Spiral FFA in Zgoubi

D. Kelliher (ISIS/RAL/STFC)

FFA22 School

Zgoubi spiral FFA magnet (FFAG-SPI)

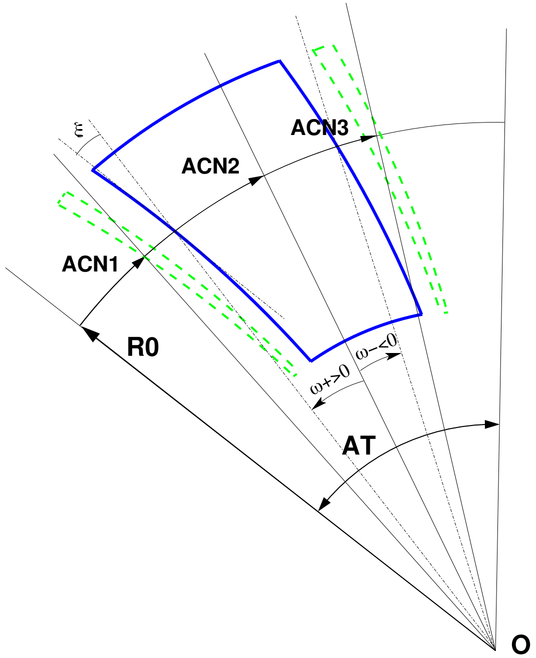


Figure 32: A N-tuple spiral sector FFAG magnet ($N = 3$ here, simulating active field clamps at entrance and exit side of a central dipole).

The dimensioning of the magnet is defined by

AT : total angular aperture
RM : mean radius used for the positioning of field boundaries

For each one of the $N = 1$ to (maximum) 5 dipoles of the N-tuple, the two effective field boundaries (entrance and exit EFBs) from which the dipole field is drawn are defined from geometric boundaries, the shape and position of which are determined by the following parameters

ACN_i : arbitrary inner angle, used for EFBs positioning
ω : azimuth of an EFB with respect to *ACN*
ξ : spiral angle

$$B_Z(r,\theta) = B_{Z0} F(r,\theta) \left(\frac{r}{r_m}\right)^k$$

FFAG-SPI **Spiral FFAG magnet, N-tuple**
UNDER DEVELOPMENT
$B_Z = \sum_{i=1}^{N} B_{Z0,i}\, \mathcal{F}_i(R,\theta)\ (R/R_{M,i})^{K_i}$

