## High-precision measurement of the W boson mass with the CDF II detector



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## Electroweak gauge boson masses

In a gauge-symmetric theory the phase of a matter field does not affect physical processes

$$
\begin{gathered}
\text { QED: } \mathscr{L}=D_{\mu} \phi D^{\mu} \phi-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}, \\
\phi^{\prime}=e^{i e \alpha(x)} \phi, \quad A^{\prime \mu}=A^{\mu}-\partial^{\mu} \alpha(x) \\
\mathscr{L}^{\prime}=\mathscr{L}
\end{gathered}
$$

The gauge field transports the matter field according to the gauge field strength (curvature)


The gauge symmetry allows a choice of axes that removes the phase

## Electroweak gauge boson masses

The weak gauge symmetry $\operatorname{SU}(2)$ transports the phase through three fields $W_{1}, W_{2}, W_{3}$

$$
\mathscr{L}=D_{\mu} \phi^{a} D^{\mu} \phi^{a}+\mu^{2} \phi^{\dagger} \phi-\lambda\left(\phi^{\dagger} \phi\right)^{2}-\frac{1}{4} F_{\mu \nu}^{a} F^{a \mu \nu}
$$




The expectation value of the scalar (Higgs) field creates an energetically favorable direction Still free to choose coordinates but perpendicular oscillations raise the gauge field energy Results in massive gauge bosons with a scalar (spin-0) component


## Higgs boson mass

Higgs field potential


$$
\begin{gathered}
m_{H}=v \sqrt{2 \lambda}=125 \mathrm{GeV} \\
\lambda \approx 0.1
\end{gathered}
$$

Quantum corrections


Naively integrating to a cutoff scale $\Lambda$ :

$$
\Delta m_{H}=\frac{3 g^{2} m_{t}^{2}}{16 \pi^{2} m_{W}^{2}} \Lambda^{2}
$$

If there is no new physics up to scale $\Lambda$ then we need 'fine-tuning' to cancel the quantum corrections

1\% fine tuning: $\Lambda=6.6 \mathrm{TeV}$
Motivates TeV-scale new physics

## Electroweak gauge boson masses

Gauge field potential
Quantum corrections

$$
\begin{aligned}
& V=-\frac{g^{2} v^{2}}{8}\left[\left(W_{\mu}^{+}\right)^{2}\right.\left.+\left(W_{\mu}^{-}\right)^{2}\right] \\
&-\frac{v^{2}\left(g^{2}+g^{2}\right)}{8} Z^{\mu} Z_{\mu} \\
& m_{W}=\frac{v}{2} g
\end{aligned}
$$



$$
m_{W}^{2}=\frac{\hbar^{3}}{c} \frac{\pi \alpha_{E M}}{\sqrt{2} G_{F}\left(1-m_{W}^{2} / m_{Z}^{2}\right)(1-\Delta r)}
$$

$$
v=246 \mathrm{GeV} \text { and } g=0.64
$$

$$
m_{W}=78.7 \mathrm{GeV}
$$

Global fit to SM measurements yields indirect W boson mass of $81354 \pm 7 \mathrm{MeV}$

## W boson mass and naturalness

The W boson mass is the most sensitive observable to sources of 'naturalness'

Classic example: Supersymmetry


Mass splittings in supersymmetric isospin doublets: different mass shifts for W \& Z bosons

## W boson mass and naturalness

Difference in corrections to $W$ and $Z$ propagators encapsulated by $\rho$ parameter

$$
\Delta \rho=\frac{\Sigma^{Z}(0)}{M_{Z}^{2}}-\frac{\Sigma^{W}(0)}{M_{W}^{2}}
$$

