Introduction to FFA Accelerators

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What is an FFA?

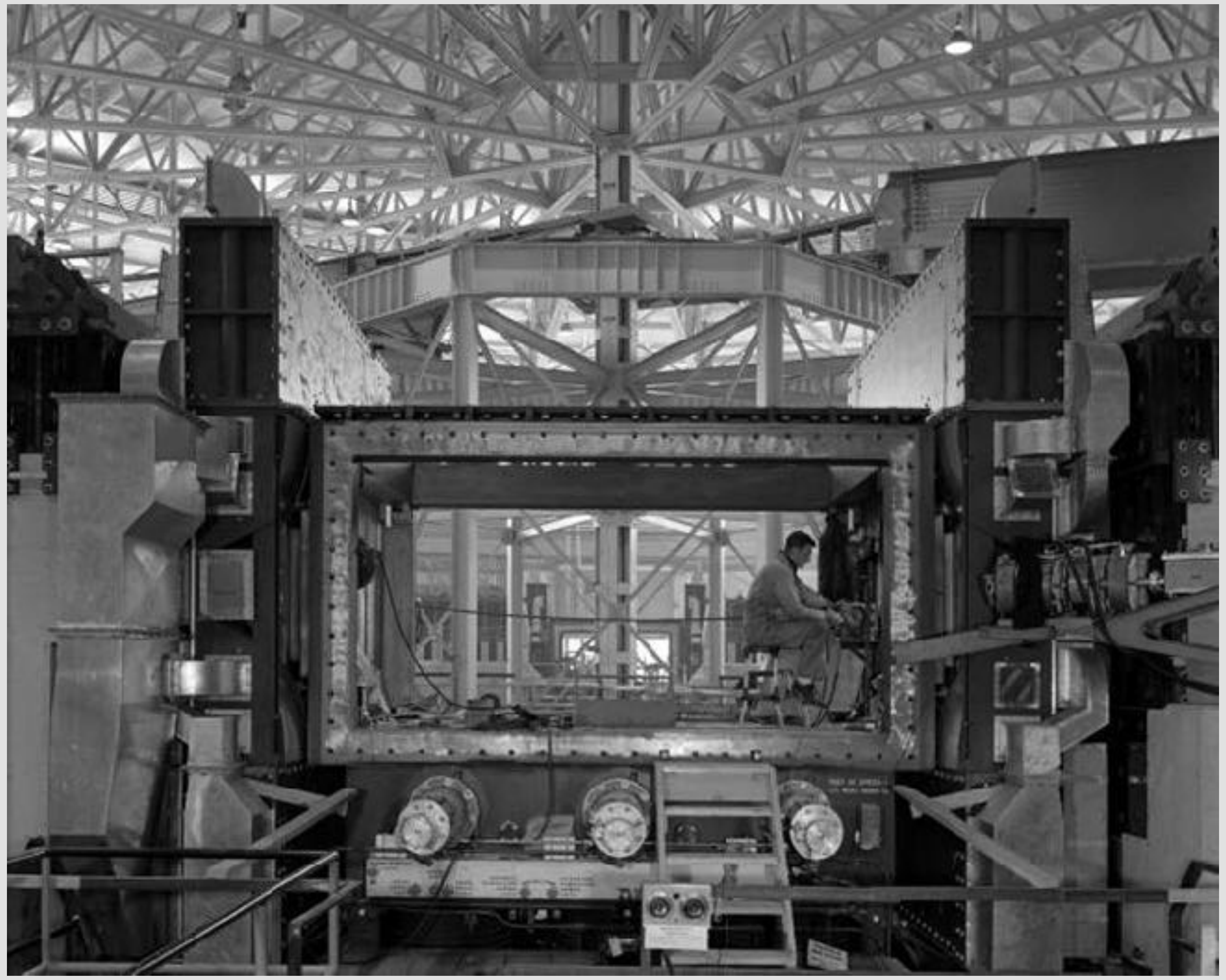
Weak focusing

Weak focusing handles the beam **weakly**

field index $n = -\frac{\rho}{B_y}\frac{\partial B_y}{\partial x}$, $0<n<1$

focus in both transverse planes at the same time

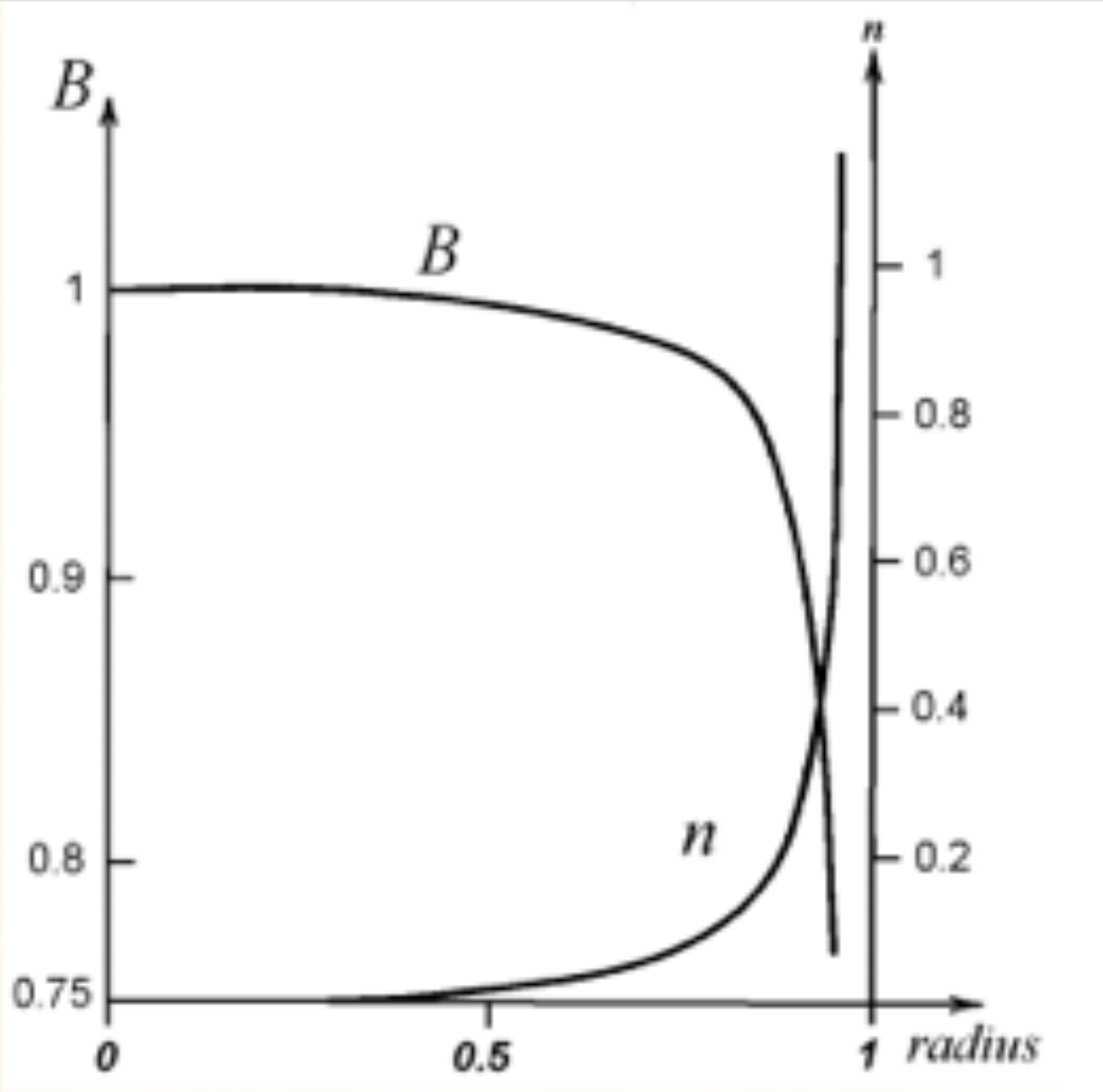
Big beam at high energy!

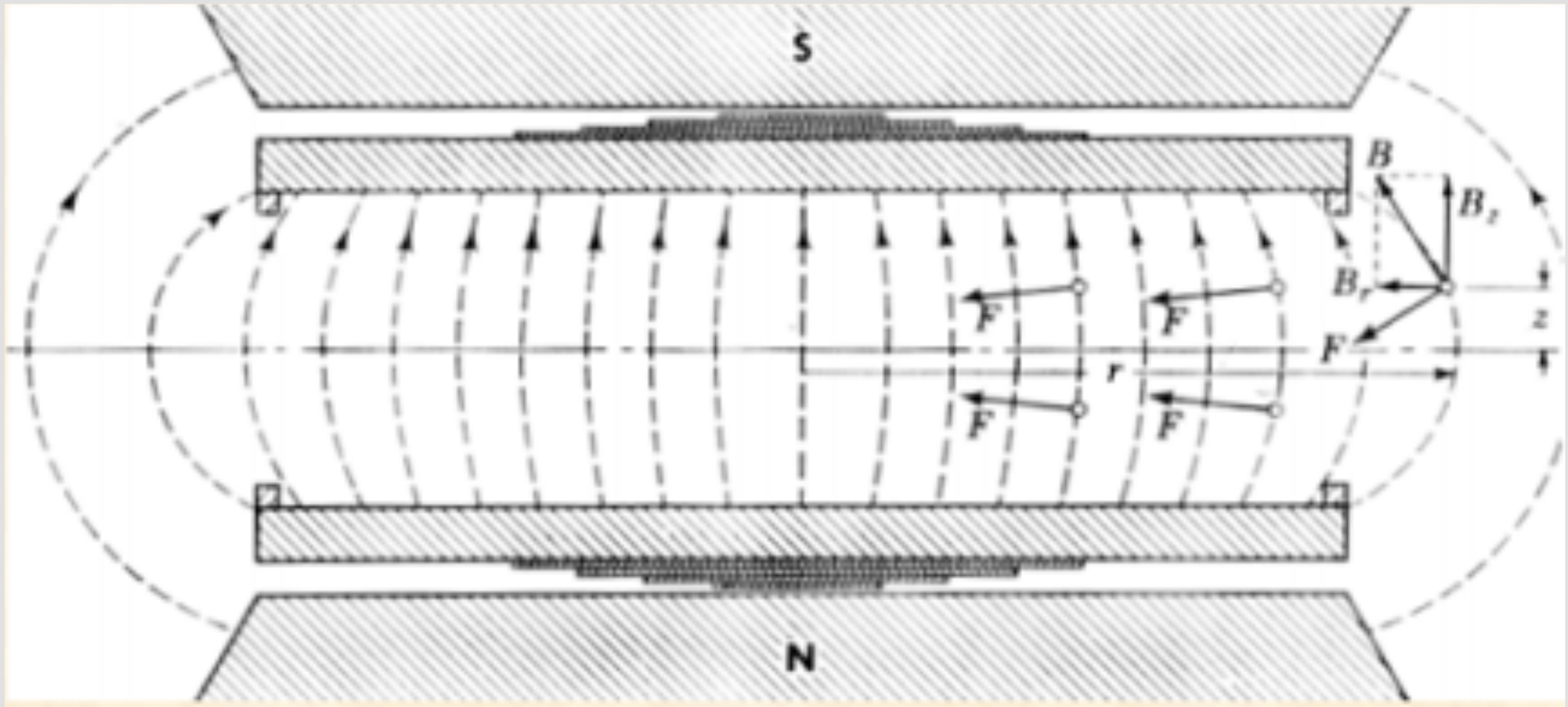


Bevatron vacuum chamber
6 GeV proton at Berkeley

Classical cyclotron

Weak focusing is still used nowadays!





