Spacetime ruler.
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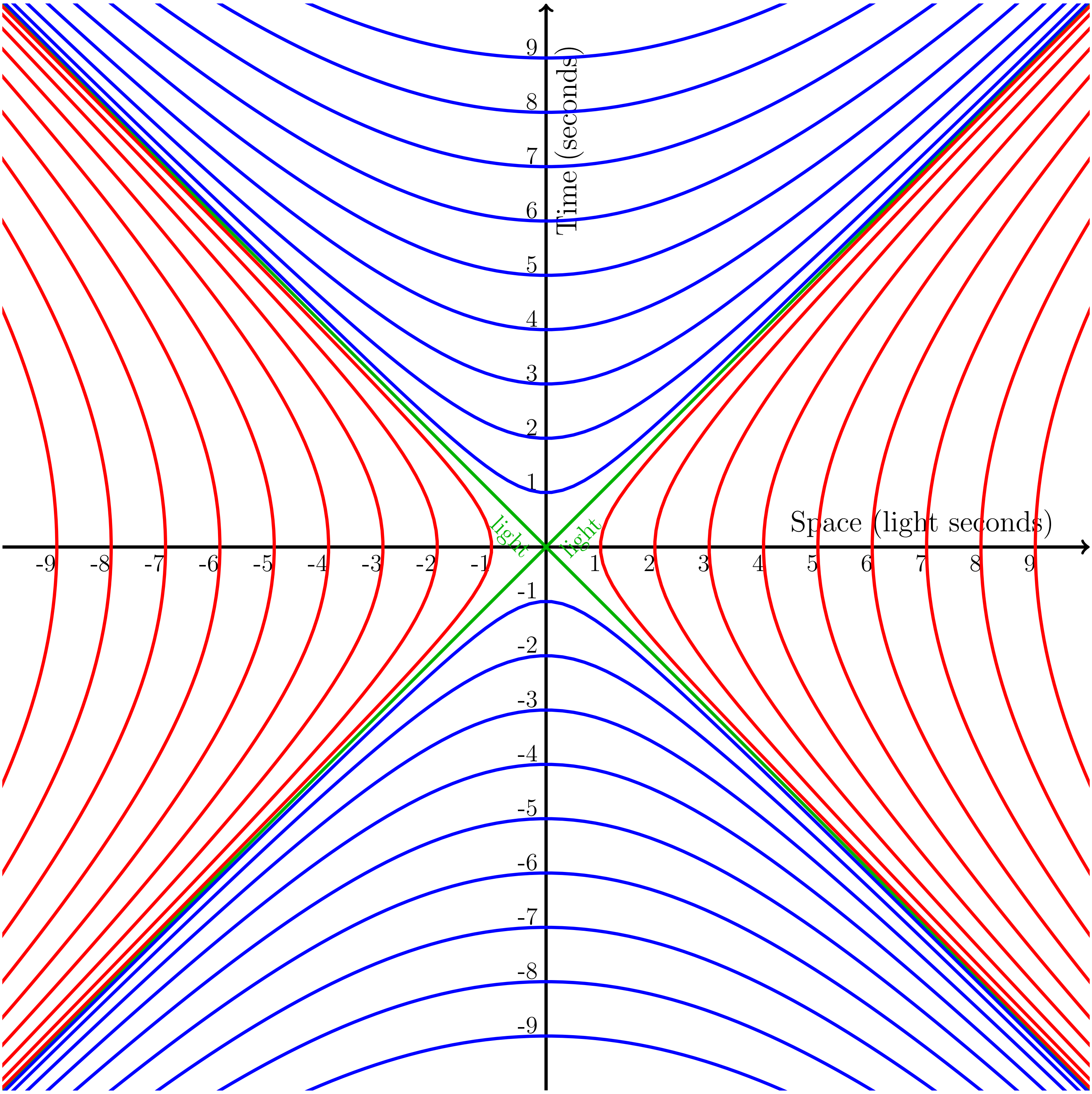


Special Relativity Worksheet.

You can try these exercises before the workshop if you wish. I will demonstrate all these exercises during the workshop.

Using a spacetime ruler: Choose one of the following options.

