Progress towards modelling of the vFFA

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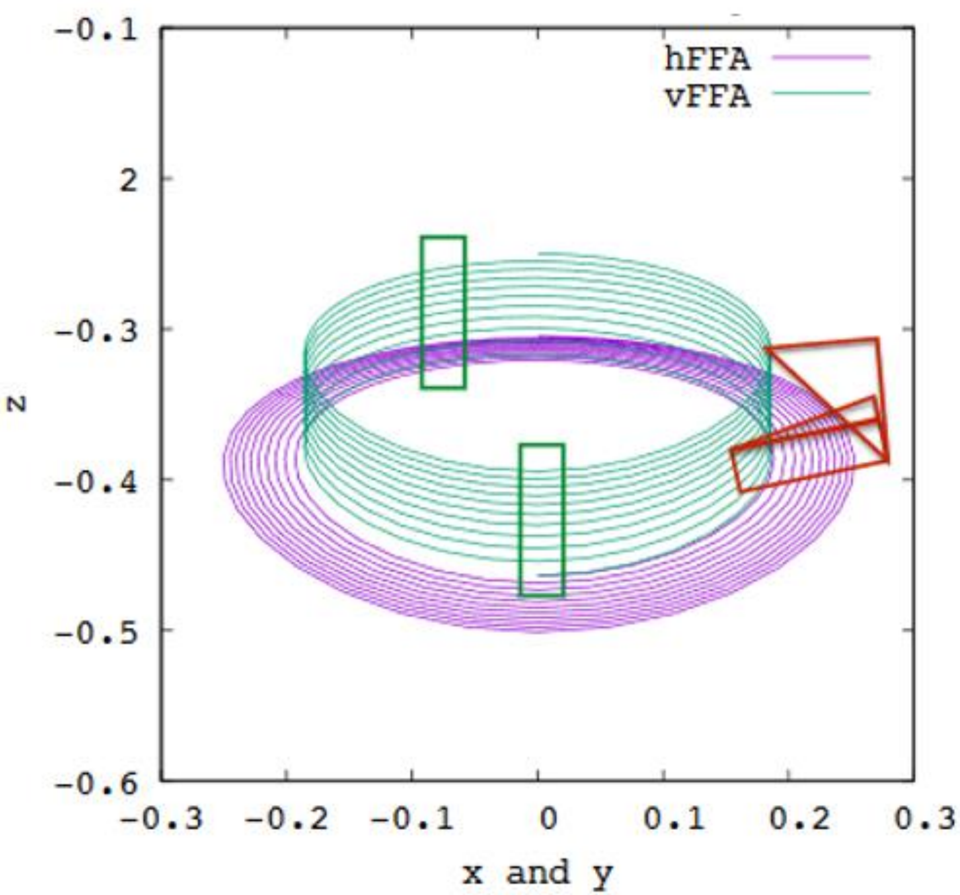
Contents

- Intro to the vFFA
  - Hamiltonians
  - Closed orbits

- Optical components & Harmonic Analysis
  - Overview
  - Numerical study

- Next steps

vFFA Concept



- Successive higher energy orbits are stacked vertically
- Constant tune achieved by condition $B(y) = B_0{}^{my}$
- Advantages
  - Strong focussing
  - Near-isochronous
- But…

$$B_X = B_0 e^{mY} \sum \frac{n+1}{m} f_{n+1} X^n$$

$$B_Y = B_0 e^{mY} \sum f_n X^n$$

$$B_Z = B_0 e^{mY} \sum \frac{1}{m} \frac{d f_n}{dZ} X^n$$

Magnetic fields from scaling law imply complicated optics

Approaching the vFFA

- For hFFA, we have the following Hamiltonian:

$$h \sim \frac{{p_x}^2}{2} + \frac{{p_y}^2}{2} + \frac{x^2}{2}\frac{1-n}{\rho^2} + \frac{y^2}{2}\frac{n}{\rho^2}$$

- Is there an equivalent for the vFFA?

(Question asked by Thomas at the FFA school)

The Fundamental vFFA Hamiltonian

$$h \sim \frac{{p_x}^2}{2} + \frac{{p_y}^2}{2} + \frac{x^2}{\rho} + xy\frac{m}{\rho}$$

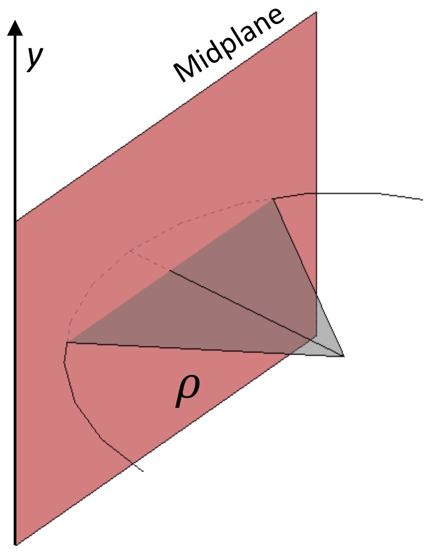
Very simple and neat, but…

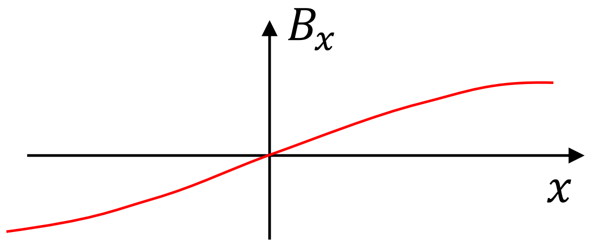
- Only applicable on the midplane
- Does not include fringe fields