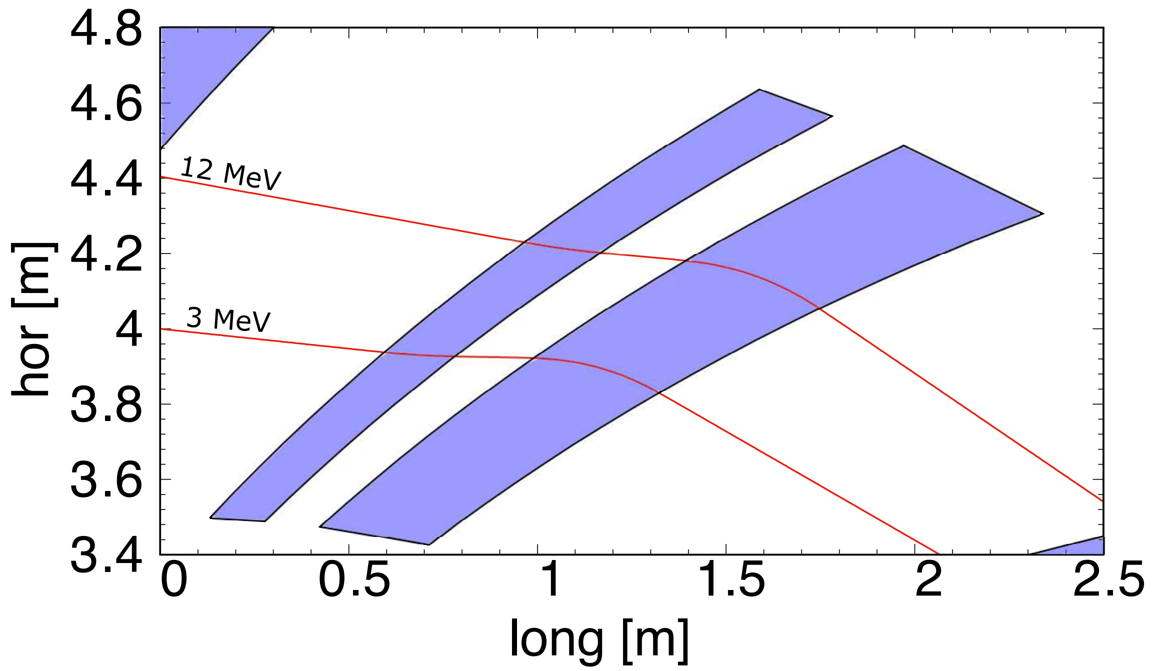
IL	$IL = 1, 2[\times 10^n],\ 7$: print coordinates along trajectories, fields, etc., into zgoubi.res (1) or zgoubi.plt ($2[\times 10^n]$) or zgoubi.impdev.out (7).	0-$2[\times 10^n]$, 7	I
N, AT, RM	Number of dipoles in the FFAG N-tuple ; total angular extent of the dipole ; reference radius.	no dim, deg, cm	I, 2*E
Repeat N times the following sequence			
ACN, δRM, B_{Z0}, K	Azimuth for dipole positioning ; $R_{M,i} = RM + \delta RM$; field at $R_{M,i}$; index.	deg, cm, kG, no dim	4*E
	ENTRANCE FIELD BOUNDARY		
g_0, κ	Fringe field extent ($g = g_0\,(RM/R)^{\kappa}$)	cm, no dim	2*E
NC, $C_0 - C_5$, shift	Unused ; C_0 to C_5 : fringe field coefficients ; EFB shift	0-6, 6*no dim, cm	I, 7*E
ω^+, ξ, 4 dummies	Azimuth of entrance EFB with respect to *ACN* ; spiral angle ; 4×unused.	2*deg, 4*unused	6*E
	EXIT FIELD BOUNDARY (See ENTRANCE FIELD BOUNDARY)		
g_0, κ	Fringe field parameters, see above	cm, no dim	2*E
NC, $C_0 - C_5$, shift		0-6, 6*no dim, cm	1, 7*E
ω^-, ξ, 4 dummies		2*deg, 4*unused	6*E
	LATERAL FIELD BOUNDARY **to be implemented - following data not used**		
g_0, κ		cm, no dim	2*E
NC, $C_0 - C_5$, shift		0-6, 6*no dim, cm	1, 7*E
ω^-, θ, R_1, U_1, U_2, R_2		2*deg, 4*cm	6*E
End of repeat			

Integration boundaries - next line is optional, starting with string `IntLim` :

`IntLim`, ID, A, B, C [, A', B', C']	Integration boundary. Line has to start with '`IntLim`'. $ID = -1$: integration in the magnet begins at entrance boundary defined by A, B, C. $ID = 1$: integration is terminated at exit boundary defined by A', B', C'. $ID = 2$: both entrance and exit boundaries.	$-1, 1, 2$; deg; cm; deg [; *id.*]	I, 3*E [,3*E]
KIRD, *Resol* [, *DNEWT*]	If KIRD=0 : analytical computation of field derivatives ; Resol = 2/4 for 2nd/4th order field derivatives computation. If KIRD $= 2, 4$ or 25 : numerical interpolation of field derivatives ; size of flying interpolation mesh is *XPAS/Resol*. KIRD=2 or 25 : second degree, 9- or 25-point grid KIRD=4 : fourth degree, 25-point grid If DNEWT is added, the distance to the magnet edge is calculated numerically (in the case KIRD≠0). In its absence, and if $\kappa = -1$, then the distance in terms of generalised azimuthal angle is used instead.	0, 2, 25 or 4 ; no dim	I, E
XPAS	Integration step	cm	E
KPOS, RE, TE, RS, TS	Positioning of the magnet, has to be 2. As follows : radius and angle of reference, respectively, at entrance and exit of the magnet.	2, 2*(cm, rad)	I, 4*E

