Linear and non-linear beam dynamics of the ISIS-II FETS vertical FFA using Zgoubi

Presented at the 2022 Workshop on Fixed Field alternating gradient Accelerators (FFA2022)

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September, 29th 2022

Content

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Linear and non-linear beam dynamics of the ISIS-II FETS **vertical FFA** using Zgoubi

Scaling law: $B = B_0 e^{kz}$, ($k = \frac{1}{B_z}\frac{\partial B_z}{\partial z}$)

→ Vertical orbit excursion

→ Strongly non-linear magnetic fields

→ Strongly coupled optics

With the fringe field function $g(x)$, the median plane field components are:

$$B_{y0}(x,0,z) = 0,$$
$$B_{z0}(x,0,z) = B_0 e^{kz} g(x),$$
$$B_{x0}(x,0,z) = \frac{B_0}{k} e^{kz} \frac{dg}{dx}.$$

Content

Linear and non-linear beam dynamics of the ISIS-II FETS **vertical FFA using Zgoubi**

The Zgoubi ray-tracing code:

- Often used to study horizontal FFAs
- Particle tracking in complex field maps
- Possibility to use its Python interface Zgoubidoo

Content

Linear and non-linear beam dynamics of the **ISIS-II FETS vertical FFA using Zgoubi**

- Presentation of the lattice and methods
  - Baseline lattice with arctan fringe field
  - Fieldmap construction and tracking with Zgoubi

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Linear and non-linear **beam dynamics** of the **ISIS-II FETS vertical FFA using Zgoubi**

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  - Baseline lattice with arctan fringe field
  - Fieldmap construction and tracking with Zgoubi

- Linear beam dynamics
  - Closed orbit search
  - Tune comparison with neighboring cells contribution

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- Presentation of the lattice and methods
  - Baseline lattice with arctan fringe field
  - Fieldmap construction and tracking with Zgoubi

- Linear beam dynamics
  - Closed orbit search
  - Tune comparison with neighboring cells contribution

- Non-linear beam dynamics - Dynamical aperture
  - Possible simple definition/computation for the 2D Dynamical aperture
  - Methods/suggestions for calculating a dynamic aperture for 4D motion in a highly non-linear and coupled lattice

Presentation of the lattice and methods

Baseline arctan lattice & Methods to track with Zgoubi in adapted field map

Lattice and methods – Arctan baseline lattice

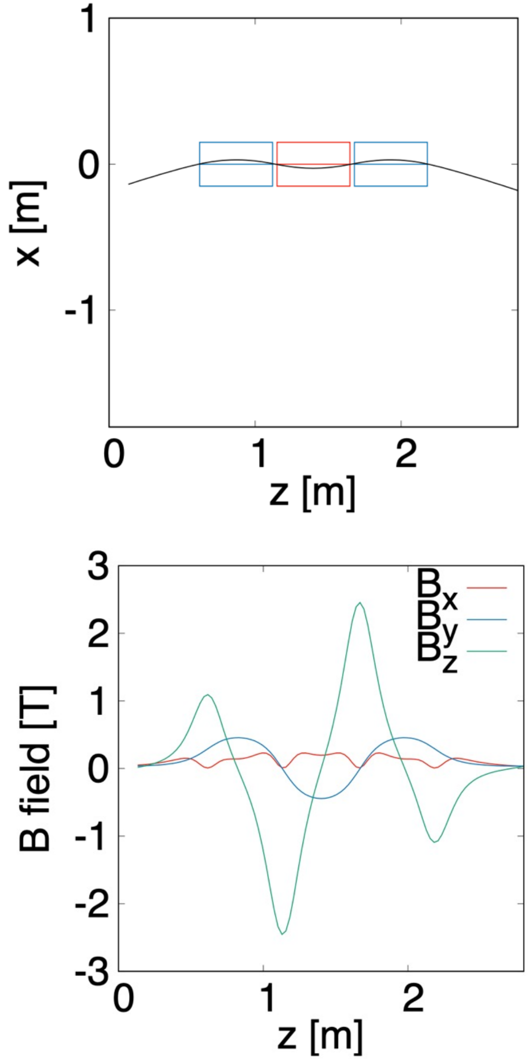
FETS-vFFA baseline lattice:

- Realistic **fringe field** fall off in Arctan(z)

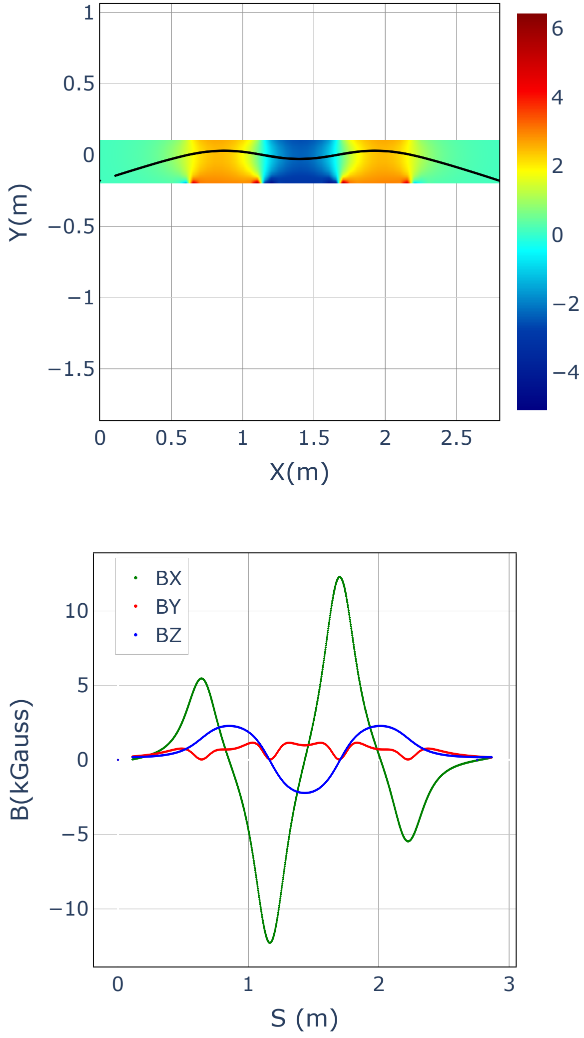
$$g\left(z\right)=\left[\arctan\left(\frac{z+M/2}{L_m}\right)-\arctan\left(\frac{z-M/2}{L_m}\right)\right]/\pi$$

- Parameters given by S. Machida:

Energy	3 to 12 MeV
Repetition	50 Hz
Number of proton per pulse	3.4×10^{11}
Focusing	FDF triplet
Circumference	28 m
Number of cell	10
Total cell length	2.8 m
Bd and Bf core length (M)	0.50 m
Straight length	1.24 m
Distance between Bd centre and Bf centre	0.53 m
Horizontal displacement between Bd and Bf	+/- 0 mm
Fringe field parameter (L)	0.15 m
Bd/Bf ratio (nominal)	1.15
m-value (nominal)	1.31
Orbit excursion	0.53 m
Tune (qu, qv, nominal)	0.243445 / 0.120023 (0756555 / 0.120023)
Dynamic aperture (normalised)	60 pi / 70 pi
Nominal 100 % emittance (normalised)	10 pi



Figures of S. Machida

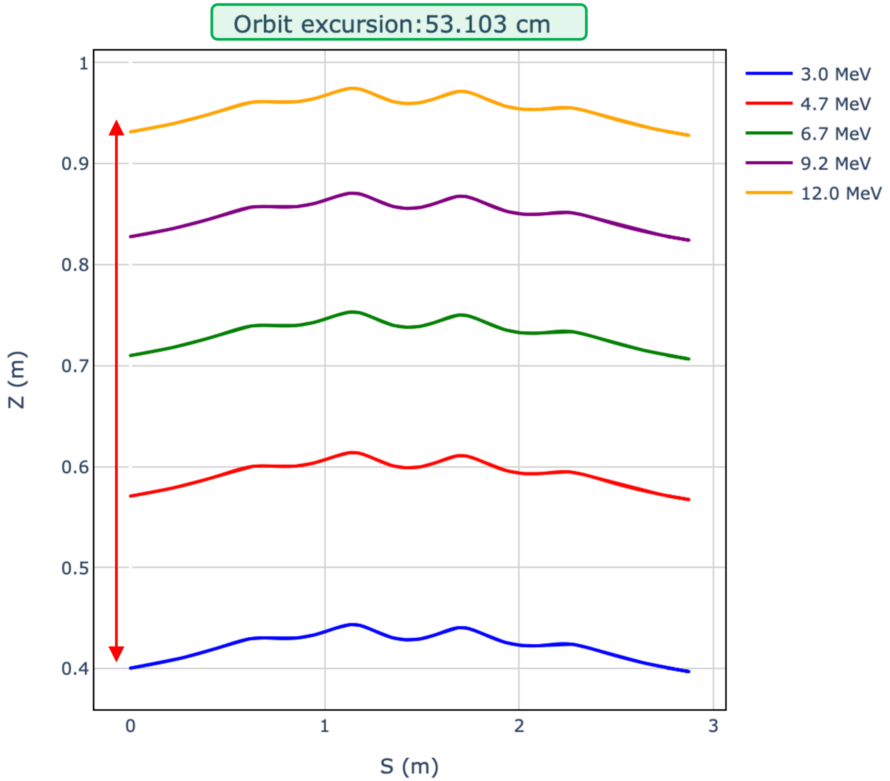


Reproduced with Zgoubi

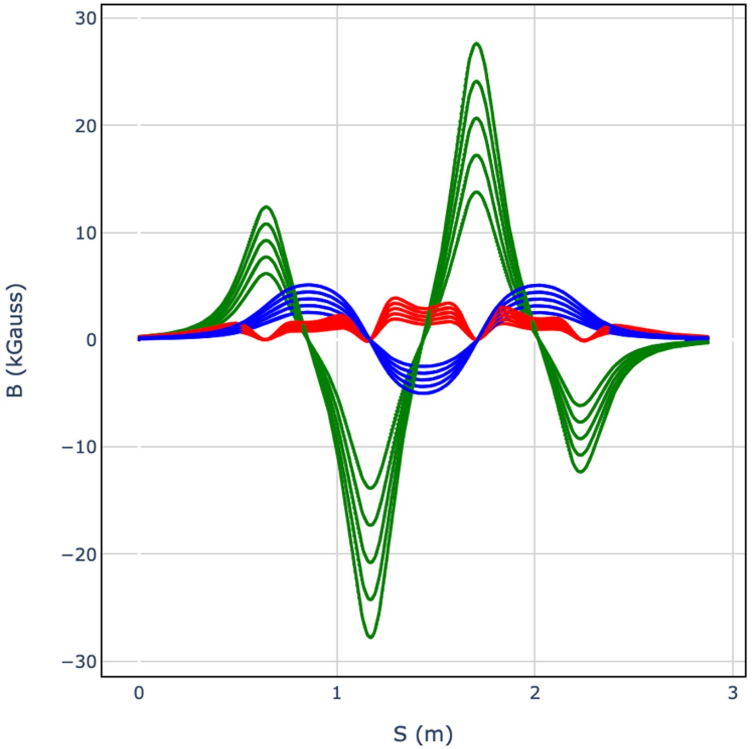
Lattice and methods – Arctan baseline lattice

- Search for **closed orbits** at different energies **for a single cell field map** without taking into account the residual field of neighboring cells:
  - Need to relax the p_0 = 0 (vertical angle) condition to find something stable
  - $\boldsymbol{p_0}$ **!=0** at the cell ends → **Not a closed orbit on the entire ring**

- The **vertical orbit excursion** is still visible and is similar to the one obtained with other codes