- The closed orbit for a vFFA cannot exist on the midplane*
- Fringe fields dominate magnets in FETS-vFFA and ISIS-II designs

A Word on vFFA closed orbits

- hFFA:
  - Plane of curvature coplanar with magnet midplane
- vFFA:
  - Plane of curvature perpendicular to magnet midplane
  - Orbit cannot be confined to magnet midplane*
    - Orbits are non-planar!





Our vFFA Hamiltonian needs to reflect these properties!

*This *may* not be the case for *certain* sector vFFAs…

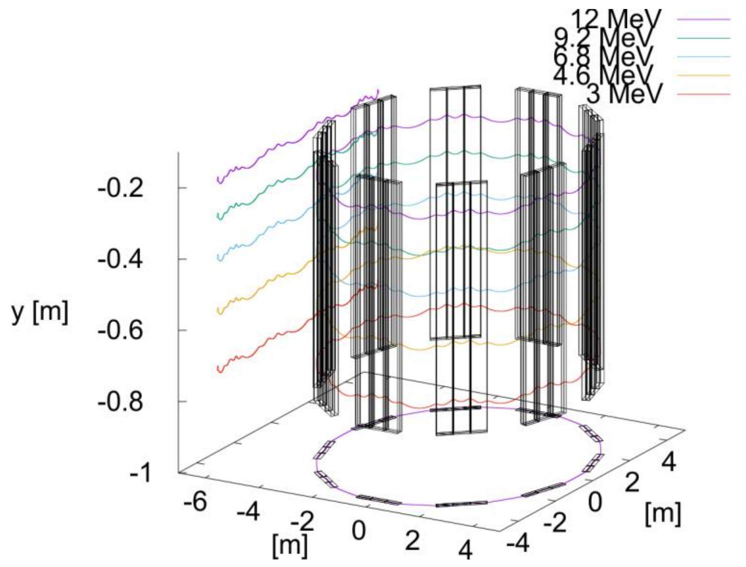
A practical vFFA Hamiltonian

$$h \sim \frac{{p_x}^2}{2} + \frac{{p_y}^2}{2} + xy\frac{m}{\rho_{3D}}\left(1 - \frac{m^2 x_0}{2}\right) + \frac{x^2}{\rho_{3D}}\left(\frac{\cos m x_0}{\rho_{3D}} + \frac{m^2 x_0}{2}\right) + \frac{y^2}{\rho_{3D}}\left(\frac{\sin m x_0}{\rho_{3D}} - \frac{m^2 x_0}{2} + \frac{m^4 {x_0}^3}{2\rho_{3D}}\right)$$

- We can use this Hamiltonian to analytically describe a vFFA if:
  - We can parametrize the cell geometry in 3d and determine:
    - x_0 horizontal offset from magnet midplane
    - ρ_{3D} Radius of curvature for a 3-dimensional bend
  - The magnets are long in comparison to the fringe
  - The radius of curvature is large
- However, FETS-vFFA and ISIS-II designs do not meet these criteria…
  - So why doesn't the model work? How must it be adapted?

Test Lattice parameters

M-value	**4.0/m**
Magnet length	0.5m
FD ratio	2
F-D offset	0.02m
Cell length	2.8m