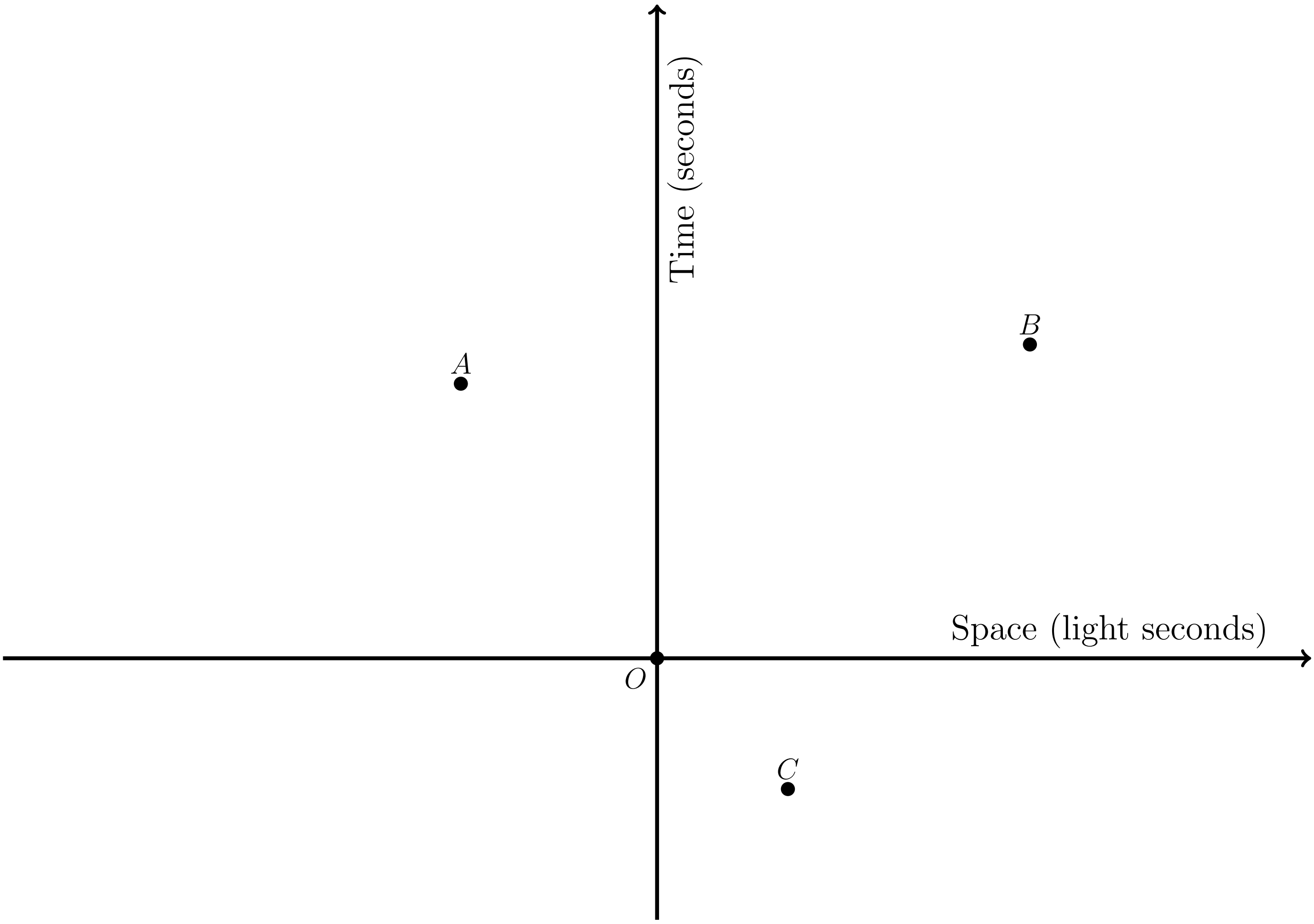
- Print out the spacetime rule and hold it against the computer monitor. Make sure page is the size of your A4 printed spacetime ruler.
- Note that when using the spacetime rulers, place the origin of the ruler on the starting point and then measure which line the second point is on.
- Print out both the spacetime ruler and the exercise. Then you can put one on top of the other and hold them up to the light.
- Use the coordinates. Then you can use the formula for time distance given by