$\Delta \rho_{0}^{\mathrm{SUSY}}=\frac{3 G_{\mu}}{8 \sqrt{2} \pi^{2}}\left[-\sin ^{2} \theta_{\hat{t}} \cos ^{2} \theta_{\hat{t}} F_{0}\left(m_{\hat{t}_{1}}^{2}, m_{\hat{t}_{2}}^{2}\right)-\sin ^{2} \theta_{\tilde{b}} \cos ^{2} \theta_{\tilde{b}} F_{0}\left(m_{\bar{b}_{1}}^{2}, m_{\bar{b}_{2}}^{2}\right)\right.$ $+\cos ^{2} \theta_{\tilde{t}} \cos ^{2} \theta_{\bar{b}} F_{0}\left(m_{\tilde{t}_{1}}^{2}, m_{\bar{b}_{1}}^{2}\right)+\cos ^{2} \theta_{\tilde{t}} \sin ^{2} \theta_{\tilde{b}} F_{0}\left(m_{\tilde{t}_{1}}^{2}, m_{\bar{b}_{2}}^{2}\right)$ $\left.+\sin ^{2} \theta_{\tilde{t}} \cos ^{2} \theta_{\bar{b}} F_{0}\left(m_{\hat{t}_{2}}^{2}, m_{\bar{b}_{1}}^{2}\right)+\sin ^{2} \theta_{\tilde{t}} \sin ^{2} \theta_{\bar{b}} F_{0}\left(m_{\hat{t}_{2}}^{2}, m_{\bar{b}_{2}}^{2}\right)\right]$.

$$
\delta M_{W} \approx \frac{M_{W}}{2} \frac{c_{W}^{2}}{c_{W}^{2}-s_{W}^{2}} \Delta \rho
$$



## W boson mass and naturalness

More generally the SM effective field theory parameterizes high-scale effects

$$
\mathcal{L}_{S M E F T}=\mathcal{L}_{S M}+\mathcal{L}^{(5)}+\mathcal{L}^{(6)}+\mathcal{L}^{(7)}+\cdots, \quad \mathcal{L}^{(d)}=\sum_{i=1}^{n_{d}} \frac{C_{i}^{(d)}}{\Lambda^{d-4}} Q_{i}^{(d)} \quad \text { for } d>4
$$



$$
\frac{\delta m_{W}}{m_{W}}=\left(0.34 c_{H D}+0.72 c_{H W B}+0.37 c_{H l 3}-0.19 c_{l l 1}\right) \frac{v^{2}}{\Lambda^{2}}
$$

$$
\begin{aligned}
\text { For } \delta m_{W} / m_{W}= & 0.1 \% \text { and } \mathrm{c}_{\mathrm{HD}}=1, \Lambda=4.5 \mathrm{TeV} \\
& \text { e.g. } \mathrm{Z}^{\prime} \text { boson }
\end{aligned}
$$

For $\delta m_{W} / m_{W}=0.1 \%$ and $\mathrm{chws}^{\prime}=1, \Lambda=6.6 \mathrm{TeV}$ e.g. compositeness

Smaller $\mathrm{c}_{\mathrm{i}} \rightarrow$ smaller $\Lambda$

## W boson mass measurements



## CDF II measurement of the W boson mass



CDF II detector consists of
silicon vertex detector
large drift chamber
coarse calorimeter towers
outer muon chambers
$\sqrt{s}=1.96 \mathrm{TeV}$ proton-antiproton collisions from the Fermilab Tevatron


## CDF II measurement of the W boson mass


$4 x$ the integrated luminosity of the previous measurement

Higher $\langle\mu\rangle$ : peaks at 3

Measurement uses complete Tevatron Run II data set


## CDF II measurement of the W boson mass



W bosons identified in their decays to $e \nu$ and $\mu \nu$

Mass measured by fitting template distributions of transverse momentum and mass

$$
m_{T}=\sqrt{2 p_{T}^{l} p_{T}(1-\cos \Delta \phi)}
$$




## Calibrations



Measurement requires precise calibrations and momentum scale and resoution

Charged lepton scale



## Calibrations

Measurement requires precise calibrations and momentum scale and resoution

$$
\vec{p}_{T}=-\left(\vec{p}_{T}^{l}+\vec{u}_{T}\right)
$$

Recoil scale



## Detector simulation

## Developed custom simulation for analysis

Models ionization energy loss, multiple scattering, bremsstrahlung, photon conversion, Compton scattering
Acceptance map for muon detectors
Parameterized GEANT4 model of electromagnetic calorimeter showers
Kotwal \& CH, NIMA 729, 25 (2013) Includes shower losses due to finite calorimeter thickness

Hit-level model of central outer tracker Layer-by-layer resolution functions and efficiencies

Material map of inner silicon detector Includes radiation lengths and Bethe-Bloch terms



## Muon momentum calibration

First step is to align the drift chamber (the "central outer tracker" or COT)
Two degrees of freedom (shift \& rotation) for each of 2520 cells made up of twelve sense wires constrained using hit residuals from cosmic-ray tracks