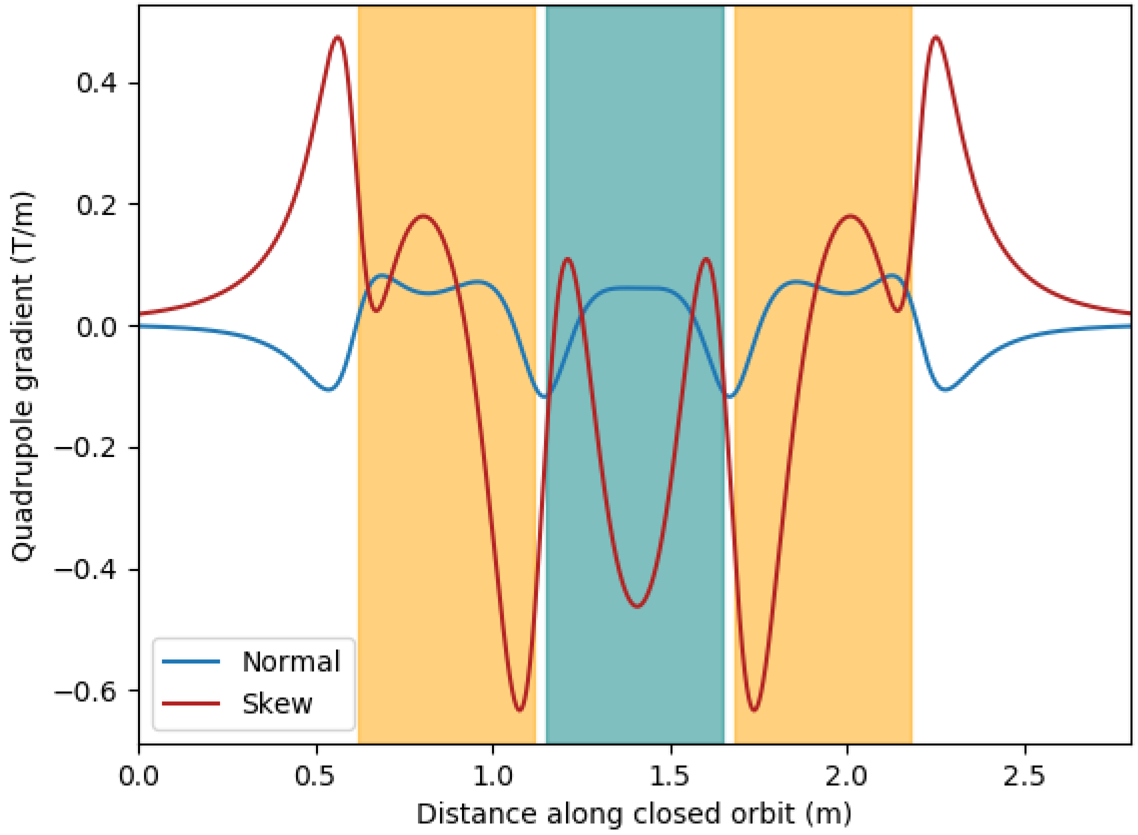


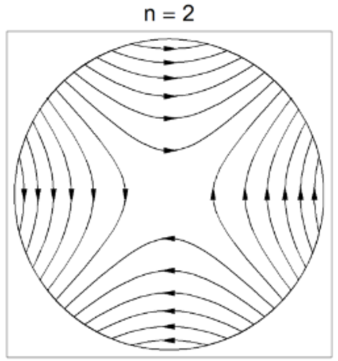
- FETS-like FDF Triplet
- Highly nonplanar closed orbit
- ‘fringe-dominated’ magnets
- Following results use a modified cell with zero bending angle

Method of Harmonic Analysis

- Scan along closed orbit
- Draw a circle at each point on orbit in a plane perpendicular to the tangent vector
- Evaluate radial field component $(\boldsymbol{B} \cdot \hat{\boldsymbol{r}})$ on a number of points around the circle
- Take Fourier transform of radial field around the circle
- Obtain multipole coefficients for each integration step
- Reconstruct optics using multipole kicks (thin lens transfer matrices) corresponding to the measured coefficient

