CI-Beam-105

Lattice Design and Computational Dynamics
Tutorial I

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In this exercise...

- Design a machine for protons at a momentum of 20 GeV/c with the following basic parameters:
  - Circumference = 1000 m
  - Quadrupole length Lq = 3.0 m
  - Ring consists of 8 FODO cells
  - Dipole length is 5 m and maximum field strength is 3 T
- Use your lecture note for lattice design:
  - Define a FODO lattice according to the boundary conditions (position of dipole and quadrupoles).
  - Find the optics (strength of dipoles and quadrupoles) so that $\beta max=300$ m.
  - Implement it in MADX format using thin lenses for all elements and verify your calculations.

- **How many dipole magnets should you use per cell?** (Hint, think about the bending angle of a single dipole first.)
- **Calculate the phase advance per cell.**
- **Calculate the quadrupole strengths and the focal lengths.**

Exercise 1

- Step 1

Given:

Circumference = 1000 m

Quad length = 3.0 m

8 FODO cells

Dipole length = 5 m

Maximum dipole field = 3 T

- How many dipole magnet should you use per cell?

Exercise 1

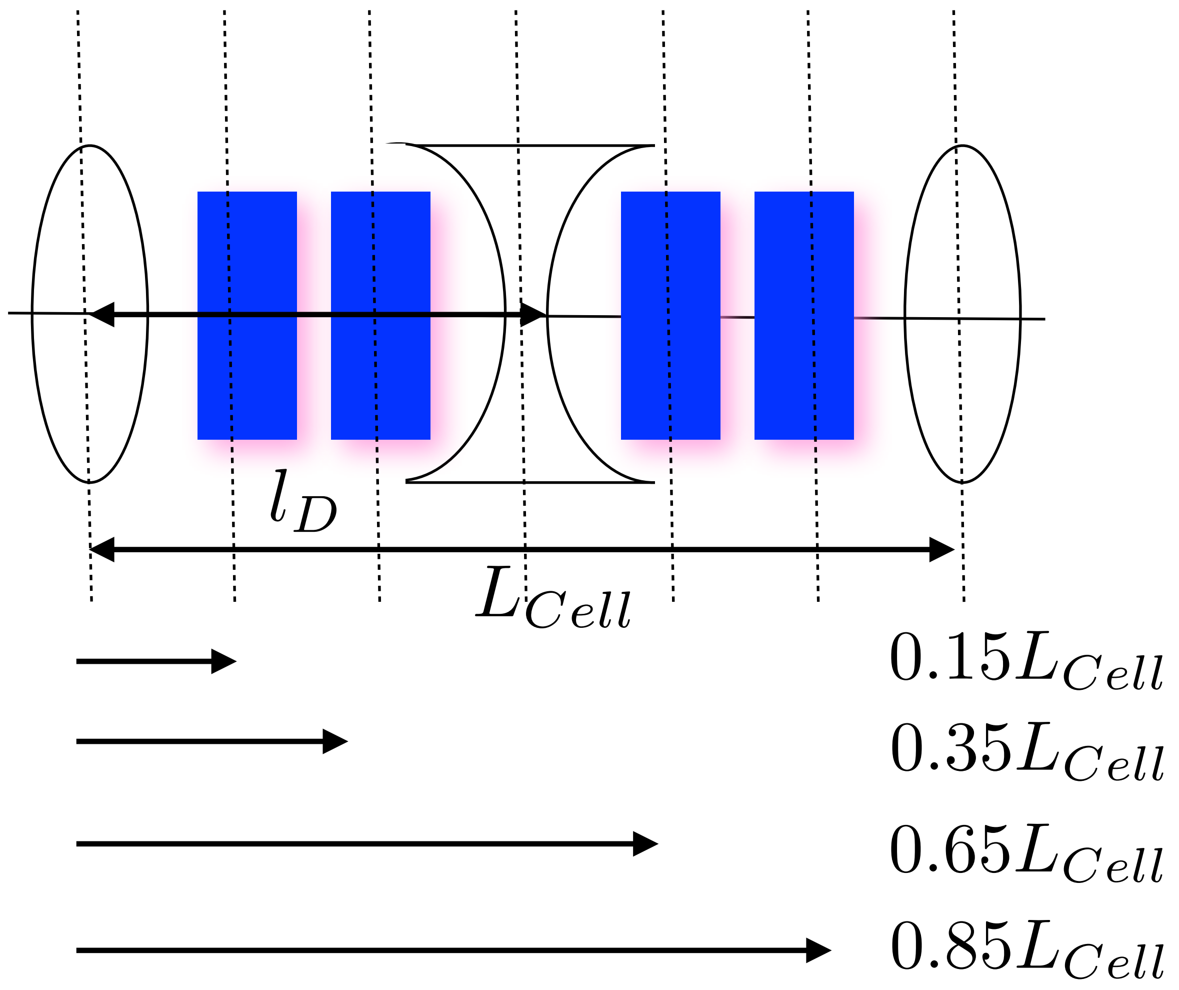
- **Step 1**