## Muon momentum calibration

First step is to align the drift chamber (the "central outer tracker" or COT)
Two parameters for the electrostatic deflection of the wire within the chamber constrained using difference between fit parameters of incoming and outgoing cosmic-ray tracks





## Muon momentum calibration

Second step is to calibrate the momentum scale using $J / \psi$ decays to muons
Simulation:
Adjust kinematics to match the data
Model resonance shape using hit-level simulation and NLO form factor for QED radiation



## Muon momentum calibration

Second step is to calibrate the momentum scale using $J / \psi$ decays to muons
Simulation corrections:
Correct the length scale of the tracker with mass measurement as a function of $\Delta \cot \theta$ Correct the amount of upstream material with mass measurement as a function of $p_{T}^{-1}$


$\xrightarrow{\sim}$

## Muon momentum calibration

Third step is to calibrate the scale using $\Upsilon$ decays to muons
Compare fit results with and without constraining the track to the collision point


with constraint
without constraint

## Muon momentum calibration

Final step is to measure the $\mathbf{Z}$ boson mass

$$
M_{Z}=91192.0 \pm 6.4_{\text {stat }} \pm 4.0_{\text {sys }} \mathrm{MeV}
$$

Result blinded with [-50,50] MeV offset until previous steps were complete Combine all measurements into a final charged-track momentum scale



## Electron momentum calibration

First step is to transfer the track calibration to the calorimeter (E/p) using W \& Z decays
Simulation:
Detailed model of bremsstrahlung and pair production upstream of the drift chamber
Parameterized calorimeter shower deposition based on GEANT4
Kotwal \& CH, NIMA 729, 25 (2013)
Tune energy loss due to material upstream of the tracker (high E/p)
Tune shower leakage due to finite calorimeter thickness (low E/p)
Correct for small non-linear calorimeter response



## Electron momentum calibration

First step is to transfer the track calibration to the calorimeter (E/p) using W \& Z decays

## Data corrections:

Use mean E/p to remove time dependence \& response variations in tower
Fit ratio of calorimeter energy to track momentum to correct each tower in $\eta$



## Electron momentum calibration

Second step is the measurement of the $\mathbf{Z}$ boson mass

$$
M_{Z}=91194.3 \pm 13.8_{\text {stat }} \pm 7.6_{\text {sys }} \mathrm{MeV}
$$

As a consistency check measure mass using only track information
e.g. $M_{Z}=91215.2 \pm 22.4 \mathrm{MeV}$ for non-radiative electrons $(E / p<1.1)$

Same blinding as for muon channel


## Recoil momentum calibration

First step is the alignment of the calorimeters
Misalignments relative to the beam axis cause a modulation in the recoil direction Alignment performed separately for each run period using minimum-bias data


## Second step is the reconstruction of the recoil

Remove towers traversed by identified leptons
Remove corresponding recoil energy in simulation using towers rotated by $90^{\circ}$ validate using towers rotated by $180^{\circ}$

| $$ | Electron Electromagnetic $E_{T}(\mathrm{MeV})$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 |  | 61 | 62 | 63 | 62 | 61 | 61 |
|  | 2 | 62 | 61 | 62 | 69 | 64 | 62 | 61 |
|  |  | 63 | 63 | 66 | 1227 | 90 | 64 | 63 |
|  |  |  | 66 | 79 | 38534 | 176 | 68 | 64 |
|  | -1 | 61 | 61 | 62 | 178 | 67 | 61 | 61 |
|  | -2 |  | 61 | 61 | 63 | 62 | 61 | 61 |
|  | -3 | 60 | 61 | 61 | 61 | 61 | 61 | 61 |
|  |  | -3 | -2 | -1 | 0 | 1 | 2 | 3 |



## Recoil momentum calibration

Third step is the calibration of the recoil response
Balance recoil against direction of $\mathrm{p}^{\mathrm{Z}}$
Check calibration using ratio of recoil magnitude to $\mathrm{p}_{\mathrm{T}} \mathrm{Z}$ along direction of $\mathrm{p}^{\mathrm{Z}}$ ( $\mathrm{R}_{\text {rec }}$ )




## Recoil momentum calibration

## Fourth step is the calibration of the recoil resolution

Includes jet-like energy and angular resolution, additional dijet fraction term, and pileup