- The **magnetic field** scales with energy

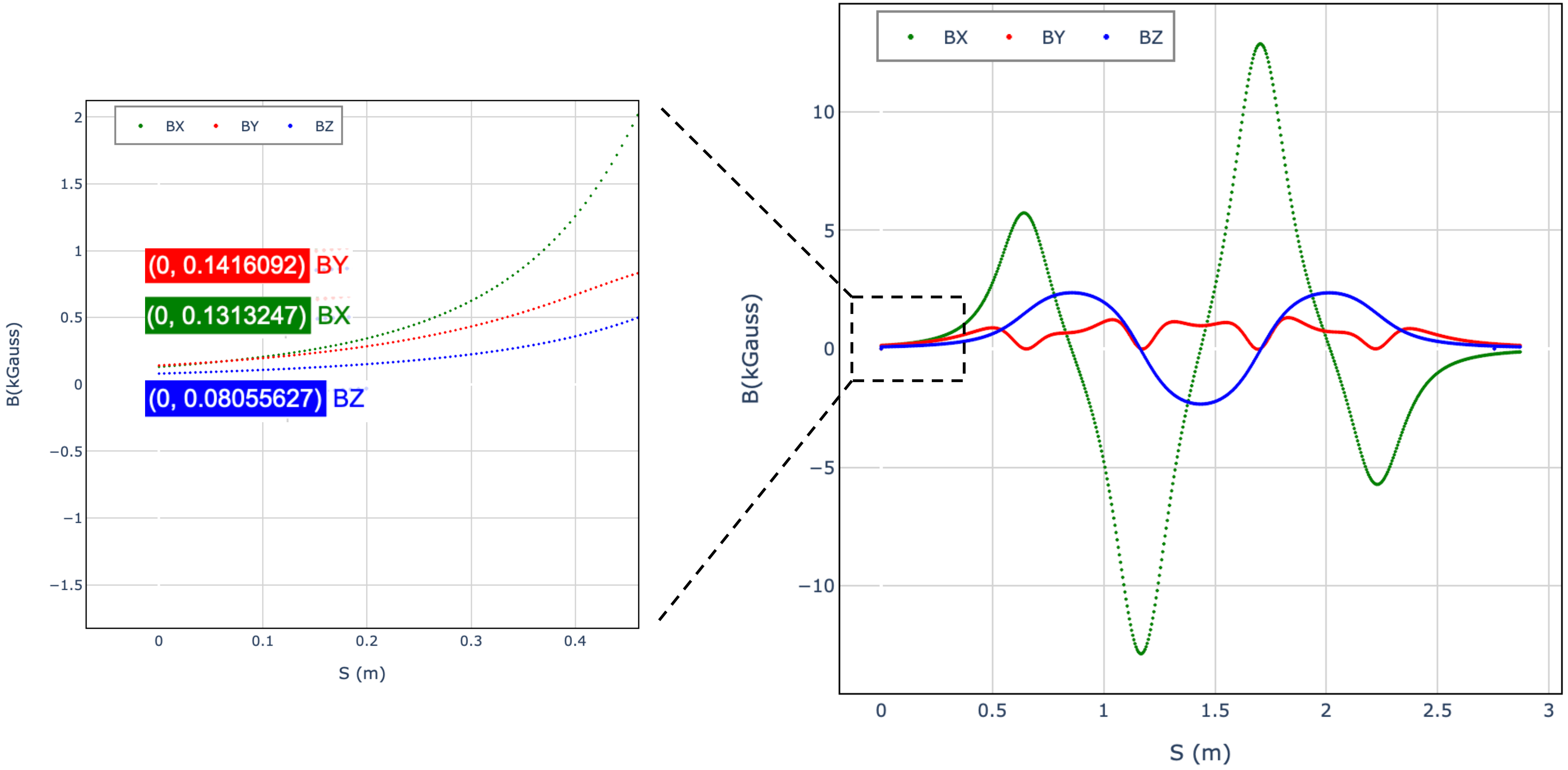


Lattice and methods – Arctan baseline lattice

Significant **neighboring cells influence** due to the **important residual field** at the cell ends

→ The residual field in the lattice in arctan **is ± 0.1kG in all directions**.

→The transverse residual field explains the non-zero vertical angle (p_0**!=0** at the cell ends)



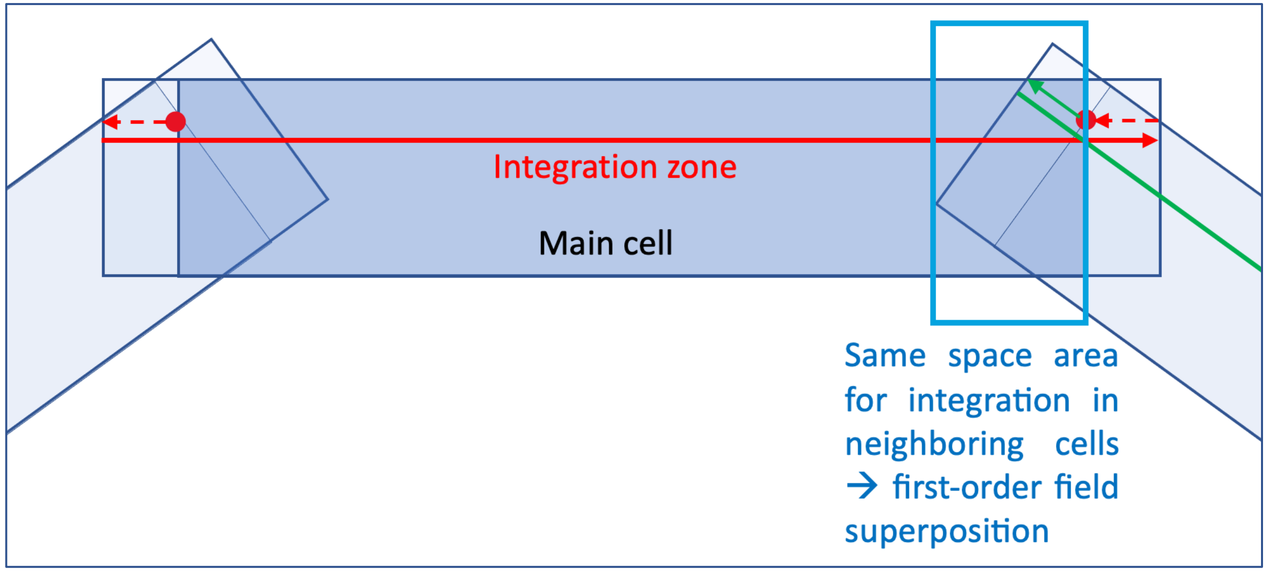
Lattice and methods – Methods

Methods to account for the residual field of neighboring cells:

1) Linear superposition by extending field maps

- Extent the integration zone to have field maps overlapping
- **Valid for small residual fields** to ensure that the trajectory deviation due to this field is limited
- In the arctan lattice, the **residual field is not negligible**

→ **Method not valid for this lattice**



Lattice and methods – Methods

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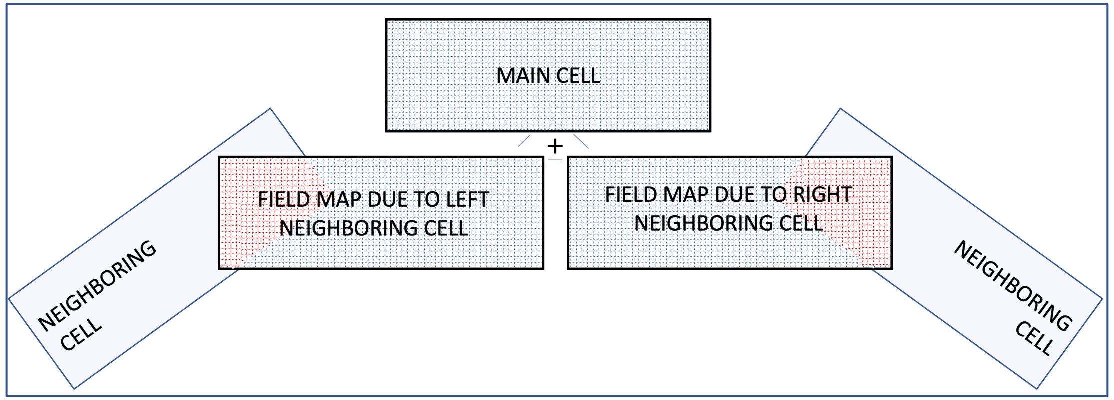
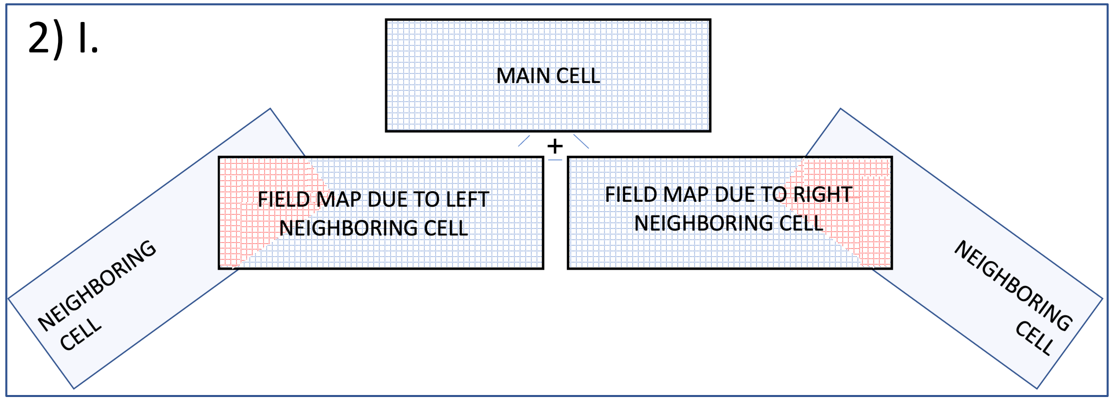


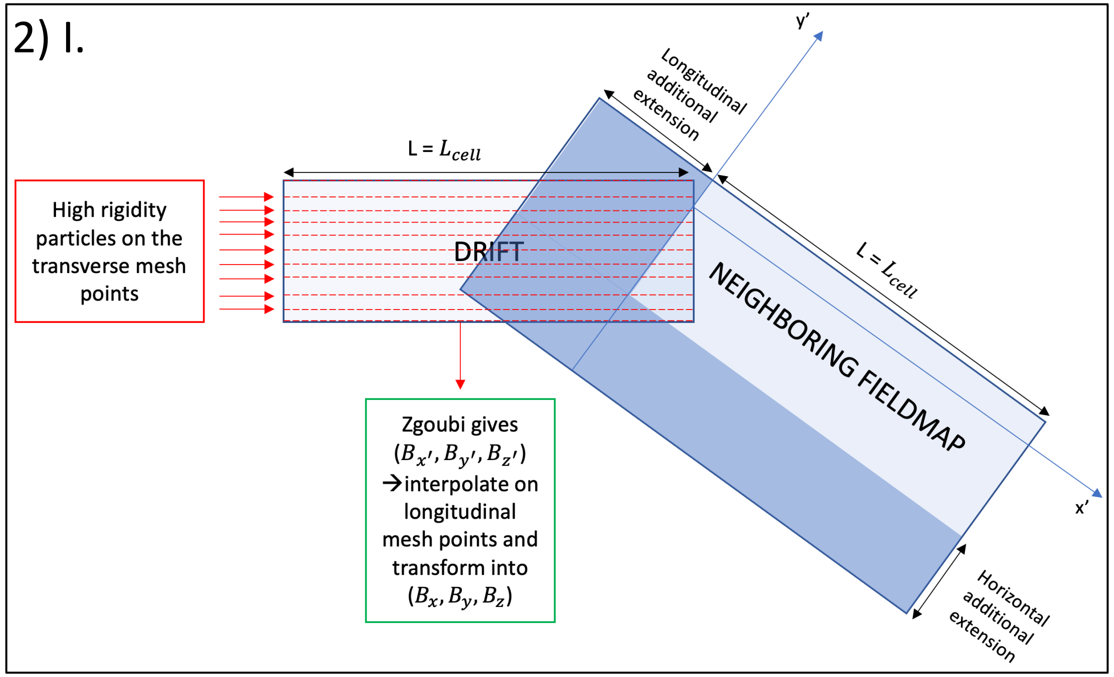
2) Actual superposition of field maps

- **Superposition of field maps with the same meshes with Zgoubi** → Construct the ‘left’ and ‘right’ field maps due to rotated neighboring cells

Lattice and methods – Methods

Methods to account for the residual field of neighboring cells:





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- **Superposition of field maps with the same meshes with Zgoubi →** Construct the 'left' and 'right' field maps due to rotated neighboring cells

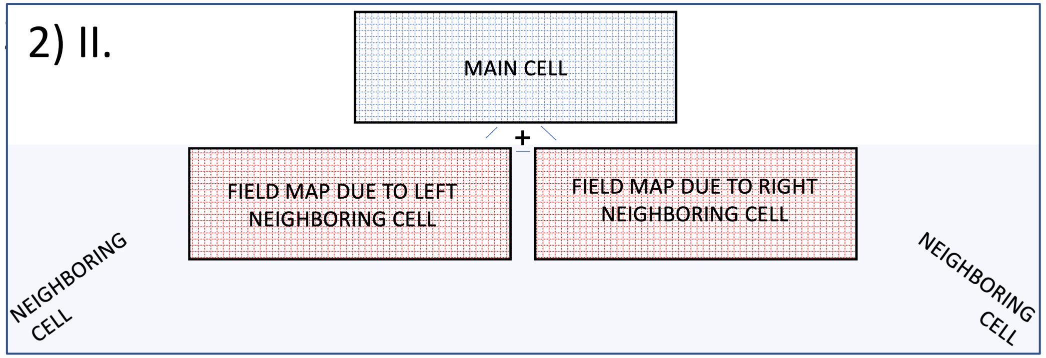
I. Construction of **truncated field maps** with 'finite' longitudinal/horizontal extents **using Zgoubi**