$$\alpha = \frac{1}{\rho}(m^{-1}) = 0.3\frac{B(T)}{p(GeV/c)}L(m)$$

$$\alpha = 0.3\frac{3(T)5(m)}{20(GeV/c)} = 0.225(rad)$$

Total number of dipoles per cell $= \dfrac{2\pi}{0.225} = 28 \rightarrow 3.5 \approx 4/Cell$

- **Step 2-** Define the lattice / positions of dipoles and quadrupoles
  - Adjust the dipoles so that they will be all at an equal distance from each other.



Exercise 1

- **Step 3-** Find the optics (dipole and quadrupole strengths) providing a maximum beta function of 300 m.

Given:
Circumference = **1000 m**
Quad length = **3.0 m**
8 FODO cells
Dipole length = **5 m**
Maximum dipole field = **3 T**

Condition:

$$\beta_{max} \equiv \hat{\beta} \approx 300m$$

- **How many dipole magnet should you use per cell??**
    - 32 dipole magnets / 4 per cell.
- **Calculate the phase advance per cell.**

- **Step 3-** Find the optics (dipole and quadrupole strengths) providing a maximum beta function of 300 m.

Cell length

$$L_{Cell} = \frac{1000m}{8} = 125$$

Max beta function for FODO cell

$$\hat{\beta} = \frac{(1 + sin\mu/2)L_{Cell}}{sin\mu} = 300m$$

Find the phase advance

$$\frac{\hat{\beta}}{L_{Cell}/2} = \frac{1 + sin\frac{\mu}{2}}{sin\frac{\mu}{2}cos\frac{\mu}{2}} = \frac{300}{62.5} = 4.8$$