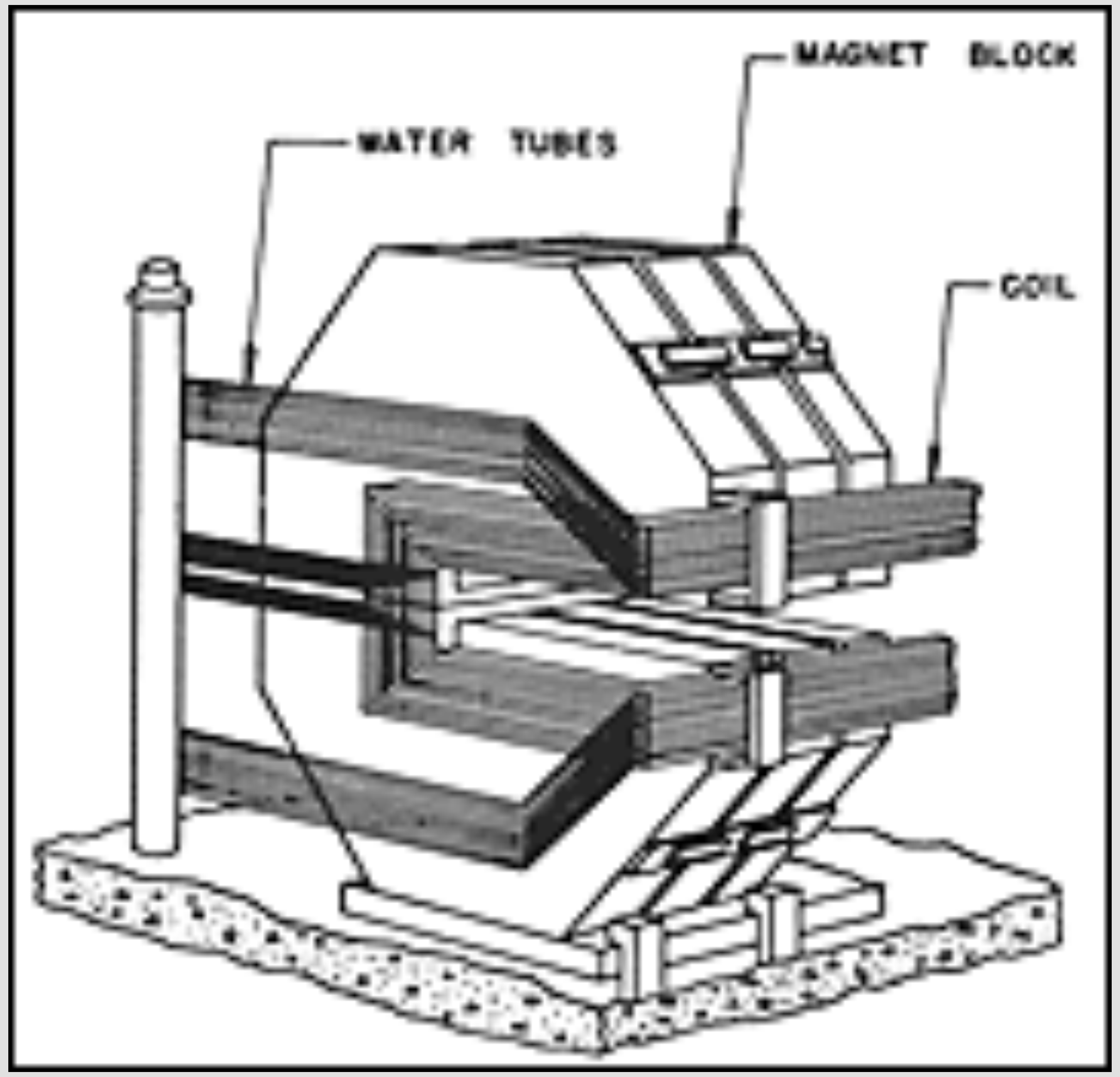
field index $n = -\frac{\rho}{B_y}\frac{\partial B_y}{\partial x}$, $0 < n < 1$

Strong focusing

In 1952, Livingston, Courant and Snyder had the idea of flipping some of the C-shape magnets of the Cosmotron at BNL to remove the limitation of the field index.

Strong focusing was born*.

*N. Christophilos had invented it before



C-shape magnet Cosmotron at Brookhaven

Definition of FFA (formerly FFAG)

FIXED FIELD ALTERNATING GRADIENT ACCELERATOR

Circular accelerator with

- fixed field (like cyclotrons),

and

- strong focusing (like synchrotrons).

Independently invented in 1950s in Japan, USSR and USA.

Advantages of FFAs

- Interesting for
  - high repetition rate (source of secondary particles, medical accelerators).
  - rapid acceleration (short-lived particles),
  - very high power machines (proton drivers, ADSR),
  - handling of big beams (muon machines, internal target),
  - reliable machines (ADSR, medical accelerators),
  - energy-efficient machines (ADSR).

Transverse motion in particle accelerators

Linearised equations of motion:

$$\frac{\partial^2 a}{\partial s^2} + K_a(s)a = 0$$

a: horizontal (x) or vertical (y)

Periodic case: Hill's equations

→ General solution: $a = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\nu\phi(s) + \phi_0)$

Betatron oscillations: pseudo-harmonic oscillation of frequency ν (tune) and varying amplitude $\sqrt{\beta(s)}$.

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Betatron resonances

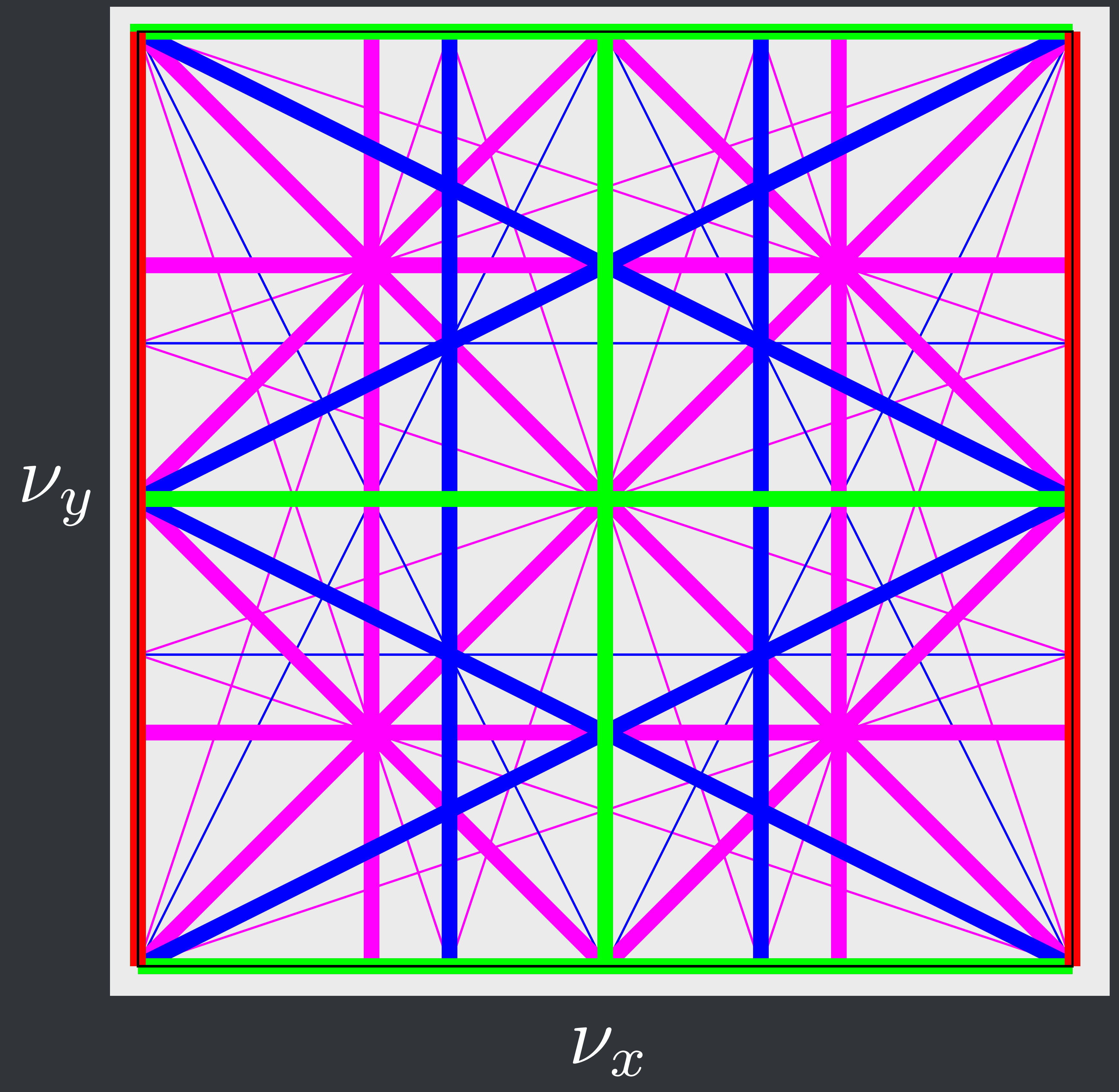
Non-linear components are considered as perturbations of the linear equations of motion.

