Constant-Tune Cyclotrons

FFA Workshop 2022

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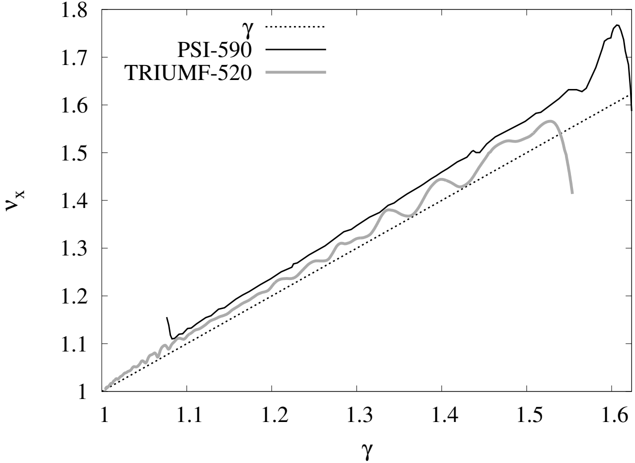
TRIUMF

Introduction



Does ν_x always follow γ? Are higher-energy cyclotrons doomed to cross many resonances?

Contents

- The isochronous condition
- How to produce an isochronous field map in a split second
- Constant-Tune 2 GeV Cyclotron
- Compact cyclotron example + magnet design

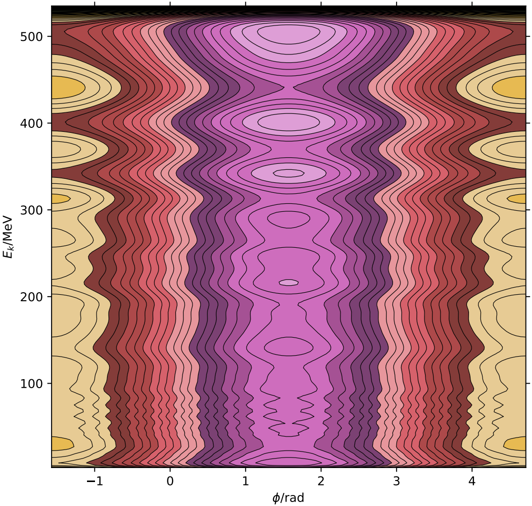
Isochronous condition

$$\mathcal{R} = \beta \mathcal{R}_\infty$$

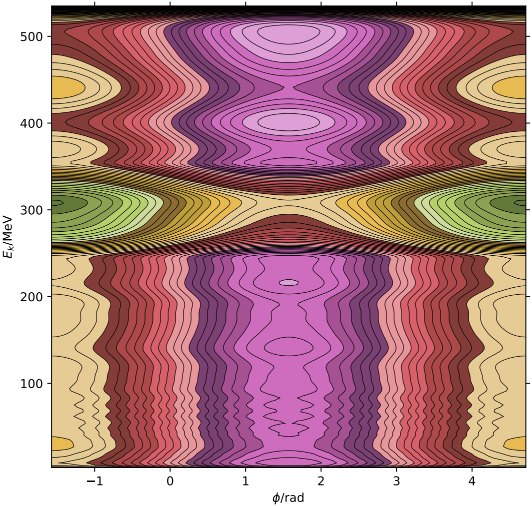
where $\mathcal{R}$ is the circumference of the closed orbit divided by 2π, and $\mathcal{R}_\infty$ is a constant.

The integral of the relative error can never exceed the ratio between a quarter of the rf period and the total time of flight from injection: on the order of 10^{-4} or less, for most cyclotrons. Stray away from this strict condition, and no more beam will come out of the machine.

Longitudinal Hamiltonian contours



Longitudinal Hamiltonian contours



Isochronous field distribution

There is no analytical solution* for an isochronous $B(r, \theta)$. Producing an isochronous field distributions is the job of the magnet designer: iterative process that takes many iterations, and on the order of days.

*Except for the case $B(r)$, which is out of question: unstable vertical motion.

