Improved Beam Dynamics Through Control of Magnetic Fringe Fields

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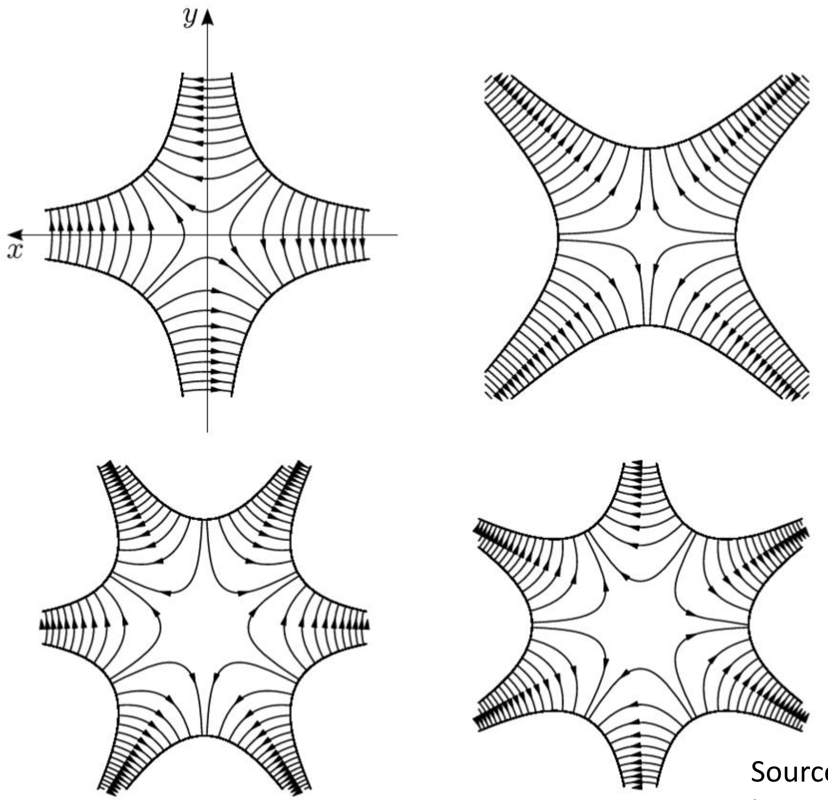
Magnets in particle accelerators

Particle accelerators use magnets that exhibit multipole fields.

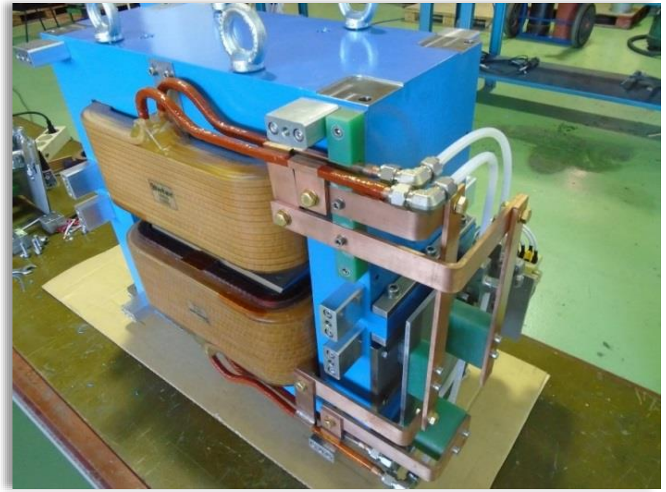
- Dipoles are used for beam steering.
- Quadrupoles are used for planar focusing.

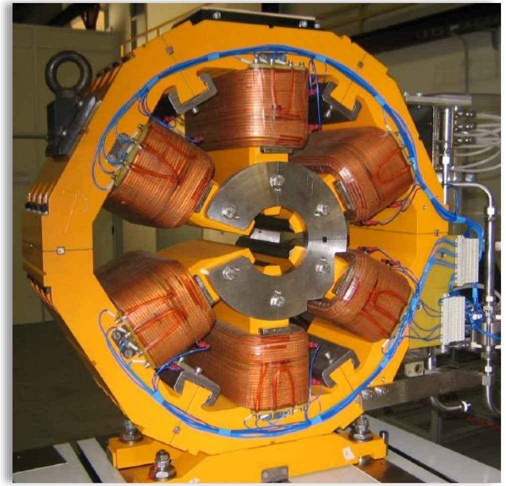


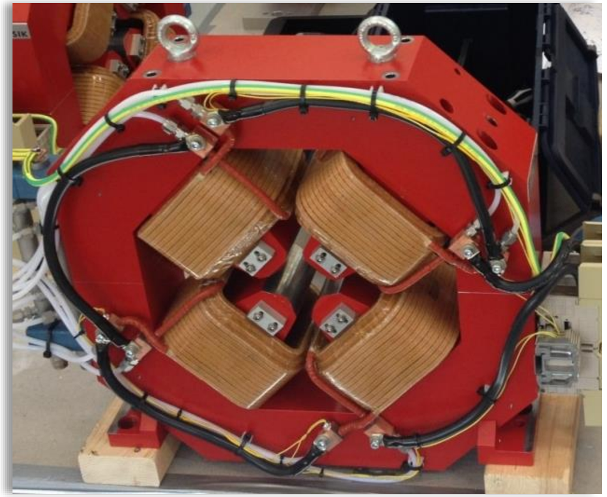
- These are pure multipole fields.
  - Quadrupole and sextupole fields in normal and skew orientation.
- Real fields are not pure and contain higher order multipole fields (allowed and forbidden).

Source – Wolski, A. Beam Dynamics in High Energy Particle Accelerators. (2014). Imperial College Press. ISBN 978-1-78326-277-9.