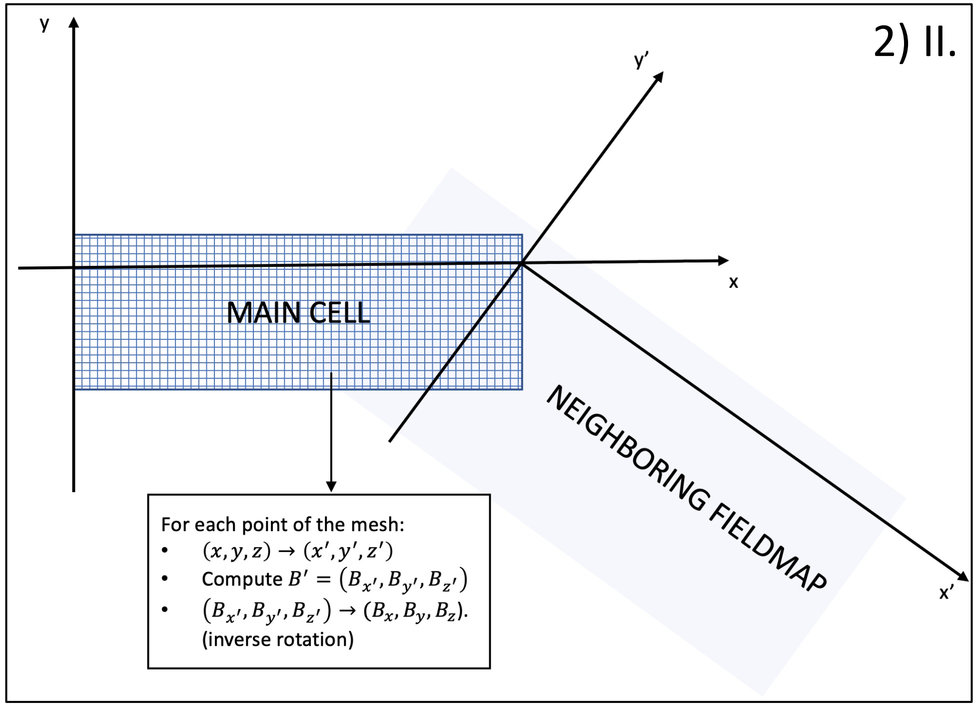
- **Influence** of the **horizontal/longitudinal** field map **extent** on the orbit

→Convergence to neighboring field maps that cover the entire orbit region

Lattice and methods – Methods

Methods to account for the residual field of neighboring cells:





2) Actual superposition of field maps

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I. Construction of **truncated field maps** with 'finite' longitudinal/horizontal extents **using Zgoubi**

- **Influence** of the **horizontal/longitudinal** field map **extent** on the orbit

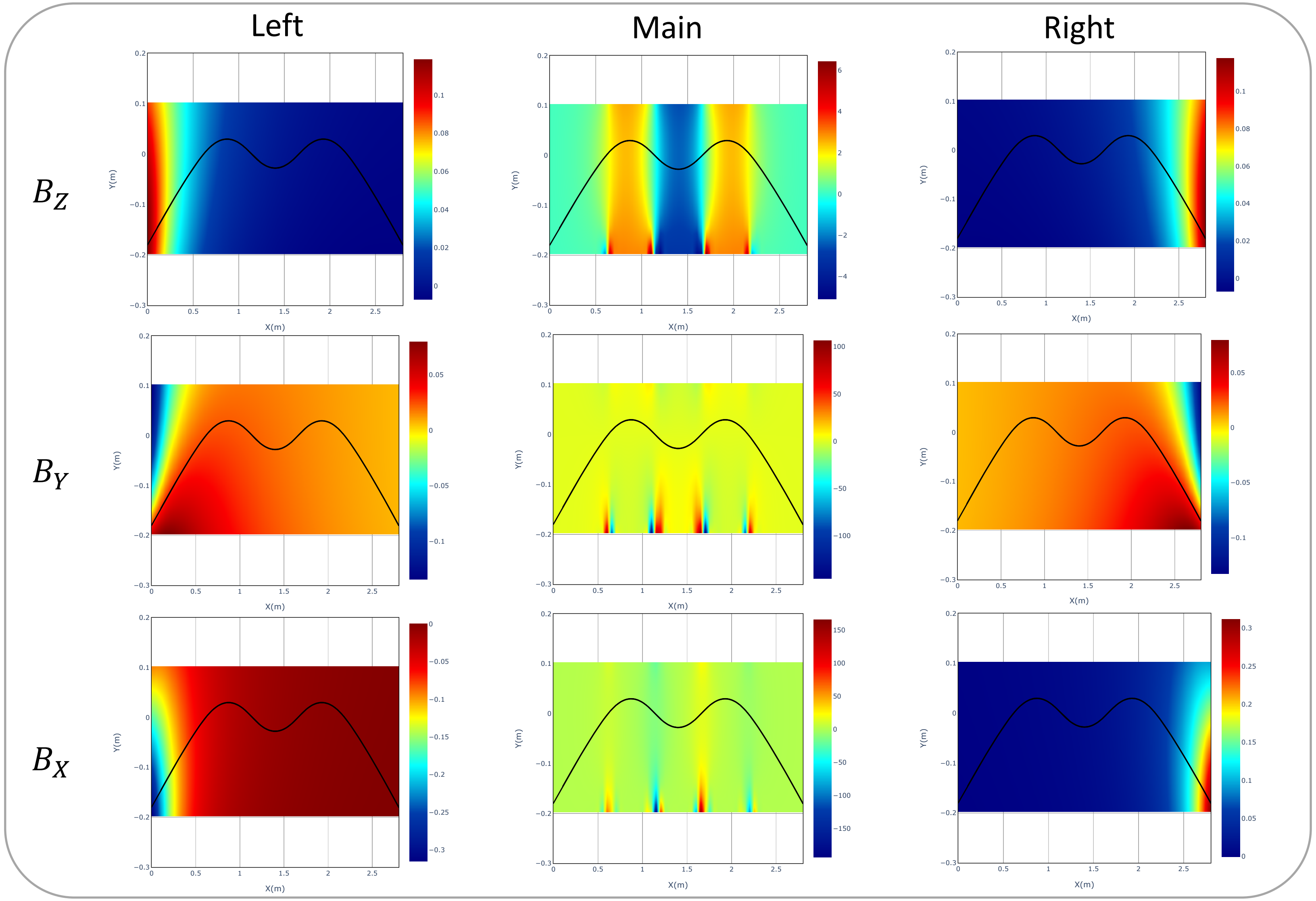
→ Convergence to neighboring field maps that cover the entire orbit region

II. Computation of the neighboring rotated fields with **analytical expressions** → No border anymore (called '**infinite**' or '**perfect**' map in the following slides)

→ Neighboring field maps that cover the entire region of the orbit

Fieldmap construction – Superposition

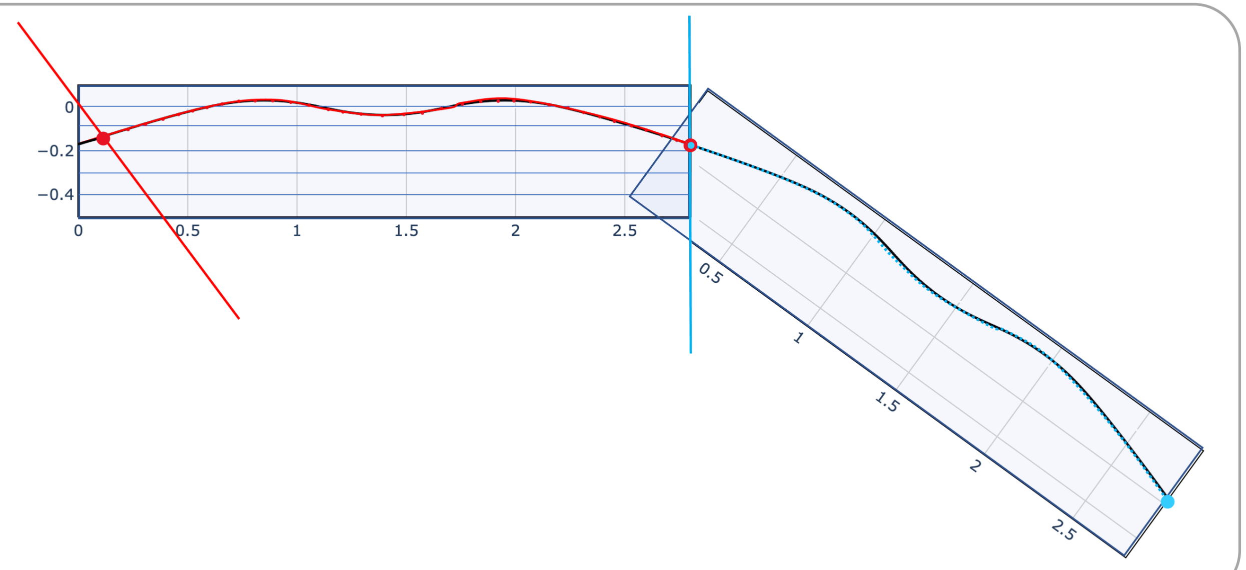
- After the construction, we use a Zgoubi Tosca option to superpose the field map with its neighboring field maps

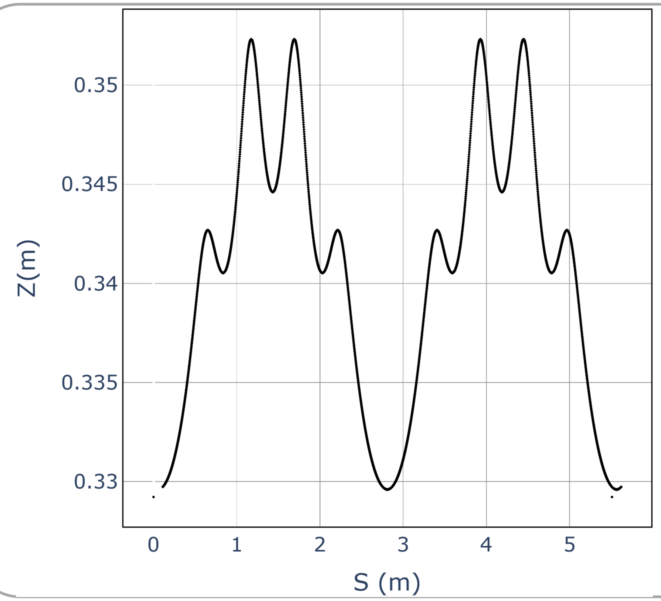


Lattice and methods – Limits of integration

Integration limits fixed by the **polar** character of the machine:

- We integrate between some ***droites de coupures*** (ddc) in Zgoubi, which are integration limits. It may not match the cartesian field map edges.
  - Entry oblique ddc: $tan(\theta)X$+Y=0
  - Exit vertical dcc: $X = 280$
- We have **no map overlapping** with well-chosen cut lines
- Perfect **trajectory continuity** between neighboring cells



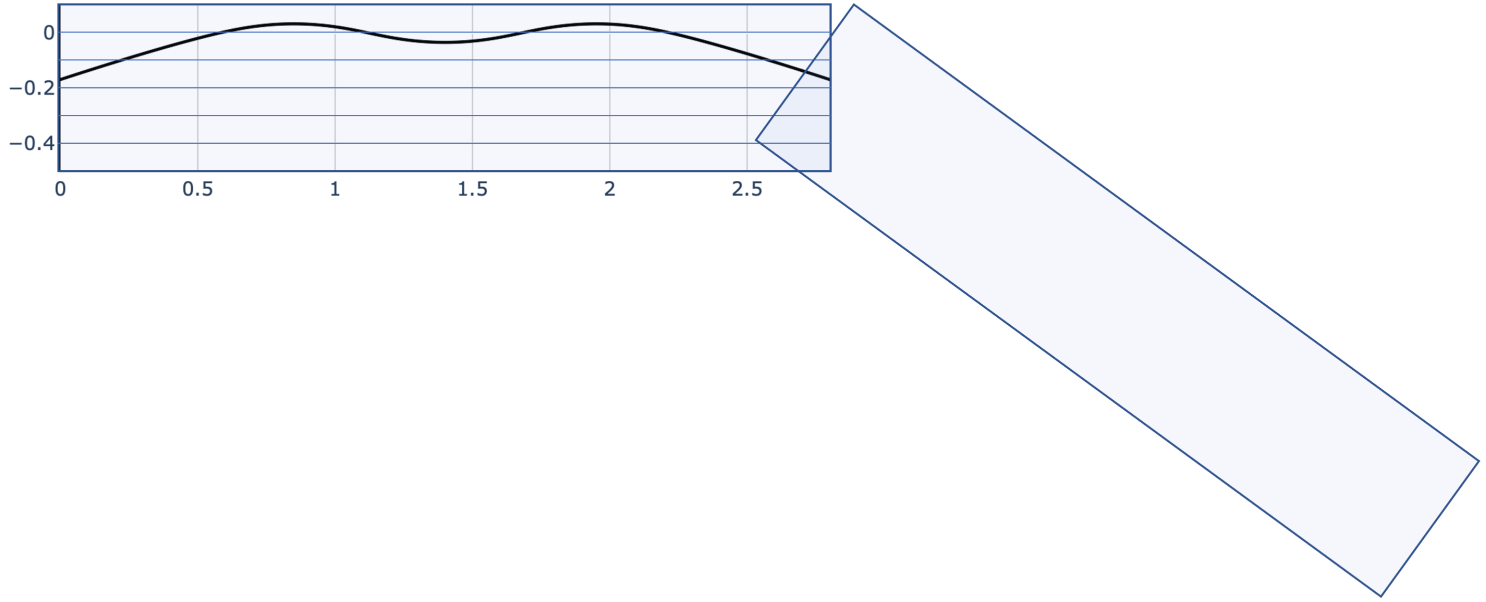


Linear beam dynamics

Closed orbit search & Computation of the tunes

Influence of the fieldmap extents – Method

For a more **systematic study of the increasing influence of neighboring cells**, we use field maps with a very large longitudinal extension (220cm) but different horizontal extensions



