Collective Effects in Particle Accelerators

Part 5

Coupled-Bunch Instabilities

There are six lectures in this course on collective effects in accelerators:

1. Space charge and scattering
2. Wake fields and impedances
3. Potential well distortion and the microwave instability
4. Head-tail instability
5. Coupled-bunch instabilities
6. Luminosity and the beam-beam effect

So far, we have looked at single-bunch instabilities, in which the charge distribution within an individual bunch in an accelerator fails to reach an equilibrium.

Wake fields can also lead to coupled-bunch instabilities, where the coherent (transverse or longitudinal) oscillations of one bunch can drive oscillations in other bunches in an accelerator.

By the end of this lecture, you should be able to:

- outline a simple model for coupled-bunch instabilities in an accelerator;
- describe some of the principal phenomena associated with the resistive-wall instability;
- explain how feedback systems can be used to suppress coupled-bunch instabilities.

In this lecture, we shall consider transverse coupled-bunch instabilities; longitudinal instabilities can also occur, and behave in much the same way.

We shall treat each bunch as a single "macroparticle" performing betatron oscillations as it moves around a storage ring. In the absence of wake fields (and effects such as synchrotron radiation and decoherence) the betatron amplitude will remain constant.

The presence of wake fields can lead to the motion of one bunch driving oscillations of other bunches, resulting in an overall growth of the oscillation amplitudes of the bunches.

Our first goal will be to derive an expression for the growth rate of coherent betatron oscillations of bunches in a storage ring, in terms of the storage ring parameters and the wake function.

In the absence of wake fields, the coherent vertical motion of a bunch in a storage ring has the equation of motion:

$$\ddot{y} + \omega_\beta^2 y = 0, \tag{1}$$

where y is the vertical co-ordinate, the double dot indicates the second derivative with respect to time, and ω_β is the betatron frequency.

Where wake fields are present, with wake function $W_\perp(-\Delta z)$, a single bunch in a storage ring will see the wake field generated by that bunch on earlier turns. The force from the wake field can be included in the equation of motion as a driving force:

$$\ddot{y} + \omega_\beta^2 y = -\frac{N_0 e^2 c^2}{E_0 C_0} \sum_{k=1}^{\infty} W_\perp(-kC_0) y(t - kC_0/c). \tag{2}$$

The bunch has total charge $N_0 e$ and energy E_0, and C_0 is the ring circumference.

If there are many bunches in the storage ring, then we need to sum the contribution to the total wake field (at any point in the ring) from every bunch on every turn.

The equation of motion then becomes:

$$\ddot{y}_n + \omega_\beta^2 y_n = -\frac{N_0 e^2 c^2}{E_0 C_0} \sum_{k=1}^{\infty} \sum_{m=0}^{M-1} W_\perp\left(-kC_0 - \frac{m-n}{M} C_0\right) y_m\left(t - k\frac{C_0}{c} - \frac{m-n}{M}\frac{C_0}{c}\right). \tag{3}$$

Here, y_n is the vertical co-ordinate of the nth bunch, k is an index for the turn number, and m is an index (in the summation for the total wake force) for each bunch.

M is the total number of bunches. Note that we assume (for simplicity) that the bunches are equally spaced around the ring, and all have the same number of particles.

To understand the dynamics of the bunches in the presence of wake fields, we need to find solutions to the equation of motion (3).