Quadrupole components

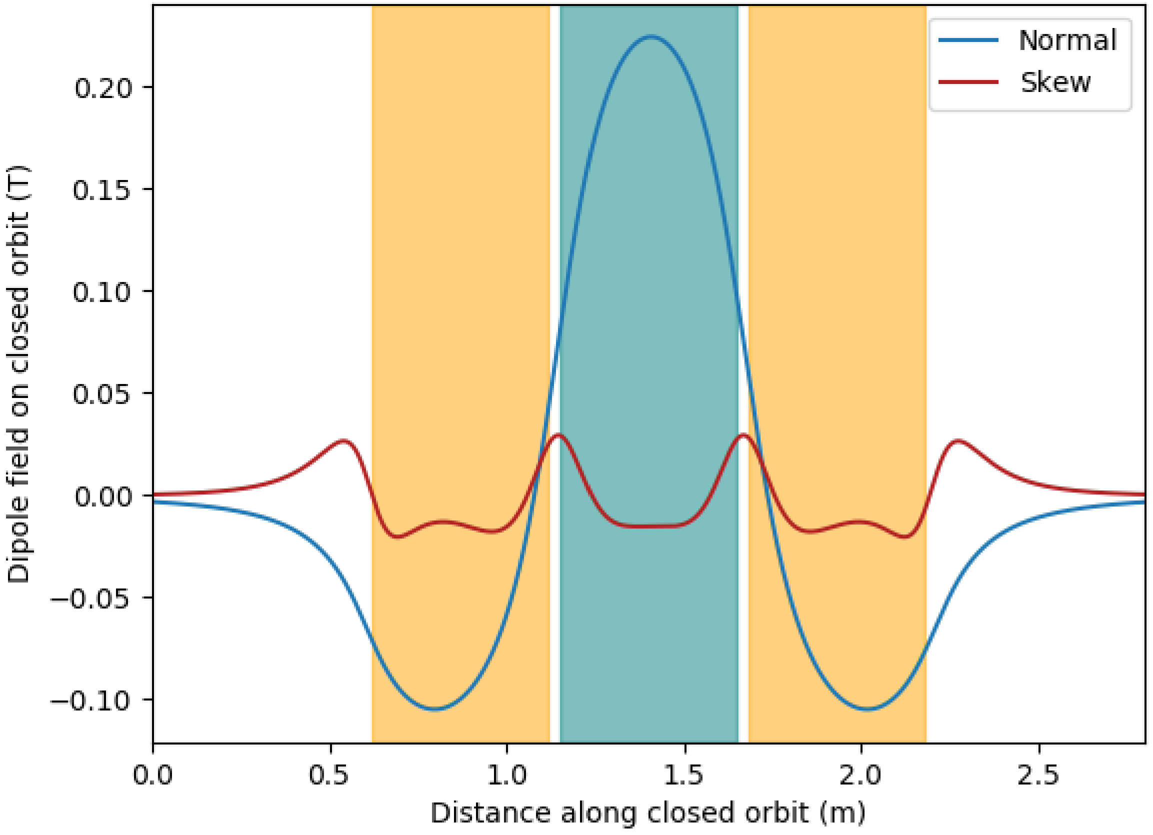


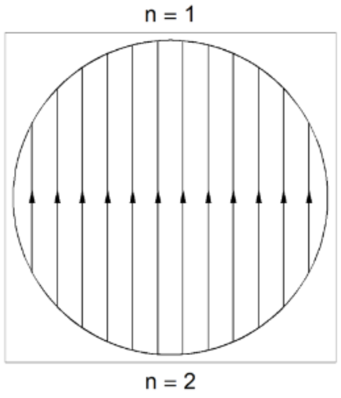


- Field focusses in one transverse direction, defocusses in the other

	Quadrupole-only tune	Full Numerical Tune (FIXFIELD)
qu	0.137	0.247
qv	0.124	0.182

Dipole components

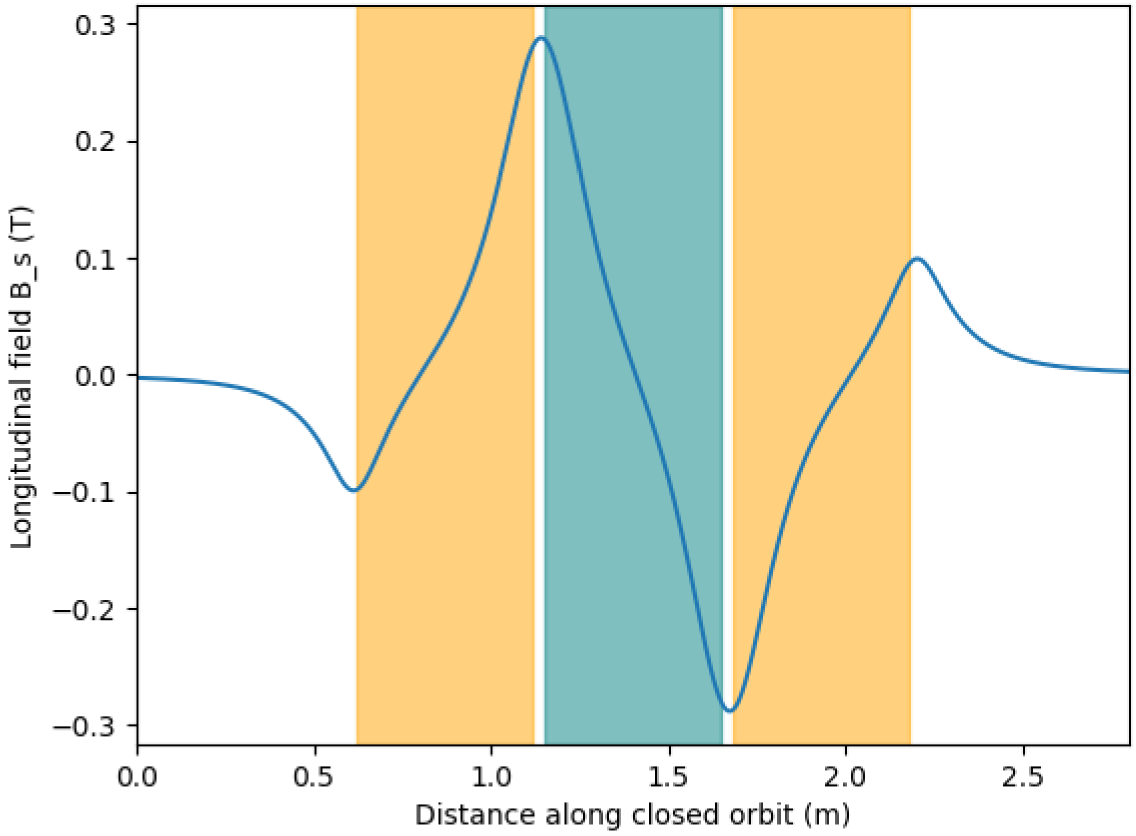




- Geometric focussing effect perpendicular to dipole field (aka weak focussing)