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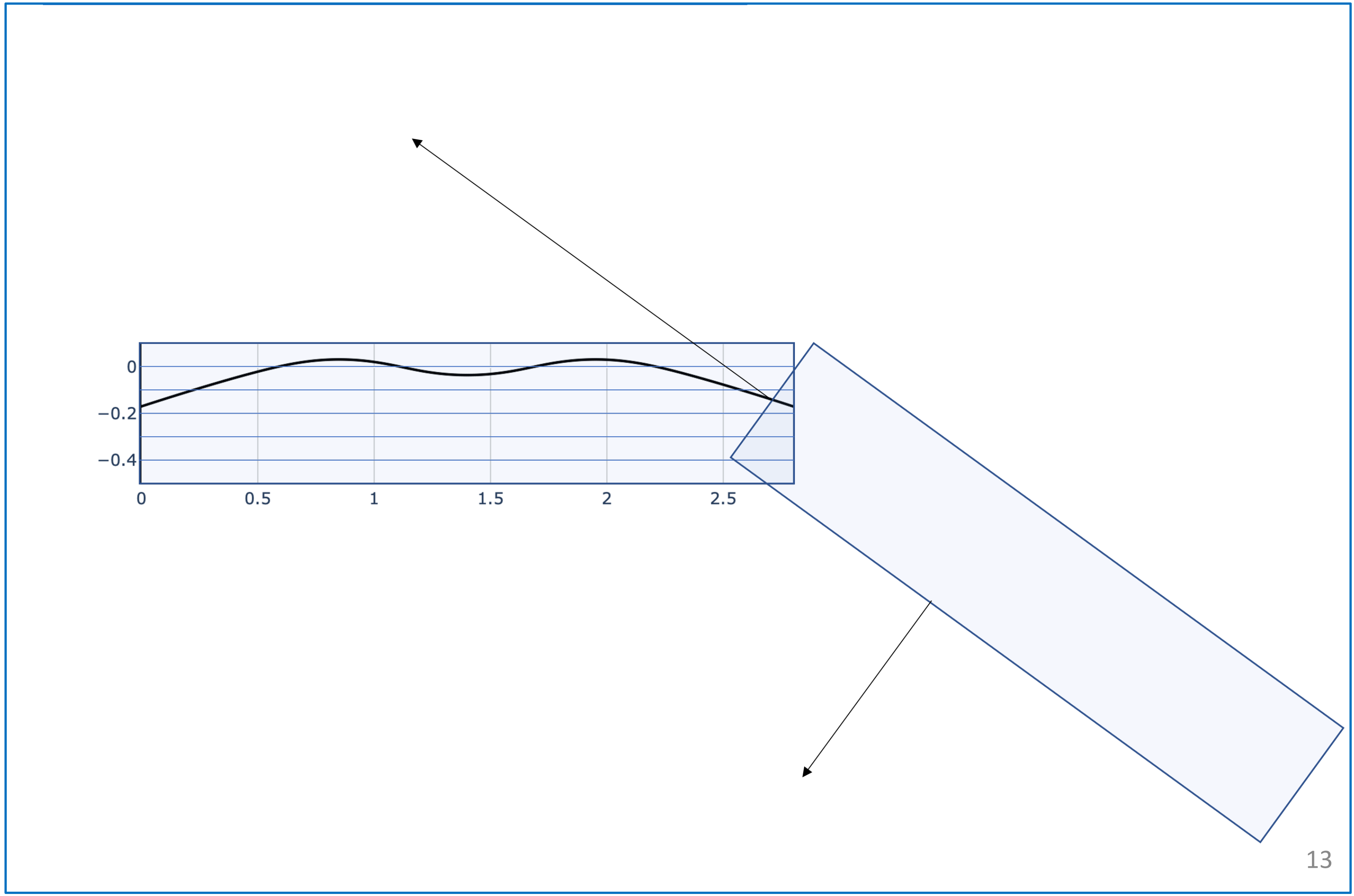

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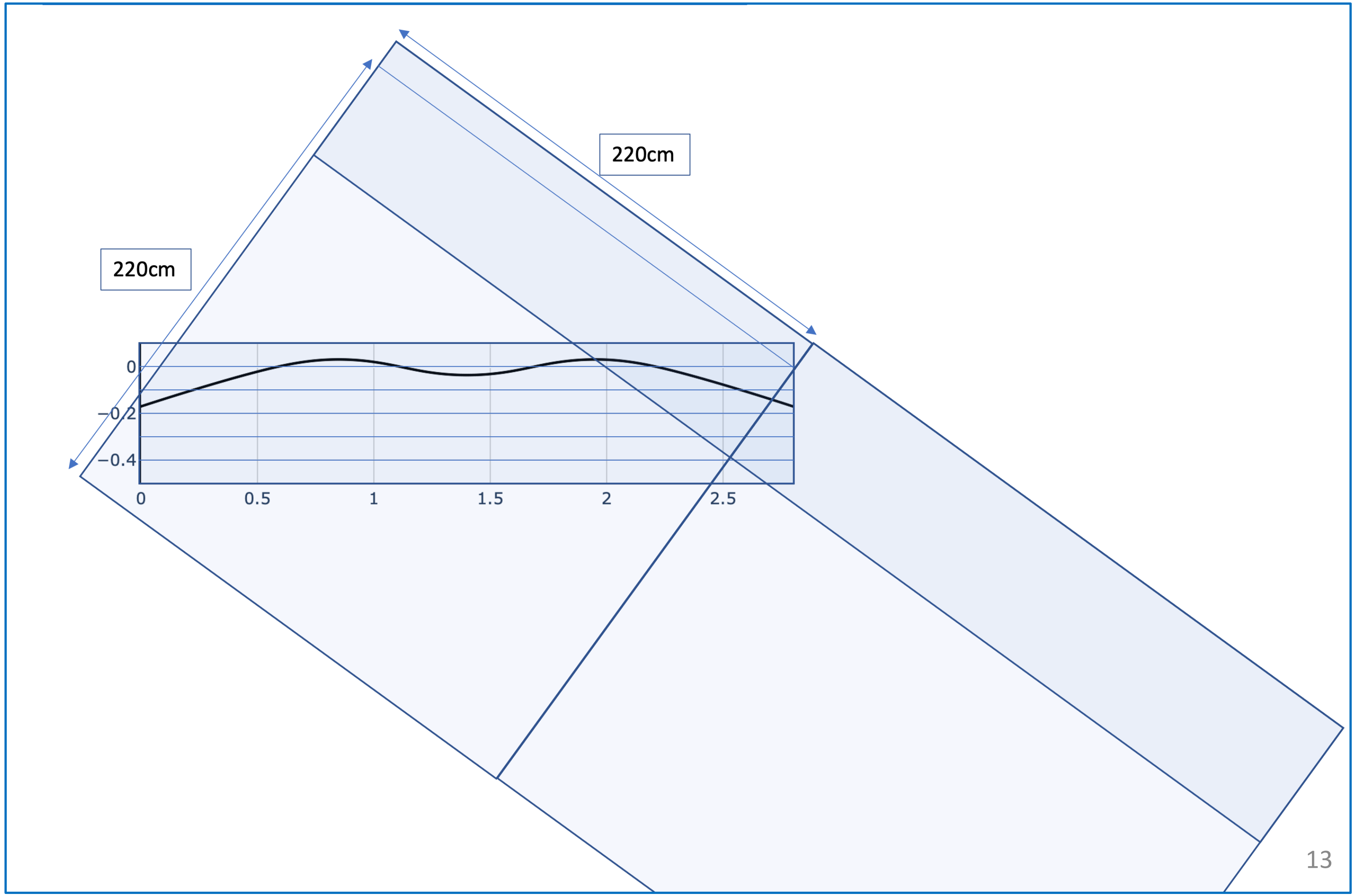
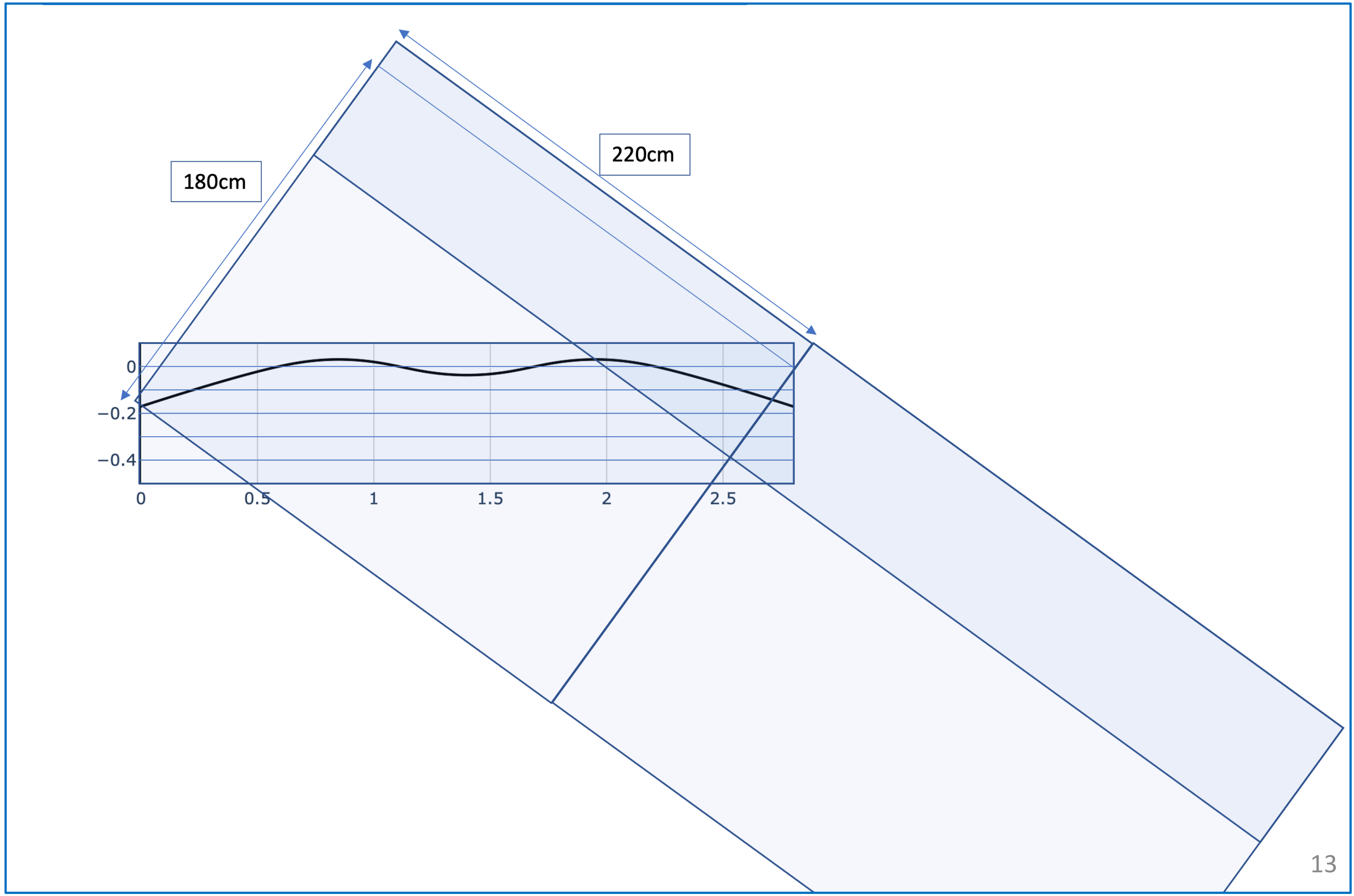


Influence of the fieldmap extents – Method

For a more **systematic study of the increasing influence of neighboring cells,** we use field maps with a very large longitudinal extension (220cm) but different horizontal extensions

- **Complete recovery of the orbit** for a horizontal extension of 180cm: we should see a **convergence**