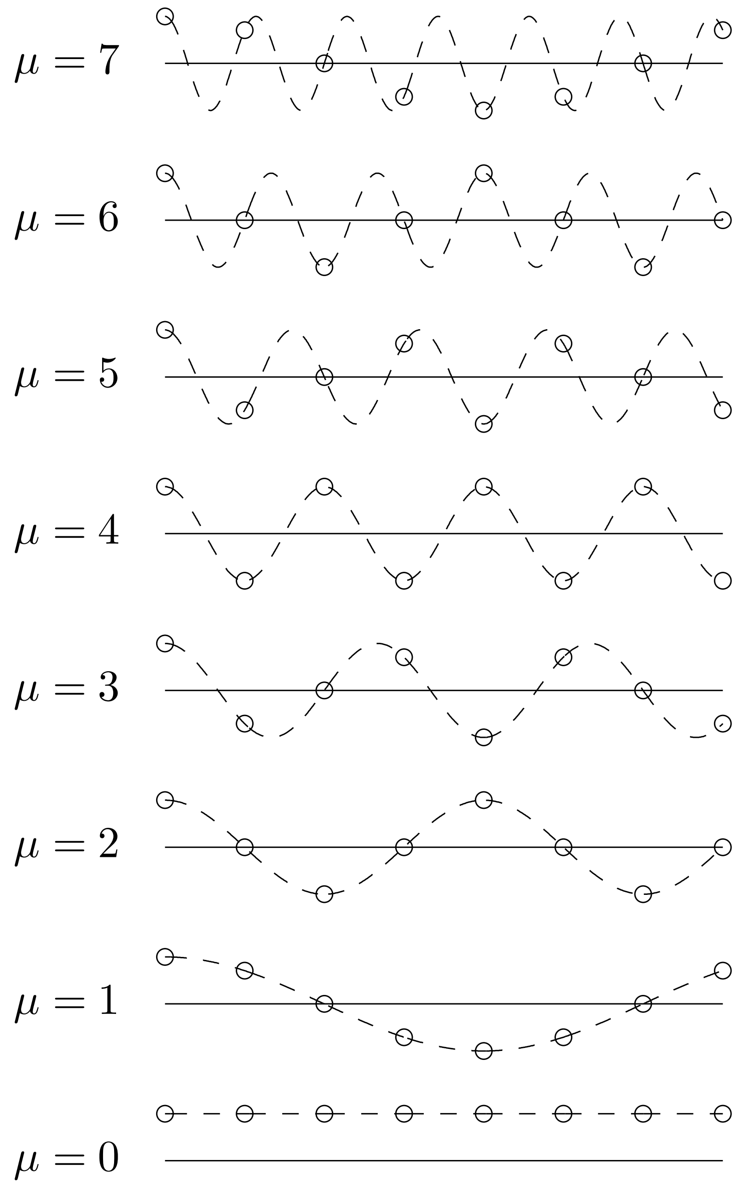
We shall assume that there are solutions of the form:

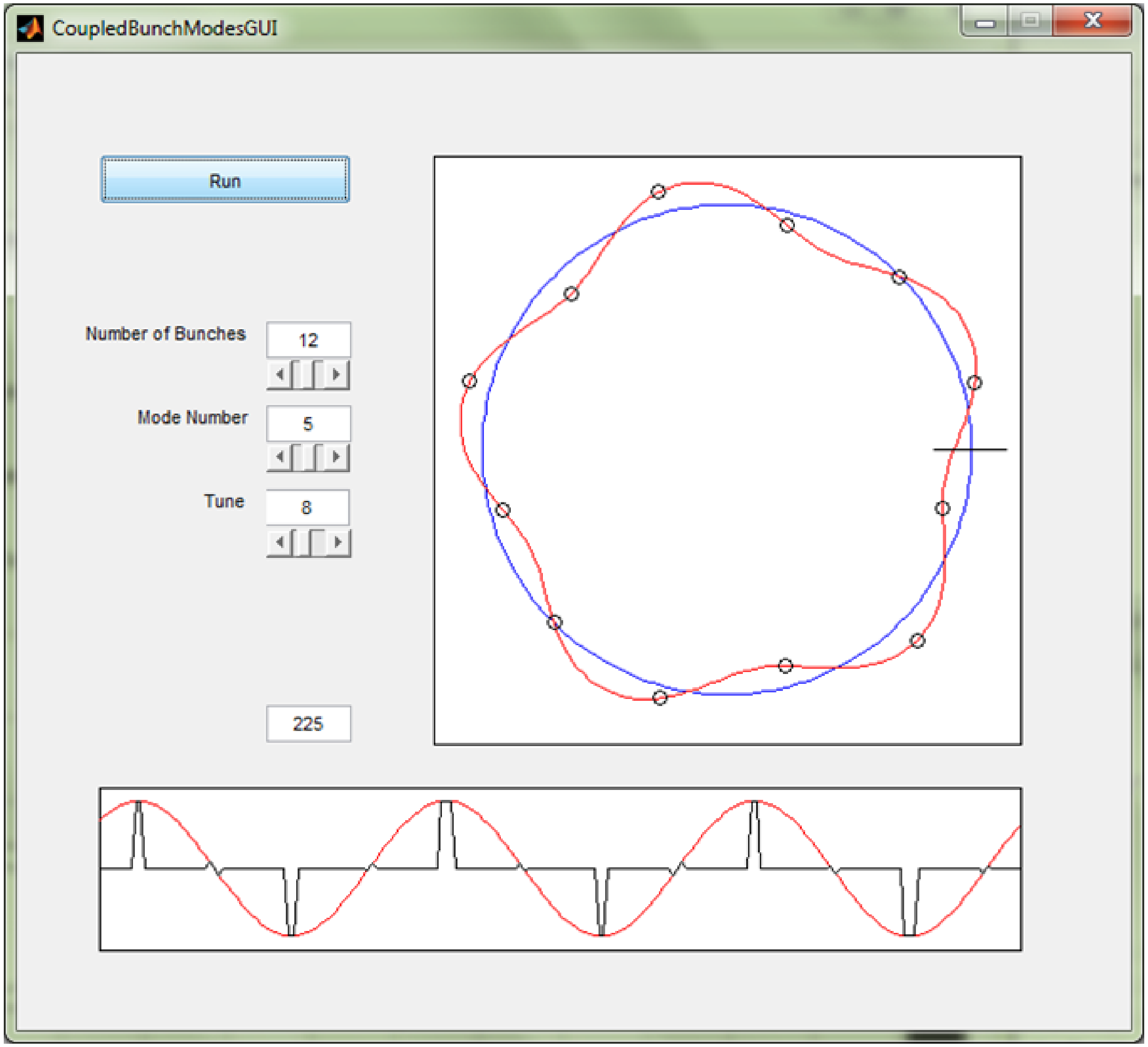
$$y_n^{\mu}(t) = A \exp\left(2\pi i \frac{\mu n}{M}\right) e^{-i\Omega_{\mu} t}, \tag{4}$$

where the integer μ is an index to distinguish different solutions.

For a given value of μ, the solution (4) takes the form of a "wave" around the ring, with wavelength C_0/μ and oscillation frequency Ω_{μ}...