FDspiral FETS ring

Energy range	3-12 MeV
Mean radius at injection	4m
Field index	8.0095
Spiral angle	45 deg.
Cells	16
Angular extent of F,D	4.5 deg., 2.25 deg.
Fringe field extent	0.07m



DFspiral example

Recent developments

- Increase order of off-midplane extrapolation, from z^3 to z^4.
- Allow different entrance and exit fringe field extents.
- In case derivatives are calculated with flying mesh (KIRD !=0), implemented fast calculation of distance to fringe field.
- In case derivatives are calculated analytically (KIRD=0), fixed some bugs that occured when extrapolating off the midplane to 4th order.

Off-midplane extrapolation

Median plane antisymmetry is assumed, which results in

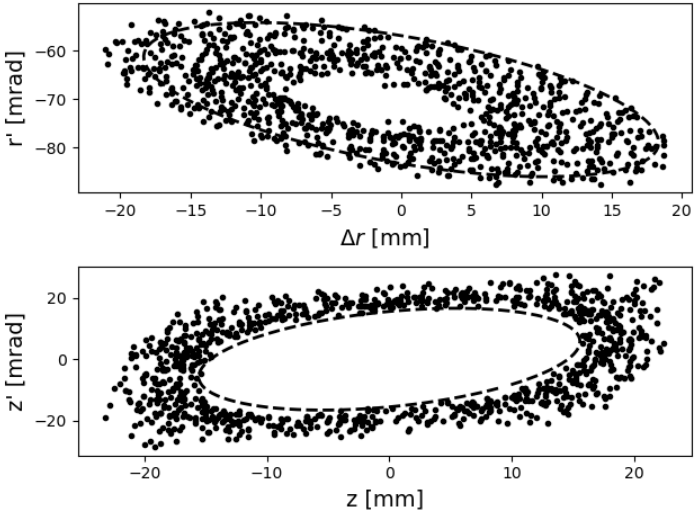
$$\begin{aligned}
B_X(X,Y,0) &= 0\\
B_Y(X,Y,0) &= 0\\
B_X(X,Y,Z) &= -B_X(X,Y,-Z)\\
B_Y(X,Y,Z) &= -B_Y(X,Y,-Z)\\
B_Z(X,Y,Z) &= B_Z(X,Y,-Z)
\end{aligned} \tag{1.3.2}$$

Accommodated with Maxwell's equations, this results in Taylor expansions below, for the three components of $\vec{B}$ (here, B stands for $B_Z(X,Y,0)$)

