam (not complete list):
  - Effect of holes in bunch trains / PACMAN bunches
  - Beam-beam deflection scan & experiments in details
  - Self consistent Vlasov equations for both beams
  - Suppression of beam-beam effects via electron lenses & wire compensation
  - Möbius lattice



Summary

- Looked at the concept of luminosity & how it is important to colliders. Specifically:
  - How luminosity is defined
  - How it changes with offsets
  - How it changes with crossing angles
  - How the hourglass effect develops for short bunches
  - How crab cavities could be used to increase it
- Derived the beam-beam parameter ξ
- Looked at the relationship between beam-beam & in particular the beam-beam parameter ξ & how it relates to luminosity

Summary

- Looked at head-on and long range beam-beam interactions and their tune shifts
- Derived beam-beam kick with & without separation – showed that this leads to an amplitude independent contribution or orbit kick
- Looked at coherent and incoherent beam-beam modes (0 & π mode) & saw how the π mode cannot be Landau damped
- Looked at various methods of ensuring the π mode can be Landau damped
- Briefly mentioned outstanding issues

Further reading

- Luminosity:
  - W. Herr & B. Muratori, Concept of luminosity, CERN Accelerator School, Zeuthen 2003, in: CERN 2006-002 (2006)
- Beam-beam:
  - A. Chao, The beam-beam instability, SLAC-PUB-3179 (1983)
  - A. Zholents, Beam-beam effects in electron-positron storage rings, Joint US-CERN School on Particle Accelerators, in Springer, Lecture Notes in Physics, 400 (1992)
  - W. Herr, Beam-beam effects, CERN Accelerator School, Zeuthen 2003, in: CERN 2006-002 (2006)

Thank you ☺