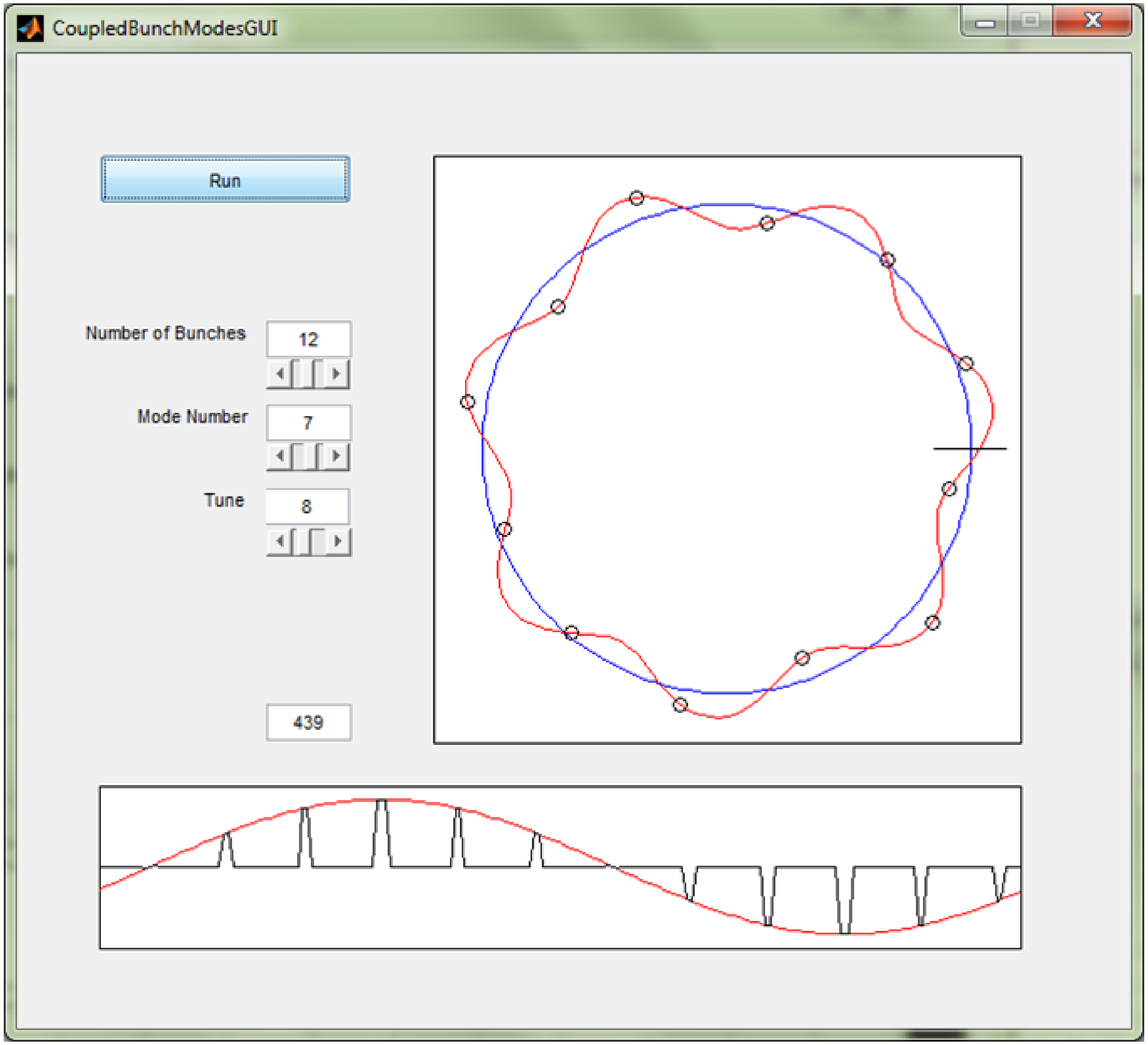
Coupled bunch modes





CoupledBunchModesGUI
Run
Number of Bunches
12
Mode Number
5
Tune
8
225

Coupled bunch modes



To find an expression for Ω_μ (in terms of the storage ring parameters and the wake function), we substitute the solution (4) into the equation of motion (3).

A positive imaginary part in Ω_μ represents an exponential growth in oscillation amplitude, and hence indicates the presence of an instability.

After some working, we find by substituting the solution (4) into the equation of motion (3):

$$\Omega_\mu \approx \omega_\beta - i\frac{MN_0e^2c^2}{4\pi\nu_y E_0 C_0} \sum_{p=-\infty}^{\infty} Z_\perp\Big((pM+\mu)\omega_0 + \omega_\beta\Big). \tag{5}$$

Here, ν_y is the betatron tune, and $Z_\perp(\omega)$ is the transverse impedance (obtained from the wake function $W_\perp(-\Delta z)$ by a Fourier transform).

For a given wake function, we can evaluate (5) to find the frequencies of the different coupled bunch modes.

The exponential growth rates of the different modes are given by the imaginary part of the corresponding frequency Ω_μ.

As an example, let us consider the wake fields generated as a consequence of the finite conductivity of the vacuum chamber, i.e. the resistive-wall wake fields.

We shall find that an instability (known as the resistive-wall instability) always exists in this case.