Realistic constraints on accelerator components





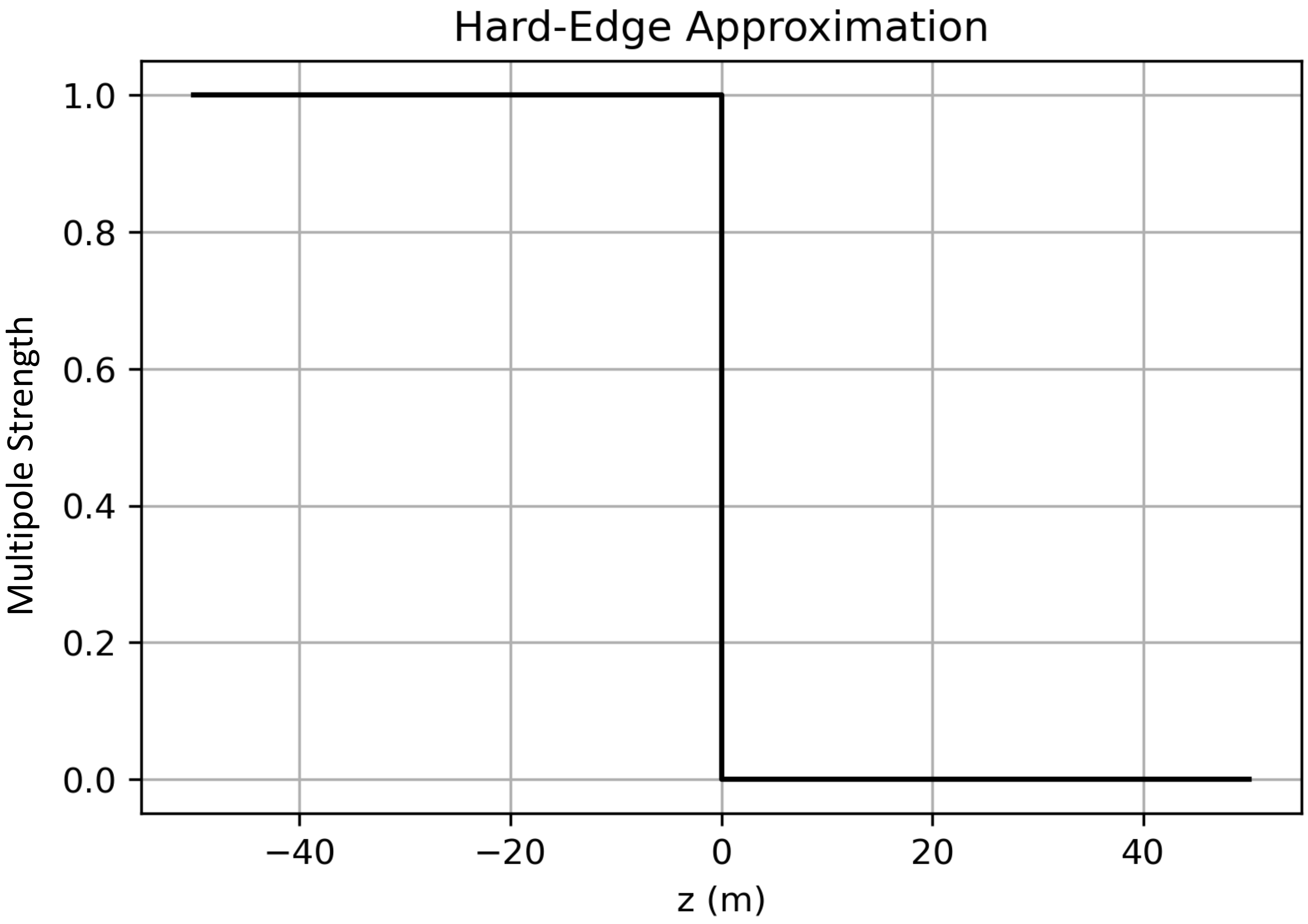


- Real magnets must have finite dimensions.
- In theory, to create pure multipole fields, magnets should not be bound, have infinite permeability and conductors placed at infinity.

Photo source - Alex Bainbridge – ASTeC – CI-MAG-106 Lectures.

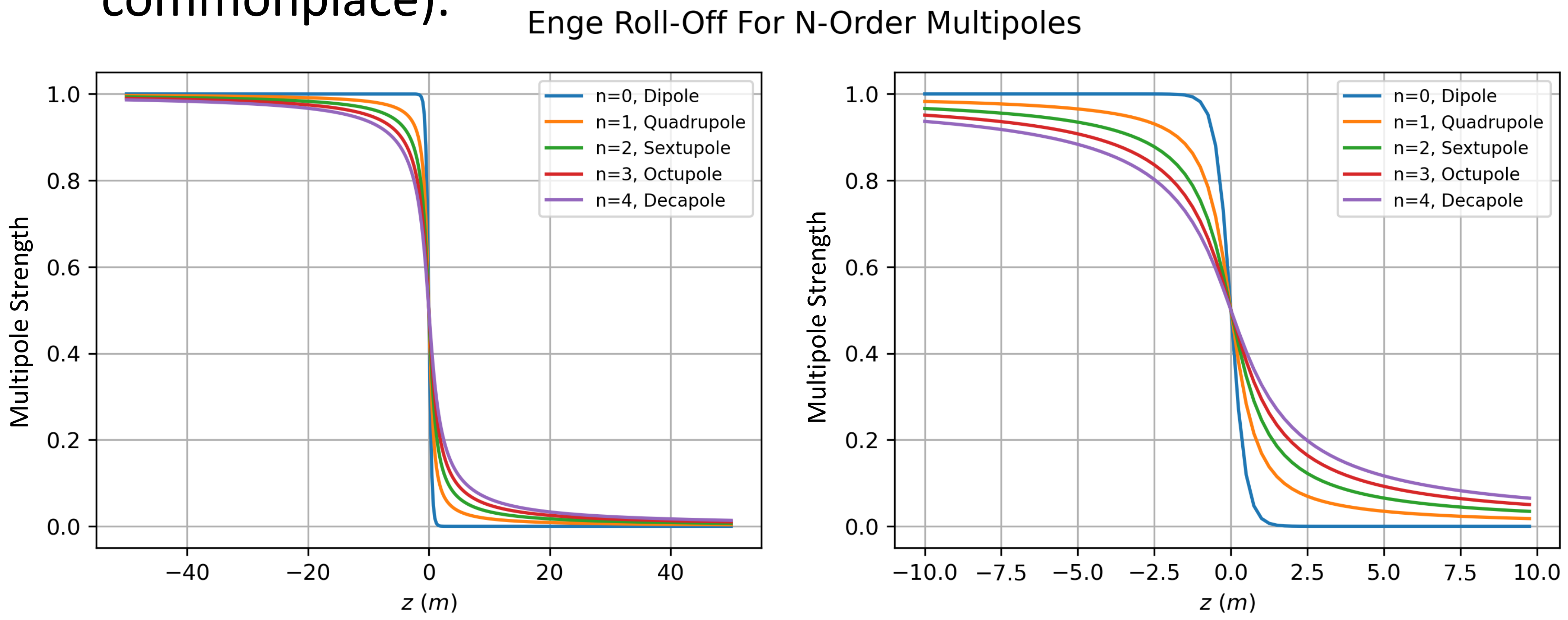
Maxwell's equations, fringe fields and modelling

- Maxwell's equations must be satisfied!
- We are interested in the fringe field regions at the edge of a magnet's influence.
- The "roll-off" will be referred to often, this is the multipole strength along the magnetic axis.
- Often, in simulations, hard-edged field models are assumed.
- These do not satisfy Maxwell's Equations.


Hard-Edge Approximation
Multipole Strength
1.0
0.8
0.6
0.4
0.2
0.0
−40
−20
0
20
40
z (m)

The Enge roll-off

- We can satisfy Maxwell's equations with the Enge roll-off.
- This is a more realistic approach (the use of the Enge is commonplace).