## Recoil momentum validation

## W boson recoil distributions validate the model

Most important is the recoil projected along the charged-lepton's momentum $\left(u_{\| \mid}\right)$

$$
m_{T} \approx 2 p_{T} \sqrt{1+u_{\|} / p_{T}} \approx 2 p_{T}+u_{\|}
$$



## W boson candidates

## W boson event selection

Triggers with low momentum thresholds ( 18 GeV ) and very loose lepton id
Offline id also loose, efficiencies vary by $2 \%$ as hadronic recoil direction changes
No lepton isolation requirement in trigger or offline selection


Background suppressed by stringent hadronic recoil requirement

$$
\mathrm{U}_{\mathrm{T}}<15 \mathrm{GeV}
$$

Other kinematic requirements
Lepton and missing $p_{T}$ in the range $30-55 \mathrm{GeV}$
Transverse mass in the range $60-100 \mathrm{GeV}$
$2.4 \mathrm{M} W \rightarrow \mu \nu$ candidates
1.8 M W $\rightarrow e \nu$ candidates

## Backgrounds

Electroweak backgrounds modelled with fast simulation tuned with data and full simulation Cross-checked with full simulation tuned to data

Largest background is $Z \rightarrow \mu \mu$ with one unreconstructed muon: 7.4\% of data sample $W \rightarrow \tau \nu$ background is $\sim 1 \%$ in each channel: largest background in electron sample

Background from hadrons misreconstructed as leptons estimated using data: 0.2-0.3\%


## W boson transverse momentum

## Boson $p_{т}$ impacts the $p_{т}$ distributions of the decay leptons

Resbos used to generate events with non-perturbative parameters and NNLL resummation to model the region of low boson $\mathrm{P}_{\mathrm{T}}$

Z boson $\mathrm{p}_{\mathrm{T}}$ used to constrain the non-perturbative parameter $\mathrm{g}_{2}$ and the perturbative coupling $\alpha_{s}$
Resbos models W boson pt well
uncertainty estimated using DYQT and constrained with data



## W boson production and decay

Parton distributions impact the measurement through lepton acceptance
Restriction in $\eta$ reduces the fraction of low- $\mathrm{p}_{\mathrm{T}}$ leptons

## Small correction applied to update to NNPDF3.1 NNLO PDF

The set with the most W charge asymmetry measurements at the time


Uncertainty determined using a principal component analysis on the replica set
Measurement sensitive to ~15 eigenvectors
Leading 25 eigenvectors used to estimate uncertainty (3.9 MeV)
Three general NNLO PDF sets (NNPDF3.1, CT18, and MMHT14) have a range of $\pm 2.1 \mathrm{MeV}$ from mean

Photos resummation with ME corrections used to model final-state photon radiation
validated by studying the average radiation in EM towers around the charged lepton,
and with the $Z$ mass measurement

## W boson mass measurement

Result blinded by [-50,50] MeV offset until all previous steps complete





## Mass measurement with $p_{T}^{\ell}$ distribution






## Mass measurement with $p_{T}^{\nu}$ distribution






## W boson mass measurement

| Combination | $m_{T}$ fit <br> Electrons Muons | $p_{T}^{\ell}$ fit Electrons Muons | $p_{T}^{\nu}$ fit Electrons Muons | Value (MeV) | $\chi^{2} /$ dof $\mid$ | Probability (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $m_{T}$ | $\checkmark \quad \checkmark$ |  |  | $80439.0 \pm 9.8$ | $1.2 / 1$ | 28 |
| $p_{T}^{\ell}$ |  | $\checkmark \quad \checkmark$ |  | $80421.2 \pm 11.9$ | 0.9 / 1 | 36 |
| $p_{T}^{\nu}$ |  |  | $\checkmark \quad \checkmark$ | $80427.7 \pm 13.8$ | $0.0 / 1$ | 91 |
| $m_{T} \& p_{T}^{\ell}$ | $\checkmark \quad \checkmark$ | $\checkmark \quad \checkmark$ |  | $80435.4 \pm 9.5$ | $4.8 / 3$ | 19 |
| $m_{T} \& p_{T}^{\nu}$ | $\checkmark \quad \checkmark$ |  | $\checkmark \quad \checkmark$ | $80437.9 \pm 9.7$ | $2.2 / 3$ | 53 |
| $p_{T}^{\ell} \& p_{T}^{\nu}$ |  | $\checkmark \quad \checkmark$ | $\checkmark \quad \checkmark$ | $80424.1 \pm 10.1$ | $1.1 / 3$ | 78 |
| Electrons | $\checkmark$ | $\checkmark$ | $\checkmark$ | $80424.6 \pm 13.2$ | $3.3 / 2$ | 19 |
| Muons | $\checkmark$ | $\checkmark$ | $\checkmark$ | $80437.9 \pm 11.0$ | $3.6 / 2$ | 17 |
| All | $\checkmark \quad \checkmark$ | $\checkmark \quad \checkmark$ | $\checkmark \quad \checkmark$ | $80433.5 \pm 9.4$ | $7.4 / 5$ | 20 |