The transverse wake function describing the wake fields arising from the finite conductivity of a circular vacuum chamber is given by:

$$W_{\perp}(z) = -\frac{2}{\pi b^3}\sqrt{\frac{Z_0 c}{4\pi}\frac{c}{\sigma}}\frac{L}{\sqrt{-z}}, \qquad z < 0, \tag{6}$$

where Z_0 is the impedance of free space, and the vacuum chamber has radius b, length L and conductivity σ.

The corresponding impedance is:

$$Z_{\perp}(\omega) \approx (1 - i\,\mathrm{sgn}(\omega))\,\frac{L}{\omega b^3}\sqrt{\frac{Z_0 c}{4\pi}\frac{2|\omega|}{\pi\sigma}}. \tag{7}$$

Note that $|Z_{\perp}(\omega)| \to \infty$ as $\omega \to 0$: the impedance becomes large at low frequencies.

The frequencies Ω_μ of different beam modes μ are given by (5). Since for resistive-wall wake fields the impedance becomes large at low frequency, the summation in (5) is dominated by terms for which:

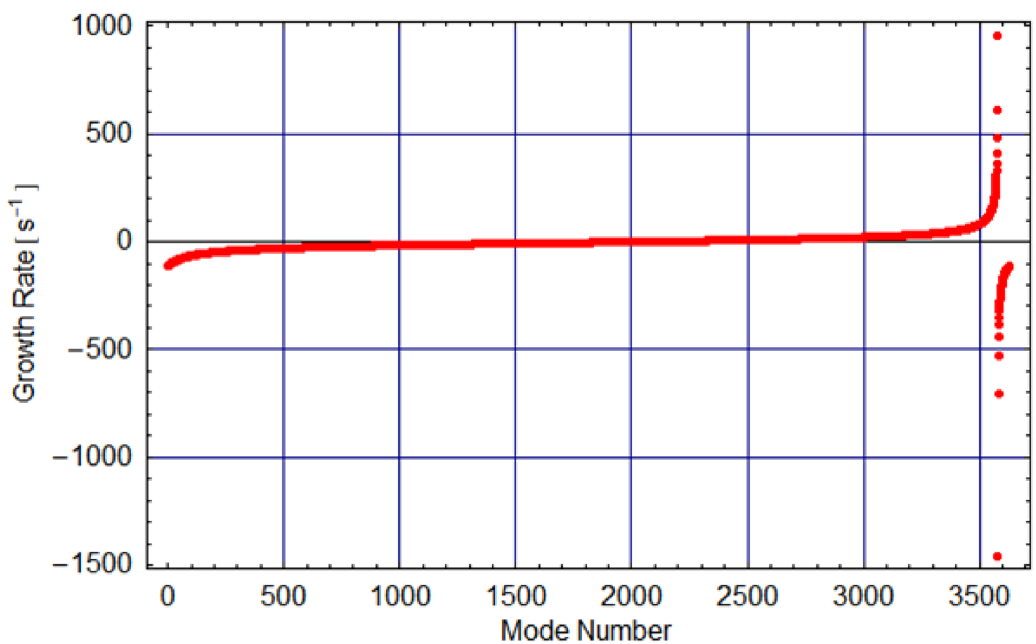
$$(pM + \mu)\omega_0 + \omega_\beta \approx 0, \quad \text{and hence:} \quad \mu \approx -pM - \nu_y. \tag{8}$$

We can restrict the mode index to $0 \leq \mu < M$: outside this range, the modes simply repeat. We then see that the strongest effects of the resistive-wall wake fields are for modes:

$$\mu \approx M - \nu_y. \tag{9}$$

These are the modes for which, at fixed points in the ring, the lowest beam oscillation frequencies are observed.

For given ring and beam parameters, we can plot the growth rates of the different modes: for resistive-wall wake fields, the plot has a characteristic shape.



Roughly half the modes are damped (are stable); but the amplitudes of the other modes grow with time (are unstable). The strongest growth rates occur for mode numbers $M - \nu_y$.