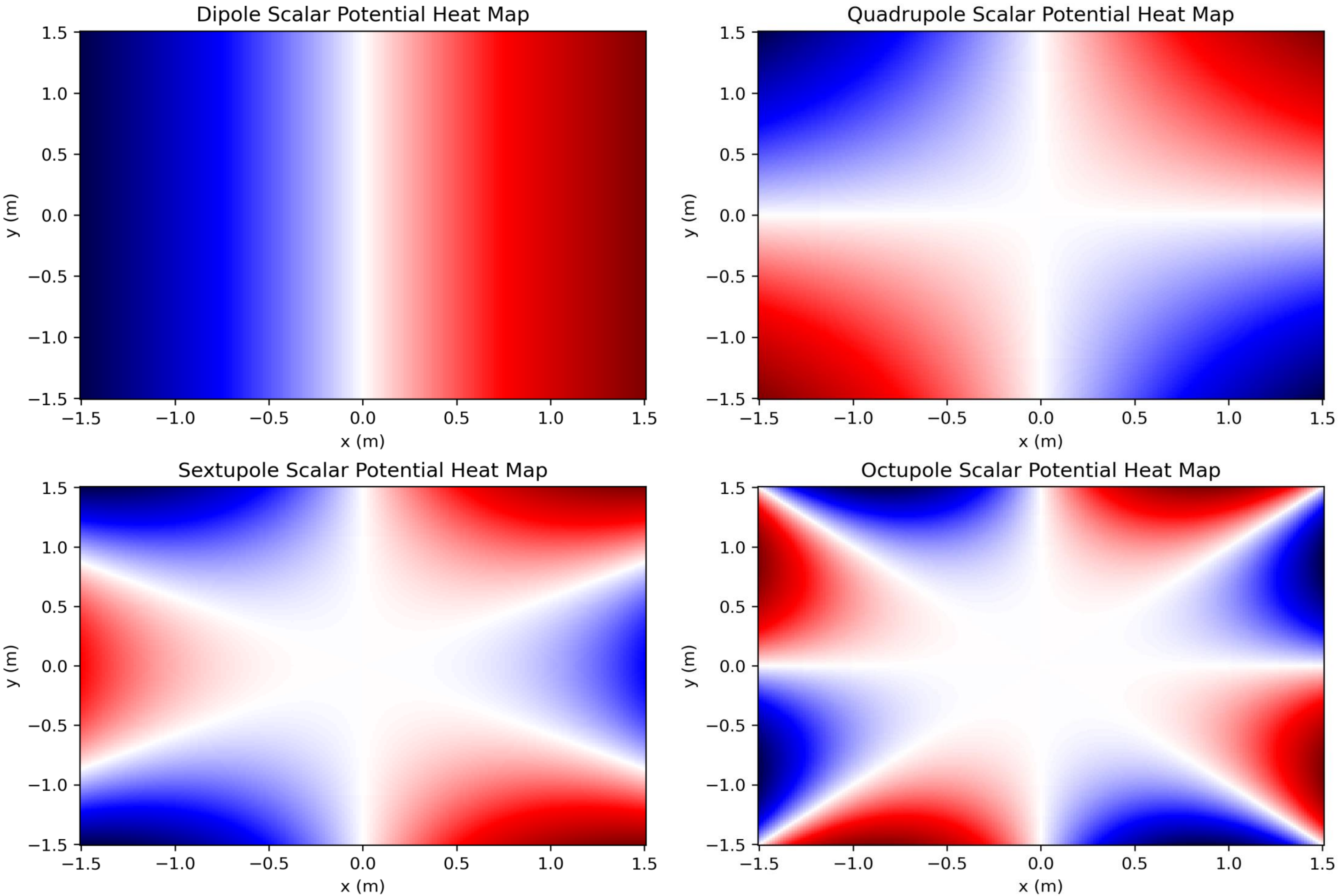


Creating a magnet

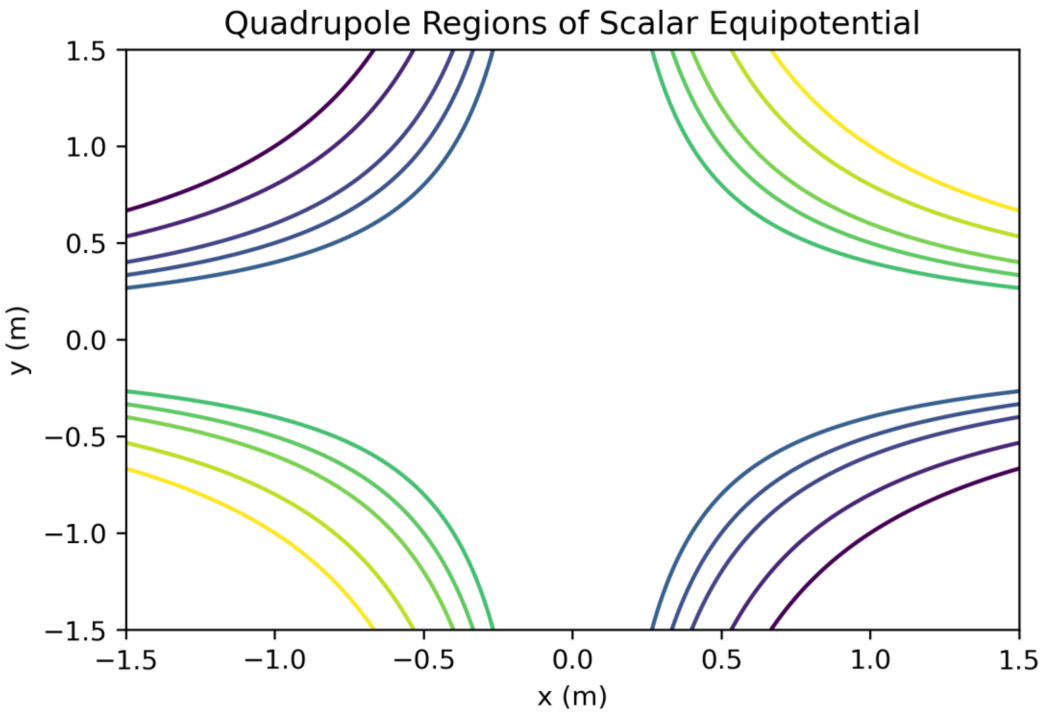
Can we customise a magnet for a specific dynamical system?

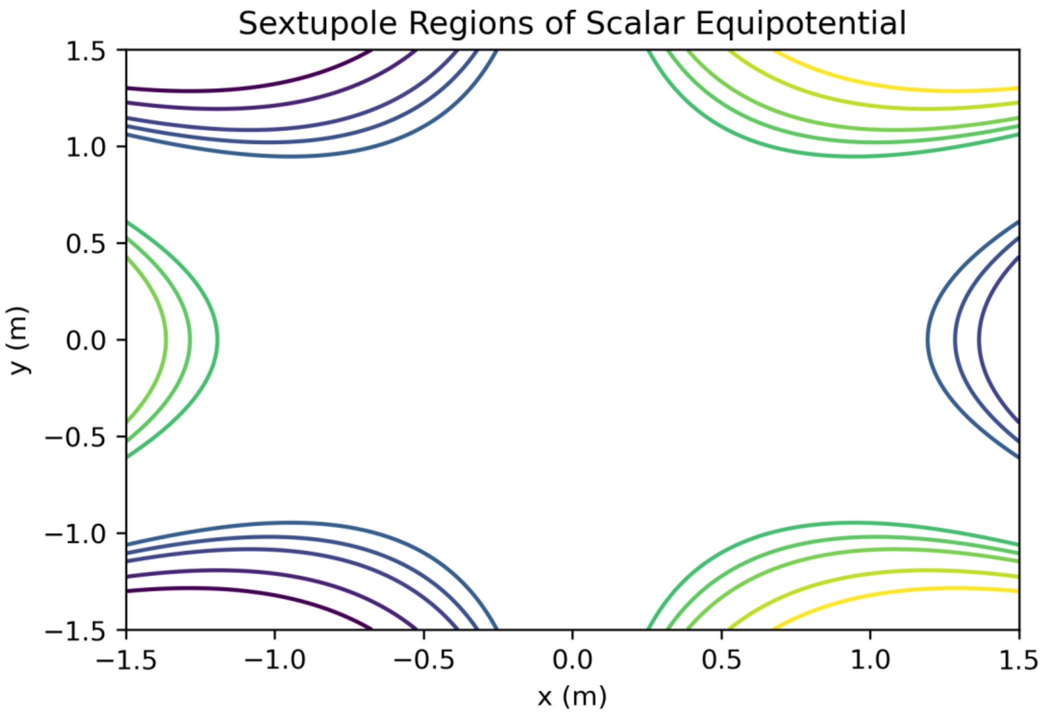
- Define the roll-off required for specific dynamics.
- Approximate the roll-off as Fourier series (for later generalisation).
- Produce the scalar potential analytically*.
- Surfaces of constant scalar potential define the pole face shape of the magnet.
  - This is the case if we assume the magnet has infinite permeability.
- Create the magnet (e.g. in OPERA)!

*Muratori, B. Wolski, A, Jones, J. Analytical Expressions for Fringe Fields in Multipole Magnets. (2014). Physical Review Special Topics – Accelerators and Beams. DOI: 10.1103/PhysRevSTAB.18.064001.