Resonance conditions:

$$m_x \nu_x + m_y \nu_y = q$$

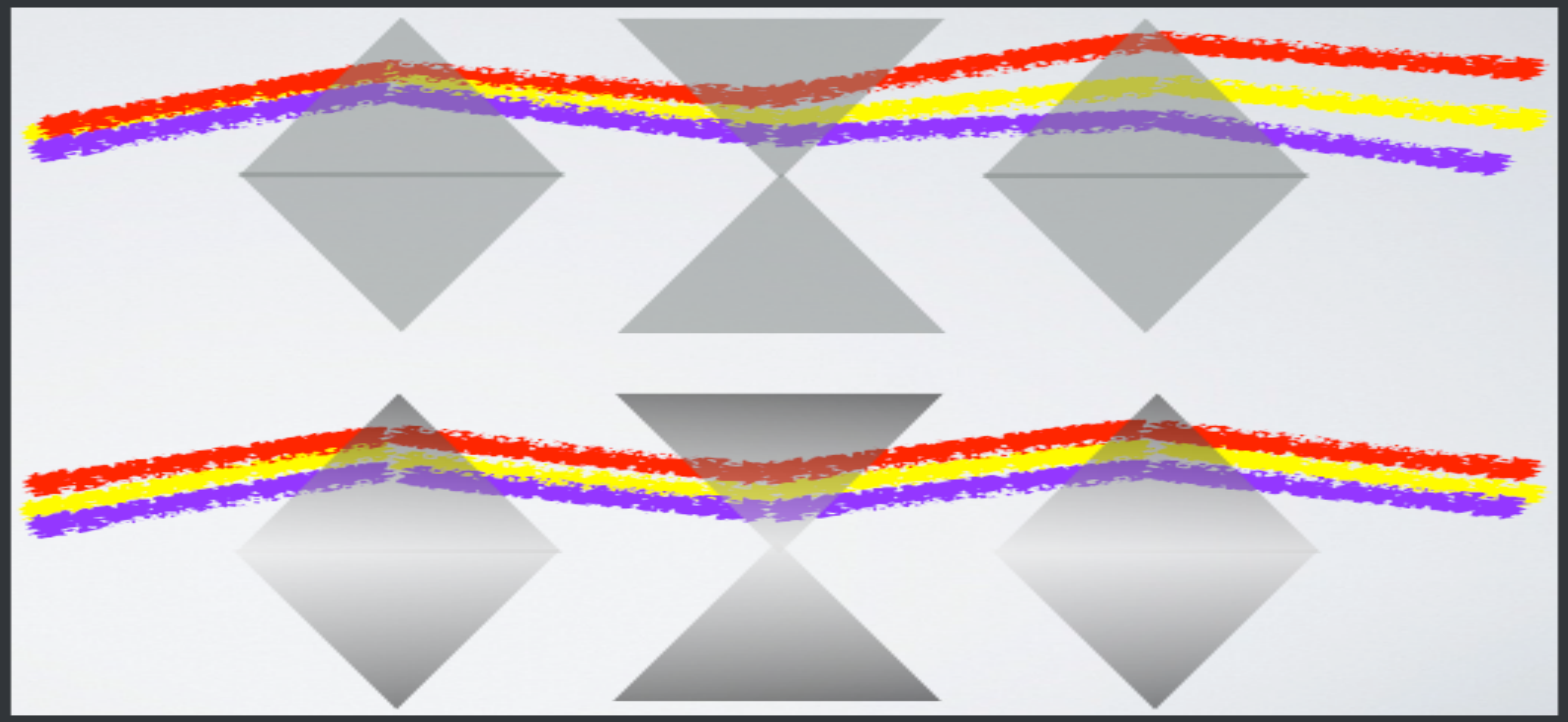
(m_x, m_y, q) integer numbers

Working point (ν_x, ν_y) positioned in the tune diagram.

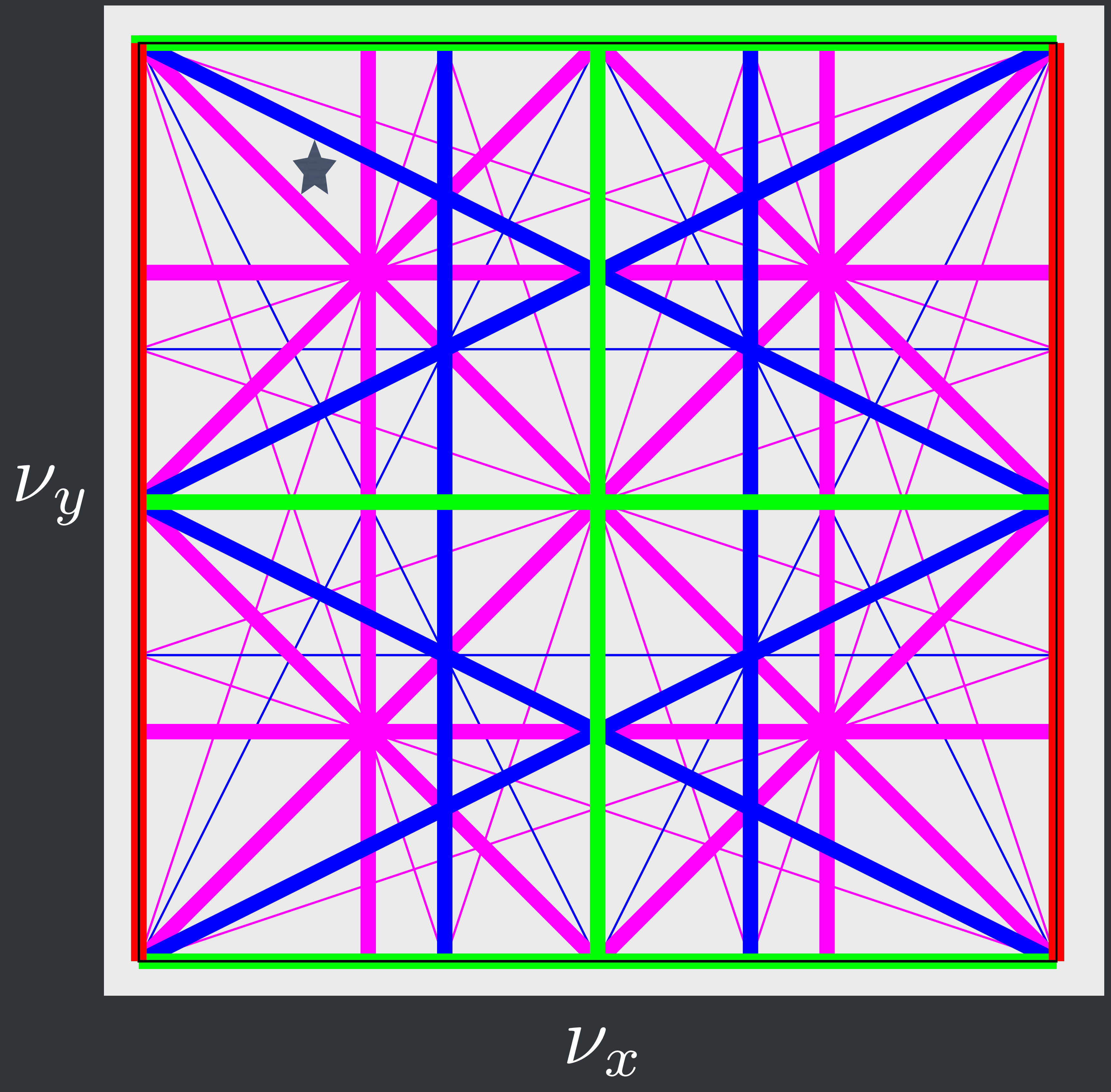


Chromaticity

Variation of tune with respect to particle energy.



$$\xi = \frac{\frac{\Delta\nu}{\nu}}{\frac{\Delta p}{p}} = -\frac{q}{4\pi\nu p}\int \beta(s)\frac{\partial B_y}{\partial x}(s)ds$$



FFA classification

2 categories:

➡ Achromatic machines

- ➡Scaling FFA
- ➡Non-linear non-scaling FFA

- ‣large momentum acceptance
- ‣large transverse acceptance
- ‣no beam loss during acceleration
- ‣fast acceleration if desirable
- ‣high repetition rate (up to 1kHz or even CW)
- ‣possibly big machines