| Fit difference | Muon channel | Electron channel |
| :--- | :---: | ---: |
| $M_{W}\left(\ell^{+}\right)-M_{W}\left(\ell^{-}\right)$ | $-7.8 \pm 18.5_{\text {stat }} \pm 12.77_{\mathrm{COT}}$ | $14.7 \pm 21.3_{\text {stat }} \pm 7.7^{\mathrm{E} / \mathrm{p}}\left(0.4 \pm 21.3_{\text {stat }}\right)$ |
| $M_{W}\left(\phi_{\ell}>0\right)-M_{W}\left(\phi_{\ell}<0\right)$ | $24.4 \pm 18.5_{\text {stat }}$ | $9.9 \pm 21.3_{\text {stat }} \pm 7.5_{\text {stat }}^{\mathrm{E} / \mathrm{p}}\left(-0.8 \pm 21.3_{\text {stat }}\right)$ |
| $M_{Z}($ run $>271100)-M_{Z}($ run $<271100)$ | $5.2 \pm 12.2_{\text {stat }}$ | $63.2 \pm 29.9_{\text {stat }} \pm 8.2_{\text {stat }}^{\mathrm{E} / \mathrm{p}}\left(-16.0 \pm 29.9_{\text {stat }}\right)$ |

## Summary

## W boson mass an important parameter for understanding naturalness

Measurement of W boson mass with $<10 \mathrm{MeV}$ precision achieved with complete CDF data set

Result of >20 years of experience with the CDF II detector
0.01\% precision required flexibility: all experimental aspects controlled by the analysis team Reconstruction, alignment, calibration, simulation, analysis

Analysis procedures approved pre-blinding and frozen

Surprising 0.1\% deviation from SM motivates expanded study of mw measurements and procedures

## Backup



CDF Components

> Muon detectors
(drift and scintillator)


## Uncertainties

| Source of systematic uncertainty | Electrons | $m_{T}$ fit <br> Muons | Common | Electrons | $p_{T}^{\ell}$ fit <br> Muons | Common | Electrons | $p_{T}^{\nu}$ fit <br> Muons | Common |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lepton energy scale | 5.8 | 2.1 | 1.8 | 5.8 | 2.1 | 1.8 | 5.8 | 2.1 | 1.8 |
| Lepton energy resolution | 0.9 | 0.3 | -0.3 | 0.9 | 0.3 | -0.3 | 0.9 | 0.3 | -0.3 |
| Recoil energy scale | 1.8 | 1.8 | 1.8 | 3.5 | 3.5 | 3.5 | 0.7 | 0.7 | 0.7 |
| Recoil energy resolution | 1.8 | 1.8 | 1.8 | 3.6 | 3.6 | 3.6 | 5.2 | 5.2 | 5.2 |
| Lepton $u_{\\| \mid}$efficiency | 0.5 | 0.5 | 0 | 1.3 | 1.0 | 0 | 2.6 | 2.1 | 0 |
| Lepton removal | 1.0 | 1.7 | 0 | 0 | 0 | 0 | 2.0 | 3.4 | 0 |
| Backgrounds | 2.6 | 3.9 | 0 | 6.6 | 6.4 | 0 | 6.4 | 6.8 | 0 |
| $p_{T}^{Z}$ model | 0.7 | 0.7 | 0.7 | 2.3 | 2.3 | 2.3 | 0.9 | 0.9 | 0.9 |
| $p_{T}^{W} / p_{T}^{Z}$ model | 0.8 | 0.8 | 0.8 | 2.3 | 2.3 | 2.3 | 0.9 | 0.9 | 0.9 |
| Parton distributions | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 |
| QED radiation | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 |
| Statistical | 10.3 | 9.2 | 0 | 10.7 | 9.6 | 0 | 14.5 | 13.1 | 0 |
| Total | 13.5 | 11.8 | 5.8 | 16.0 | 14.1 | 7.9 | 18.8 | 17.1 | 7.4 |