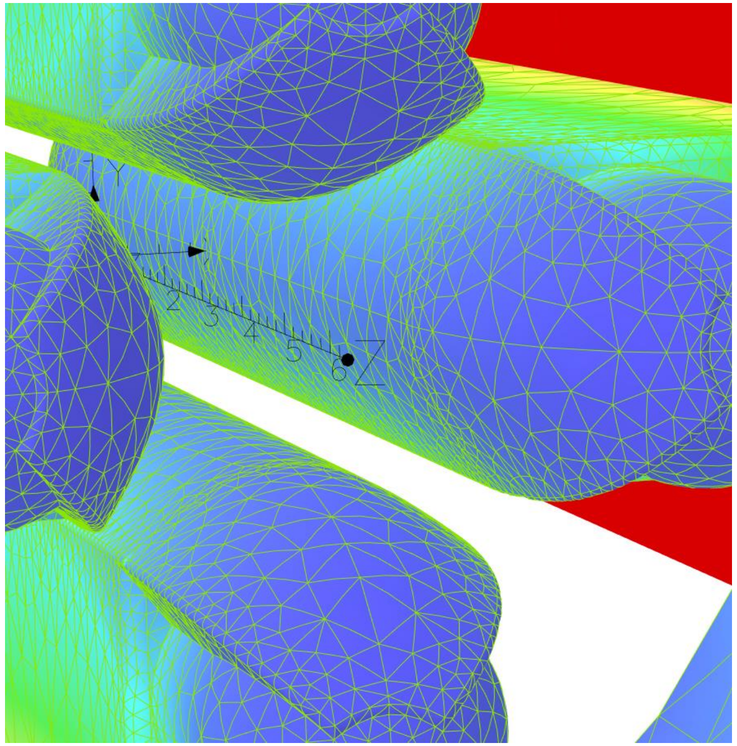

Dipole Scalar Potential Heat Map
Quadrupole Scalar Potential Heat Map
Sextupole Scalar Potential Heat Map
Octupole Scalar Potential Heat Map
x (m)
y (m)
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

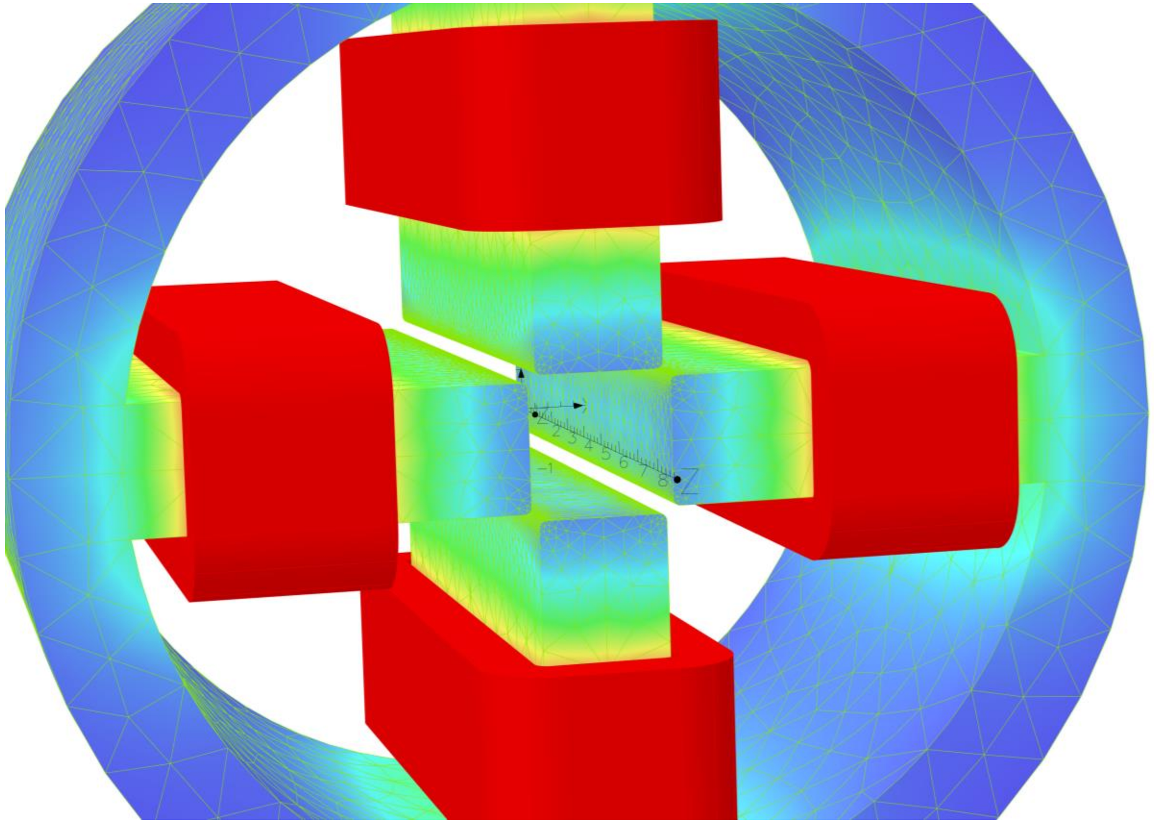
Scalar equipotential





Create and test the magnets





*Work in progress.

Progress so far

- We've now progressed from the roll-off function (multipole strength) to the fields, the scalar potential and finally to the pole face shape.
- Process is slow, a faster solution from roll-off to pole face would have significant benefits.