➡ Chromatic machines

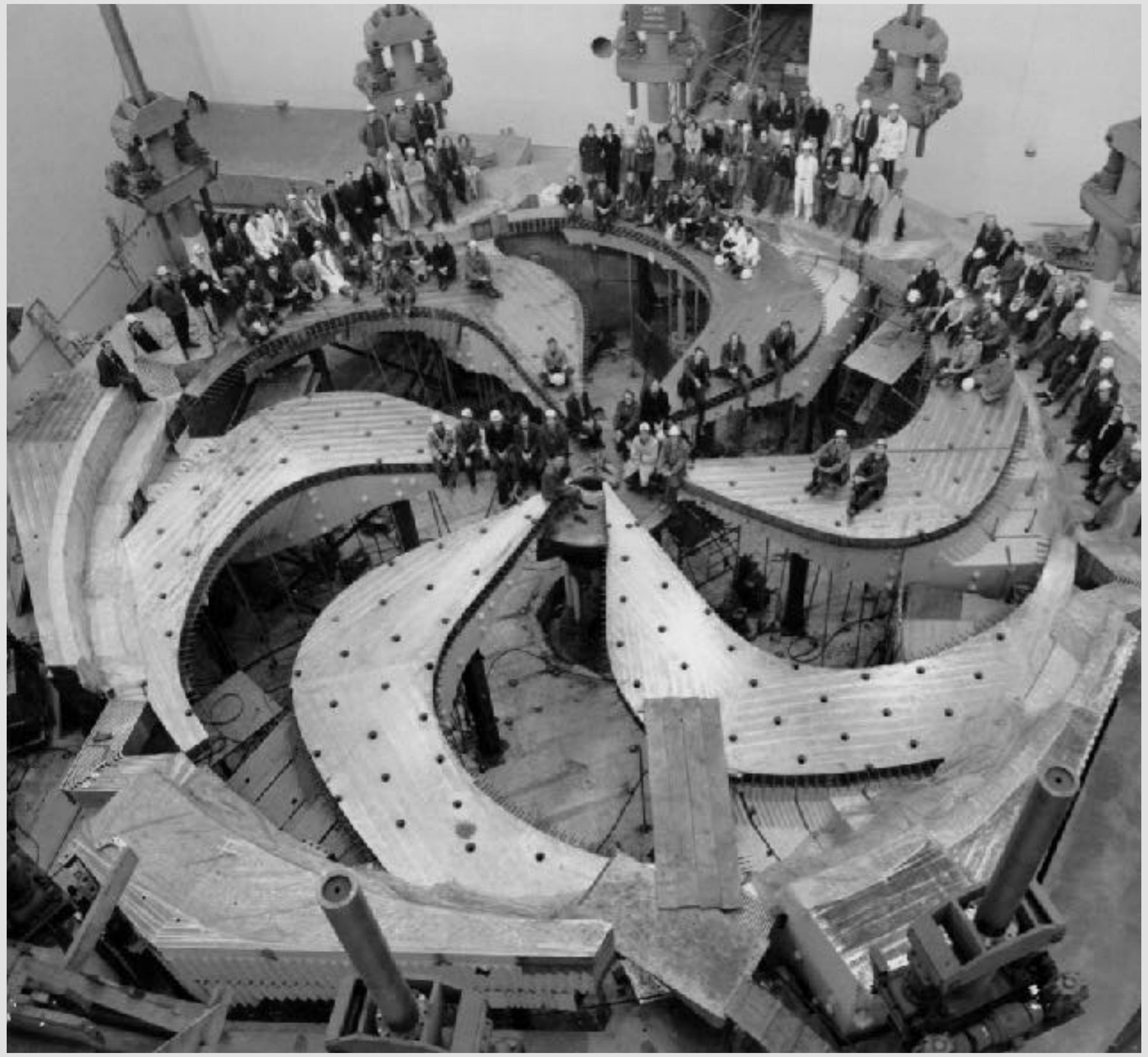
- ➡Isochronous cyclotrons
  - ‣efficient acceleration
  - ‣CW (Continuous Wave) acceleration
  - ‣big machines
  - ‣need to treat resonances one by one
  - ‣limited energy
- ➡Linear non-scaling FFA
  - ‣very strong focusing
  - ‣CW (Continuous Wave) acceleration
  - ‣rapid acceleration necessary

FFAs covered here

- Isochronous cyclotrons
- Linear FFA
- Scaling FFA:
    - Horizontal scaling FFA
    - Straight scaling FFA
    - Vertical scaling FFA

Isochronous cyclotrons

Cyclotrons benefitted from spiral sectors invented for FFAGs to go to higher energies.



TRIUMF cyclotron 520 MeV H⁻

PSI cyclotron 590 MeV proton

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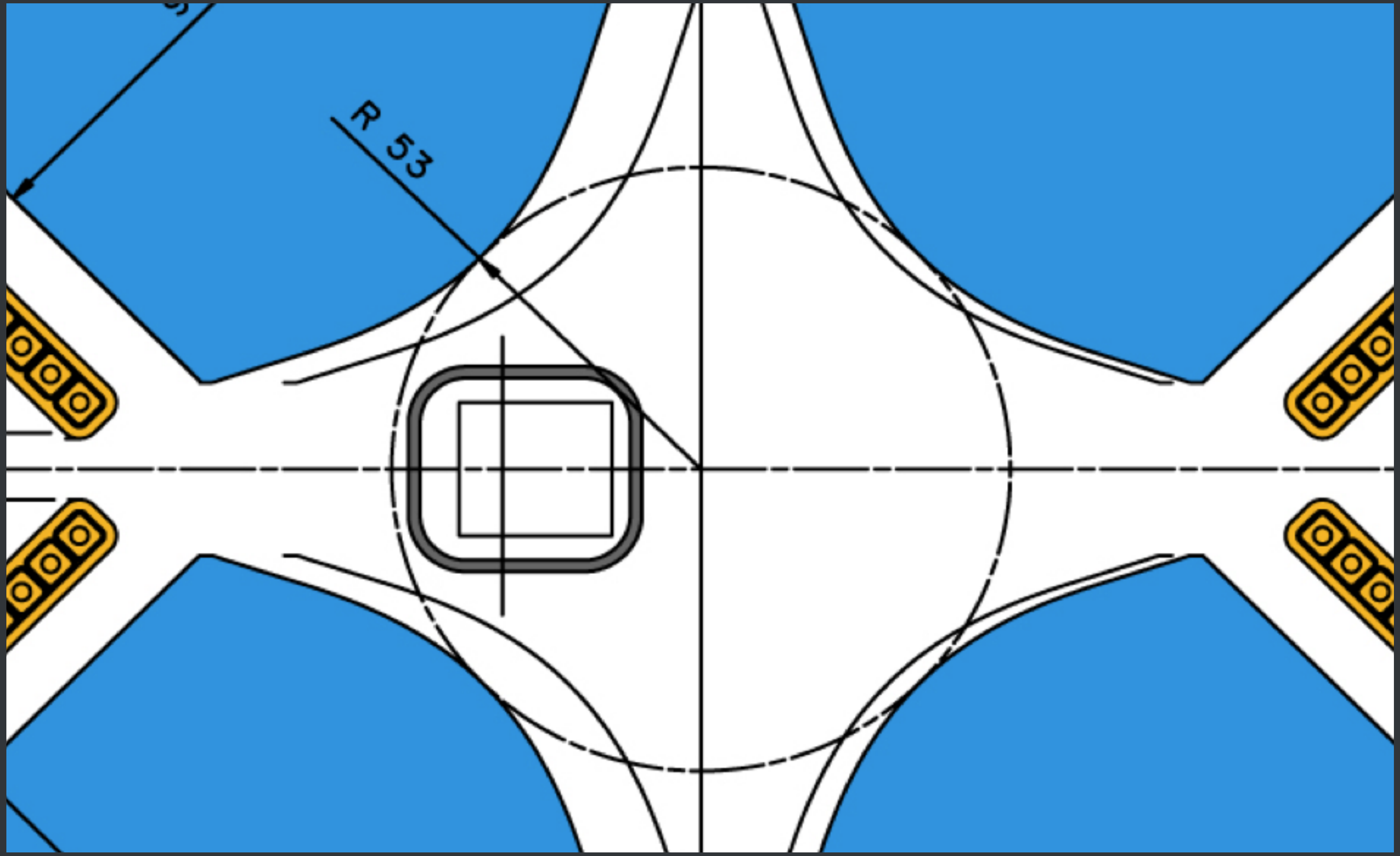
Linear focusing FFA

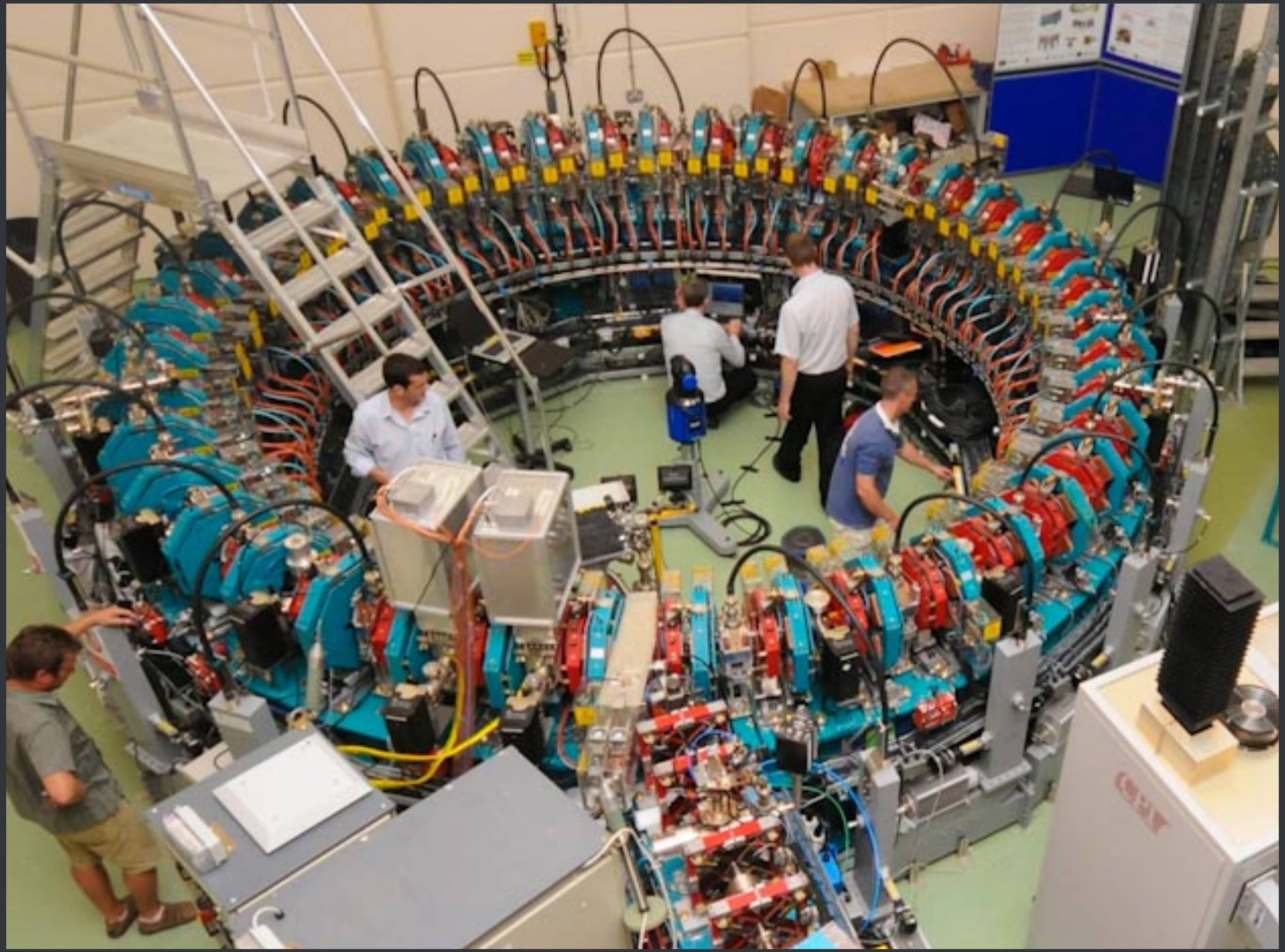
Use of offset quadrupoles to bend the beam:

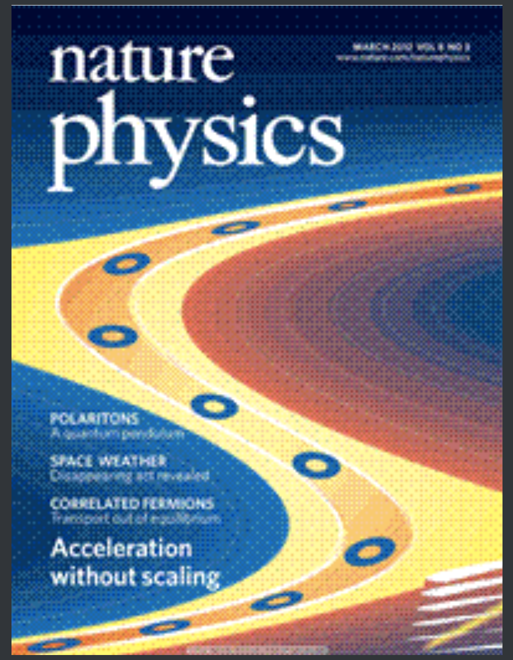


Behaves like a linear combined function magnet

Linear focusing FFA

EMMA (Electron Model for Many Applications)
at Daresbury Laboratory, UK.