Isochronous field distribution

For lack of an analytical formula, there is a way to generate a perfectly isochronous (and Maxwellian) field distribution in a split second using a trick I presented at the cyclotron conference in 2019 [Planche, 2019].

`from_orbit` in a nutshell

Any set of closed orbits can be written in the form of a Fourier series:

$$r(a, \theta) = aC_0(a) + a \sum_{j=0}^{\infty} C_j(a) \cos(jN\theta) + S_j(a) \sin(jN\theta)$$

where N is the number of sectors, and a is the average radius of the orbit. You can calculate the local curvature of the orbit $\rho(\theta)$, the circumference of the orbit, etc... directly from $r(a, \theta)$ and its partial derivatives [Planche, 2019]. Once you impose that the circumference of the orbit scales exactly with β, you can the magnetic field in the median plane from:

$$B(r, \theta) = \frac{\beta(a)}{\sqrt{1 - \beta^2(a)}} \frac{m}{q\rho(a, \theta)}$$

And you've got a perfectly isochronous field map.

`from_orbit` in a nutshell

Bonus: you also get the transverse tunes directly from the geometry of the orbits, since the Hamiltonian for linear transverse motion in an FFA with mid-plan symmetry [Courant and Snyder, 1958, Baartman, 2005]:

$$h = \frac{x^2}{2}\frac{1-n}{\rho^2} + \frac{y^2}{2}\frac{n}{\rho^2} + \frac{p_x^2}{2} + \frac{p_y^2}{2}$$

depends only on n and ρ. You can calculate both directly form $r(a, \theta)$.

Digression

I do this with a little bit of python code that is available here:
https://gitlab.triumf.ca/tplanche/from-orbit

To simplify things a little I ensure that:

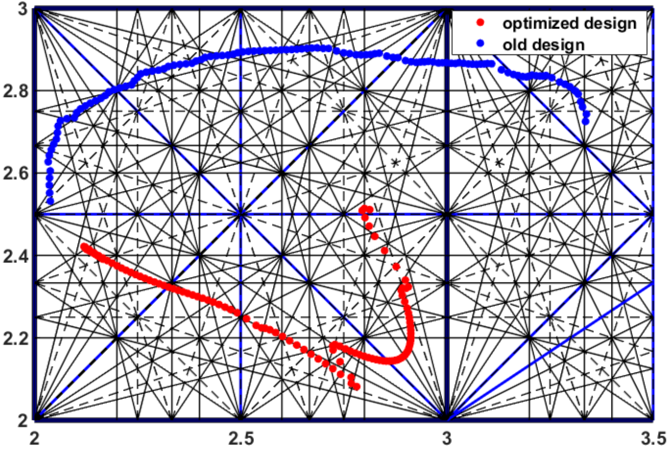
$$\frac{\partial r}{\partial a} > 0$$

so that orbit don't cross. It is not a necessary condition: the only real constraint is that the magnetic field must be uniquely defined where 2 orbits cross (application: gantries).

Constant-tune isochronous field maps

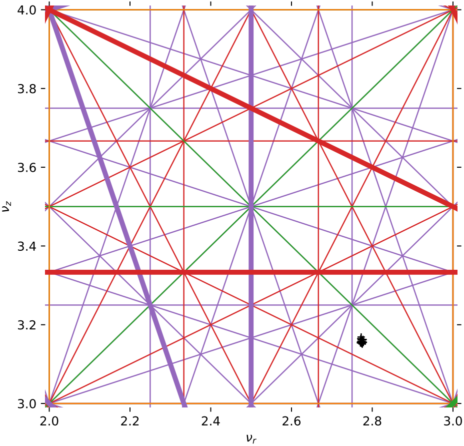
Standard optimization problem: parametrize $r(a, \theta)$ is a small number of parameters, and vary the parameters to minimize the rms variation of ν_x and ν_z. And instead of this:

CIEA preliminary 800 MeV to 2 GeV cyclotron design



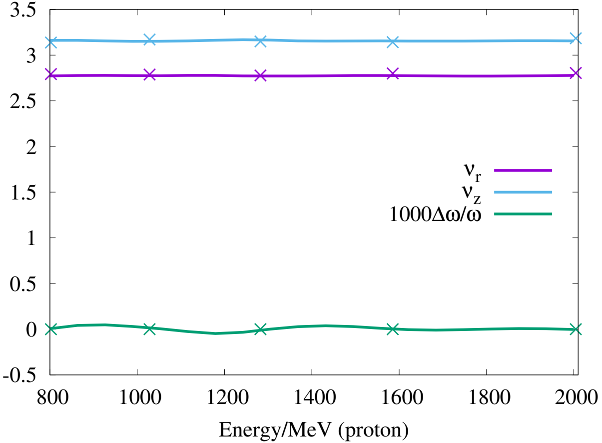
From Tianjue's talk at FFA'20, you get this:

Constant-tune 800 MeV to 2 GeV field map

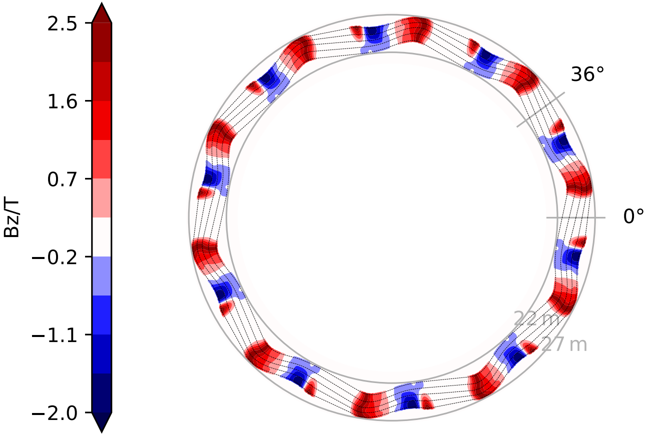


for the same basic machine parameters.

Constant-tune 800 MeV to 2 GeV field map



Constant-tune 800 MeV to 2 GeV field map

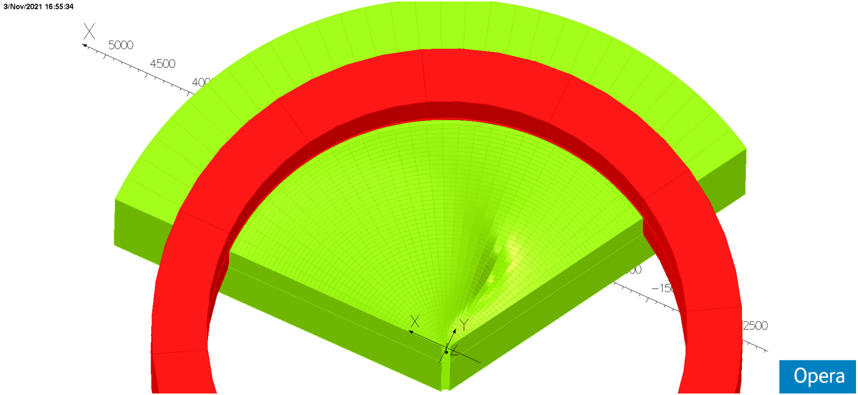


$r(a, \theta)$ is parametrized so as to have drift sections between sectors.

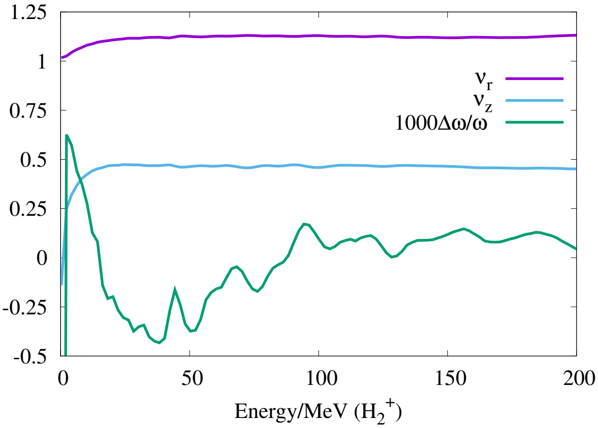
Magnet Design?