## Background fractions

|  | Fraction |  | $\delta M_{W}(\mathrm{MeV})$ |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Source | $(\%)$ | $m_{T}$ fit | $p_{T}^{\mu}$ fit | $p_{T}^{\nu}$ fit |  |
| $Z / \gamma^{*} \rightarrow \mu \mu$ | $7.37 \pm 0.10$ | $1.6(0.7)$ | $3.6(0.3)$ | $0.1(1.5)$ |  |
| $W \rightarrow \tau \nu$ | $0.880 \pm 0.004$ | $0.1(0.0)$ | $0.1(0.0)$ | $0.1(0.0)$ |  |
| Hadronic jets | $0.01 \pm 0.04$ | $0.1(0.8)$ | $-0.6(0.8)$ | $2.4(0.5)$ |  |
| Decays in flight | $0.20 \pm 0.14$ | $1.3(3.1)$ | $1.3(5.0)$ | $-5.2(3.2)$ |  |
| Cosmic rays | $0.01 \pm 0.01$ | $0.3(0.0)$ | $0.5(0.0)$ | $0.3(0.3)$ |  |
| Total | $8.47 \pm 0.18$ | $2.1(3.3)$ | $3.9(5.1)$ | $5.7(3.6)$ |  |


|  | Fraction |  | $\delta M_{W}(\mathrm{MeV})$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Source | $(\%)$ | $m_{T}$ fit | $p_{T}^{e}$ fit | $p_{T}^{\nu}$ fit |  |
| $Z / \gamma^{*} \rightarrow e e$ | $0.134 \pm 0.003$ | $0.2(0.3)$ | $0.3(0.0)$ | $0.0(0.6)$ |  |
| $W \rightarrow \tau \nu$ | $0.94 \pm 0.01$ | $0.6(0.0)$ | $0.6(0.0)$ | $0.6(0.0)$ |  |
| Hadronic jets | $0.34 \pm 0.08$ | $2.2(1.2)$ | $0.9(6.5)$ | $6.2(-1.1)$ |  |
| Total | $1.41 \pm 0.08$ | $2.3(1.2)$ | $1.1(6.5)$ | $6.2(1.3)$ |  |

## Initial state LO \& NLO

| W+ initial | Type | Pythia LO | Madgraph LO | Madgraph NLO |
| :---: | :---: | :---: | :---: | :---: |
| u dbar | v-v | $81.7 \%$ | $82.0 \%$ | $82.7 \%$ |
| dbar u | s-s | $8.9 \%$ | $9.0 \%$ | $8.8 \%$ |
| u sbar | v-s | $1.6 \%$ | $1.9 \%$ | $1.8 \%$ |
| sbar u | s-s | $0.3 \%$ | $0.3 \%$ | $0.3 \%$ |
| c sbar | s-s | $2.9 \%$ | $2.9 \%$ | - |
| sbar c | s-s | $2.9 \%$ | $2.9 \%$ | - |
| c dbar | s-v | $0.7 \%$ | $0.7 \%$ | - |
| dbar c | s-s | $0.2 \%$ | $0.2 \%$ | - |
| u g | v-g |  | - | $3.7 \%$ |
| g dbar | g-v |  | - | $1.8 \%$ |
| g u | g-s |  | - | $0.4 \%$ |
| dbar g | s-g |  | - | $0.5 \%$ |
| g sbar | g-s |  | - | $0.02 \%$ |
| sbar g | s-g |  | - | $0.02 \%$ |