	Quad+dipole tune	Full Numerical Tune (FIXFIELD)
qu	0.229	0.247
qv	0.129	0.182

Longitudinal field components

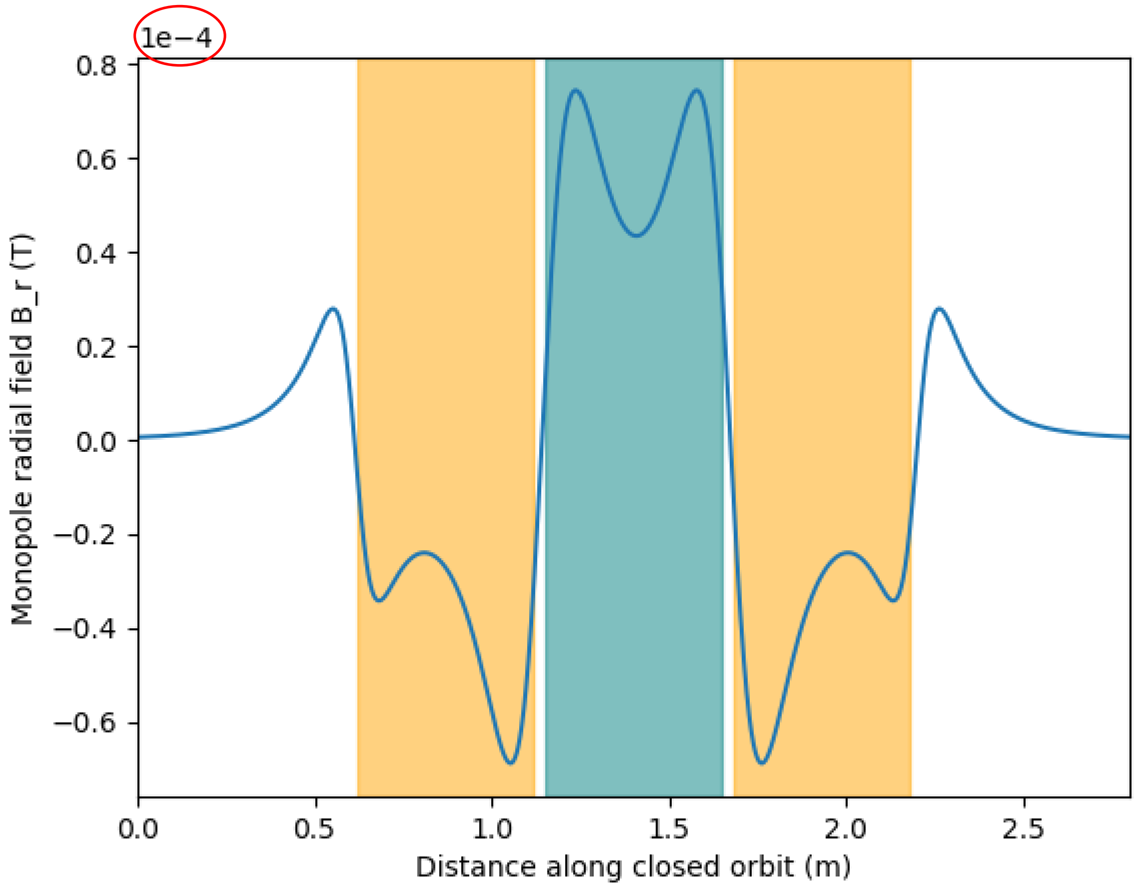


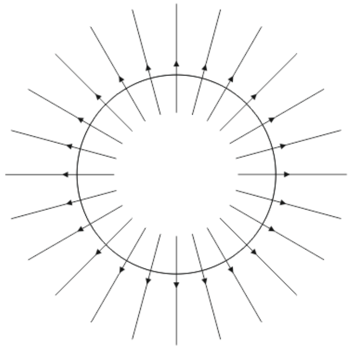
- Fields collinear with orbit
  - Induce a rotation of the transverse optical planes around the closed orbit – 'Larmor rotation'

$$\theta = \int \frac{qB_z}{2P} \cdot dz$$

	Quad+dipole+longitudinal tune	Full Numerical Tune (FIXFIELD)
qu	0.211	0.247
qv	0.154	0.182