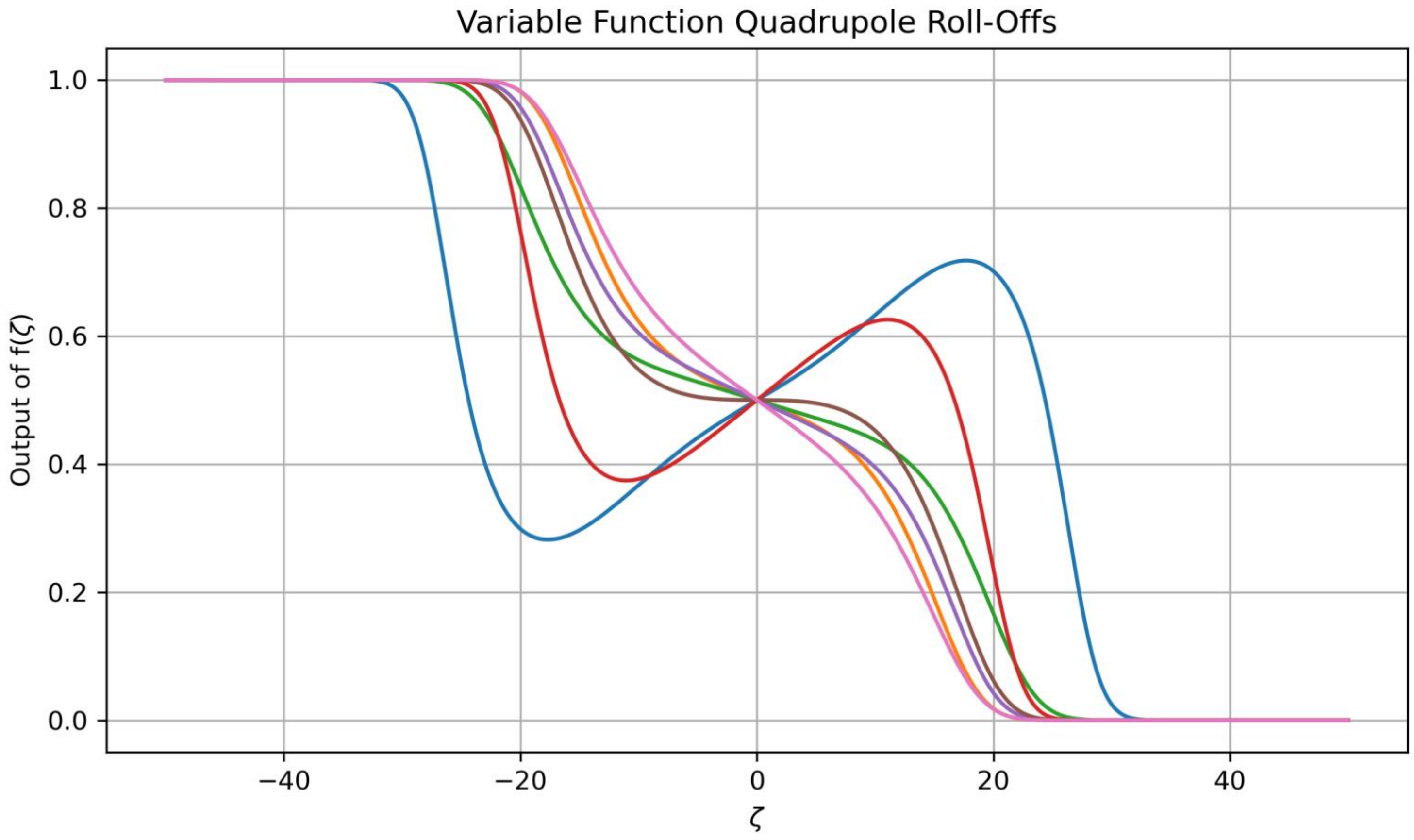
Machine learning applications

The goal, use machine learning to move directly between the roll-off function and the pole face shape.

1. Modify the Enge function to produce many field roll-offs that take different forms.
2. Create the associated scalar potential for each given roll-off.
3. Train a neural network to learn and apply the underlying patterns of this relationship.

Using machine learning saves time and allows for quick computation of roll-offs beyond the Enge.

The variable Enge-type roll-off



The shape of these roll-offs are manipulated by altering three Enge coefficients, A, B and C.

Order of machine learning processes

Enge coefficients, A, B and C.

Order of machine learning processes

Enge coefficients, A, B and C.

ML model ↓

Roll-off shape (field gradient at entrance / exit).

Order of machine learning processes

Enge coefficients, A, B and C.

ML model ↓ ↑ ML model

Roll-off shape (field gradient at entrance / exit).

Order of machine learning processes

Enge coefficients, A, B and C.

ML model ↓ ↑ ML model

Roll-off shape (field gradient at entrance / exit).

ML model ↓

Scalar potential.

Order of machine learning processes

Enge coefficients, A, B and C.

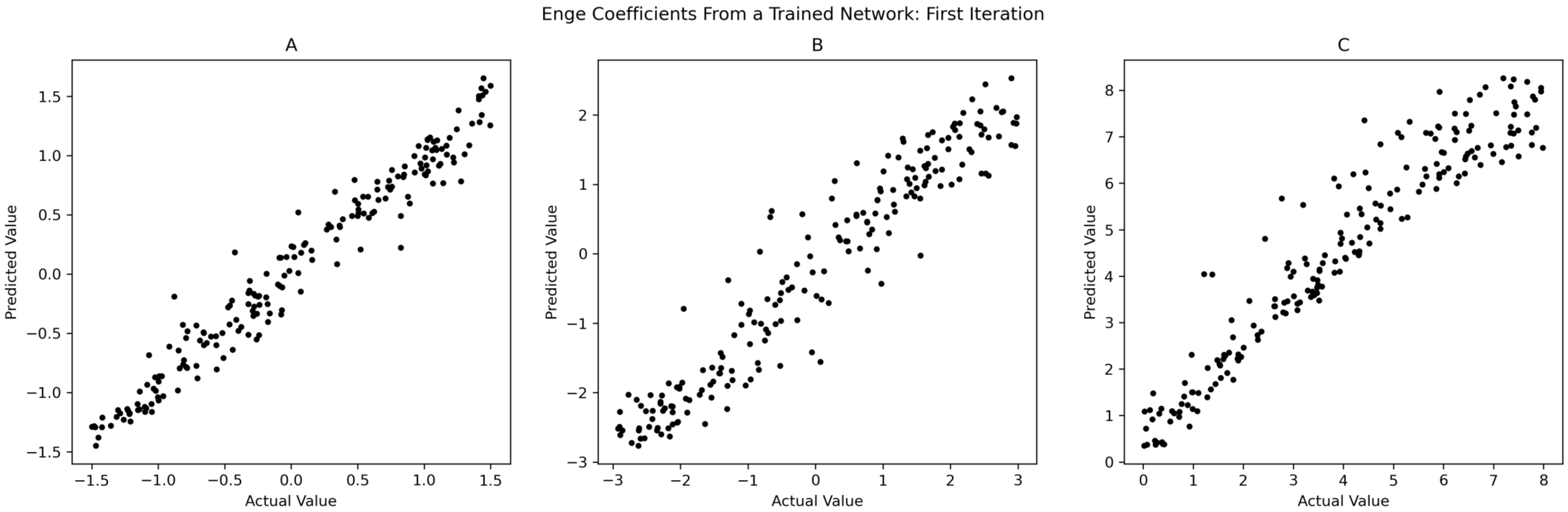
ML model ↓ ↑ ML model

Roll-off shape (field gradient at entrance / exit).

ML model ↓ ↑ ML model

Scalar potential.

Predicting Enge coefficients from the roll-off

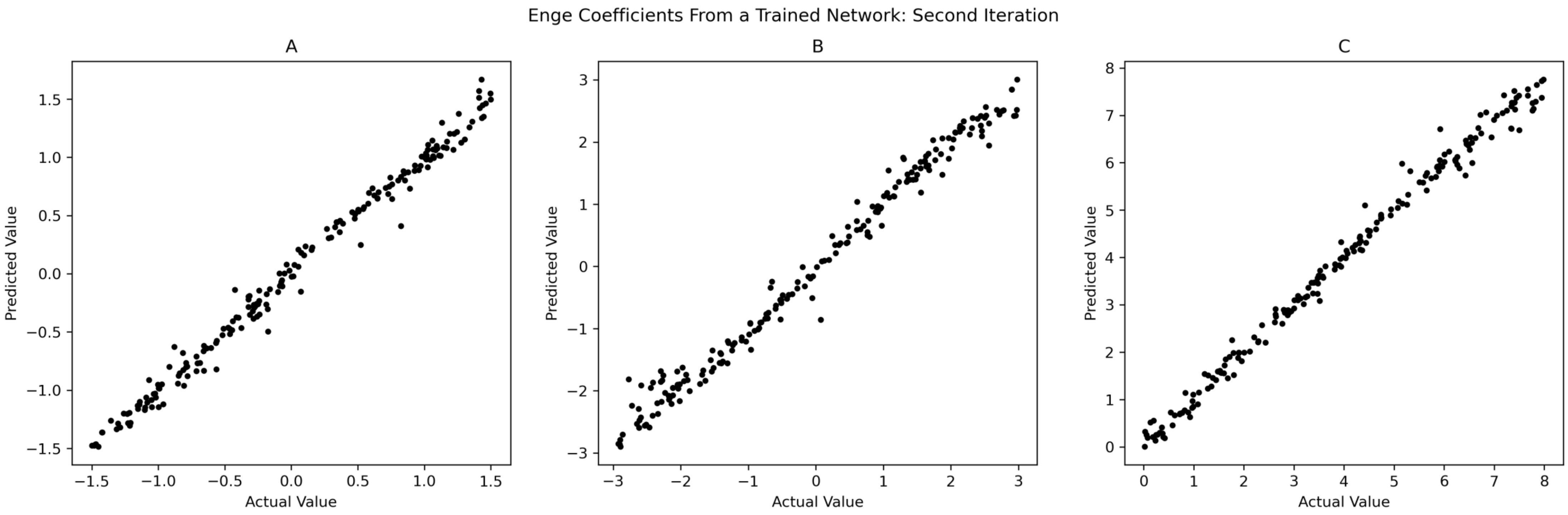


*Work in progress.