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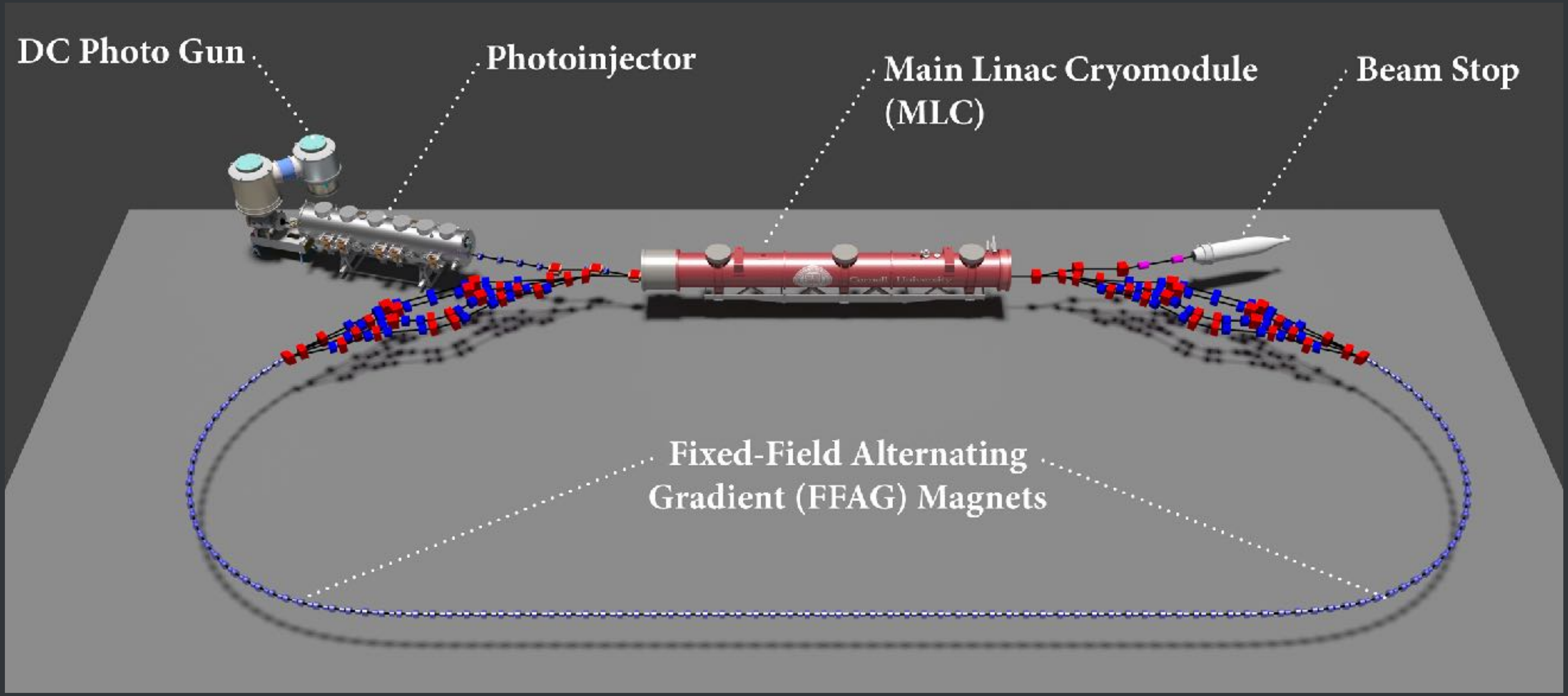
Linear FFA arcs for ERL

Cost effective arcs for Energy Recovery Linacs



CBETA at Cornell University, USA

Scaling FFA

keep independent of momentum the transverse linearised equations of motion.

- Analytical solution
- Achromatic system for any momentum range

There are also numerical ways to keep the tune constant over a certain momentum range (“non-linear non-scaling” FFA).

Circular scaling FFA

Linearised equations of motion around the closed orbit for a momentum p:

$$\begin{cases} \dfrac{d^2x}{d\Theta^2} + \dfrac{R^2}{\rho^2}(1-n)x = 0, \\ \dfrac{d^2y}{d\Theta^2} - \dfrac{R^2}{\rho^2}ny = 0. \end{cases}$$

(x, s, y): curvilinear coordinates.

New system of coordinates (x, Θ, y)

$\Theta = s/R$ with $R = \dfrac{1}{2\pi}\oint ds$

n: field index

ρ: curvature radius

Independent of momentum p:

$$\begin{cases} \left(\dfrac{\partial(R/\rho)}{\partial p}\right)_\Theta = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_\Theta = 0. \end{cases}$$

➡ Similarity of the reference trajectories.

➡ Invariance of the focusing strength.

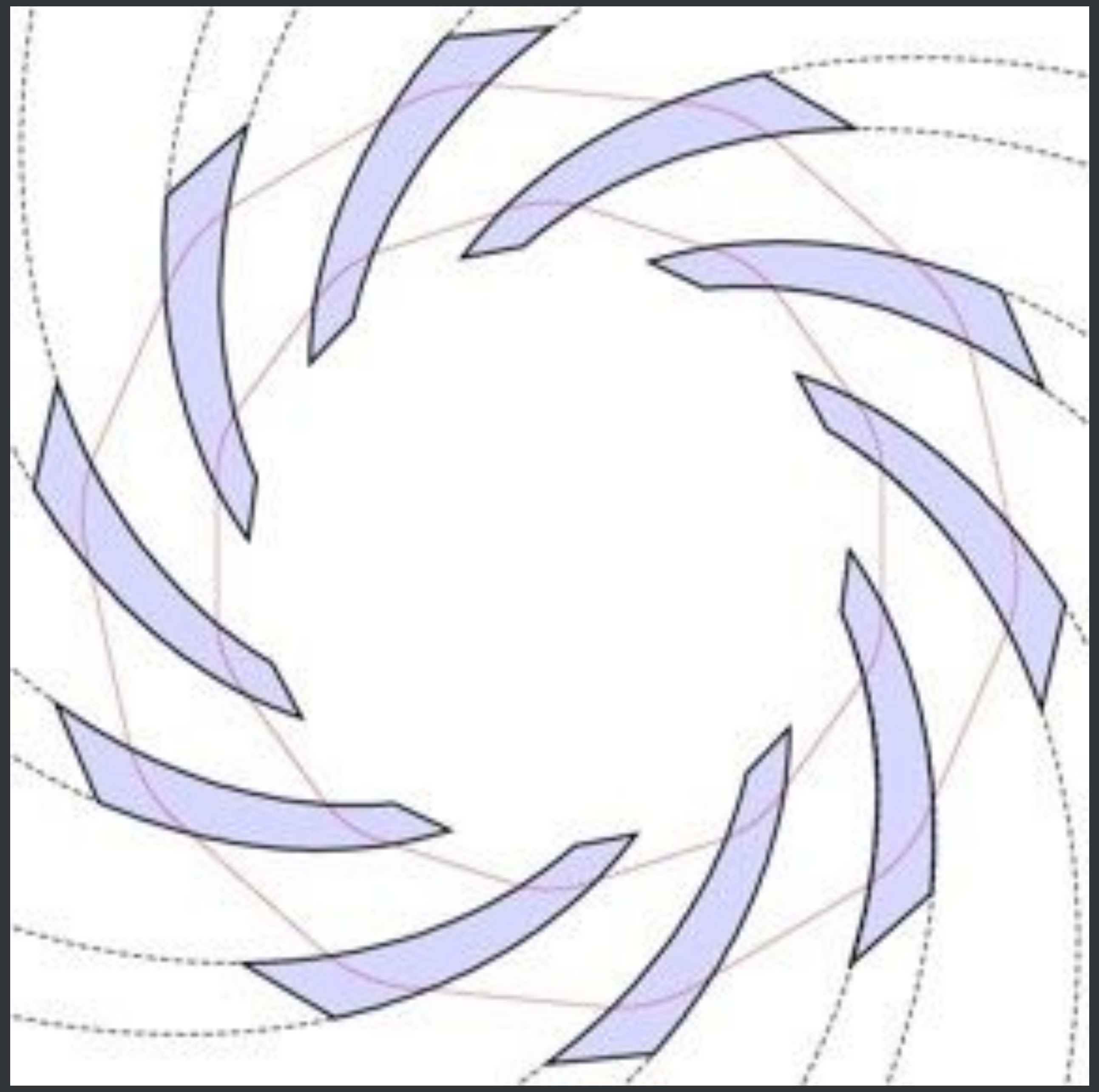
Circular scaling FFA

Invariance of the betatron oscillations ⟷ Similarity of the closed orbits <u>and</u> invariance of the field index