The growth rate of a given mode can be found by taking the imaginary part of the frequency for that mode.

In the case of resistive-wall wake fields, the mode for which the amplitude grows most rapidly has growth rate:

$$\frac{1}{\tau} = \frac{MN_0 e^2 c^2}{4\pi \nu_y E_0} \frac{1}{\pi b^3} \sqrt{\frac{Z_0 c}{4\pi} \frac{1}{\sigma}} \sqrt{\frac{2\pi}{\omega_0 (1 - \Delta_y)}}, \qquad (10)$$

where the betatron tune is written:

$$\nu_y = N_y + \Delta_y \qquad (11)$$

for integer N_y and $0 \leq \Delta_y < 1$.

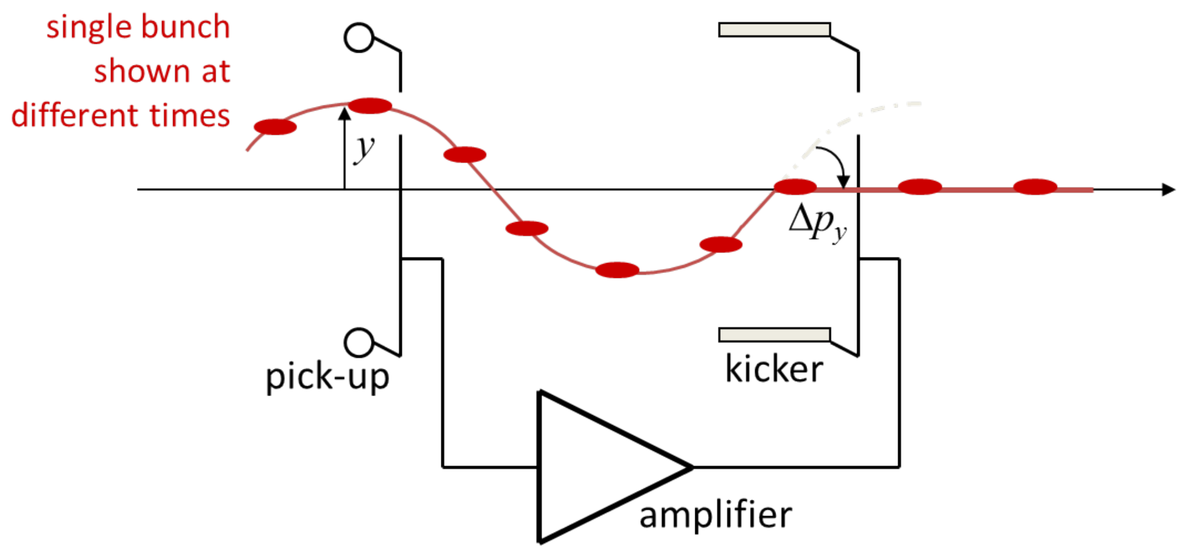
Resistive-wall wake fields can drive coupled-bunch modes with growth times of hundreds or tens of turns.

In practice, the beam can be stable at low currents because natural damping effects (synchrotron radiation damping, and decoherence) can suppress modes with slow growth rates.

As more current is injected into a ring, at some point a mode will become unstable. Bunch oscillations will grow until some current is lost from the beam, and the beam becomes stable again.

The instability appears as a "current limit" in the storage ring. To keep the beam stable at higher currents, it is possible to use bunch-by-bunch feedback systems.

Bunch-by-bunch feedback systems add to the damping from natural mechanisms. Modern feedback systems can achieve damping times of order 20 turns.



An ideal feedback system would detect the position of a single bunch, and apply a single kick to restore the bunch to its correct trajectory.