Radial field components



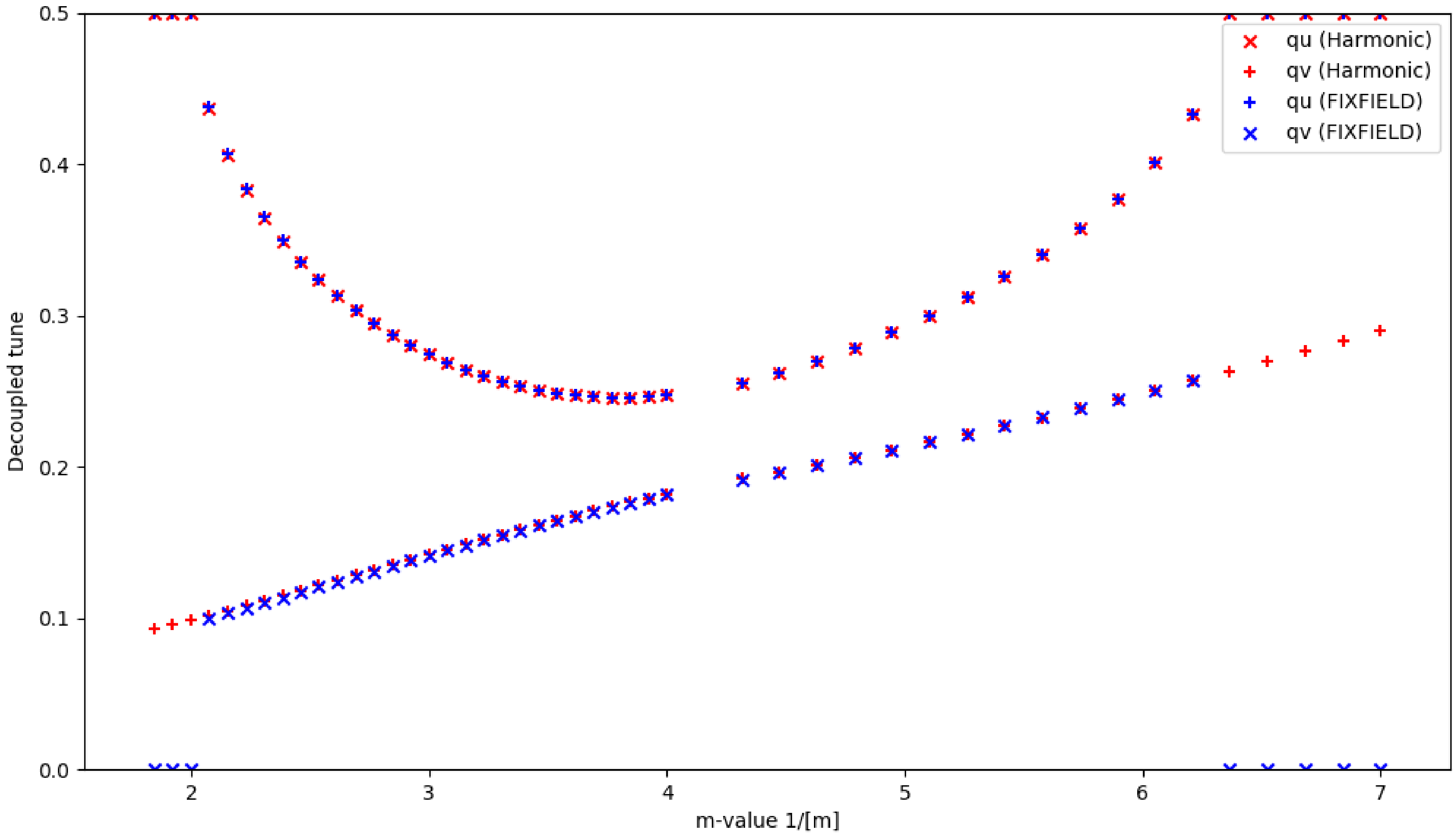


$$B_r = -\frac{r}{2}B'(z)$$

- Imparts angular momentum to the beam

	Quad+dipole+longitudinal +radial tune	Full Numerical Tune (FIXFIELD)
qu	0.247	0.247
qv	0.182	0.182

- To demonstrate robustness of method across parameter space:



	Q	D	L	R	Q+D	Q+L	Q+R	D+L	D+R	S+R	Q+D+L	Q+D+R	Q+L+R	D+L+R	Q+D+L+R
qu	0.137	0.173	0.000	0.006	0.229	0.095	0.079	0.172	0.137	0.102	0.211	0.196	0.141	0.203	0.247
qv	0.124	0.026	0.000	0.006	0.129	0.156	0.079	0.018	0.000	0.102	0.154	0.079	0.192	0.104	0.182

- *All* field elements critical to reconstructing accurate tune in both planes

- Can we quantify the importance of each element?
- Can we explore how this changes across the parameter space?