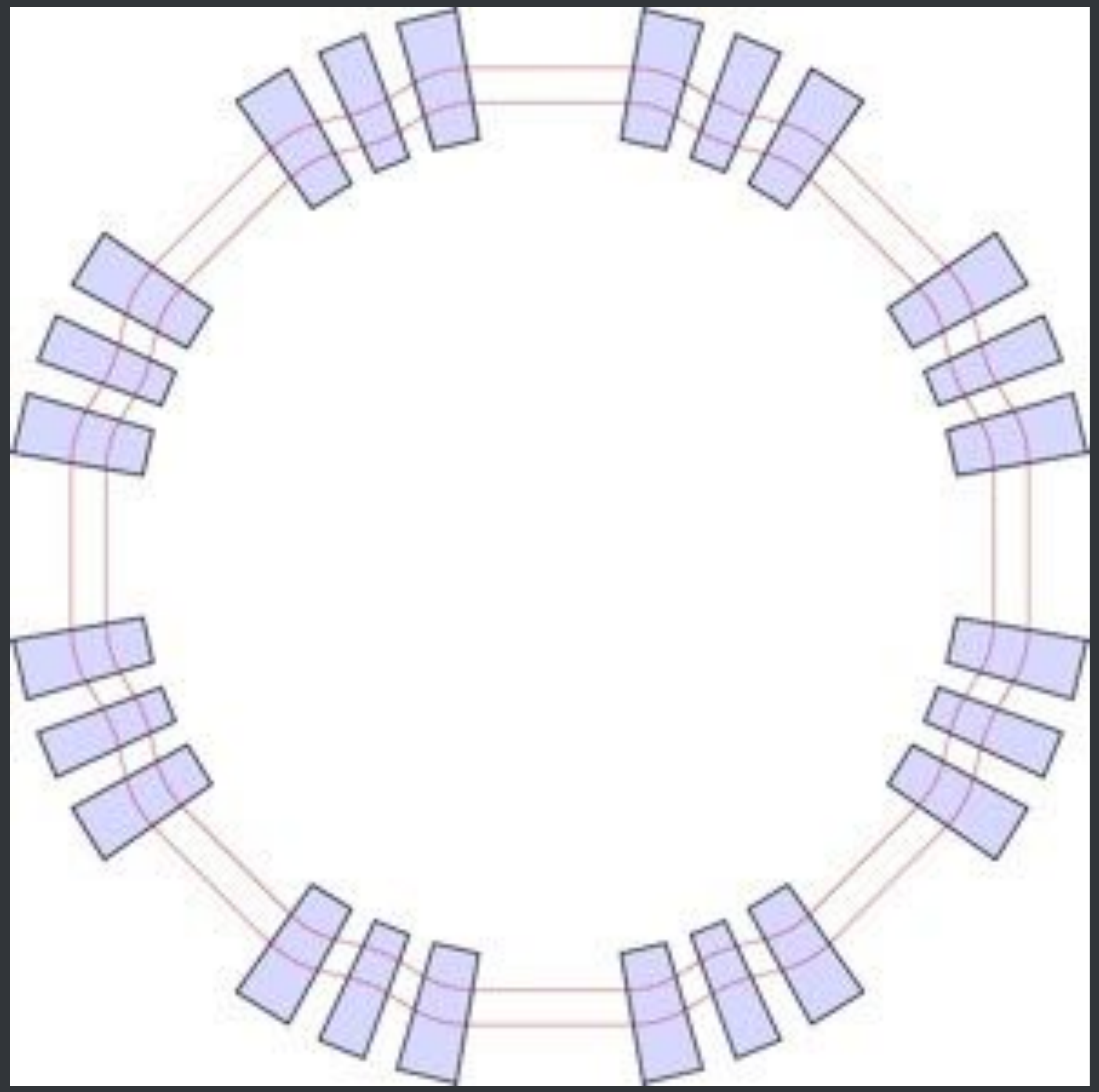
Constant geometrical field index:

$$k = \frac{R}{\bar{B}}\frac{d\bar{B}}{dR}$$

$$B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k \cdot \mathcal{F}(\theta - \tan\zeta \ln \frac{r}{r_0})$$

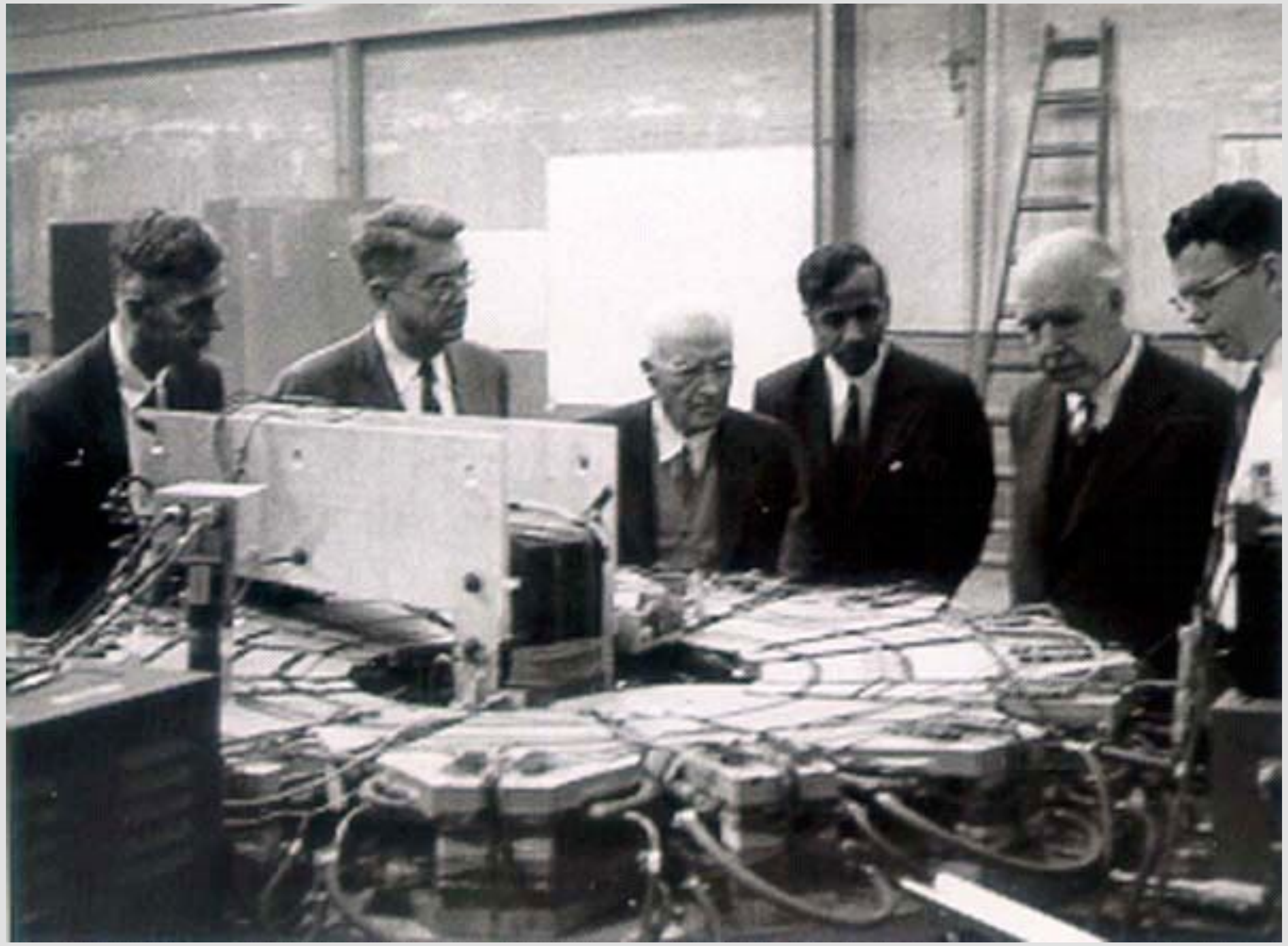


Spiral sector: $\zeta = const.$

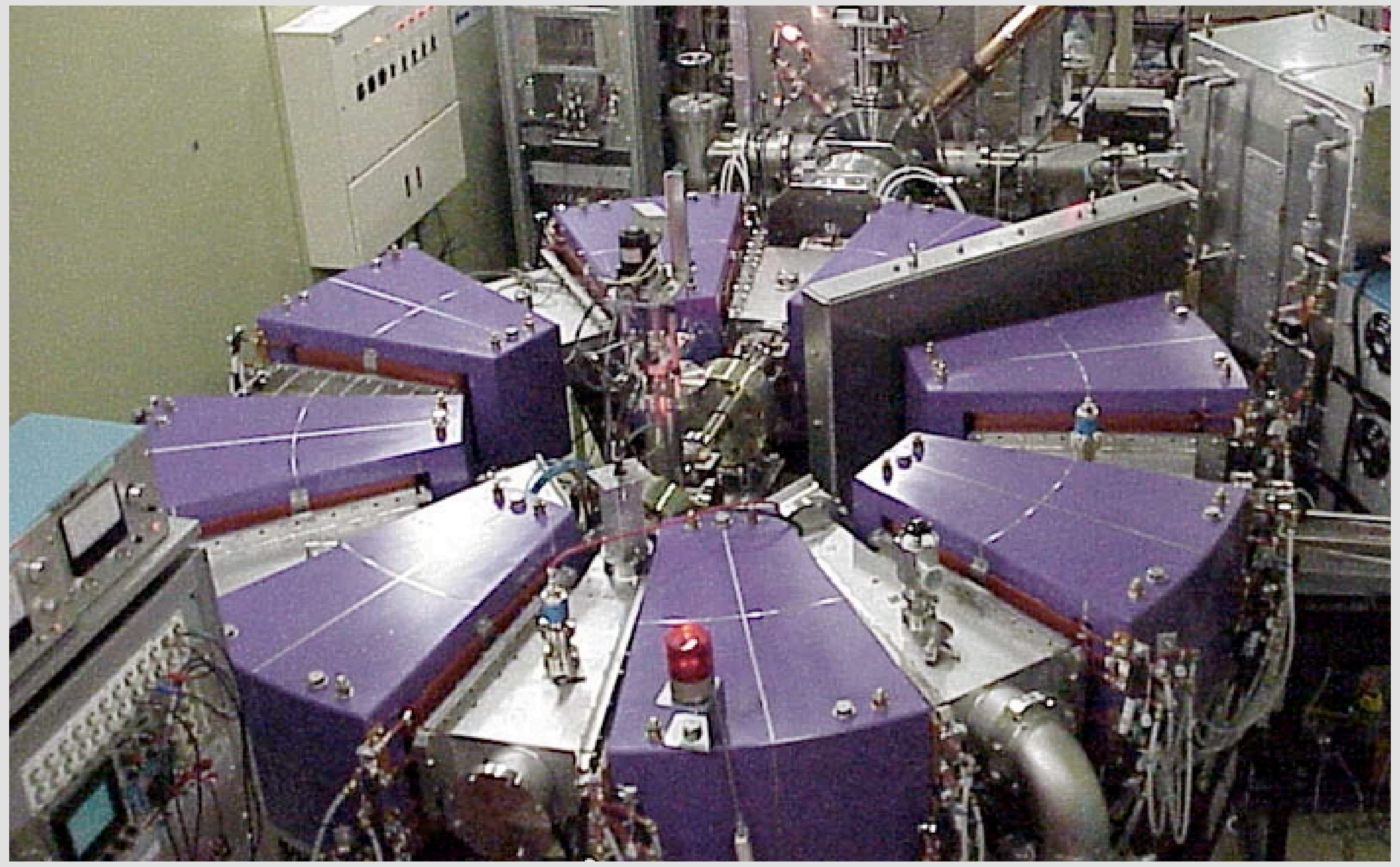


Radial sector: $\zeta = 0$

Horizontal scaling FFA



Mark II at MURA



POP machine at KEK

Proton FFA complex at Kyoto University

Straight scaling FFA

Linearised equations of motion around the closed orbit for a momentum p:

$$\begin{cases} \dfrac{d^2x}{ds^2} + \dfrac{(1-n)}{\rho^2}x = 0, \\ \dfrac{d^2y}{ds^2} - \dfrac{n}{\rho^2}y = 0. \end{cases}$$