→No more change in the optics if we further increase the map



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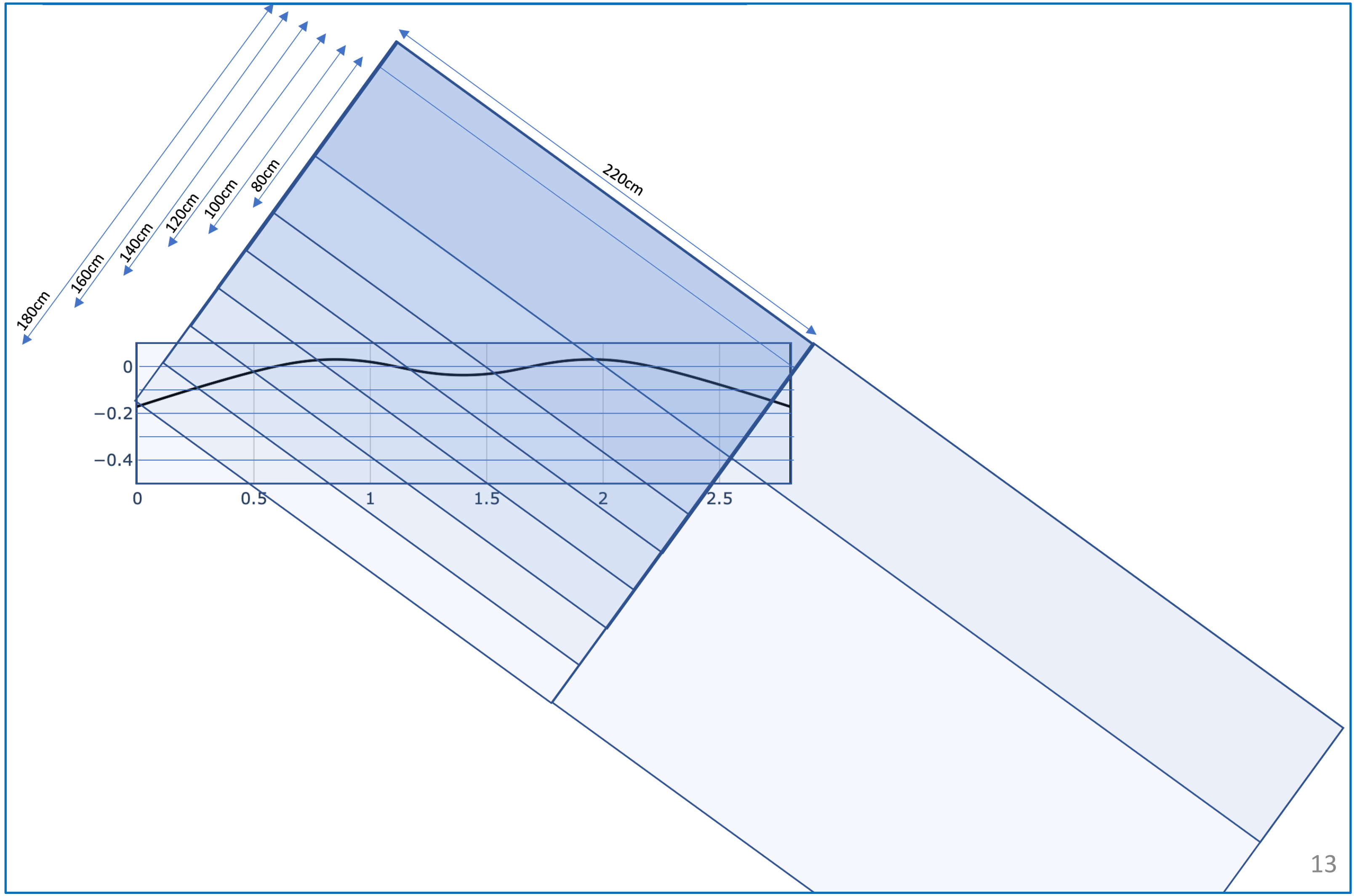
- For **smaller horizontal extension**, we cross the closed orbit at different places→ **More or less influence of neighboring cells**



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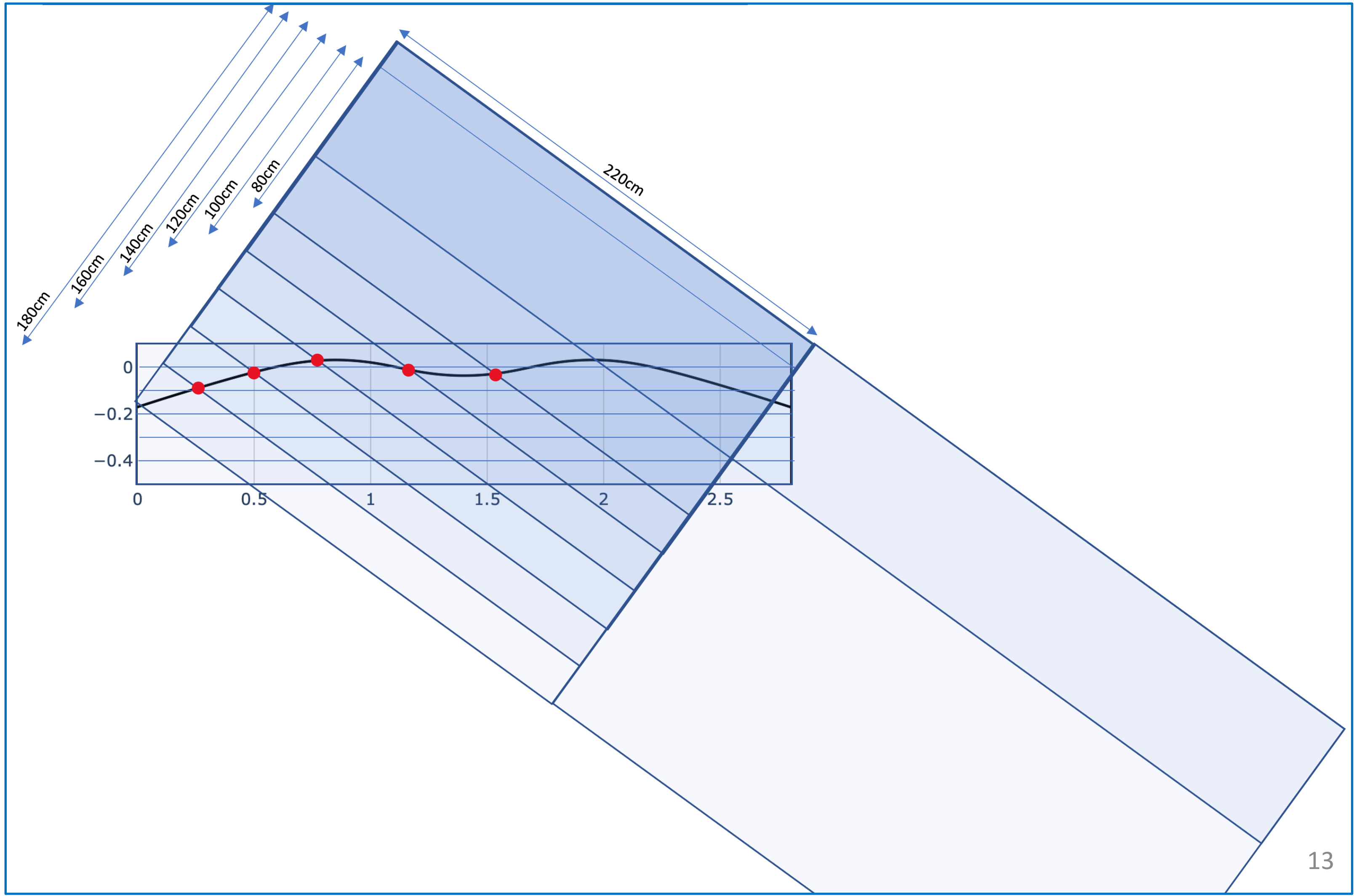
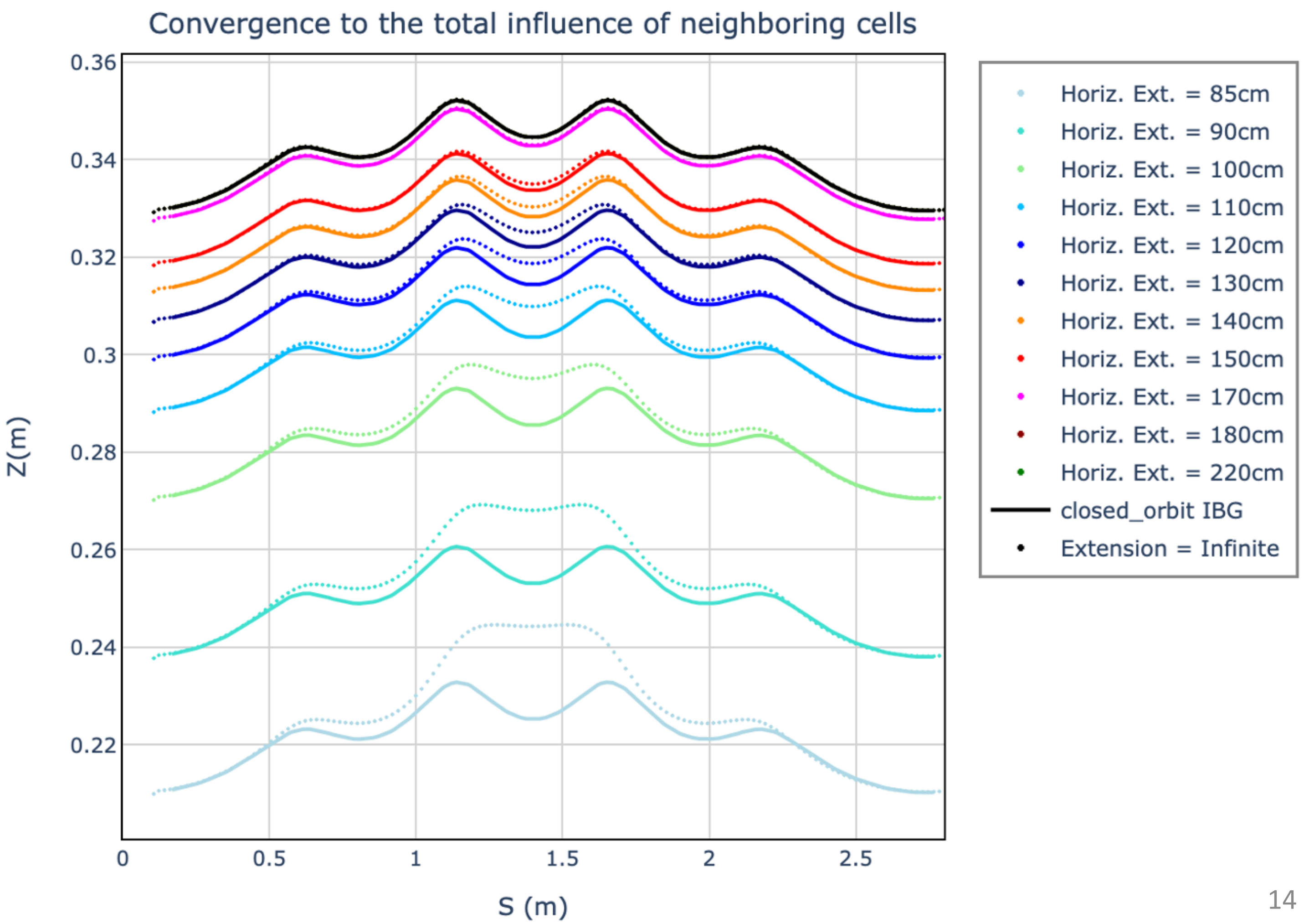


Closed orbits for different neighboring fieldmap extents

Orbit convergence with the field map extension from rotated truncated field maps computed with Zgoubi to « perfect » field map analytically computed

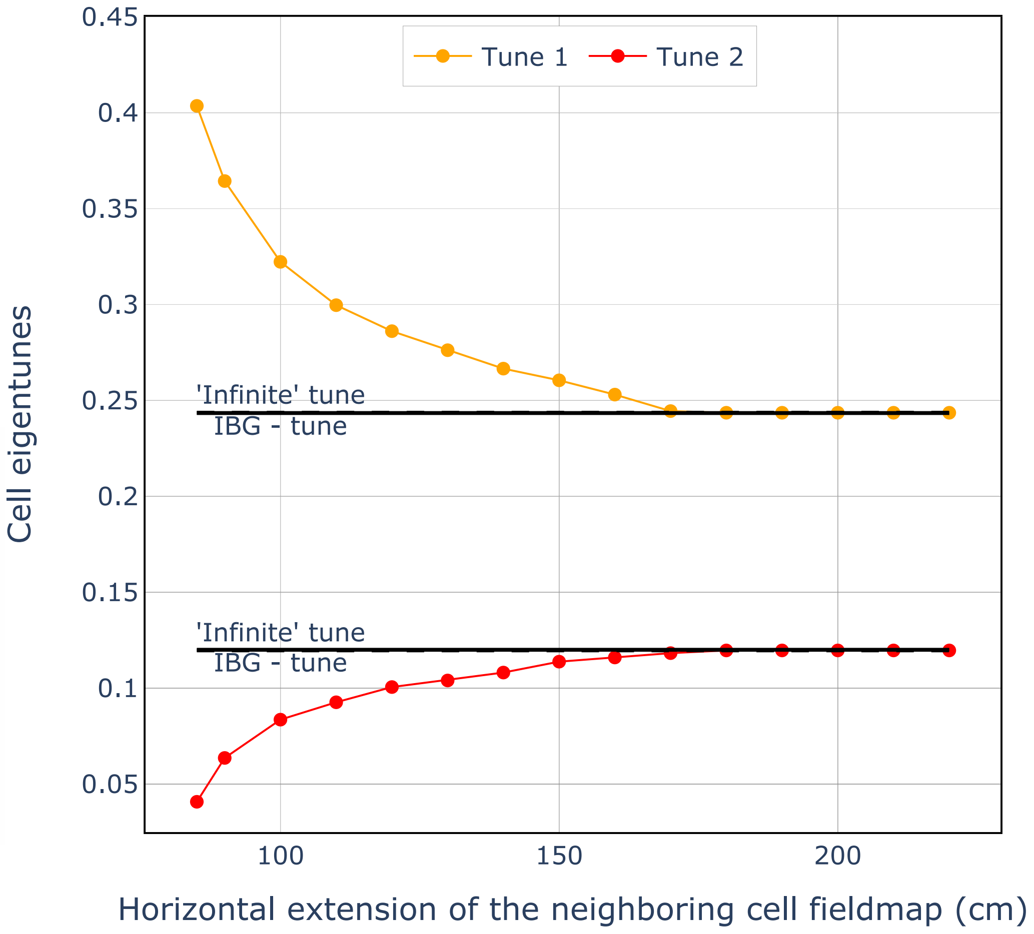
- The closed orbit given by S. Machida (IBG) is put at an arbitrary vertical coordinate to compare the shape and the vertical extension
- **Orbit convergence towards IBG orbit**, which is the same as the orbit found with the perfect field map
- **Vertical excursion** of the orbit with an increasing neighboring field map extension