Multipole Element Significance Study

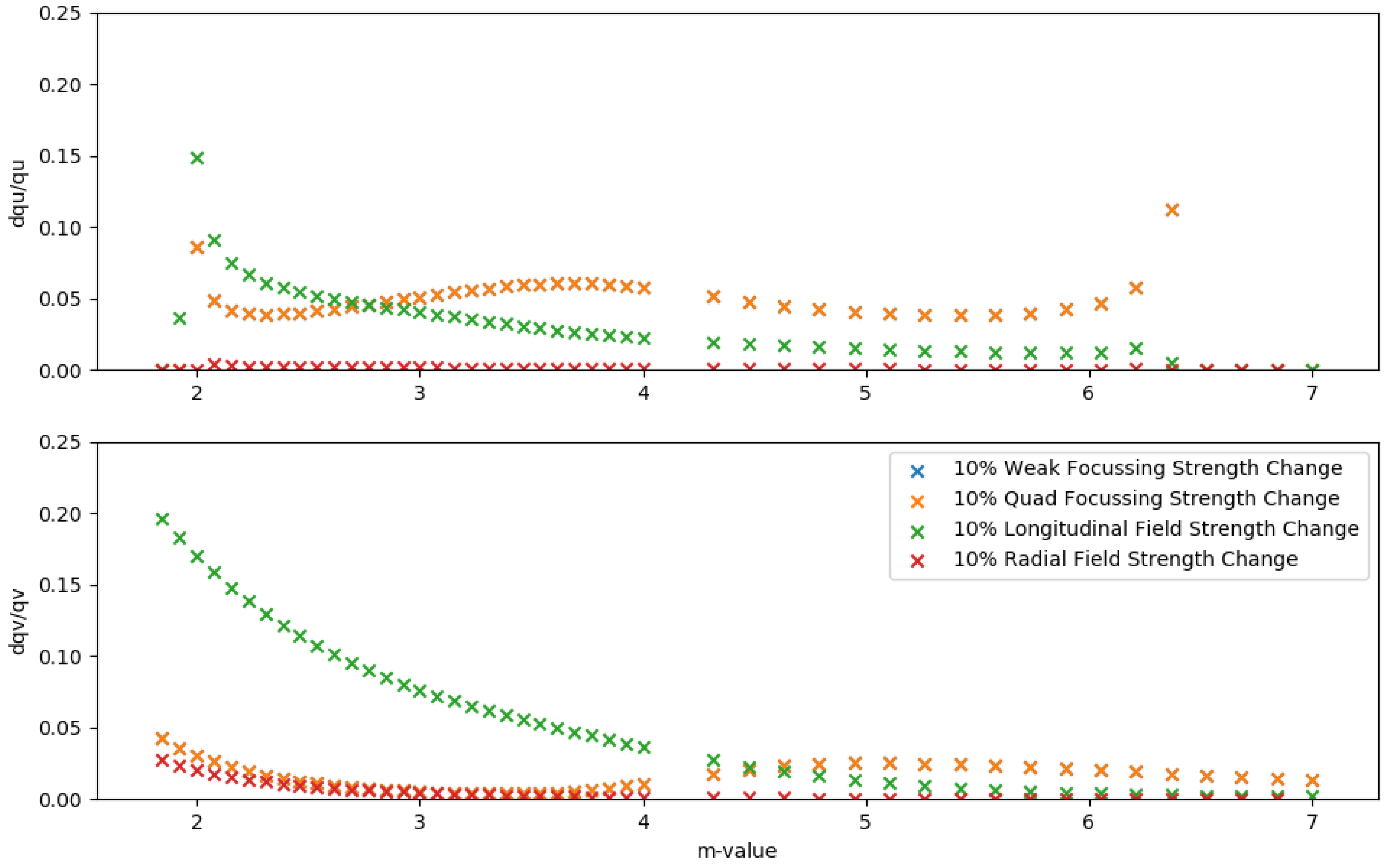
- Measure multipoles and construct optics as in Harmonic Analysis scheme outlined earlier
- Take optical model and adjust strength of individual multipole by given amount
    - E.g. rerun kick-drift integration with quadrupole strength tweaked by 10%
- Measure change in u and v tune and normalise ($\frac{\Delta q_i}{q_i}$)
- Record data across the parameter space

Rough way of 'quantifying' importance of each field element

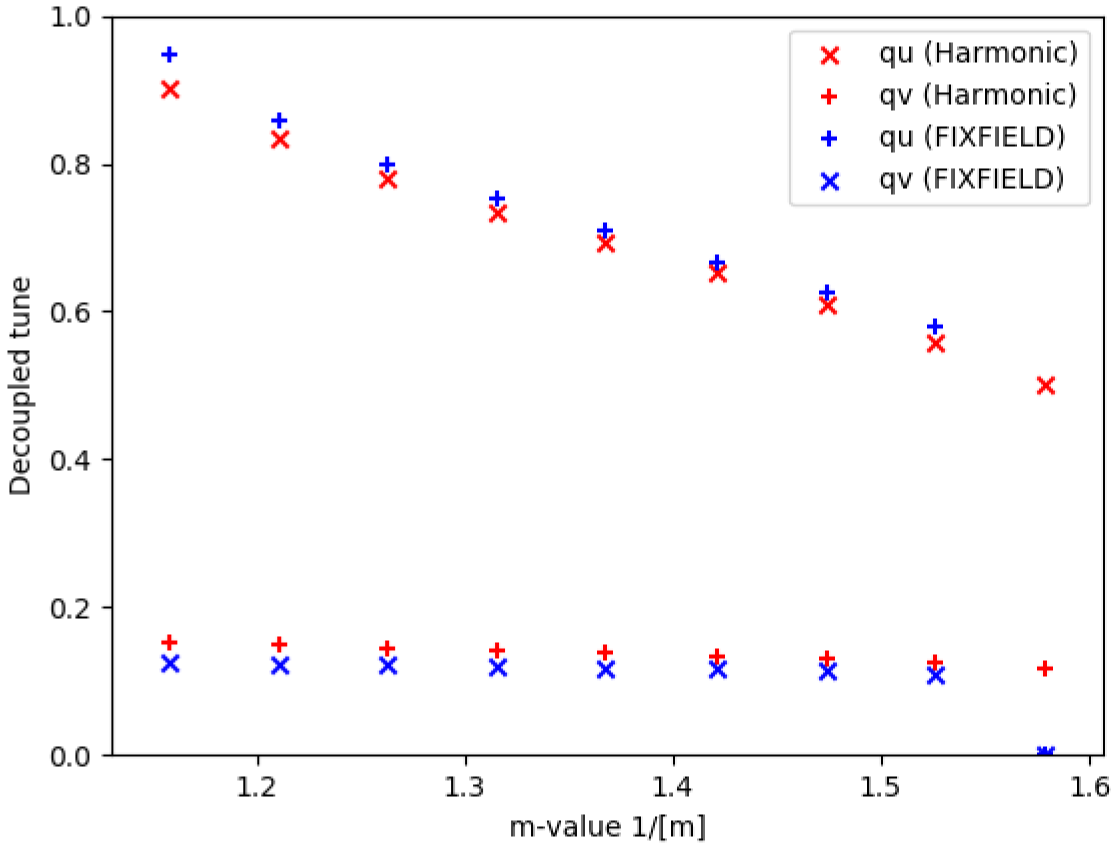
Multipole Element Significance Study

- Evolution of harmonic analysis for straight triplet vFFA across upper stability region