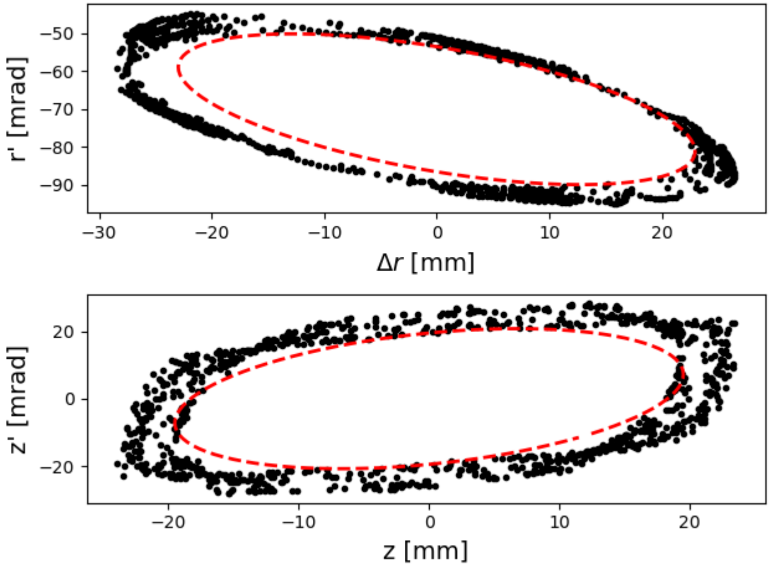
$$\begin{aligned}
B_X(X,Y,Z) &= Z\frac{\partial B}{\partial X} - \frac{Z^3}{6}\left(\frac{\partial^3 B}{\partial X^3} + \frac{\partial^3 B}{\partial X \partial Y^2}\right)\\
B_Y(X,Y,Z) &= Z\frac{\partial B}{\partial Y} - \frac{Z^3}{6}\left(\frac{\partial^3 B}{\partial X^2 \partial Y} + \frac{\partial^3 B}{\partial Y^3}\right)\\
B_Z(X,Y,Z) &= B - \frac{Z^2}{2}\left(\frac{\partial^2 B}{\partial X^2} + \frac{\partial^2 B}{\partial Y^2}\right) + \frac{Z^4}{24}\left(\frac{\partial^4 B}{\partial X^4} + 2\frac{\partial^4 B}{\partial X^2 \partial Y^2} + \frac{\partial^4 B}{\partial Y^4}\right)
\end{aligned} \tag{1.3.3}$$

which are then differentiated one by one with respect to X, Y, or Z, up to second or fourth order (depending on optical element or *IORDRE* option, see section 1.4.2) so as to get the expressions involved in eq. (1.2.10).

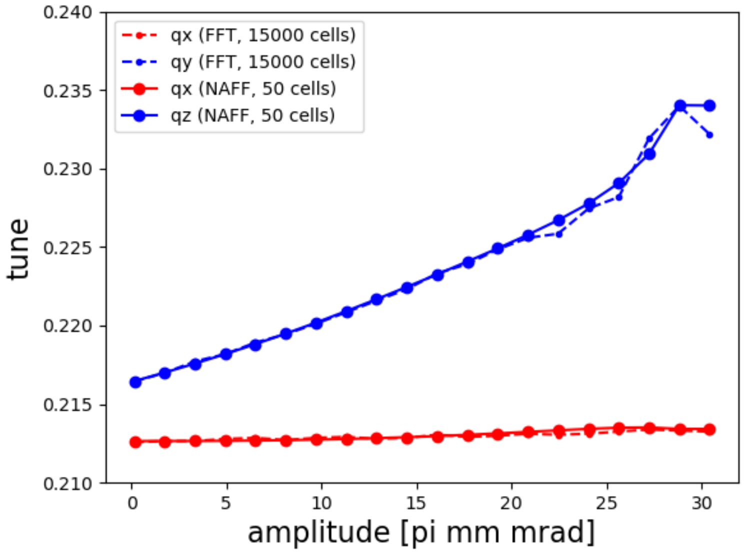
Flying mesh : expansion off midplane to z^4



ε_n= 19.36 π mm mrad
Step size: 10mm
Order: z^3



DA, ε_n= 30.4 π mm mrad
Step size: 10mm
Order: z^3



- Note - Expansion off midplane to 4^{th} order (fix implemented in code)
- 1000 turns.
- KIRD=4 (5*5 mesh to calculate derivatives)

Fringe field model

- The fringe field (Enge) is a function of distance to the magnet edge (d) divided by the extent λ. The distance d is a function of both (r,θ) while λ depends on r only.

$$F(r,\theta) = \frac{1}{1 + \exp P(d(r,\theta), \lambda(r))}$$

- The polynomial P is given by

$$P(d,\lambda) = \sum_{n=0,5} c_n * \left(\frac{d(r,\theta)}{\lambda(r)}\right)^n$$

- The fringe field of the entrance, edge and lateral (unused) are combined by taking the product.

$$F(r,\theta) = F_e * F_s * F_l$$

- The fringe field extent λ extent varies with radius according to the exponent -κ. Set κ=-1 to meet the scaling condition that λ increases with radius.

$$\lambda(r) = \lambda_m(\frac{r_m}{r})^{\kappa}$$

Fringe field calculation in flying mesh case

- Distance to edge d_S and d_E found by Newton-Raphson root finding.
- Around 50% of CPU time spent on this!

