Cockcroft Lectures: Intermediate Beam Dynamics for Particle Accelerators

Problem Set 10

A. Wolski & B. Muratori

May, 2022

1. Show that, for uncorrelated quadrupole vertical misalignments in an uncoupled storage ring:

$$\left\langle \frac{y_{co}^2}{\beta_y} \right\rangle \approx \frac{\langle \Delta Y^2 \rangle}{8 \sin^2 \pi \nu_y} \sum_i \beta_{yi} (k_1 L)_i^2 \tag{1}$$

where y_{co} is the vertical closed orbit distortion at a point in the ring where the vertical beta function is β_y; $\langle \Delta Y^2 \rangle$ is the mean square quadrupole vertical misalignment; ν_y is the vertical tune; the sum extends over all quadrupoles; β_{yi} is the beta function at the ith quadrupole; and $(k_1 L)_i$ is the integrated normalised strength of the ith quadrupole. The brackets $\langle \cdot \rangle$ indicate an average over many sets of alignment errors.

2. The luminosity in a collider increases with decreasing beam size at the interaction point. During a collision where particles in one bunch have the opposite electric charge to the particles in the other bunch, the electromagnetic field from one bunch provides a (generally small) focusing force on the other bunch. Discuss the most appropriate betatron tune for the machine, to get maximum benefit from this "dynamic focusing" effect, and explain some of the difficulties that might be involved in operating the machine with this tune. How would the situation change in the case of colliding bunches of particles of the same sign of charge?