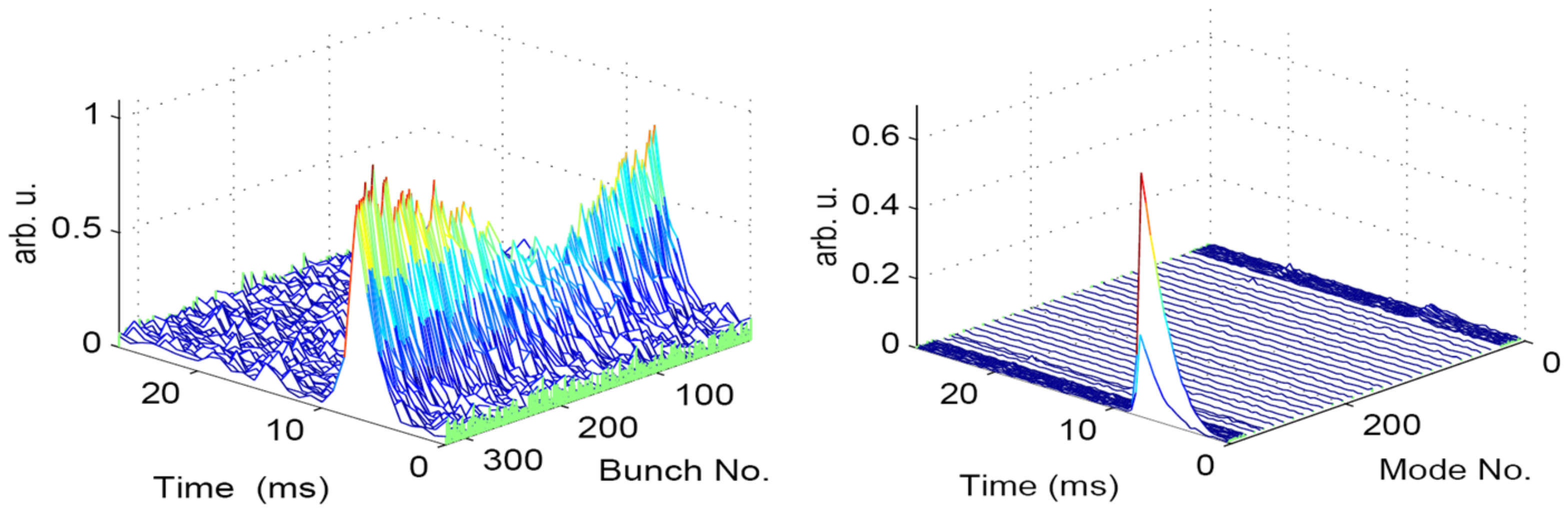
Because of technical limitations, feedback systems apply a series of small kicks over several turns to correct bunch trajectories.

Feedback systems can be used to measure the growth rates of coupled bunch modes and the damping rate achieved by the feedback system: these measurements are sometimes known as “grow-damp” measurements.

A beam is stored at high current with the feedback system turned on to maintain beam stability.

The feedback system is then turned off, and the transverse position of each bunch is recorded turn-by-turn using a suitable pick-up.

Turning the feedback system back on before the beam is lost allows the damping rate of the feedback system to be measured.



Grow-damp measurements from the Advanced Light Source.

J. Fox et al, "Multi-bunch instability diagnostics via digital feedback systems at PEP-II, DAΦNE, ALS and SPEAR", Proceedings of the 1999 Particle Accelerator Conference, New York, 1999.

Coupled bunch instabilities in a storage ring can be studied using a model in which each bunch is represented as a single macroparticle performing coherent betatron (or synchrotron) oscillations.

Long-range wake fields couple the motion of each bunch to the motion of all other bunches in the ring.

The system of coupled equations of motion for all bunches in the ring has an (approximate) solution in which the bunch displacements travel as a wave around the ring.

Each mode (wavelength) of oscillation has a frequency that can be found by substituting the wave-like solution into the equation of motion: the growth (or damping) rate of each mode is given by the imaginary part of the frequency.

In the presence of resistive-wall wake fields, a beam in a storage ring always has modes that are unstable.

However, the stability of a beam in a storage ring can be maintained by means of a bunch-by-bunch feedback system.