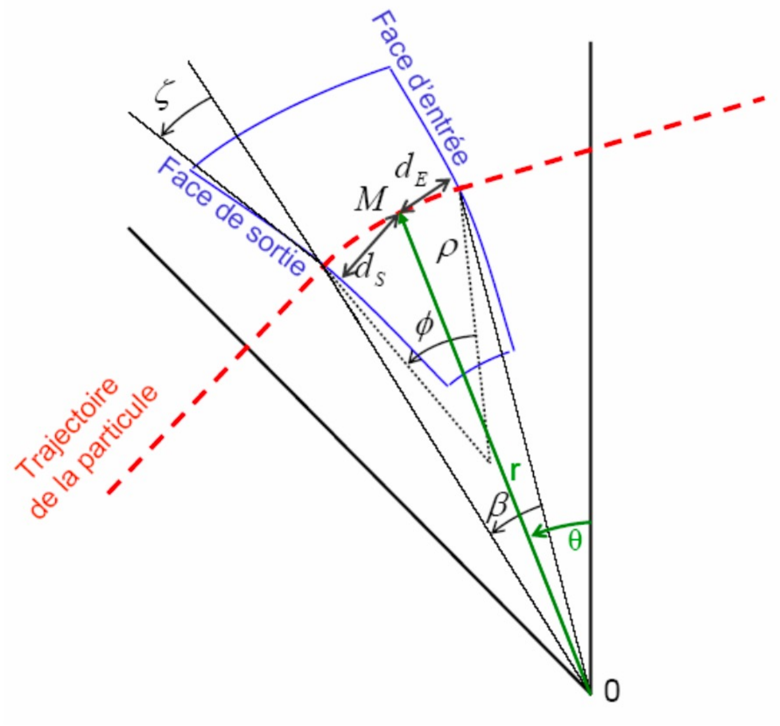
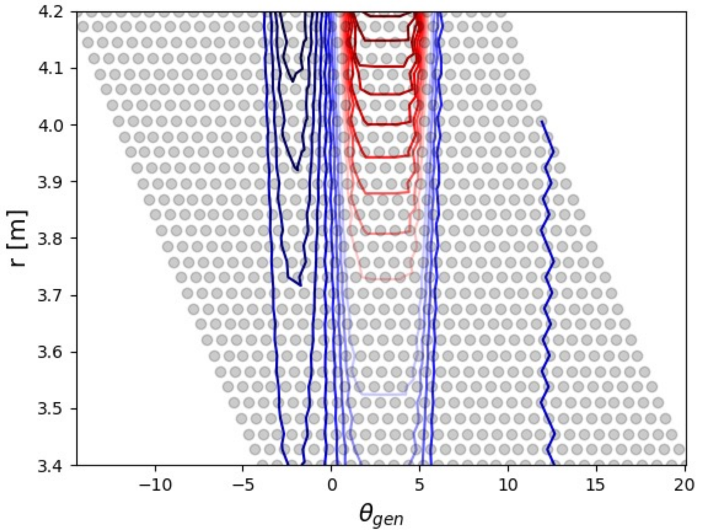


Figure 4.3 : **Schéma de principe d'un aimant FFAG spiral dans un repère de coordonnées polaires et ingrédients entrants dans le calcul du champ magnétique $B_z(r,\theta)$ dans le modèle « FFAG-SPI ».**