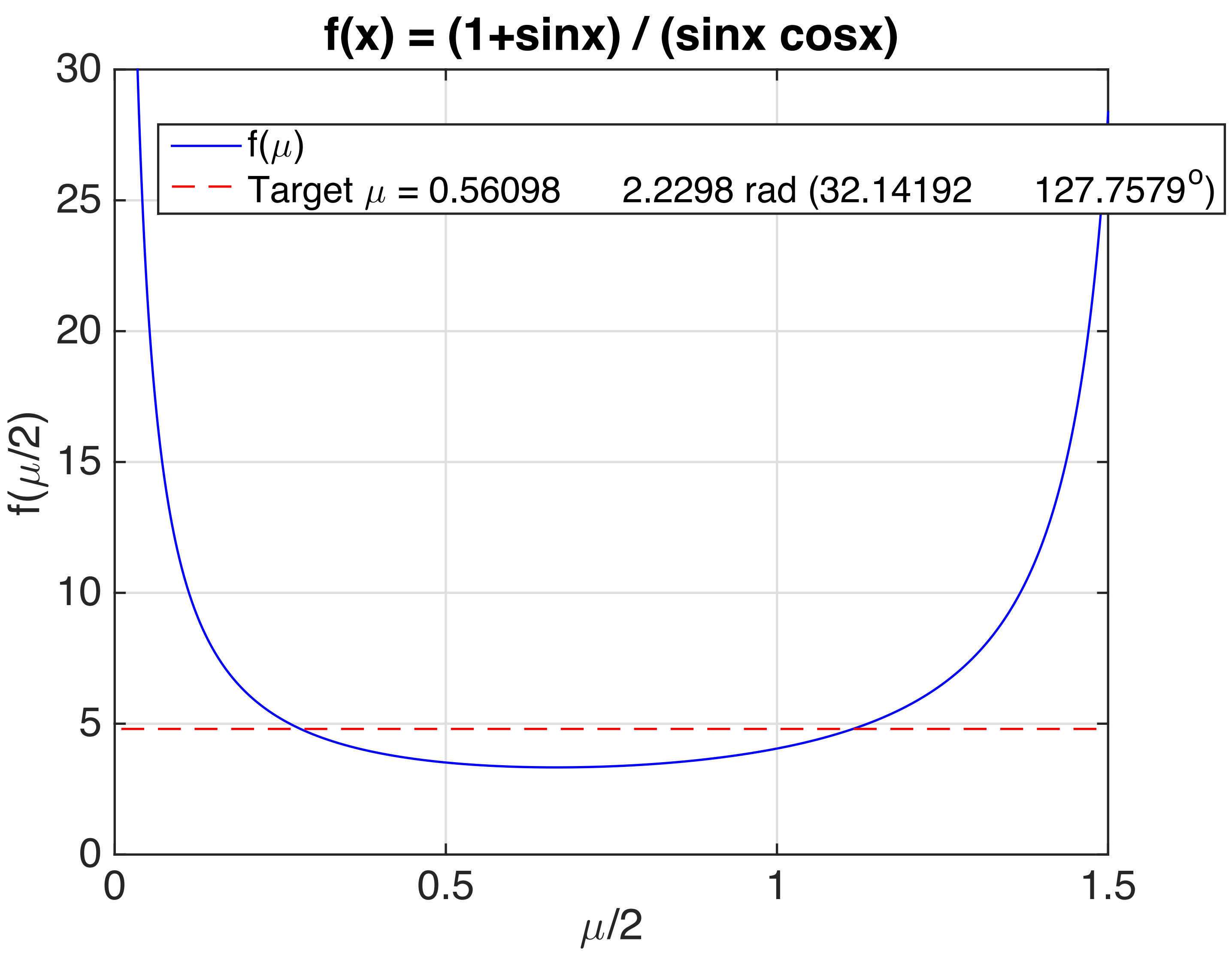
But how?

- **Step 3-** Find the optics (dipole and quadrupole strengths) providing a maximum beta function of 300 m.

Calculate the phase advance.

$$\frac{\hat{\beta}}{L_{1/2}} = \frac{1+sin\frac{\mu}{2}}{sin\frac{\mu}{2}cos\frac{\mu}{2}}$$

- Following an indirect method, find the solution for f(μ/2) for a μ/2 satisfying βmax/L1/2.



Phase advance for one cell

$$\mu = 32.1^o$$

- **Step 3-** Find the optics (dipole and quadrupole strengths) providing a maximum beta function of 300 m.

Given:

Circumference = **1000 m**

Quad length = **3.0 m**

8 FODO cells

Dipole length = **5 m**

Maximum dipole field = **3 T**

Condition:

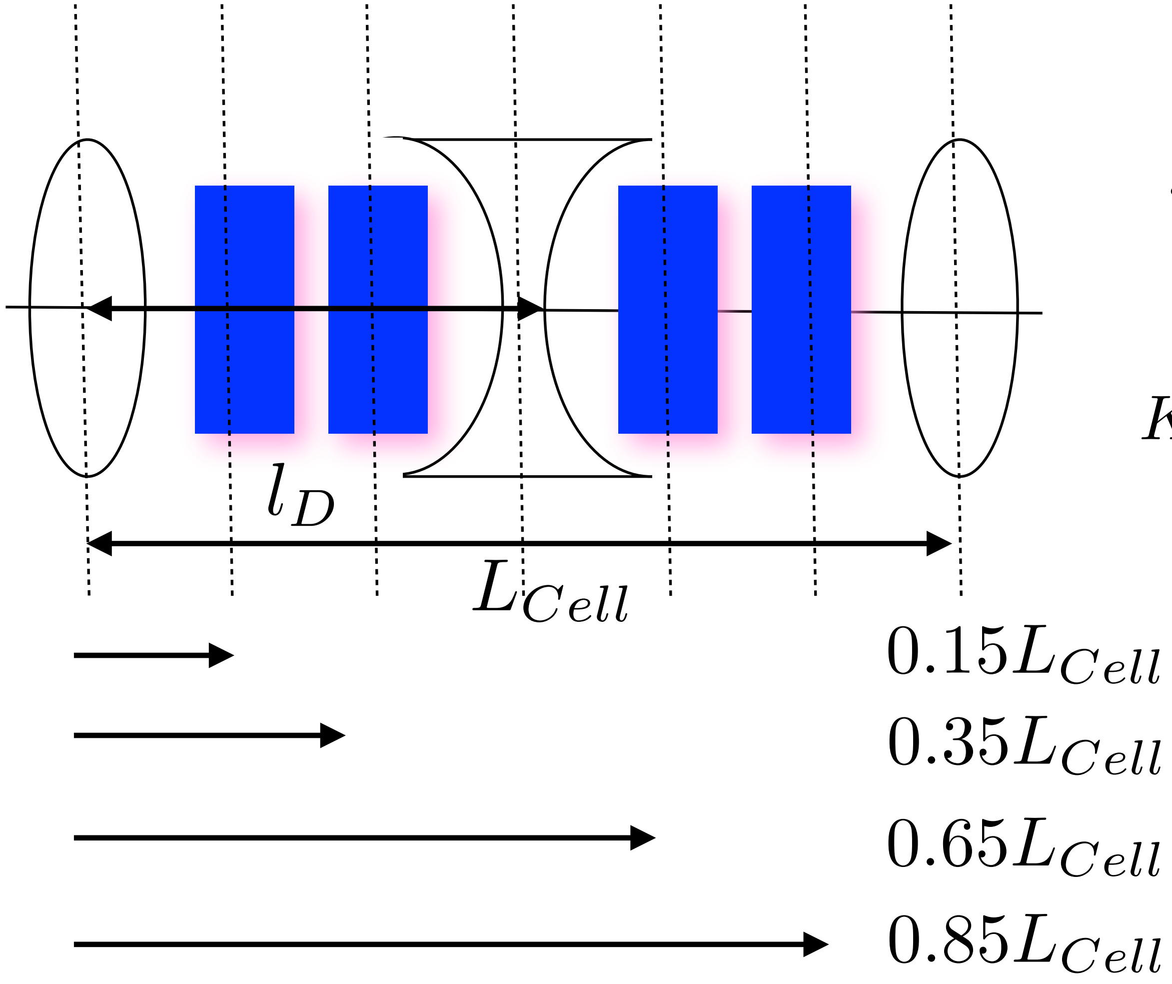
$$\beta_{max} \equiv \hat{\beta} \approx 300m$$

- **How many dipole magnet should you use per cell??**
  - 32 dipole magnets / 4 per cell.
- **Calculate the phase advance per cell.**
  - Cell length 125m and the phase advance is 32.14°.
- **Calculate the quadrupole strengths and the focal lengths.**

- **Step 3-** Find the optics (dipole and quadrupole strengths) providing a maximum beta function of 300 m.

- **Remember the equation relating the phase advance and the focal length of a quadrupole magnet for a FODO cell:**

$$sin\frac{\mu}{2} = \frac{L_{Cell}}{4f_Q} \qquad f_Q = \frac{1}{k_Q l_Q}$$



$$f_Q = \frac{L_{Cell}}{4sin\frac{\mu}{2}} = \frac{125m}{4 * sin(16.07^o)} = 112.9$$

$$K_Q = \frac{1}{f_Q lQ} = \frac{1}{112.9m * 3m} = 2.9 * 10^{-3}\, m^{-2}$$

- **Step 3-** Find the optics (dipole and quadrupole strengths) providing a maximum beta function of 300 m.