- FETS-vFFA operates in lower stability region



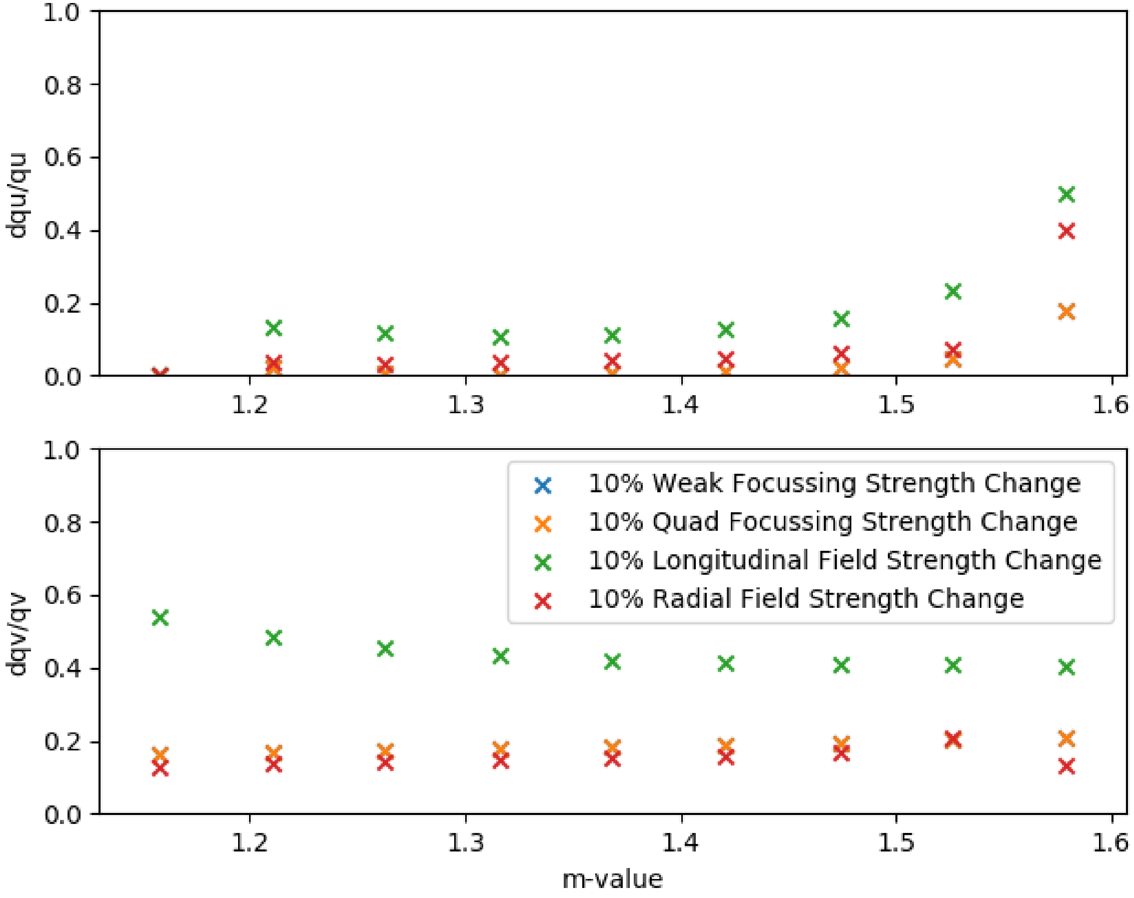
Harmonic analysis of FETS baseline



- FETS baseline parameter scan
- Lower stability region

- Agreement isn't quite as good as the previous results

Preliminary results from FETS baseline



- Dominated by longitudinal fields
- Radial fields have similar order of effect to quadrupole
- Dqu appears to decrease with increasing longitudinal field!
  - Interesting effect of Larmor angle?

- Lattice dominated by longitudinal fields at low m

- This dq/q metric is limited:
  - Useful for comparing regions of parameter space
  - Could be used to classify regimes of behaviour for a machine
  - Doesn’t give the full picture of how the optics behave

- Effect of dipole and quadrupole focussing well-understood
- Effect of longitudinal field can be expressed in terms of Larmor rotation
- How do we understand the influence of the radial field?

- Further exploration of vFFA parameter space needed
  - Can we define 'fringe-dominated' and 'magnet-dominated' regimes?
- Can we design a planar vFFA?
  - What is the influence of longitudinal and radial fields in this case?

- Apply harmonic study generally in FFA machines (hFFA etc…)
- Further coupling study in vFFA
- Use results of vFFA study to inform further modelling from an analytic perspective
- Possible future study of harmonic elements on other properties of the accelerator
  - E.g. dynamic aperture