## Recoil in W \& Z events



## Recoil projections in W events



## Recoil model parameters

| Parameter | Description | Source | $m_{T}$ | $p_{T}^{\ell}$ | $p_{T}^{\nu}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a | average response | Fig. S23 |  |  | -0.2 |
| b | response non-linearity | Fig. S23 | -0.8 | -2.0 | 0.7 |
| Response |  |  | 1.8 | 3.5 | 0.7 |
| $N_{V}$ | spectator interactions | Fig. S24 | 0.5 | -3.2 | 3.6 |
| $s_{\text {had }}$ | sampling resolution | Fig. S24 | 0.3 | 0.3 | 0.8 |
| $f_{\pi^{0}}^{4}$ | EM fluctuations at low $u_{T}$ | Fig. S25 | -0.3 |  | -1.0 |
| $f_{\pi^{0}}^{15}$ | EM fluctuations at high $u_{T}$ | Fig. S25 | -0.3 |  | -0.2 |
| $\alpha$ | angular resolution at low $u_{T}$ | Fig. S26 | 1.4 | 0.1 | 2.5 |
| $\beta$ | angular resolution at intermediate $u_{T}$ | Fig. S26 | 0.2 | 0.1 | 0.7 |
| $\gamma$ | angular resolution at high $u_{T}$ | Fig. S26 | 0.3 | 0.3 | 0.7 |
| $f_{2}^{a}$ | average dijet component | Fig. S27 | 0.1 | -1.1 | 0.8 |
| $f_{2}^{s}$ | variation of dijet component with $u_{T}$ | Fig. S27 | -0.1 |  | -0.1 |
| $k_{\xi}$ | average dijet resolution | Fig. S28 | -0.1 | 0.1 | -0.3 |
| $\delta_{\xi}$ | fluctuations in dijet resolution | Fig. S28 | -0.2 | 0.2 | -1.1 |
| $A_{\xi}$ | higher-order term in dijet resolution | Fig. S28 | 0.1 | -1.0 | 0.7 |
| $\mu_{\xi}$ | -"- | Fig. S28 | -0.5 |  | -0.9 |
| $\epsilon_{\xi}$ | -" | Fig. S28 | 0.1 | -0.2 | 0.4 |
| $S_{\xi}^{+}$ | -" | Fig. S28 | 0.5 | -0.4 | 1.4 |
| $S_{\xi}^{-}$ | -"- | Fig. S28 | -0.3 |  | -0.5 |
| $q_{\xi}$ | -"- | Fig. S28 | -0.2 | 0.0 | 0.2 |
| Resolution |  |  | 1.8 | 3.6 | 5.2 |

## Z mass fits using tracker or calorimeter



## Recoil reconstruction in muon channel






## Electron momentum calibration





## Electroweak observables at dimension 6

$$
Q_{H W B}, Q_{H D}, Q_{H \ell}^{(1)}, Q_{H \ell}^{(3)}, Q_{H q}^{(1)}, Q_{H q}^{(3)}, Q_{H e}, Q_{H u}, Q_{H d}, Q_{\ell \ell}
$$

| Parameter | Input Value |
| :---: | :---: |
| $\hat{m}_{Z}$ | $91.1875 \pm 0.0021$ |
| $\hat{G}_{F}$ | $1.1663787(6) \times 10^{-5}$ |
| $\hat{\alpha}_{e w}$ | $1 / 137.035999074(94)$ |


| Observable | Experimental Value | Ref. | SM Theoretical Value | Ref. |
| :---: | :---: | :---: | :---: | :---: |
| $\hat{m}_{Z}[\mathrm{GeV}]$ | $91.1875 \pm 0.0021$ | $[19]$ | - | - |
| $\hat{m}_{W}[\mathrm{GeV}]$ | $80.385 \pm 0.015$ | $[49]$ | $80.365 \pm 0.004$ | $[50]$ |
| $\Gamma_{Z}[\mathrm{GeV}]$ | $2.4952 \pm 0.0023$ | $[19]$ | $2.4942 \pm 0.0005$ | $[48]$ |
| $R_{\ell}^{0}$ | $20.767 \pm 0.025$ | $[19]$ | $20.751 \pm 0.005$ | $[48]$ |
| $R_{c}^{0}$ | $0.1721 \pm 0.0030$ | $[19]$ | $0.17223 \pm 0.00005$ | $[48]$ |
| $R_{b}^{0}$ | $0.21629 \pm 0.00066$ | $[19]$ | $0.21580 \pm 0.00015$ | $[48]$ |
| $\sigma_{h}^{0}[\mathrm{nb}]$ | $41.540 \pm 0.037$ | $[19]$ | $41.488 \pm 0.006$ | $[48]$ |
| $A_{\mathrm{FB}}^{\ell}$ | $0.0171 \pm 0.0010$ | $[19]$ | $0.01616 \pm 0.00008$ | $[32]$ |
| $A_{\mathrm{FB}}^{c}$ | $0.0707 \pm 0.0035$ | $[19]$ | $0.0735 \pm 0.0002$ | $[32]$ |
| $A_{\mathrm{FB}}^{b}$ | $0.0992 \pm 0.0016$ | $[19]$ | $0.1029 \pm 0.0003$ | $[32]$ |