Given:

Circumference = **1000 m**

Quad length = **3.0 m**

8 FODO cells

Dipole length = **5 m**

Maximum dipole field = **3 T**

Condition:

$$\beta_{max} \equiv \hat{\beta} \approx 300m$$

- **How many dipole magnet should you use per cell??**
  - 32 dipole magnets / 4 per cell.
- **Calculate the phase advance per cell.**
  - Cell length 125m and the phase advance is 32.14°.
- **Calculate the quadrupole strengths and the focal lengths.**
  - Quadrupole focal length is 112.9m and quad strength is 2.9×10^{-3} m^{-2}.

- **Step 4-** Implement it in MADX format using thin lenses for all elements and verify your calculations.

- Prepare the sequence file (**ex1.seq**).
- Prepare MADX commands file (**ex1.madx**).
- Run your file from the command line: **madx < ex1.madx**

- **Step 4-** Implement it in MADX format using thin lenses for all elements and verify your calculations.

```
circum=1000.0;
ncell =  8; // Number of cells
lcell = circum/ncell;
lq = 3.00; // Quadrupole length

// Define the elements.

// Define the bending magnets as multipoles.
// There will be 4 bending magnets per cell.
mb: multipole,knl={2.0*pi/(4*ncell)};

// Define the quadrupoles as multipoles.
qf: multipole,knl={0,2.9e-3*lq};
qd: multipole,knl={0,-2.9e-3*lq};

// Define the sequence.
ci_cell1: sequence, refer=centre, l=circum;
start_machine: marker, at = 0;
!
   n = 1;
   while (n < ncell+1) {
   qf: qf,    at=(n-1)*lcell;
   mb: mb,    at=(n-1)*lcell+0.15*lcell;
   mb: mb,    at=(n-1)*lcell+0.35*lcell;
   qd: qd,    at=(n-1)*lcell+0.50*lcell;
   mb: mb,    at=(n-1)*lcell+0.65*lcell;
   mb: mb,    at=(n-1)*lcell+0.85*lcell;
!
   n = n + 1;
}
end_machine: marker, at=circum;
endsequence;
```

- Prepare the sequence file (**ex1.seq**).

- **Step 4-** Implement it in MADX format using thin lenses for all elements and verify your calculations.

  - Prepare MADX commands file (**ex1.madx**).