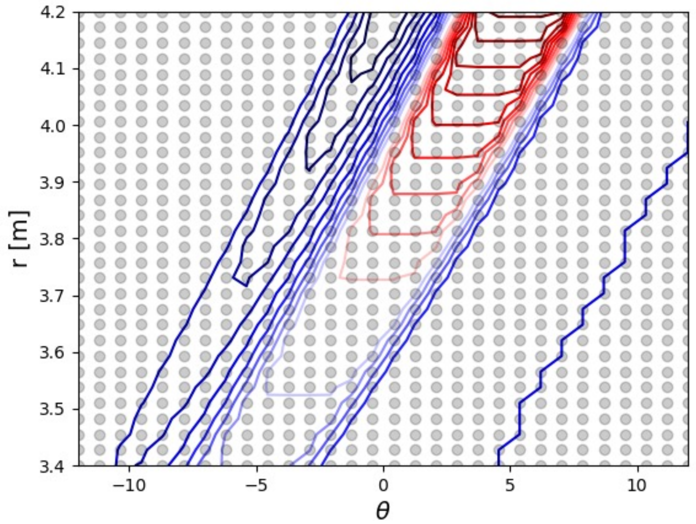
Field maps in various coordinate

Generalised polar mesh

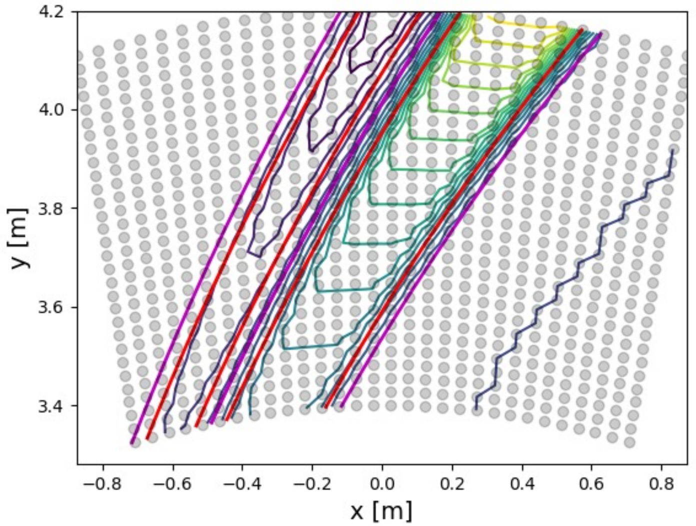
$$\theta_{gen} = \theta - \xi \log(r/r_0)$$



Polar mesh



Cartesian mesh



- Create regular mesh in polar coordinates (r, θ). Transform to (r, θ_{gen}) and (x,y).
- Evaluate midplane field in generalised polar coordinates (r, θ_{gen}) -> $B_z = B_0 (r/r_0)^k F(\theta_{gen})$.
- The magnet edge is at a fixed generalized polar coordinate.

Distance ratio found in generalised polar coordinates

$$\Delta\theta_{fringe} = \tan^{-1}\frac{\lambda}{r_m \cos\xi}$$

$$\frac{d}{\lambda} = \frac{\theta_{gen} - \theta_{edge}}{\Delta\theta_{fringe}}$$

```
IF(DNEWT.OR.QAPPAE(KMAG).NE.-1.D0) THEN
  D=DSTEFB(X,Y,RO1,B1,AMIN,AMAX,XACC,TTA1,YN)
  IF( Y .GT. YN ) D = -D
  D = D/GAP
ELSE
  D = -(TTAIG - TTA1)/DTHE
ENDIF
```

- Significant speed up by direct calculation of distance in terms of generalized polar coordinates.
- Include DNEWT to revert to old method.