  $$\tau = \sqrt{t^2 - x^2}$$

  and for space distanced given by

  $$s = \sqrt{x^2 - t^2}$$

Exercise 1. Using your spacetime rulers

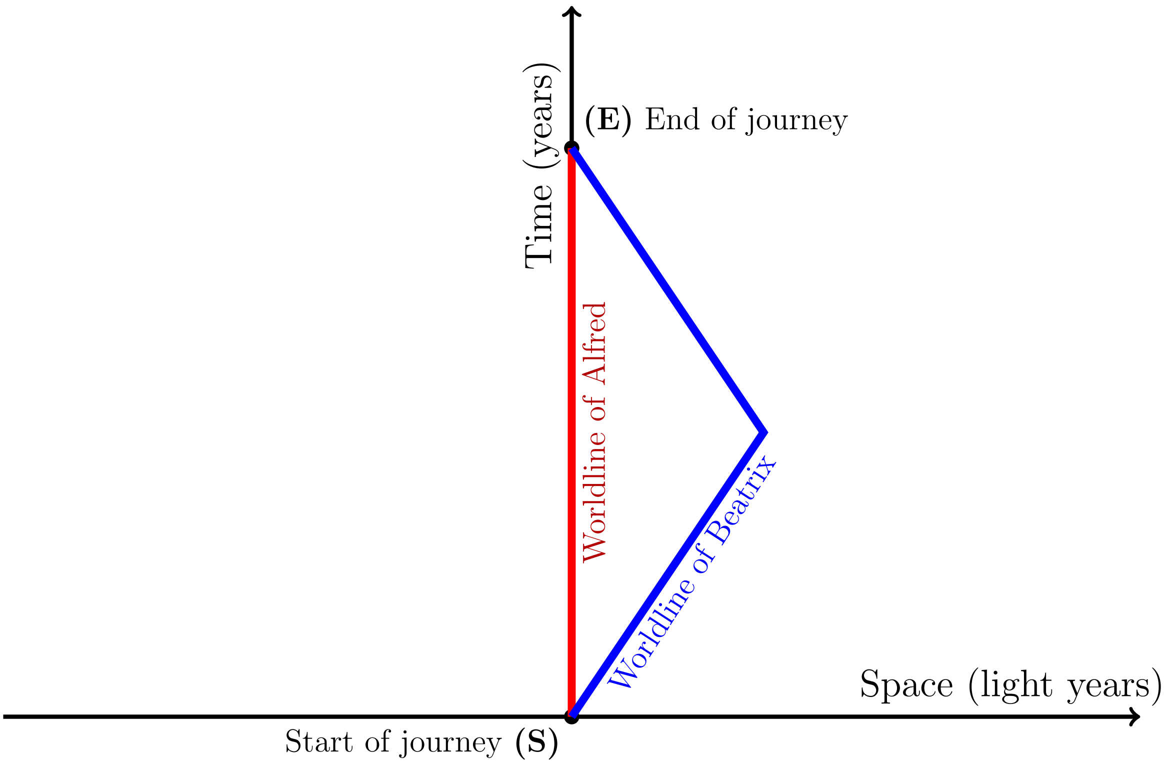


Measure the lengths of the following vectors.
Are they timelike, spacelike or lightlike?
Do the first point lie in the future or past of the second?

From	To	Timelike? Lightlike? or Spacelike?	Future? or Past?	Length
O	A			
O	B			
O	C			
A	B			
B	C			
A	C			

If you want to use the formula. The positions of the points are A (-3,4.2), B (5.7,4.8) and C (2,-2).

Exercise 2. Alfred and Beatrix



(a) Measure the time Alfred has aged.

(b) Measure the time Beatrix has aged.

(c) Draw alternative worldlines connecting the start and end of the journey. How do the times compare to that to Alfred?

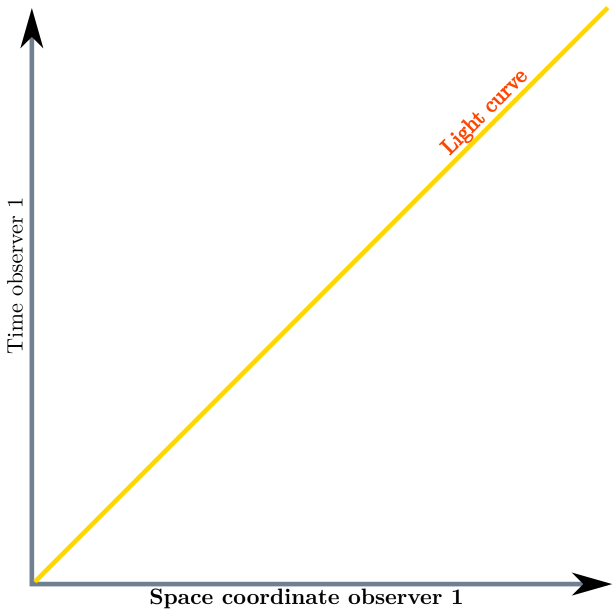
(Do no go faster than light!)