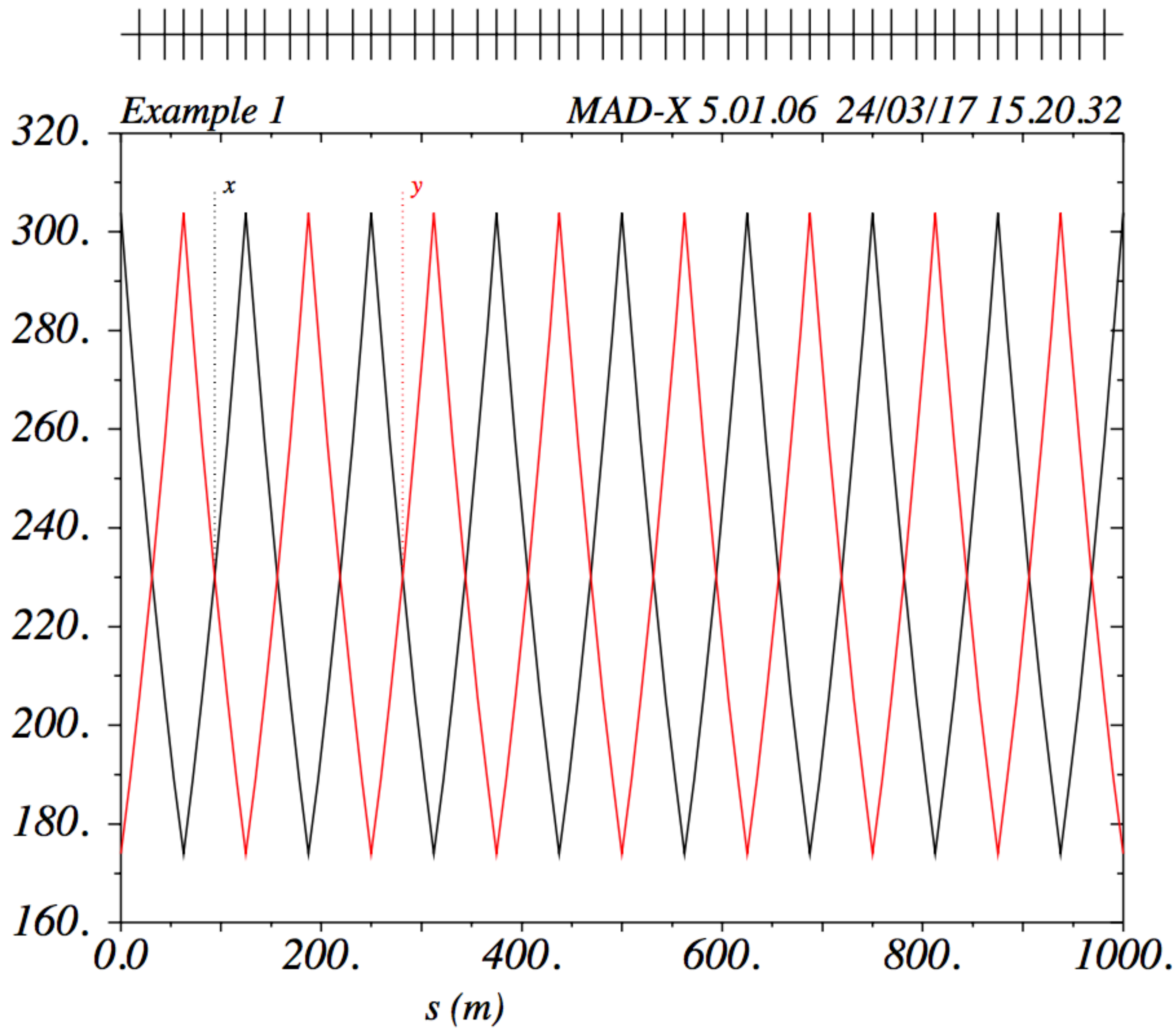
```
TITLE, 'Example 1';
call file="ex1.seq";
option,-echo;

Beam, particle = proton, sequence= ci_cell1, energy =  20.0;

use, sequence= ci_cell1;
!select,flag=twiss,pattern="^q.*",column=name,s,x,y,mux,betx,
!                          muy,bety,dx,dy;
select,flag=twiss,column=name,s,betx,bety;

twiss,save,centre,file=twiss.out;
plot, haxis=s, vaxis=betx, bety, colour=100;

Survey,file=survey.out;

stop;
```

- **Step 4-** Implement it in MADX format using thin lenses for all elements and verify your calculations.

```
++++++ table: summm

           length            orbit5              alfa           gammatr
             1000                -0       2.063189563       0.696194253

               q1               dq1           betxmax             dxmax
     0.7011498128     -0.7202158003       303.8295337       377.6019552

            dxrms            xcomax            xcorms                q2
      333.2951861                 0                 0      0.7011498128

              dq2           betymax             dymax             dyrms
    -0.7202158003       303.8295337                 0                 0

           ycomax            ycorms            deltap           synch_1
                0                 0                 0                 0

          synch_2           synch_3           synch_4           synch_5
                0                 0                 0                 0
```

command line output

- **Step 4-** Implement it in MADX format using thin lenses for all elements and verify your calculations.