Derivative of field

- The derivative of the magnetic field includes the fringe field contribution.

$$\frac{dB_z(r,\theta)}{dr} = B_{z0}\frac{\partial F(r,\theta)}{\partial r}\left(\frac{r}{r_m}\right)^k + \frac{k}{r}B_{z0}F(r,\theta)\left(\frac{r}{r_m}\right)^{k-1}$$

- Assuming F(r,θ) = Fe*Fs (product of entrance and and exit fringes)

$$\frac{dB_z(r,\theta)}{dr} = B_{z0}\left(\frac{\partial F_e(r,\theta)}{\partial r}F_s + F_e\frac{\partial F_s(r,\theta)}{\partial r}\right)\left(\frac{r}{r_m}\right)^k + \frac{k}{r}B_{z0}F(r,\theta)\left(\frac{r}{r_m}\right)^{k-1}$$

- The derivative of an individual Enge fringe field (e.g. entrance) is given by

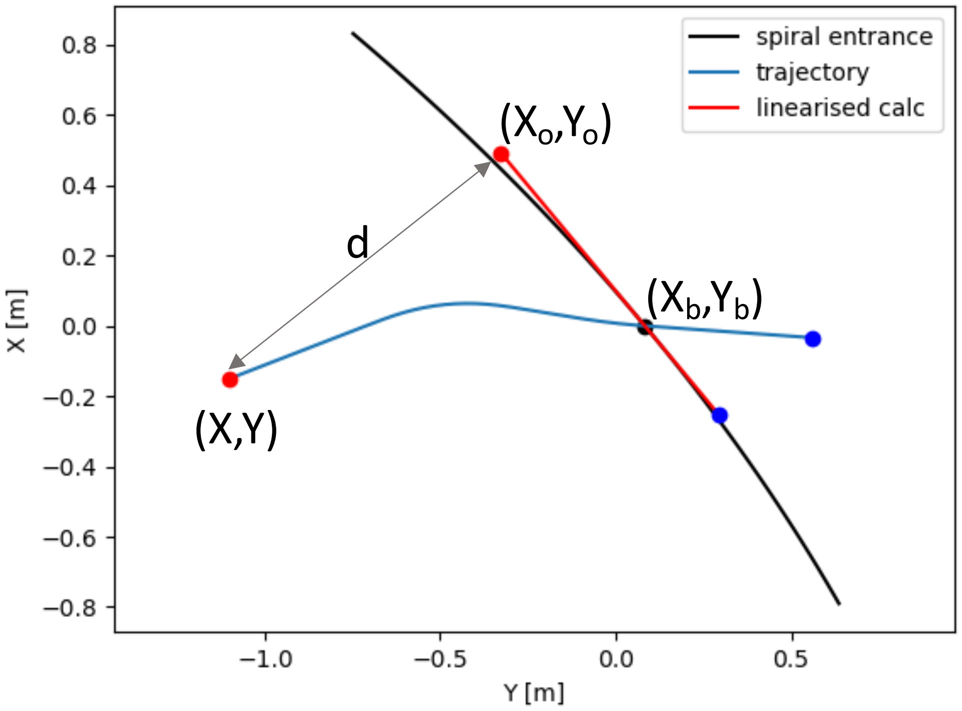
$$\frac{\partial F_e(r,\theta)}{\partial r} = -F_e(r,\theta)^2 e^{P(d,\lambda)}\frac{\partial P}{\partial r}$$

- The derivative of the polynomial P follows

$$\frac{\partial P}{\partial r} = \left(\frac{\pm d'\lambda - d\lambda'}{\lambda^2}\right)\sum_{n=1,5} nc_n * \left(\frac{d}{\lambda}\right)^{n-1}$$

where $d' = \frac{\partial d}{\partial r}$ $\quad \lambda' = -\frac{\kappa}{r}\lambda$