(x, s, y): curvilinear coordinates
n: field index
ρ: curvature radius

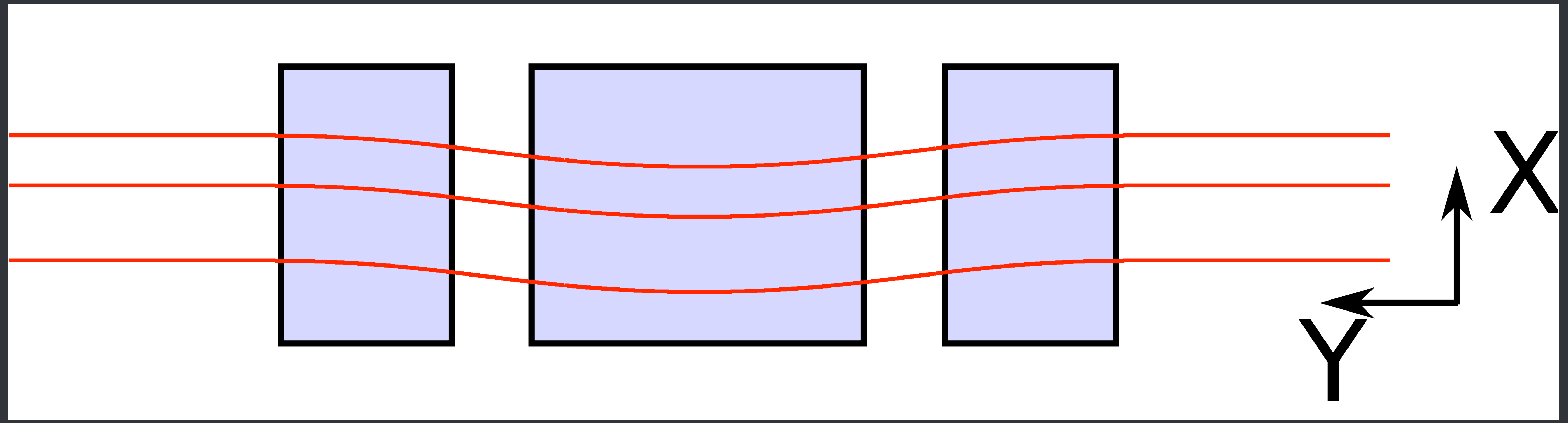
Independent of momentum p:

$$\begin{cases} \left(\dfrac{\partial\rho}{\partial p}\right)_s = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_s = 0. \end{cases}$$

➡ Similarity of the reference trajectories.
➡ Invariance of the focusing strength.

Straight scaling FFA

Invariance of the betatron oscillations ⟷ Similarity of the closed orbits and invariance of the field index

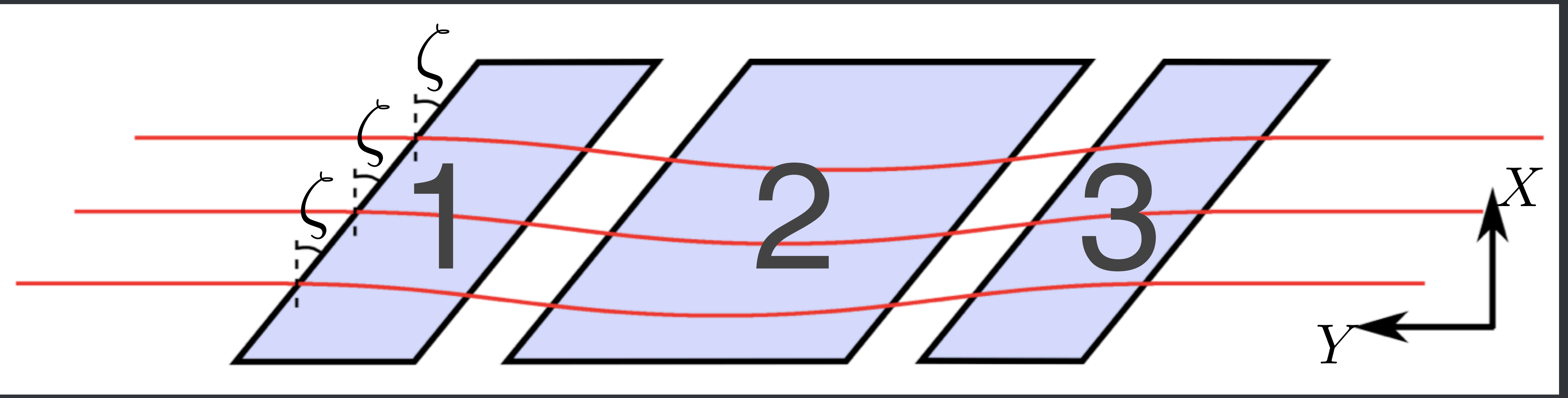


Rectangular case: $\zeta = 0$

Constant normalised field gradient:

$$m = \frac{1}{B_y}\frac{dB_y}{dx}$$

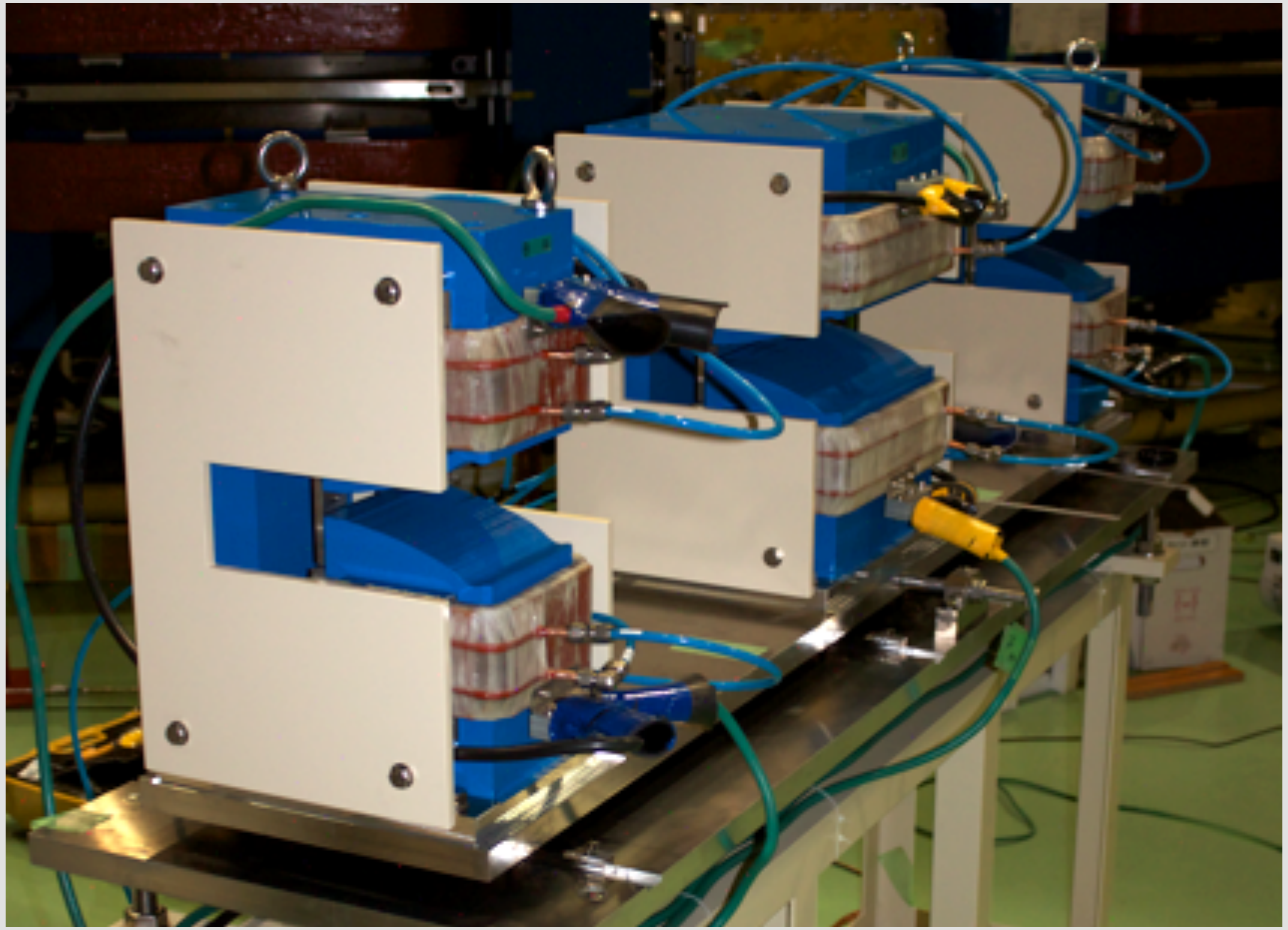
$$\boxed{B(X,Y) = B_0 e^{m(X-X_0)} \mathcal{F}\left(Y - (X - X_0)\tan\zeta\right)}$$