Can you design such magnets? Have have not tried yet for this high-energy machine. But I have tried for another design, lower energy, because that is what my team has been working on recently.

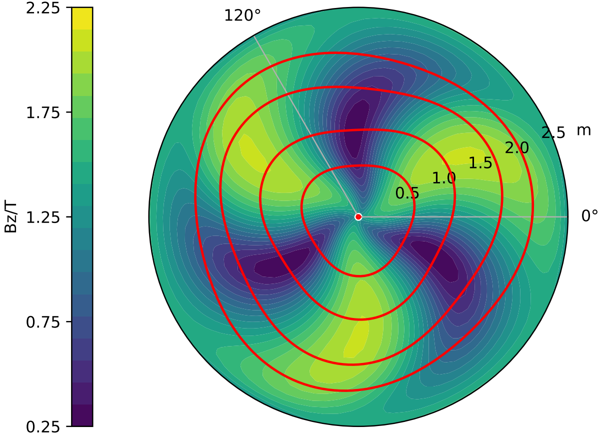
Magnet Design: 200 MeV H_2^+ cyclotron



Magnet Design: 200 MeV H_2^+ cyclotron

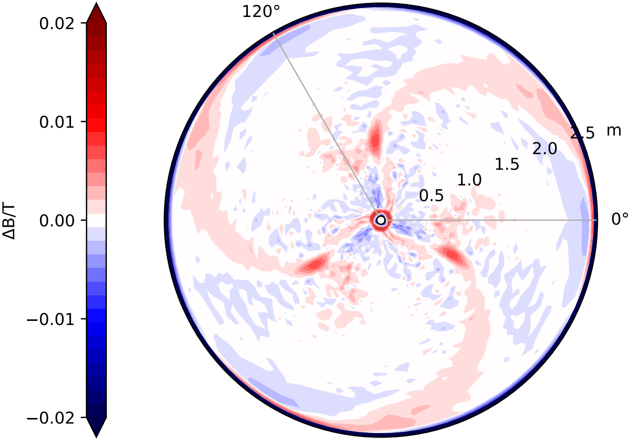


Magnet Design: 200 MeV H_2^+ cyclotron



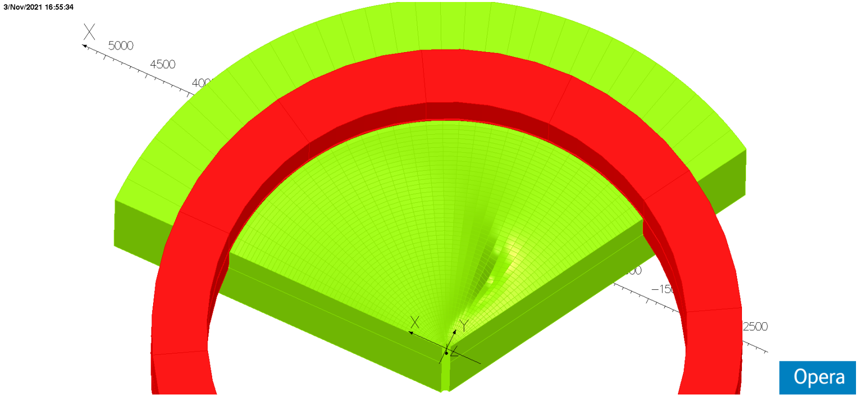
Desired field and corresponding orbits.

Magnet Design: 200 MeV H_2^+ cyclotron



Desired field - OPERA-3d calculated field.

Magnet Design: 200 MeV H_2^+ cyclotron



Conclusion

I have demonstrated that cyclotron magnetic fields with constant tunes can be designed. Except for the central few orbits in a compact cyclotron, this avoids crossing betatron resonances. In ring cyclotrons, the tunes can be made completely constant, while being isochronous. Future work is required to convert the fields into actual steel and coil configurations.

Bibliography:

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