Enge coefficients, A, B and C.

Roll-off shape (field gradient at entrance / exit).

Predicting Enge coefficients from the roll-off



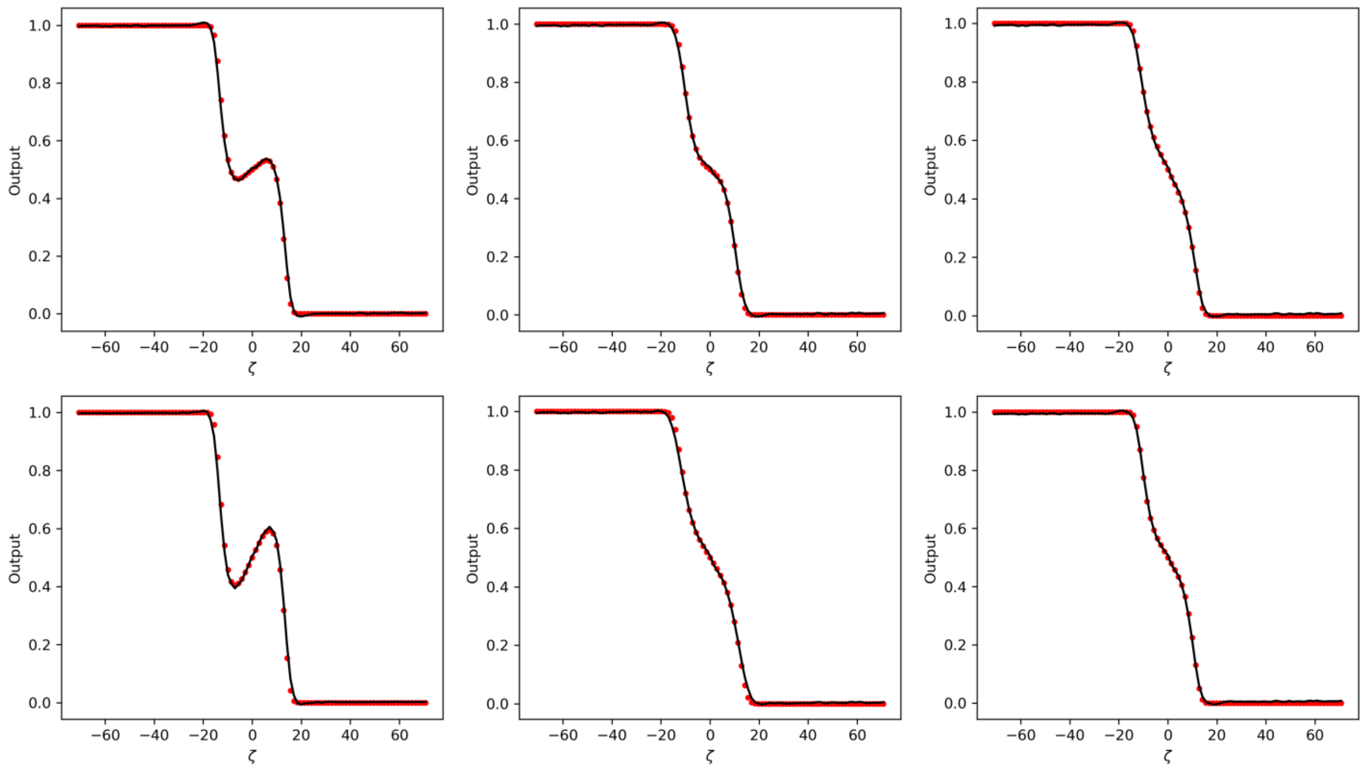
*Work in progress.

Enge coefficients, A, B and C.

ML model

Roll-off shape (field gradient at entrance / exit).

Predicting the roll-off from the coefficients



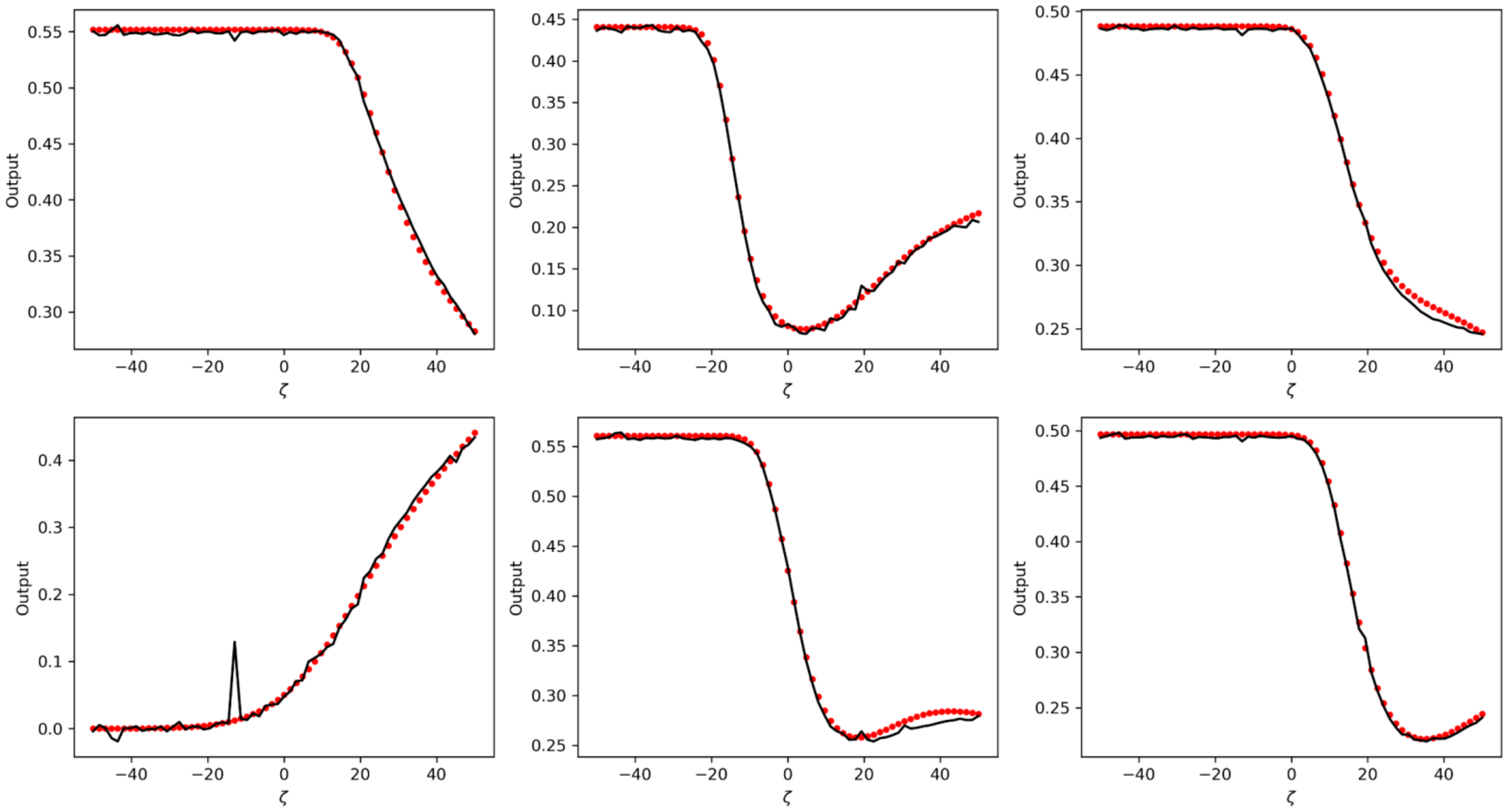
*Work in progress.

Enge coefficients, A, B and C.

ML model

Roll-off shape (field gradient at entrance / exit).