Tilted straight case: $\zeta = const.$

Straight scaling FFA

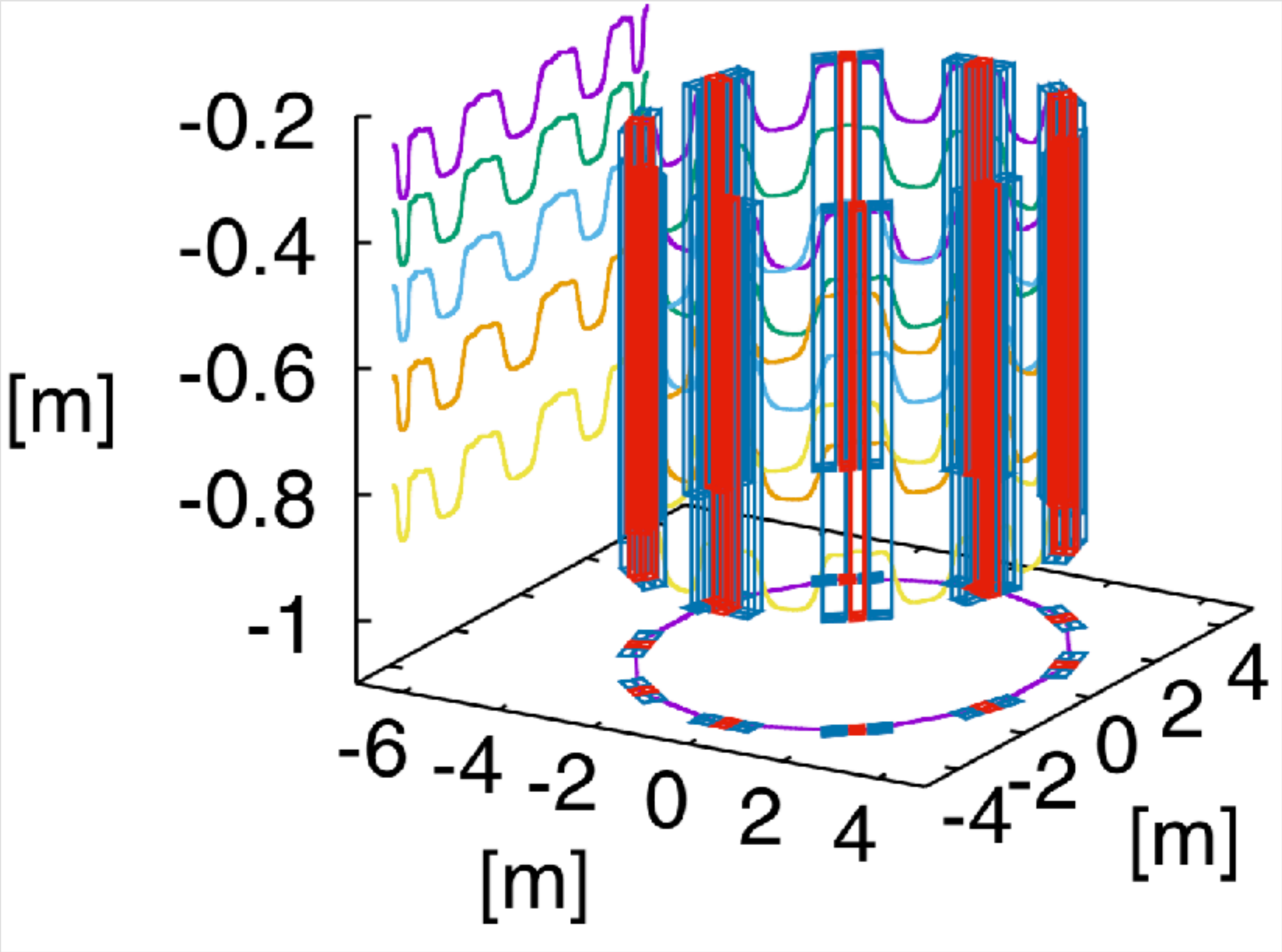
Achromatic FFA cell with no overall bend.



Straight scaling FFA prototype at Kyoto University (2011)

Vertical FFA

- "Electron cyclotron"
- Beam orbit rises vertically.
- Circumference constant with beam energy.
- Magnetic field increases exponentially vertically.
- Zero-chromatic for any momentum range.



Vertical scaling FFA

Linearised equations of motion around the closed orbit for a momentum p:

$$\begin{cases} \dfrac{d^2x}{d\theta^2} + \dfrac{x}{\rho^2} - n_y y = 0, \\ \dfrac{d^2y}{d\theta^2} + n_y x = 0. \end{cases}$$

⚠ vertical gradient, so coupling between horizontal and vertical plane.

(x, s, y): curvilinear coordinates.

New system of coordinates (x, θ, y)

$\Theta = s/R$ with $R = \dfrac{1}{2\pi} \oint ds$

ρ: curvature radius

n_y: vertical field index $n_y = -\dfrac{\rho}{B_y} \dfrac{\partial B_y}{\partial y}$

Independent of momentum p:

$$\begin{cases} \left(\dfrac{\partial \rho}{\partial p}\right)_s = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_s = 0. \end{cases}$$

➡ Similarity of the reference trajectories.

➡ Invariance of the focusing strength.

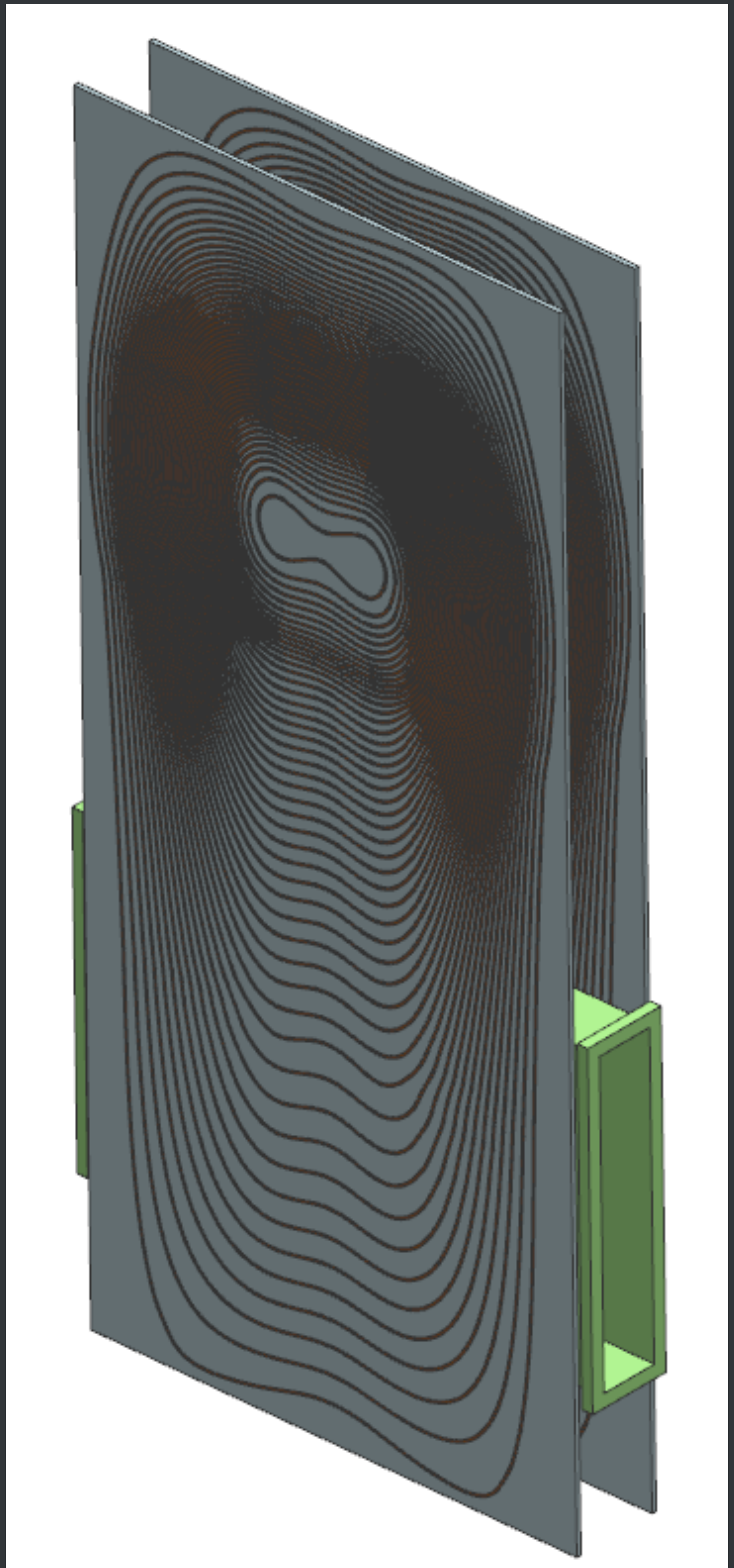
Vertical scaling FFA

Invariance of the betatron oscillations ↔ Similarity of the closed orbits <u>and</u> invariance of the field index

Constant normalised field gradient:

$$m_y = \frac{1}{B_y}\frac{dB_y}{dy}$$

$$\boxed{B_y(y,\theta) = B_0 e^{m_y(y-y_0)}\mathcal{F}(\theta)}$$



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Scaling FFA cells - Summary

	horizontal circular scaling	straight scaling	vertical scaling
Field law	$B_y = B_0 \left(\frac{r}{r_0}\right)^k$	$B_y = B_0 e^{m(x-x_0)}$	$B_y = B_0 e^{m_y(y-y_0)}$
Dispersion	$D_x = p_0 \left(\frac{\partial r_{co}}{\partial p}\right)_{p_0} = \frac{r}{k+1}$	$D = p_0 \left(\frac{\partial x_{co}}{\partial p}\right)_{p_0} = \frac{1}{m}$	$D_y = p_0 \left(\frac{\partial y_{co}}{\partial p}\right)_{p_0} = \frac{1}{m_y}$
momentum compaction factor	$\alpha = \frac{\Delta r_{co}/r_{co}}{\Delta p/p} = \frac{1}{k+1}$	$\alpha = \frac{\Delta x_{co}/x_{co}}{\Delta p/p} = 0$	$\alpha = \frac{\Delta r_{co}/r_{co}}{\Delta p/p} = 0$

Conclusion

FFA at the boundary of cyclotrons and synchrotrons

- Pros: Very useful for specific applications
  - high power machines,
  - handling of big beams (muon, internal target),
  - high repetition rate,
  - Energy efficient and reliable.

- Cons:
  - potentially big and complex magnets,
  - difficult to adjust parameters once machine is built,
  - Bigger machine circumference when reverse bend is used for vertical stability.