Single-Particle Linear Dynamics

Problem Set 1

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1. A region of space has zero magnetic field, and an electric field given by:

$$E_x = -E_0 x, \qquad E_y = E_0 y, \qquad E_z = 0 \tag{1}$$

where E_0 is a constant. From the Lorentz force and Newton's equation, write down the equations of motion for a non-relativistic particle of charge q and mass m in this field. Write down the solution for the initial coordinates:

$$x(0) = x_0, \qquad y(0) = y_0, \qquad z(0) = 0 \tag{2}$$

where x_0 and y_0 are constants, and the initial velocity is:

$$\dot{x}(0) = 0, \qquad \dot{y}(0) = 0, \qquad \dot{z}(0) = v_z \tag{3}$$

Describe the trajectory of the particle, distinguishing between the cases $qE_0 < 0$ and $qE_0 > 0$.

2. Write down an electric potential that gives the electric field (1). Thus, write down a Hamiltonian for a non-relativistic particle moving in the electric field (1). Using Hamilton's equations, write down the equations of motion for a particle moving in the electric field (1). Find the *general* solution to the equations of motion. Write the general solution in the form of a transfer matrix, and show (by explicit matrix multiplication) that the transfer matrix is symplectic.

3. Consider cartesian coordinates (x, z) in a plane. A new set of "curvilinear" coordinates (X, S) is defined as shown in Figure 1. Note that ρ is a constant.

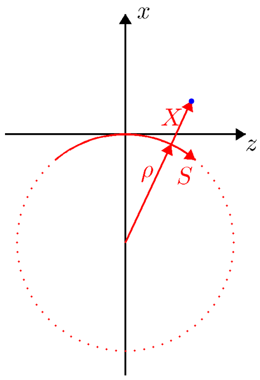

Figure 1: Cartesian coordinates (x, z) and curvilinear coordinates (X, S).

Write down expressions for the "old" (cartesian) coordinates (x, z) in terms of the "new" (curvilinear) coordinates (X, S). Show that the expressions relating the old and new coordinates can be derived from a generating function of the third kind, given by:

$$F_3(X, p_x, S, p_z) = -\left[(\rho + X)\cos\frac{S}{\rho} - \rho\right] p_x - \left[(\rho + X)\sin\frac{S}{\rho}\right] p_z \tag{4}$$

Hence write down expressions for the new momenta (P_X, P_S) conjugate to the new coordinates (X, S), in terms of the old momenta (p_x, p_z).