Linear beam dynamics - Computation of the tunes

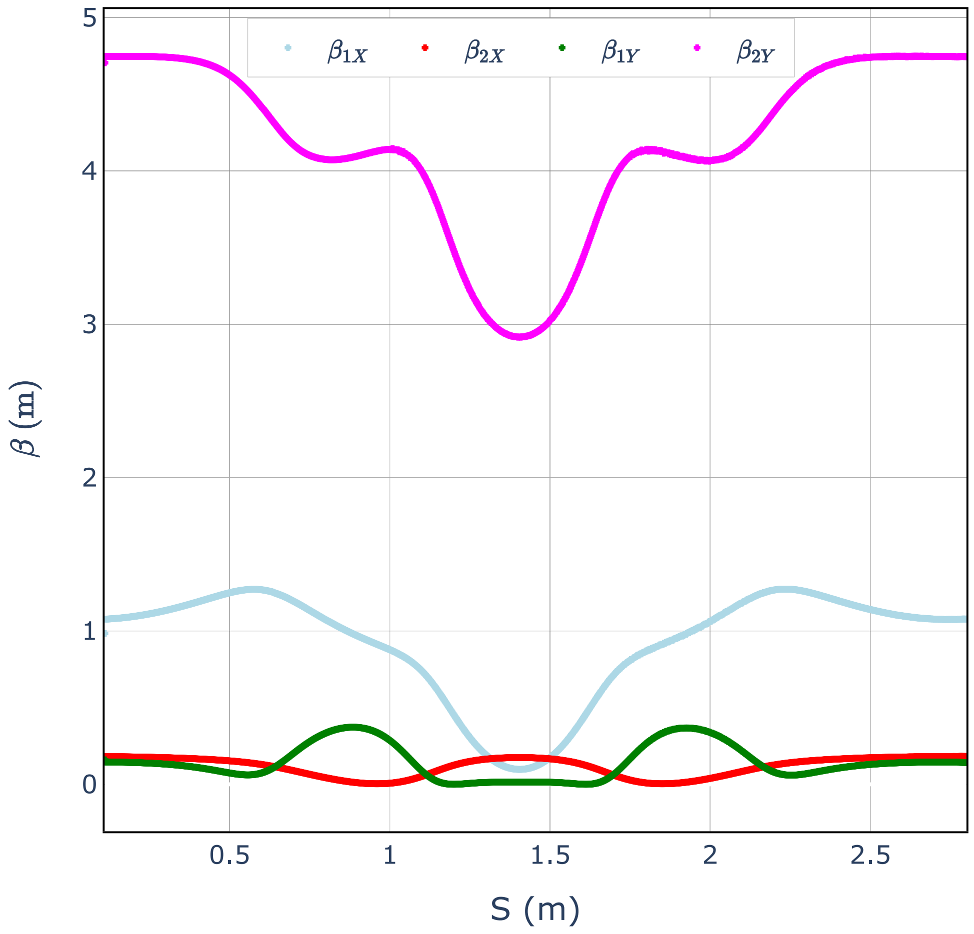
Convergence of the tunes:

- Tunes computed with the perfect map are **(0.24362, 0.119732)**, compared with IBG tunes: **(0.243445, 0.12002)**



Computation of the lattice functions:

- Correct computation of the lattice function with the Lebedev and Bogacz parametrization



Non-linear beam dynamics

2D Dynamic aperture - 4D Dynamic aperture

Dynamic aperture – 2D

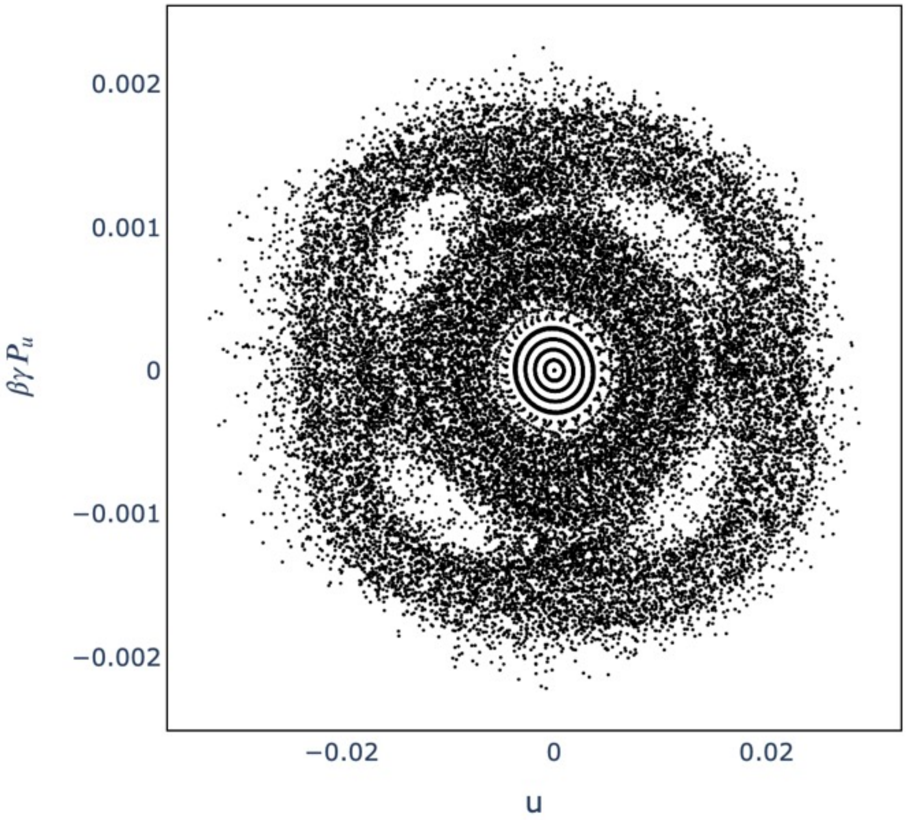
Tune (qu, qv, nominal)	0.243445 / 0.120023 (0756555 / 0.120023)
Dynamic aperture (normalised)	60 pi / 70 pi

Normalized dynamic aperture calculated with Zgoubi in eigenplanes on **100 turns**

- Assumed ellipse-shaped phase space
- Small "stable" region, then a more diffuse region, but where the particles are not lost
- Islands → fixed points of order 4

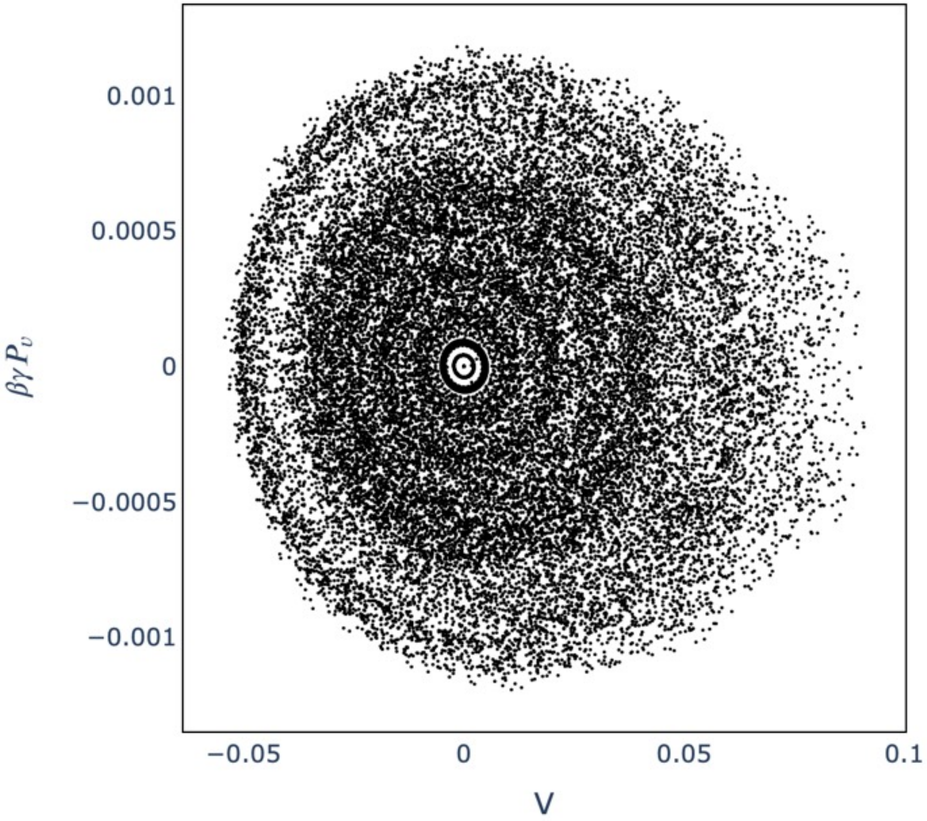
$Neighboring\ cell\ fieldmap\ extension : Infinite$

$e_u = 53.966\pi\ mm\ mrad$



$Neighboring\ cell\ fieldmap\ extension : Infinite$

$e_v = 74.81\pi\ mm\ mrad$



Dynamic aperture – 2D

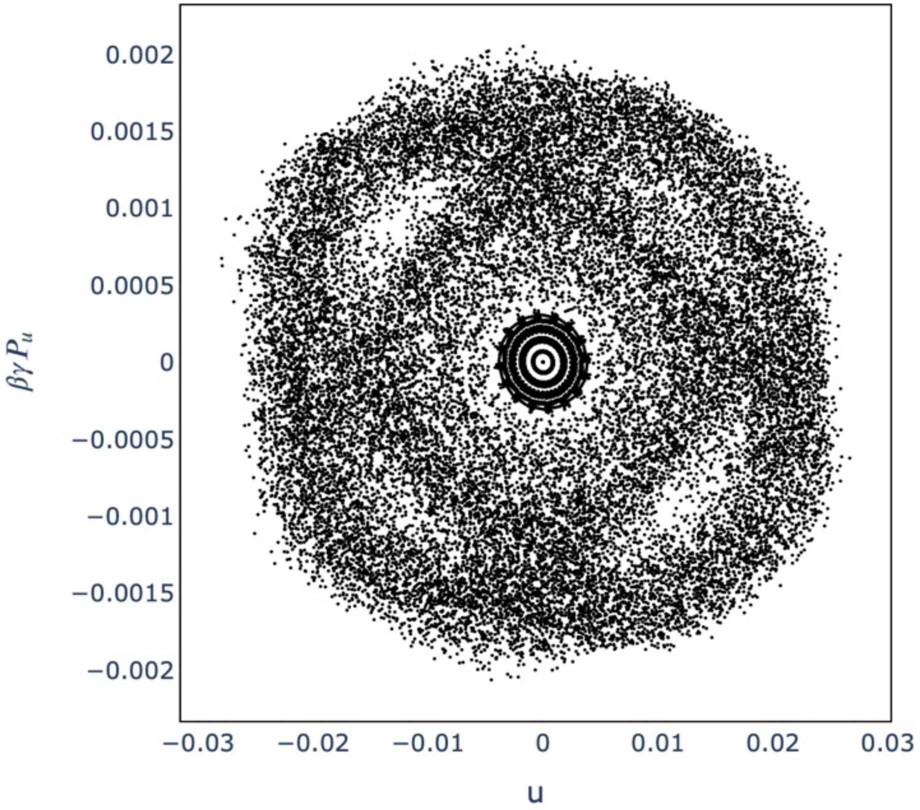
Tune (qu, qv, nominal)	0.243445 / 0.120023 (0756555 / 0.120023)
Dynamic aperture (normalised)	60 pi / 70 pi

Normalized dynamic aperture calculated with Zgoubi in eigenplanes on **1000 turns**

- The phase space is even more “diffuse”
- We still observe the islands (4th order fixed points)
- The 2D-dynamic aperture (assuming ellipse formula) in u and v spaces decrease