(d) What is the shortest worldline you can draw which connect **(S)** and **(E)**?

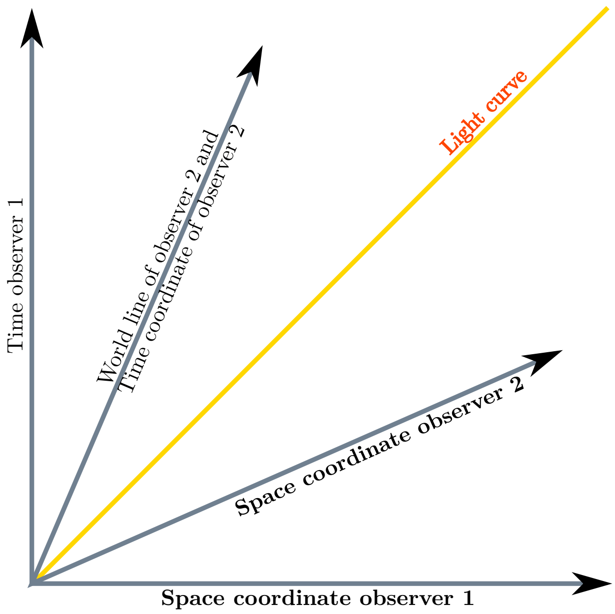
For those wanting to use formula, the positions of the points are: The position of the top of the red line (0,8) and the position of the bend (2.7,4).

Drawing a spacetime diagram

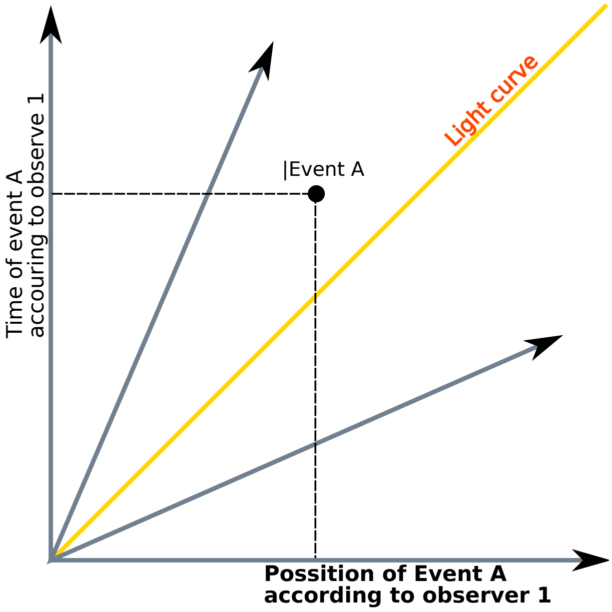
- – Draw horizontal and vertical axes.
  – The vertical axis represents the worldline of a stationary particle, called observer 1.
  – The horizontal line is considered the space axis of observer 1.
  – Draw a diagonal line at 45°. This represents the motion of a beam of light.



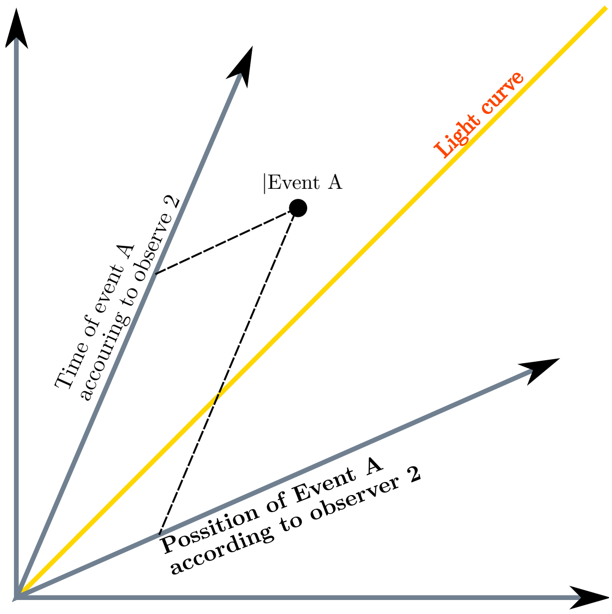
- – Draw a timelike line. This is a line more vertical than the diagonal line at 45°. This represents the worldline of a moving particle, called observer 2. (Moving with respect to the stationary particle). This is called the time axis of observer 2.
  – Draw another line by reflecting the worldline of the moving particle about the 45° lightline. This is the called the space axis of observer 2.