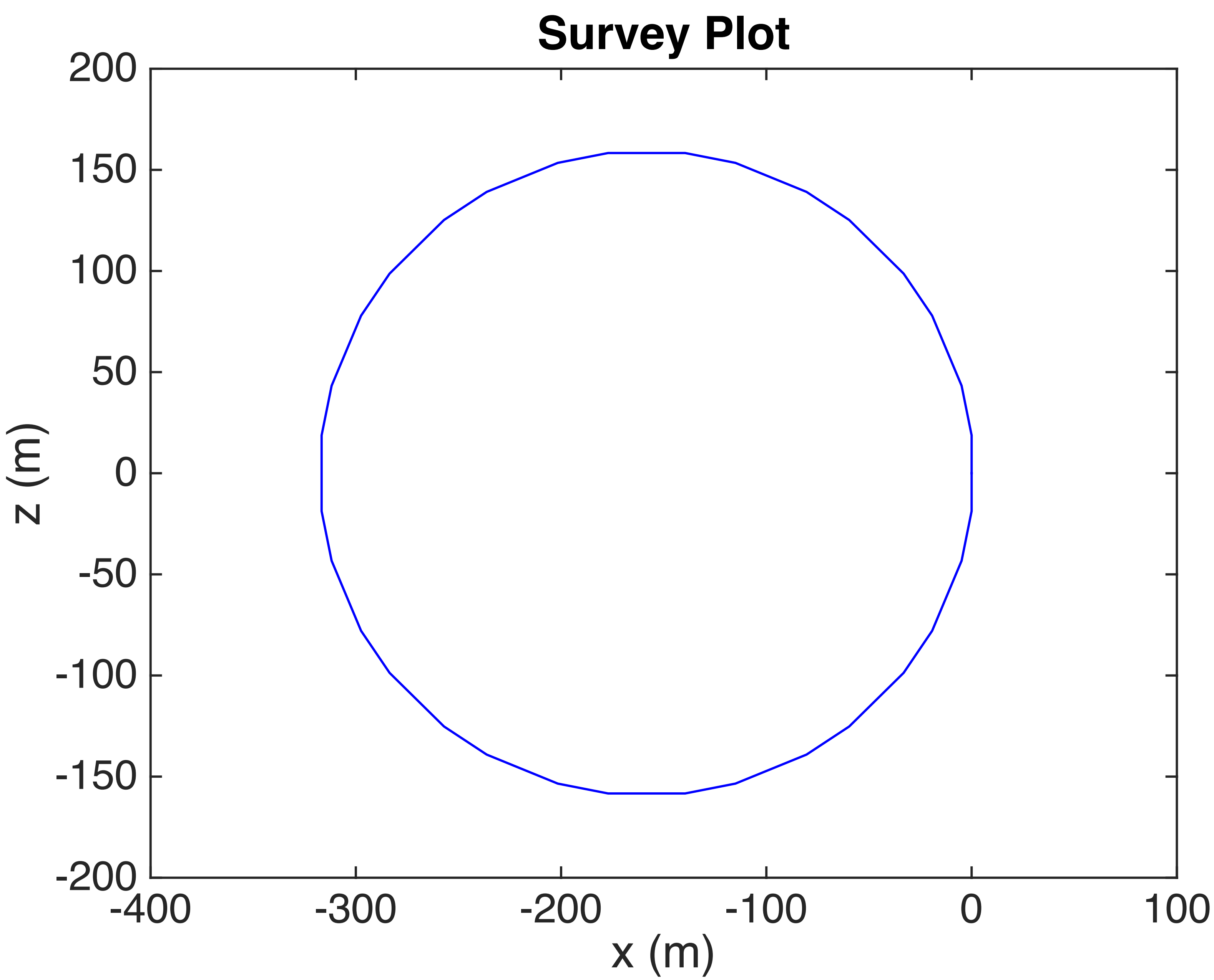


Plot the dispersion function for your ring.

madx.ps

- **Step 4-** Implement it in MADX format using thin lenses for all elements and verify your calculations.