Predicting the roll-off from the scalar potential



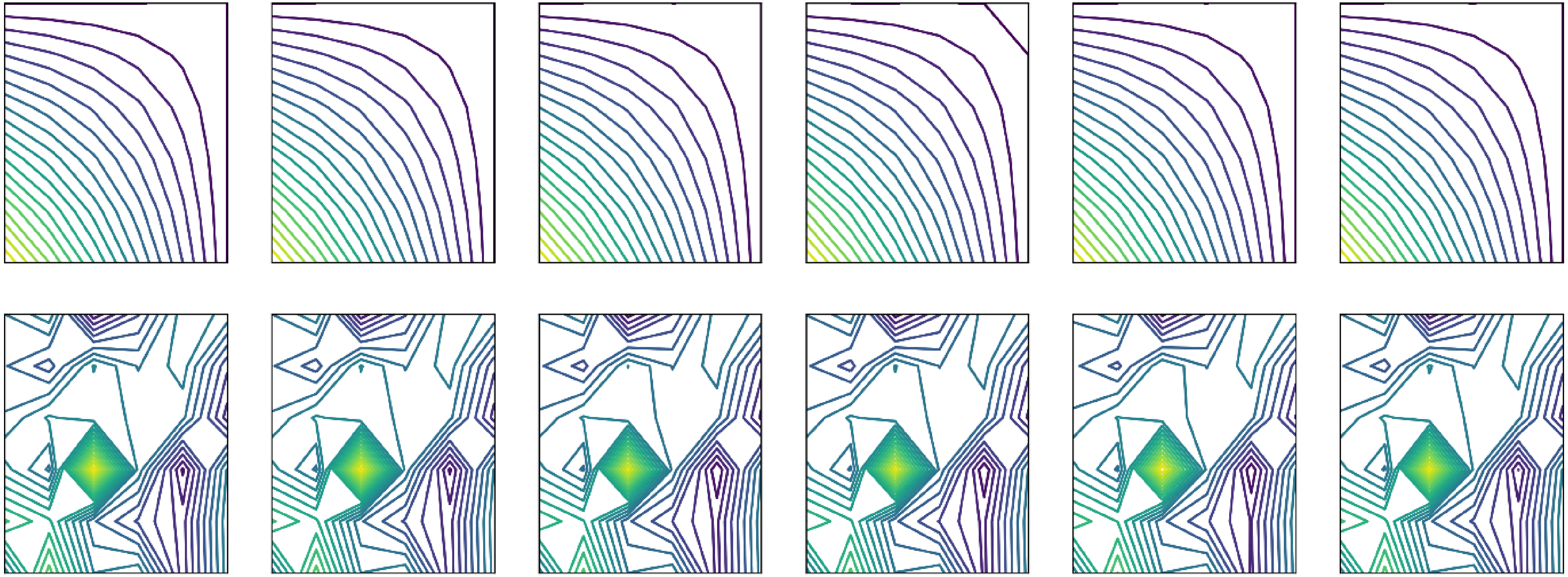
*Work in progress.

Roll-off shape (field gradient at entrance / exit).

↑ ML model

Scalar potential.

Optimising the scalar potential model



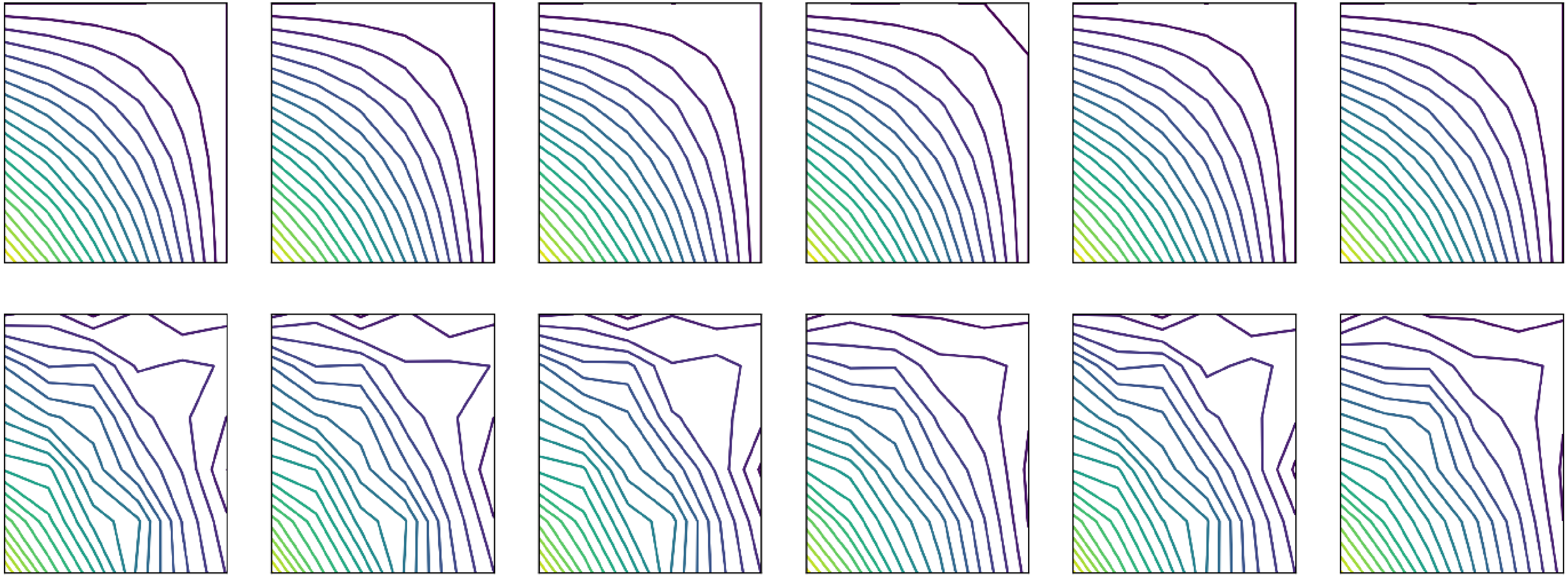
*Work in progress.

Roll-off shape (field gradient at entrance / exit).

ML model

Scalar potential.