- – A point on the spacetime diagram is called an event. This is a point in space at a specific moment in time.
  – The vertical value of this event is the time as measured by observer 1.
  – The horizontal value event is the position of the event as measured by observer 1.



  – Take a line from the event, parallel to the space axis of observer 2. The value on the time axis where this line crosses it is the time of the event as measured by observer 2.
  – Take a line from the event, parallel to the time axis of observer 2. The value on the space axis where this line crosses it is the position of the event as measured by observer 2.



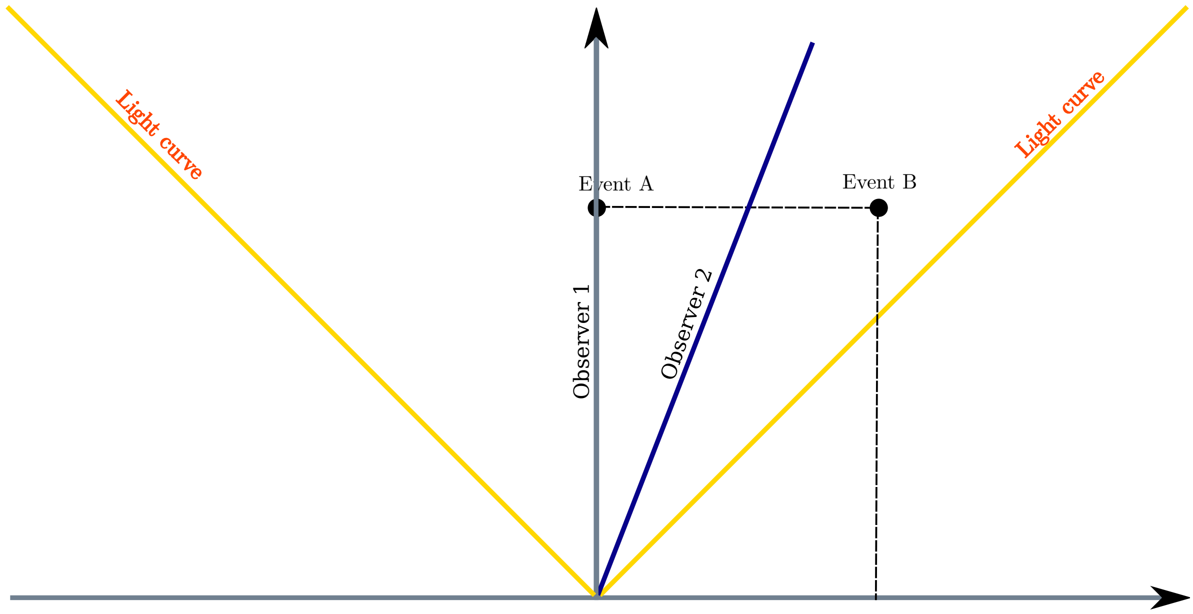


Figure 1: The spacetime diagram representing simultaneous events.

Exercise 3.

Figure 1, shows a spacetime diagram. According to observer 1, events A and B are simultaneous.

(a) Show that according to observer 2, event B occurs before event A.

(b) Draw the axes of another observer who would measure event A before event B.

