## Exercise 1. Using your spacetime rulers



Measure the lengths of the following vectors.
Are they timelike, spacelike or lightlike?
Do the first point lie in the future or past of the second?

| From | To | Timelike? <br> Lightlike? <br> or Spacelike? | Future? <br> or Past? | Length |
| :--- | :--- | :--- | :--- | :--- |
| O | A |  |  |  |
| O | B |  |  |  |
| O | C |  |  |  |
| A | B |  |  |  |
| B | C |  |  |  |
| A | C |  |  |  |

If you want to use the formula. The positions of the points are $\mathrm{A}(-3,4.2), \mathrm{B}(5.7,4.8)$ and $\mathrm{C}(2,-2)$.

## Exercise 2. Alfred and Beatrix. The two twin paradox



Start of journey (S)
(a) Measure the time Alfred has aged.
(b) Measure the time Beatrix has aged.
(c) Draw alternative worldines connecting the start and end of the journey. How do the times compare to that to Alfred?
(Do no go faster than light!)
(d) What is the shortest worldline you can draw which connect (S) and (E)?

For those wanting to use formula, the positions of the points are: The position of the top of the red line $(0,8)$ and the position of the bend $(2.7,4)$.


Figure 1: The spacetime diagram representing simultaneous events.

## Exercise 3.

Figure 1, shows a spacetime diagram. According to observer 1, events A and B are simultaneous.
(a) Show that according to observer 2 , event B occurs before event A .
(b) Draw the axes of another observer who would measure event A before event B.


Figure 2: The spacetime diagram represents the "paradox" of time dilation

## Exercise 4.

In figure 2 observer 2 , is moving at a speed $\frac{\sqrt{5}}{9} c \approx .745 c$ where $c$ is the speed of light. The axes measure seconds with respect to both observers.
(a) Show that when observer 2 measures 1 second on her timeline, observer 1 measures 1.5 seconds. Therefore observer 1 thinks observer 2 clock is slow.
(b) When observer 1 measures 1 second on his timeline, what time does observer 2 measure. Does observer 2 think observer 1 clock is fast or slow.
Let's say observer 1 and observer 2 want to directly compare clocks. Both send a light signal to the other after 1 second.
(c) Show that the signal from observer 2 reaches observer 1 at time 2.62 seconds, according to observer 1 clock. (The actual value is $\frac{3}{2}\left(1+\frac{\sqrt{5}}{3}\right.$.)
(d) What time, according to observer 2 clock, does the signal from observer 1 arrive?

## Exercise 5.

In figure 3:
(a) Identify the events on the diagram $\mathrm{A}, \ldots, \mathrm{J}$ which represents the following events.

- The moment when the front of the car passes side 1 of the garage.
- The moment when the front of the car passes side 2 of the garage.
- The moment when the rear of the car passes side 1 of the garage.
- The moment when the rear of the car passes side 2 of the garage.
(b) Does the front of the car pass side 2 of the garage before or after rear of the car passes side 1 of the garage:
- According to the garage observer.
- According to the car observer.
(c) Identify two events which represents the length of the garage according to the garage observer. Identify two events which represents the length of the car according to the garage observer. Show that the garage observer measures the car as shorter than the garage.
(d) Identify two events which represents the length of the garage according to the car observer. Identify two events which represents the length of the car according to the car observer. Show that the car observer measures the garage as shorter than the car.


Figure 3: The spacetime diagram represents the "paradox" of the car and the garage.