survey.out

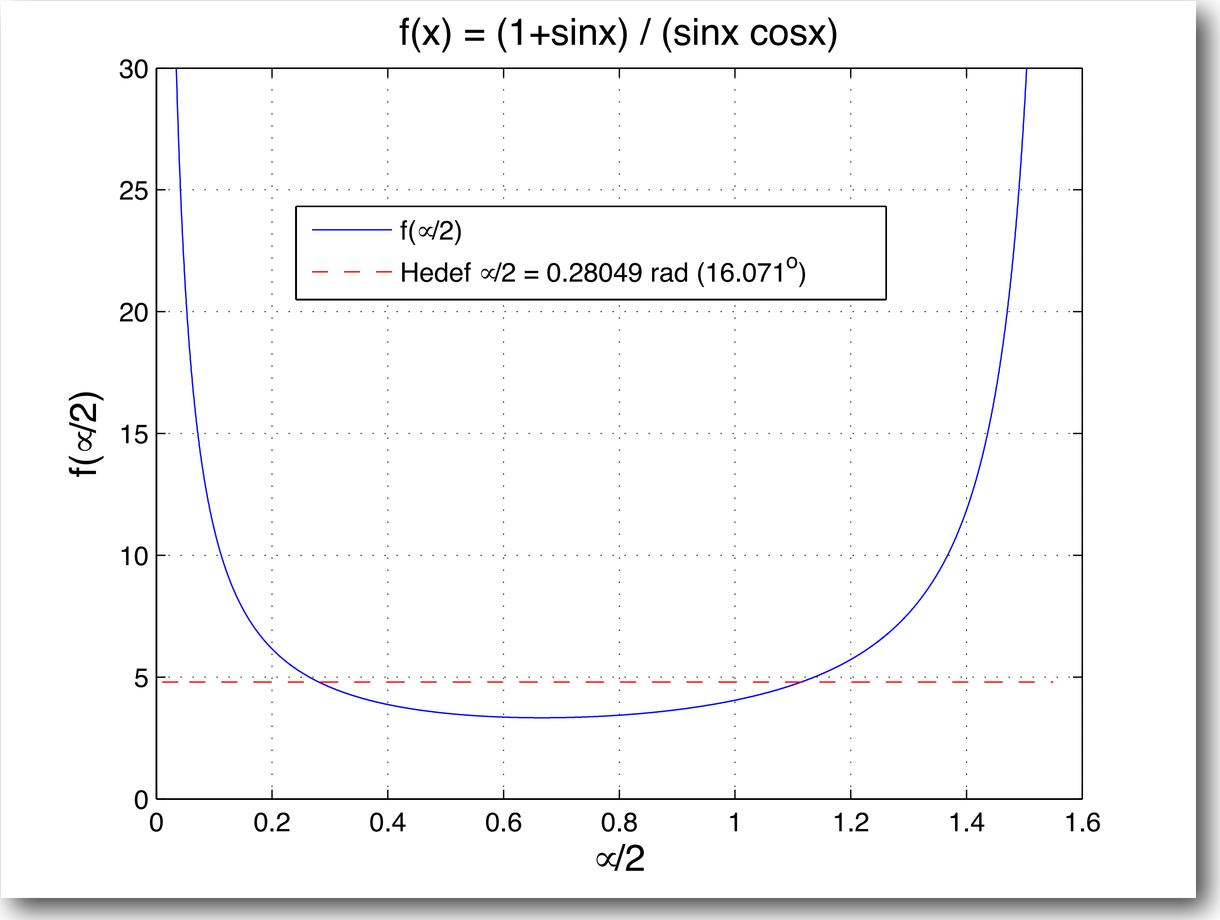


- **Step 5-** Let's change beta function from 300m to 100m. Hint: use 20 cells.

$$\hat{\beta} = 300m \rightarrow \hat{\beta} = 100m$$

$$L_{Cell} = \frac{1000m}{20} = 50m$$

$$\frac{\hat{\beta}}{L_{1/2}} = \frac{1 + sin\frac{\mu}{2}}{sin\frac{\mu}{2}cos\frac{\mu}{2}} = \frac{100m}{25m} = 4$$



$$f_Q = \frac{L_{Cell}}{4sin\frac{\mu}{2}} = \frac{50m}{4sin(21.5^o)} = 34m$$

$$k_Q = \frac{1}{f_Q l_Q} = \frac{1}{34m3m} = 9.8 * 10^{-3}$$

Exercise 1

- **Step 5-** Let's change beta function from 300m to 100m. Hint: use 20 cells.

```
++++++ table: summm

          length            orbit5              alfa           gammatr
            1000                -0      0.1789932276       2.363642011

              q1               dq1           betxmax             dxmax
     2.395724244      -2.518384148       100.0267292       34.45748961

           dxrms            xcomax            xcorms                q2
     29.03430831                 0                 0       2.395724244

             dq2           betymax             dymax             dyrms
    -2.518384148       100.0267292                 0                 0

          ycomax            ycorms            deltap           synch_1
               0                 0                 0                 0

         synch_2           synch_3           synch_4           synch_5
               0                 0                 0                 0
```

command line output