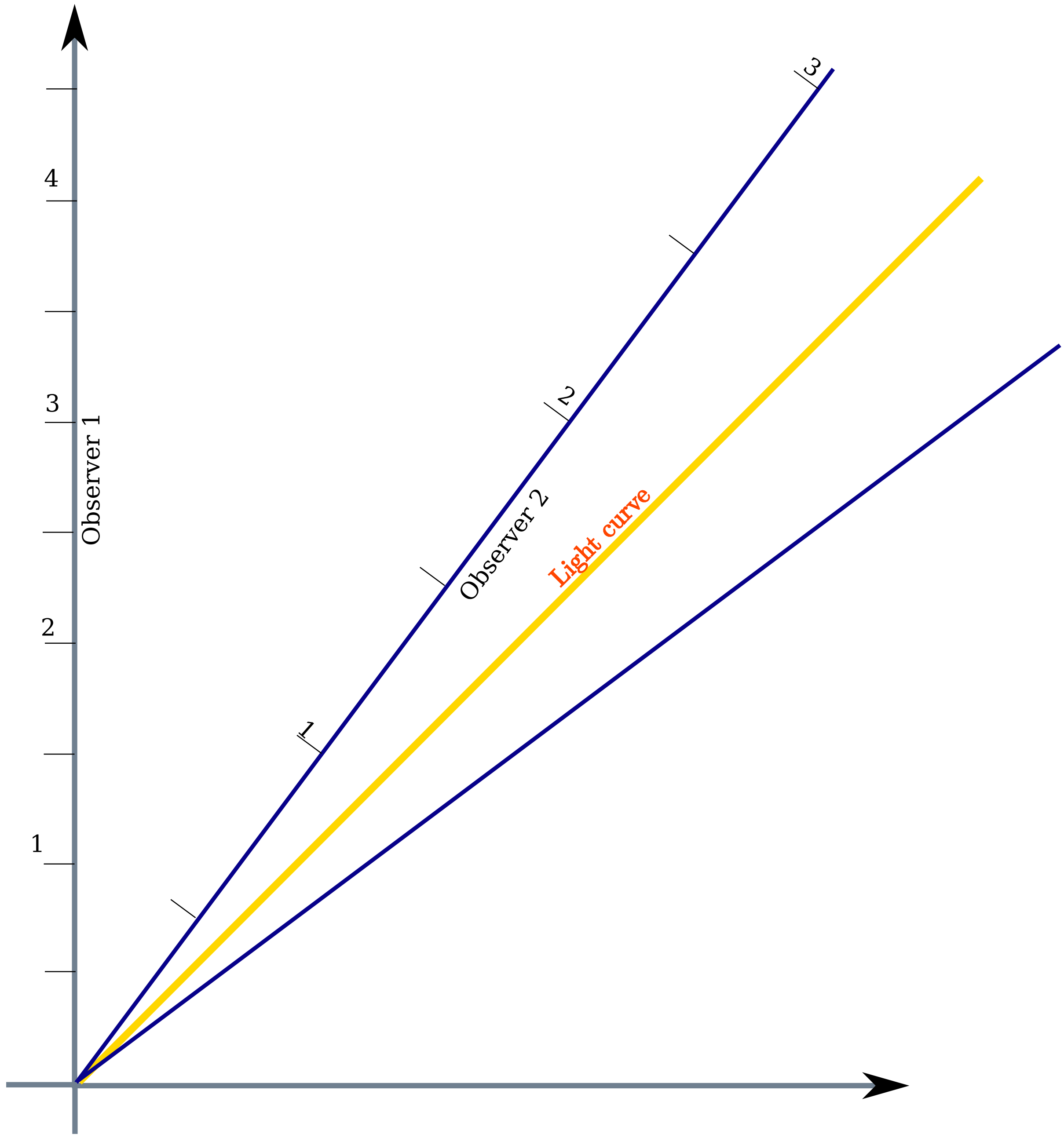


Figure 2: The spacetime diagram represents the “paradox” of time dilation

Exercise 4.

In figure 2 observer 2, is moving at a speed $\frac{\sqrt{5}}{9}c \approx .745c$ where c is the speed of light. The axes measure seconds with respect to both observers.

(a) Show that when observer 2 measures 1 second on her timeline, observer 1 measures 1.5 seconds. Therefore observer 1 thinks observer 2 clock is slow.

(b) When observer 1 measures 1 second on his timeline, what time does observer 2 measure. Does observer 2 think observer 1 clock is fast or slow.

Let's say observer 1 and observer 2 want to directly compare clocks. Both send a light signal to the other after 1 second.

(c) Show that the signal from observer 2 reaches observer 1 at time 2.62 seconds, according to observer 1 clock. (The actual value is $\frac{3}{2}(1 + \frac{\sqrt{5}}{3}.)$

(d) What time, according to observer 2 clock, does the signal from observer 1 arrive?

Exercise 5.