After some more optimisation

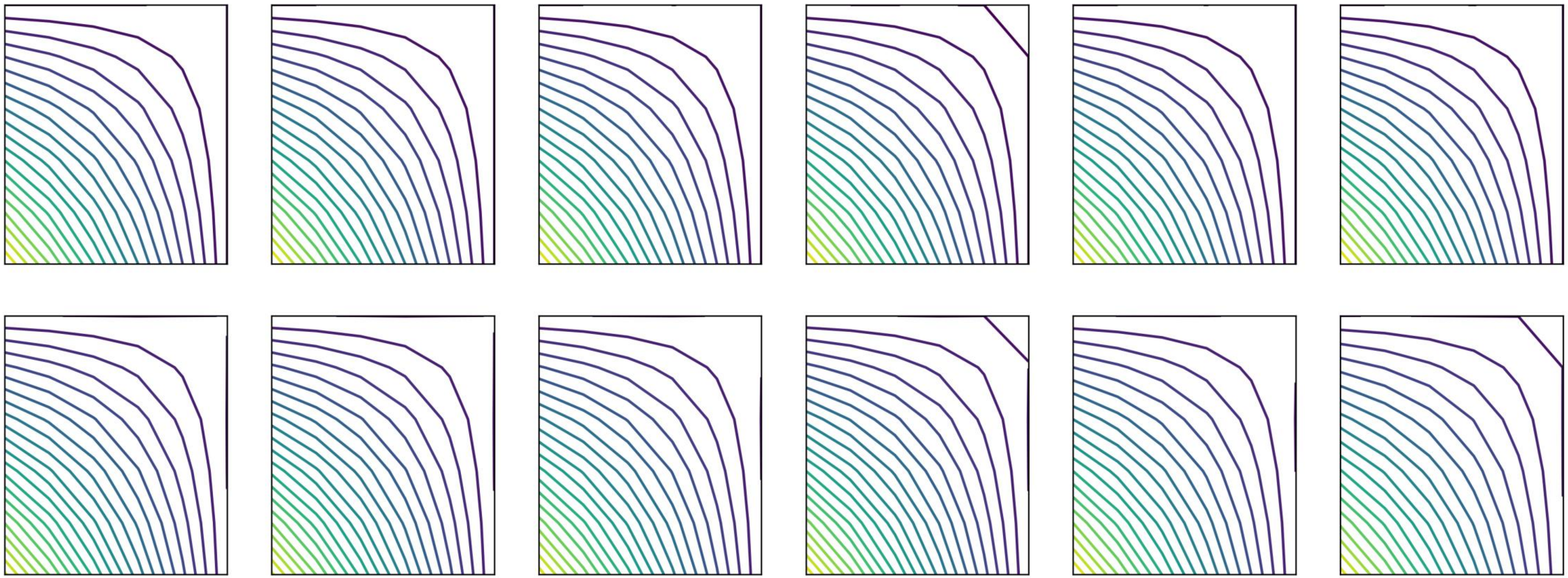


*Work in progress.

Roll-off shape (field gradient at entrance / exit).

ML model

Scalar potential.



*Work in progress.

Roll-off shape (field gradient at entrance / exit).

ML model

Scalar potential.

Beam dynamics and future study

The next step is to model the beam dynamics.

Including beam dynamics in the set of machine learning tools will allow rapid optimisation of magnet geometry for specified dynamical properties.

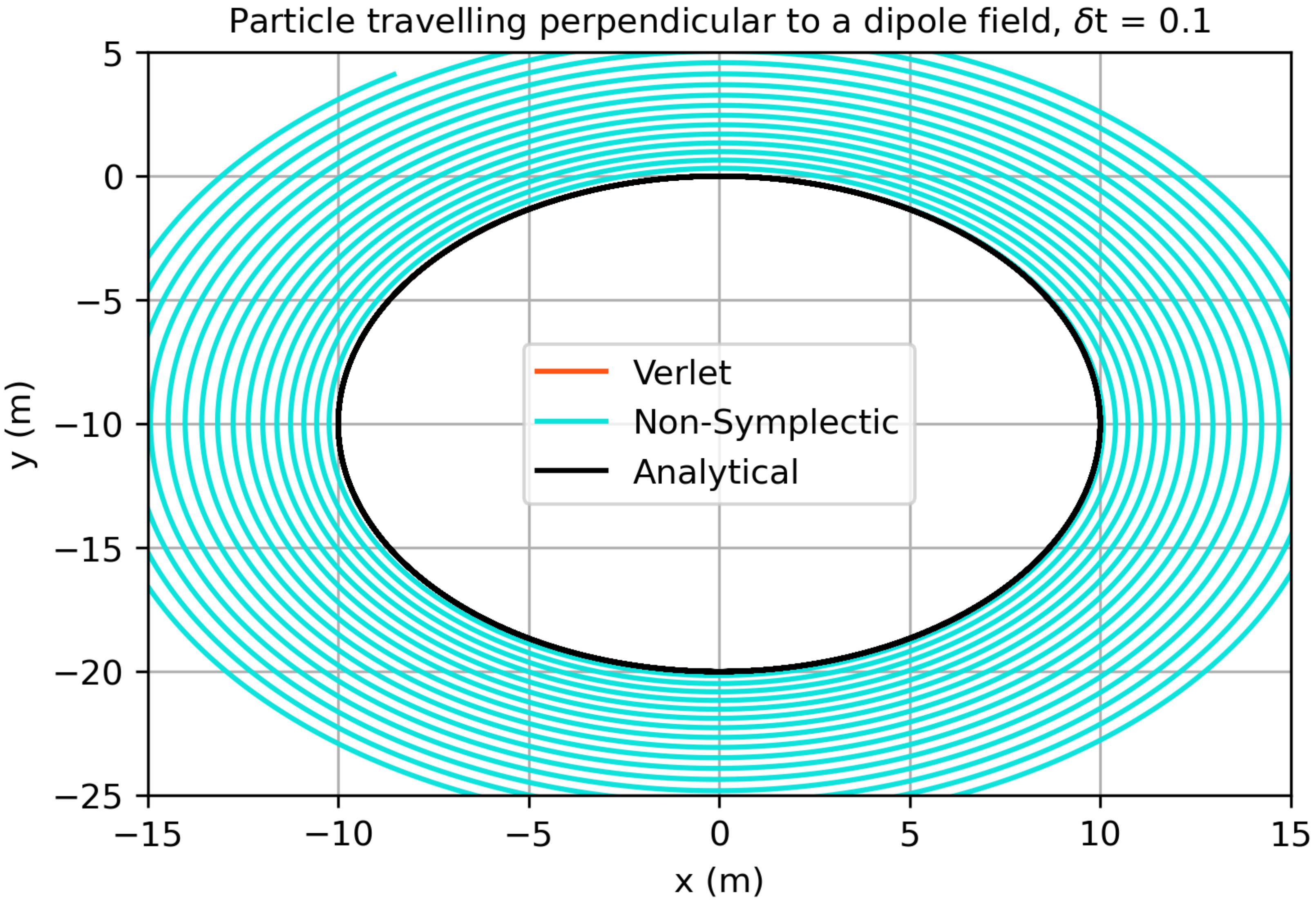
Modelling the beam dynamics requires implementation of an integrator (particle tracker).

Magnetic velocity Verlet method

- The force exerted by a magnetic field on a charged particle is velocity dependent. Think back to the Lorentz force equation.

- An integrator has been specifically created for this situation.
  - "A Magnetic Velocity Verlet Method." Chambliss, A. Franklin, J (2020).

- The magnetic velocity Verlet method is symplectic.



*Work in progress.

Future study

- Use of the particle tracker is yet to be fully implemented and optimised.

- The transfer functions through the fringe fields are to be constructed from the particle tracker.

- Ultimate goal – create a magnet quickly that satisfies specified beam dynamics.

Thank you for your time!

A special thank you to

Andrzej Wolski
Bruno Muratori
Tessa Charles
Alex Bainbridge
Amelia Pollard

Appendix A – Non symplectic integrator

$$x_{n+1} = x_n + \dot{x}_n \Delta t$$

$$a_n = \frac{q}{m}(\dot{x}_n \times B)$$

$$\dot{x}_{n+1} = \dot{x}_n + a_n \Delta t$$

Appendix B – Verlet integrator

$$x_{n+1} = x_n + \dot{x}_n \Delta t + \frac{1}{2m} F_n \Delta t^2$$

$$\dot{x}_{n+1} = \dot{x}_n + \frac{1}{2m} F_{n+1} \Delta t^2$$

We don’t have access to the force value at the new position without velocity (Lorentz force).

Appendix C – Velocity Verlet integrator

Consider,

$$\dot{x}_{n+1} = \dot{x}_n + \frac{q\Delta t}{2m}((\dot{x}_n \times B_n) + (\dot{x}_{n+1} \times B_{n+1}))\Delta t^2$$

It can be shown that,

$$\dot{x}_{n+1} = \frac{1}{1 + (\frac{q\Delta t}{2m})^2 C \cdot C}[d + \frac{q\Delta t}{2m} d \times C + \left(\frac{q\Delta t}{2m}\right)^2 C(d \cdot C)]$$

Where,

$$C = B_{n+1}$$

$$d = (\dot{x}_n + \frac{q\Delta t}{2m}\dot{x}_n \times B_n)$$