Analytic calculation of distance to edge



Coordinate system

$$X = r\cos(\theta_d - \theta) - r_m, Y = r\sin(\theta_d - \theta)$$

Find point on straight edge passing through (Xb,Yb) with slope (ω-ξ)

$$X_o = \sin(\omega - \xi)\cos(\omega - \xi)(Y - Y_b) + X_b\sin^2(\omega - \xi) + X\cos^2(\omega - \xi)$$

$$Y_o = \sin(\omega - \xi)\cos(\omega - \xi)(X - X_b) + Y_b\cos^2(\omega - \xi) + Y\sin^2(\omega - \xi)$$

Distance to magnet edge

$$d = \sqrt{(X - X_o)^2 + (Y - Y_o)^2}$$

Define

$$\Delta X = X - X_o \qquad \Delta Y = Y - Y_o$$

Derivative of distance

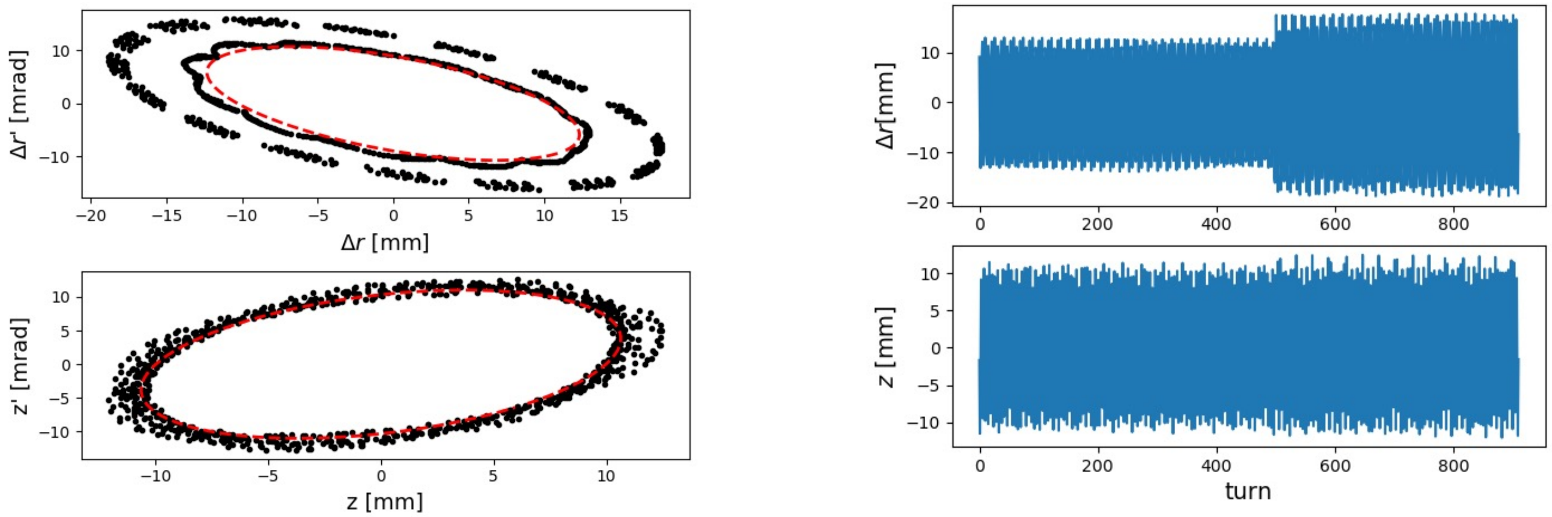
$$d' = \frac{(X' - X'_o)\Delta X + (Y' - Y'_o)\Delta Y}{d}$$

$$X' = \cos(\theta_d - \theta) \quad Y' = \sin(\theta_d - \theta)$$

$$X'_o = \sin(\omega - \xi)\cos(\omega - \xi)Y' + X'\cos^2(\omega - \xi)$$

$$Y'_o = \sin(\omega - \xi)\cos(\omega - \xi)X' + Y'\sin^2(\omega - \xi)$$

Issue



- Jump in coordinate caused by some numerical error.
- This occurs if the derivatives are calculated analytically up to 4th order in z (KIRD=0, RESOL=4).