In figure 3:

(a) Identify the events on the diagram A,. . . ,J which represents the following events.

- The moment when the front of the car passes side 1 of the garage.
- The moment when the front of the car passes side 2 of the garage.
- The moment when the rear of the car passes side 1 of the garage.
- The moment when the rear of the car passes side 2 of the garage.

(b) Does the front of the car pass side 2 of the garage before or after rear of the car passes side 1 of the garage:

- According to the garage observer.
- According to the car observer.

(c) Identify two events which represents the length of the garage according to the garage observer.

Identify two events which represents the length of the car according to the garage observer.

Show that the garage observer measures the car as shorter than the garage.

(d) Identify two events which represents the length of the garage according to the car observer.

Identify two events which represents the length of the car according to the car observer.

Show that the car observer measures the garage as shorter than the car.

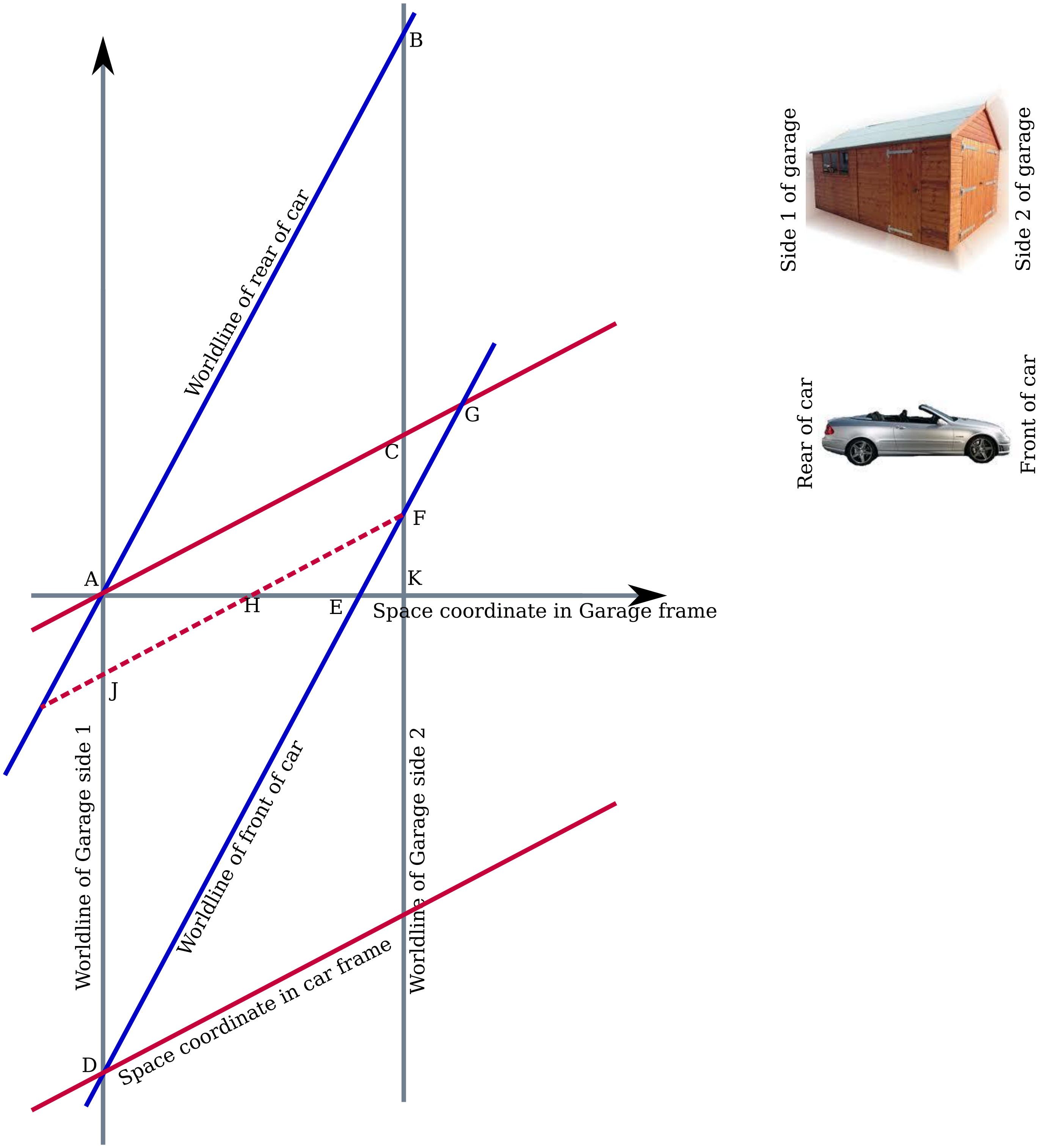
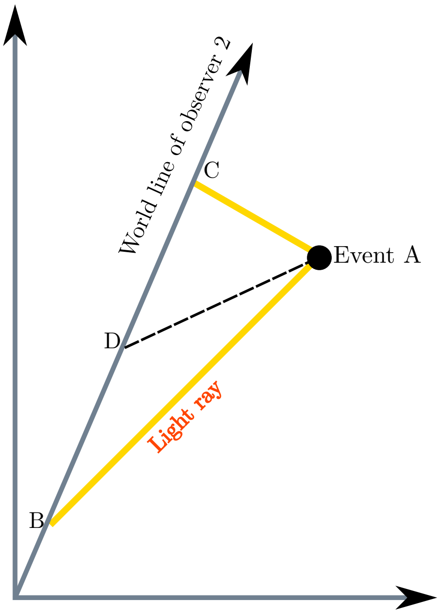