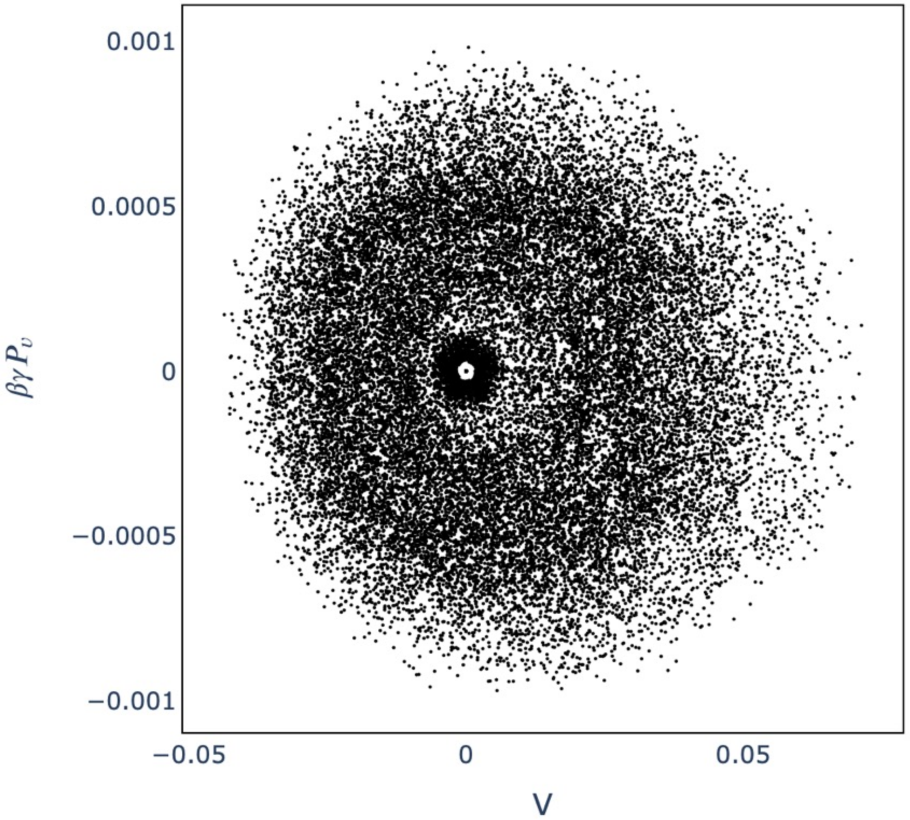
Neighboring cell fieldmap extension : Infinite

$e_u = 43.466\pi$ mm mrad



Neighboring cell fieldmap extension : Infinite

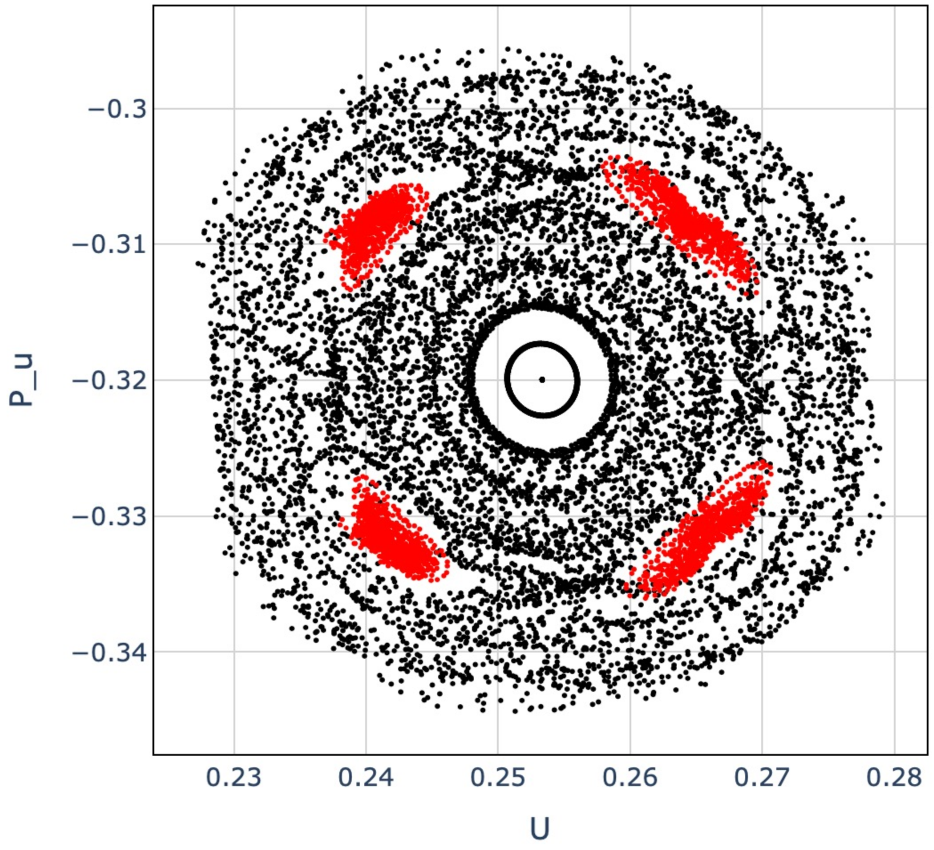
$e_v = 41.208\pi$ mm mrad

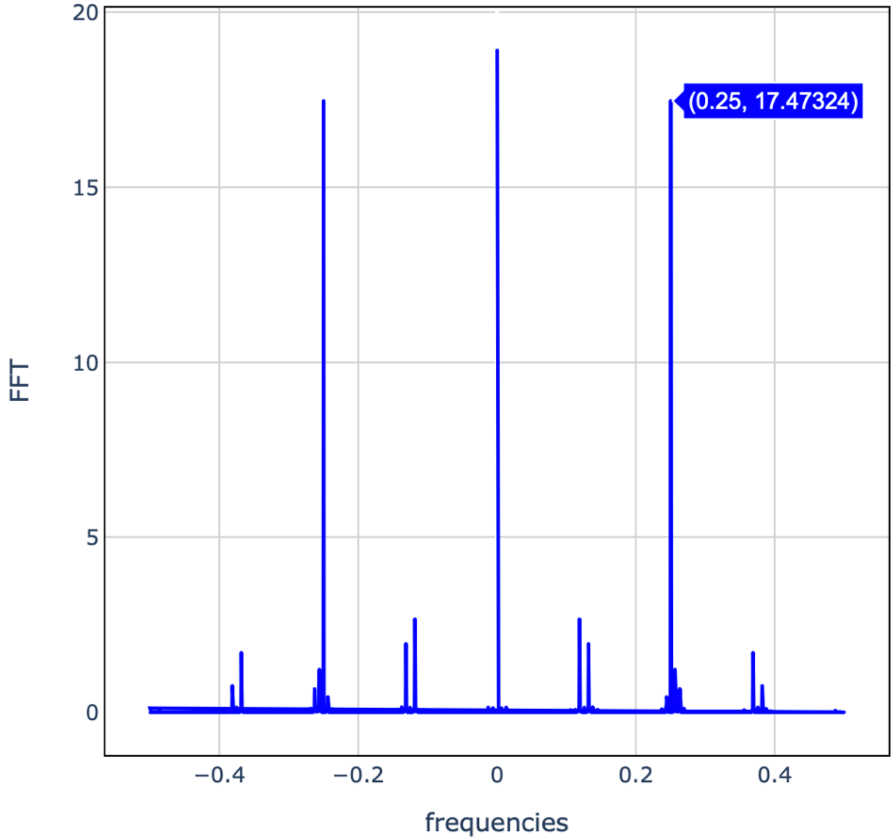


Dynamic aperture – 2D

Normalized dynamic aperture calculated with Zgoubi in eigenplanes on **1000 turns**

- Tracking of 3 particles into the islands
- The tune is 0.25, as expected





Dynamic aperture – Non-linearity considerations

The **magnetic field** in vFFA is **highly non-linear**:

- **Non-elliptical shapes** in the linearly decoupled phase spaces → The **"2D-DA" need to be refined** to take into account the non-linearity

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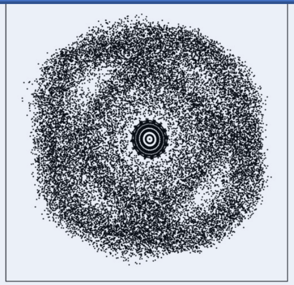
- **Non-elliptical shapes** in the linearly decoupled phase spaces → The **"2D-DA" need to be refined** to take into account the non-linearity

- Different computation methods for the 2D-DA:
    - Assumed **ellipse-shaped** phase space
    - Computation of the **average of linear invariants**
    - **Integration** around the innermost points of the phase space area

Non-elliptical shape; 2D-DA



Dynamic aperture – Non-linearity considerations

The **magnetic field** in vFFA is **highly non-linear**:

- **Non-elliptical shapes** in the linearly decoupled phase spaces → The **“2D-DA” need to be refined** to take into account the non-linearity
- The non-linearity couples the ‘linearly decoupled’ planes → Need to account for the **non-linear coupling** and to define/compute a **more general “4D-DA”**

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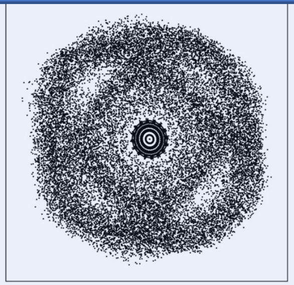
Non-elliptical shape; 2D-DA



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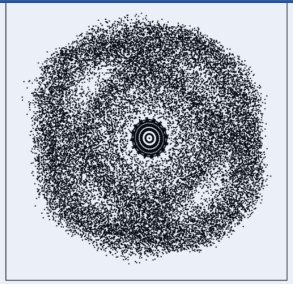
Non-elliptical shape; 2D-DA



- Different computation methods for the 2D-DA:
  - Assumed **ellipse-shaped** phase space
  - Computation of the **average of linear invariants**
  - **Integration** around the innermost points of the phase space area

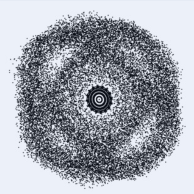
Non-linear coupling; "4D-DA"

- More robust definition for the 4D betatron motion:
  - Literature review for the 2D and 4D betatron motion
  - Computation of 2D-DA for different amplitude ratios between decoupled planes
  - Computation of a "theoretical" 4D-DA (average distance) to compare working points
  - Computation of a "practical" DA based on precautionary principle for better interpretation

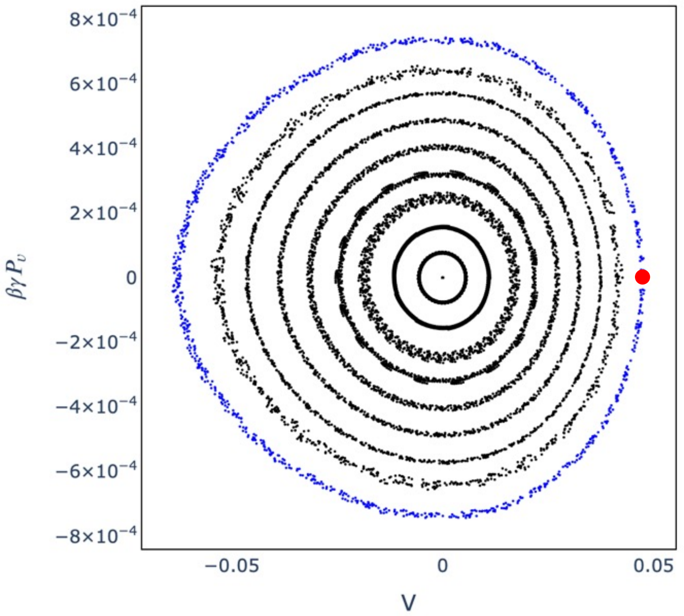
Dynamic aperture – 2D

Non-elliptical shape; 2D-DA

- Different computation methods

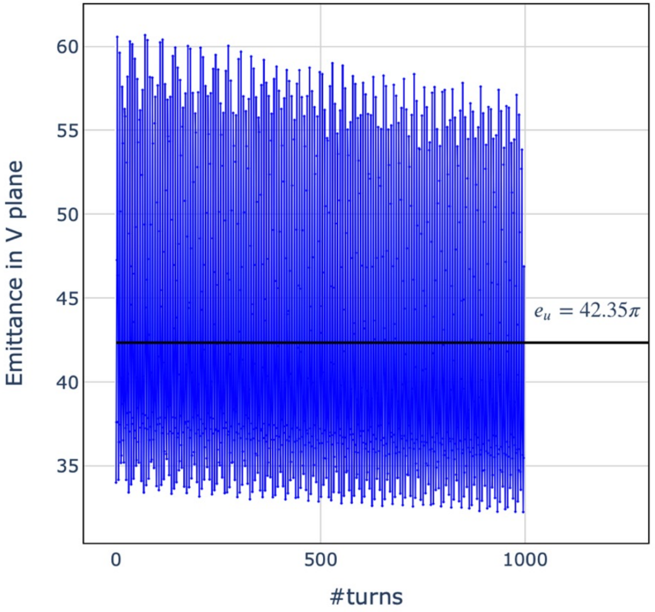


Possible definitions for 2D-DA:



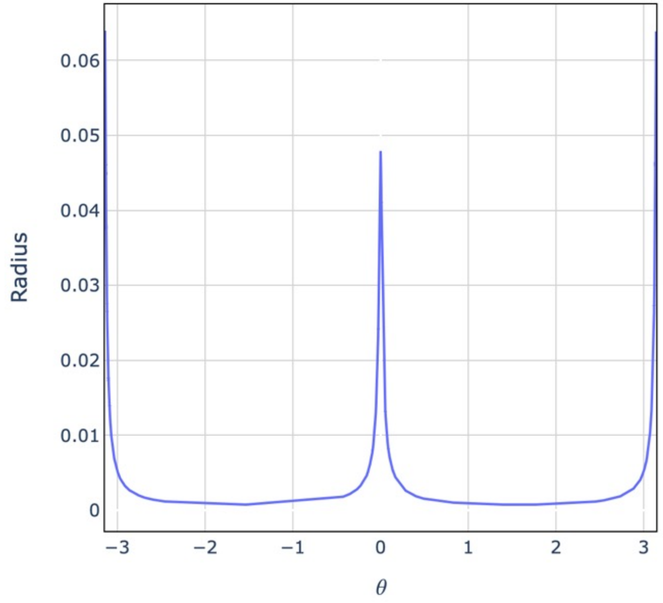
Assume ellipse-shaped space

- Computation of the Courant-Snyder invariant at a **given point**



Average of linear invariants

- **Average** of the emittances computed with the ellipse formula for **each** blue **point in the cloud** (each turn)