Figure 3: The spacetime diagram represents the "paradox" of the car and the garage.

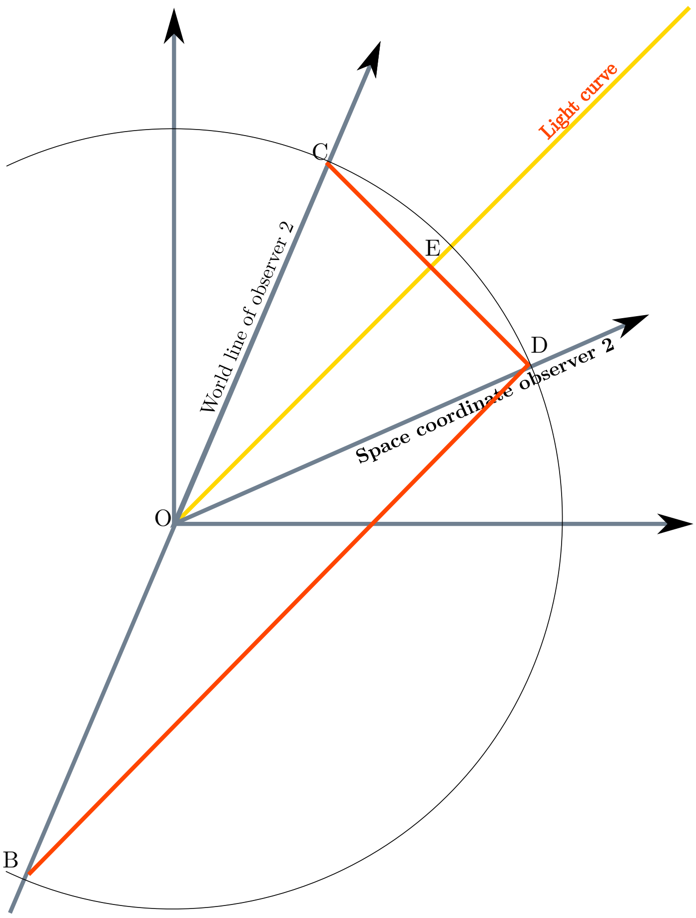
Appendix: Why is the space coordinate axis where it is?



We start with **radar time**. Observer 2 sends a light signal at event B to event A. The signal is immediately returned and intersects the worldline of observer 2 at C. Observer 2 defines the the time of event A as the same time as event D which is **half way** between B and C.

We define the space coordinate axis of observer 2 to be all the points with time 0 according to observer 2.

It is now simple geometry to show that the space coordinate axis of observer 2 is the reflection of the worldline of observer 2 through the light curve.



In the diagram on the right, BD and CD are at 45°, therefore BCD is a right angled triangle.

Thus the circle which circumscribes BCD must have BC as a diameter. Since O bisects BC then O is the origin of the circle.

Thus the length OC equals OD so the triangle OCD is an isosceles triangle.

However the light curve OE is at right angles to C and therefore must bisect the triangle. Thus OD is the reflection of OC through OE.