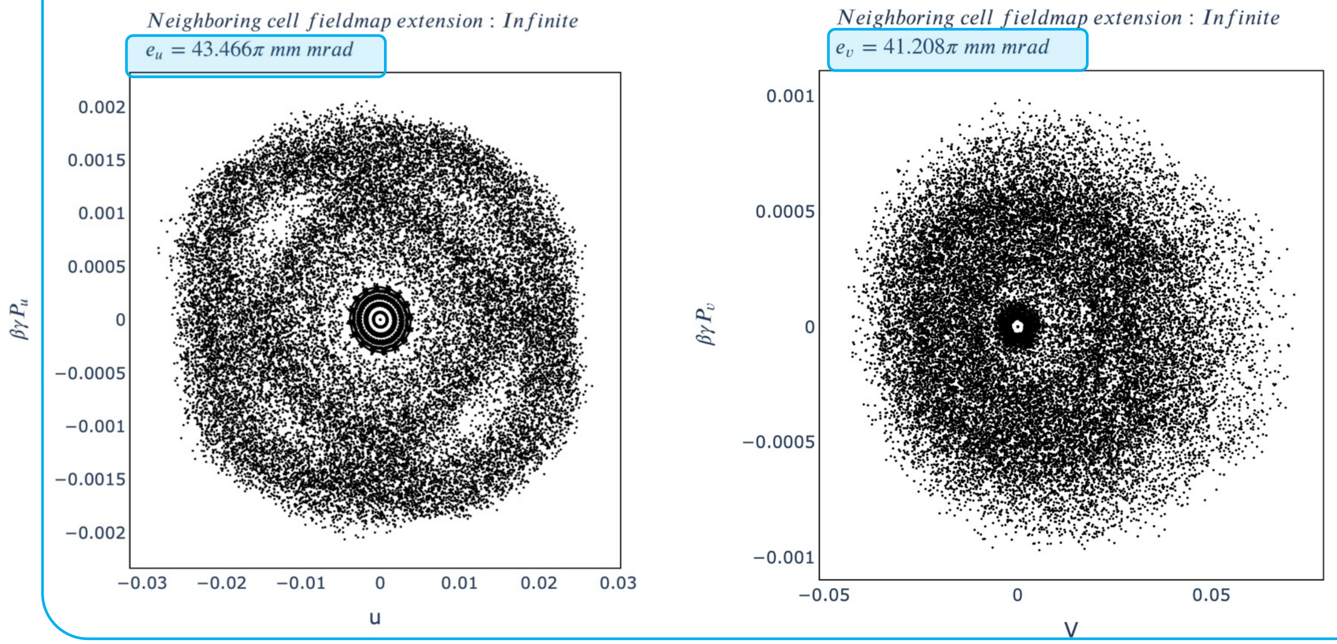


Integration over phase space

- Points cloud in $\boldsymbol{R-\theta}$ **form** and **integrate** around the innermost points of the phase space area

Dynamic aperture 2D – FETS-vFFA

Ellipse formula

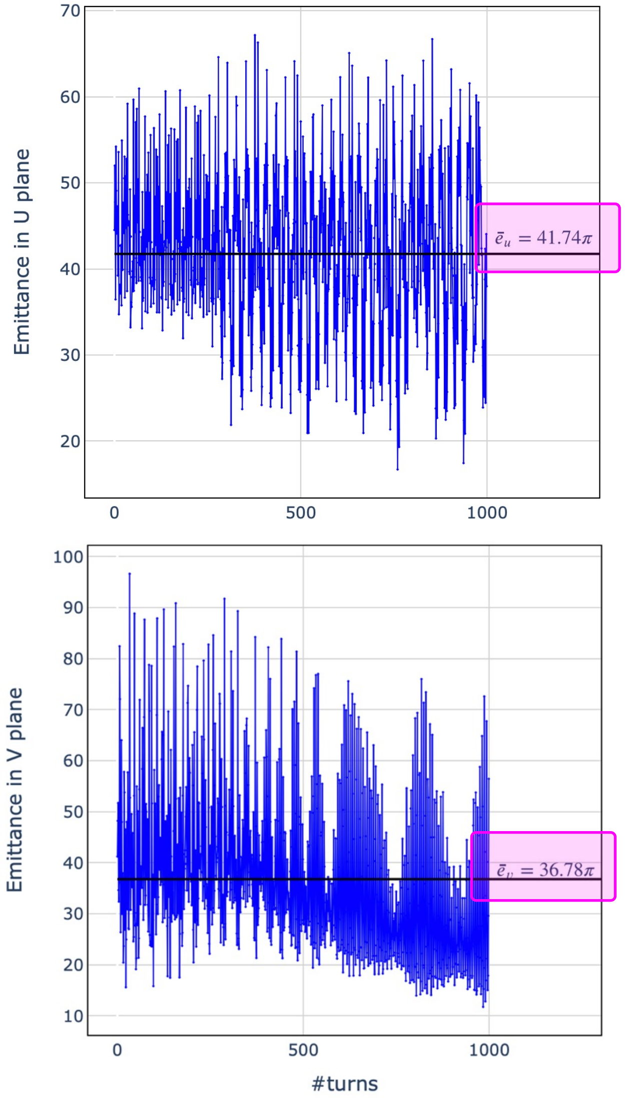


Integration over phase space

- $e_u = 42.016\ \pi\ mm\ mrad,$
- $e_v = 35.412\ \pi mm\ mrad$

We obtain something very **similar** between the DA calculated with the **average of the linear invariants** and the DA calculated with the **integration over the phase space**

Average of linear invariants



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Dynamic aperture – 4D

Non-linear coupling; "4D-DA"

- More robust definition for the 4D motion:
  - Litterature review
  - 2D-DA (various decoupled planes amplitude ratios)
  - "Theoretical" 4D-DA (average distance)
  - "Practical" DA (precautionary principle)

- Examples of papers that extensively discuss Dynamic Aperture:
  - **E. Todesco and M. Giovannozzi, 'Dynamic aperture estimates and phase-space distortions in nonlinear betatron motion', Phys. Rev. E 53(4), 4067 (1996).**
  - M. Giovannozzi and E. Todesco, Numerical methods to estimate the dynamic aperture, Part. Accel. 54, 203 (1996).
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Dynamic aperture – General considerations

- **« General » definition**: **stability domain** – particles bounded after **N turns**

- **Dependent on N**, and N depends on the application

  →How many turns do we need for the full acceleration cycle?

  →For hadron storage ring: predict « long-term » stability

- **Dependent on the motion** we look at:

  - **2D betatron motion**, without coupling

    →Stability domain = phase space **area** of initial conditions that survive N turns

    →Border between stable/unstable motion (1D KAM torus) → stability domain enclosed by the last connected stable invariant curve

  - **4D betatron motion**, including coupling

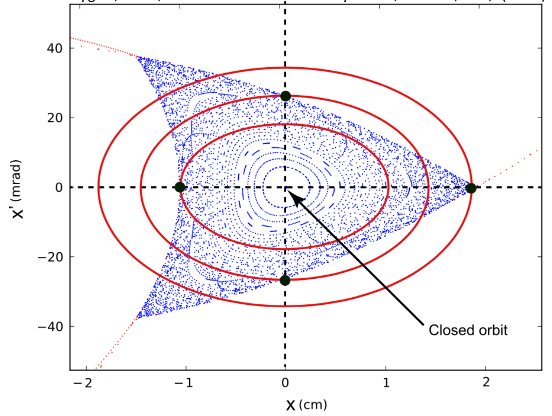
    → Stability domain = phase space **volume** of initial conditions that survive N turns

    → Volume may be irregular/have holes but generally not the case

    → Dynamic aperture: **radius of the hypersphere** with the same volume as the stability domain

Dynamic aperture – 2D betatron motion

S. Tygier, et al., Nucl.Instrum.MethodsPhys. Res., Sect. A , 775, (2015).



- **Methods without averaging - precautionary principle**:
  - With the linear definition, the ellipse depends on the direction because of the phase distortion

  → Choose the smaller possible ellipse.

- **Methods** that give an **average** distance to stability border:
  - Direct integration: $\int\int \chi(x,p_x)\,dx\,dp_x \quad A_\vartheta = \int_0^{2\pi}\int_0^{r(\vartheta)} r\,dr\,d\vartheta = \frac{1}{2}\int_0^{2\pi}[r(\vartheta)]^2 d\vartheta \quad r_\vartheta = \left[\frac{A_\vartheta}{\pi}\right]^{1/2}$
    - Scan on all phase space variables needed
  - Integration over the dynamics $\frac{1}{2\pi}\int_0^{2\pi}[r(\vartheta)]^2 d\vartheta \rightarrow \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}[r^{(n)}(\bar{\vartheta})]^2$
    - Fix θ and replace the space average with average over the N iterates
    - Uniform distribution of the phases of the iterates needed
  - Normal form method: $\rho(\bar{\vartheta}) = |\Psi(r(\bar{\vartheta})\cos\bar{\vartheta}, r(\bar{\vartheta})\sin\bar{\vartheta})|^2$
    - Compute the NL invariant with the truncated inverse conjugating function
    - Not valid close to a resonance

Dynamic aperture – 4D betatron motion

- **Coupling** between 'linearly decoupled' planes **due to non-linearities**

  → Ratio between the amplitudes in the different planes; Use of α such that $x = r\,cos(\alpha)$ and $y = rsin(\alpha)$

- «**Fast DA estimates**», commonly used : $\frac{x}{y} = 1$ ($\alpha = 45^o$) with $\theta_x = \theta_y = 0$

  → Unprecise results, can not be used with non-negligible phase space distortion or 'ratio-dependent' dynamics

- <u>**Methods** that give an **average** distance to stability border:</u>
  - Direct integration:
    - Scan on all phase space variables

    → 4 variables; CPU time consuming
  - Integration over the dynamics
    - Generalization of the 2D case
  - Normal form method
    - Generalization of the 2D case

- <u>**Methods without averaging**</u>:
  - Scan different α and take the minimum DA

$$\int\int\int\int \chi(x, p_x, y, p_y)\, dx\, dp_x\, dy\, dp_y$$

↓

$$A_{\alpha,\vartheta_1,\vartheta_2} = \frac{1}{8}\int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi/2} [r(\alpha,\vartheta_1,\vartheta_2)]^4 \sin(2\alpha)\, d\alpha\, d\vartheta_1\, d\vartheta_2.$$

↓

$$r_{\alpha,\vartheta_1,\vartheta_2} = \pi^{-1/2}(2A_{\alpha,\vartheta_1,\vartheta_2})^{1/4}$$

Dynamic aperture – 4D motion – « roadmap »

Methods for 4D motion dynamic aperture computation in a strongly non-linear and coupled lattice

- Computation of **2D-DA** (ϵ_1, ϵ_2) for **different amplitude ratios** between the decoupled planes to account for the non-linearity
  - Evolution of the ϵ_1 et ϵ_2 invariants as a function of α and computation of the average of ϵ_1 and ϵ_2; ϵ_1 and ϵ_2 can be related to measurable parameters with appropriate lattice functions (linear coupling).

- Computation of a **“theoretical” 4D-DA estimate** (average distance to the stability border) to **compare working points**
  - Direct integration: scan all the phase space variables
  - Integration over the dynamics
  - The 4th order islands of stability are taken into account, and a metric representing the « filling factor » is being defined to account for the topology of the phase space

- Computation of a **“practical” DA** based on precautionary principle for **better interpretation**
  - **Launch an elliptical shape bunch** → Every particle needs to survive N turns

→ Define the DA in the **coupled space**

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Summary

- Fieldmaps for the **FETS-vFFA arctan lattice** have been generated, and particles tracked with Zgoubi
  - There is an **important influence of neighboring cells** due to the significant cell ends residual fields

  → The actual superposition of field maps is needed

- The detailed study of the **linear transverse beam dynamics** has been performed
  - The **influence** of the neighboring cell **field map extents** on the orbit and tunes have been studied, and **convergence** towards the 'infinite' map has been found
  - The **tunes and lattice functions** have been computed and are **similar** to those obtained with other codes

- An **in-depth study of the non-linear dynamics** of the lattice is **in progress**
  - The **2D normalized dynamic apertures** were calculated in the decoupled planes with Zgoubi, without non-linearities; **Islands of stability** appear in phase space.
  - Different **definitions and methods to calculate the DA** exist in the literature for **2D and 4D** betatron motion; it includes definitions based on a **phase space variable average** or **the 'precautionary principle'**.
  - **Methods to study completely the 4D** motion **dynamic aperture** in a **strongly non-linear and coupled lattice** have been suggested. An in-depth study of dynamic aperture in the full 4D phase space is still in progress.