Electromagnetic Questions

Question 1. A spatially dispersive material has $\mu = \mu_0$ and

$$\epsilon(\omega, \boldsymbol{k}) = \epsilon_0 + \frac{\chi_0}{-\omega^2 + i\lambda\omega + \omega_P^2 + k_z^2}$$

Calculate the dispersion relation for both Transverse $\boldsymbol{E}(t, \boldsymbol{x}) = E(t, z)\boldsymbol{i}$ and Longitudinal waves $\boldsymbol{E}(t, \boldsymbol{x}) = E(t, z)\boldsymbol{k}$ in the z-direction. I.e. the relationship between ω and $\boldsymbol{k}$ such that Maxwell's equations are solved by $\boldsymbol{E}(t, \boldsymbol{x}) = e^{i\omega t + \boldsymbol{k}\cdot\boldsymbol{x}}\boldsymbol{E}_0$

Question 2. A solenoid and a cylindrical magnet are designed to produce the same B field in a vacuum (As me to draw a picture).

If I place a rod of iron in the middle $\mu = 1000\mu_0$ describe the resulting B field near the rod.

Question 3. (a) Show that the two Maxwell's equations involving $\boldsymbol{D}$ and $\boldsymbol{H}$ are invariant under the "gauge" transformation

$$\boldsymbol{H} \to \boldsymbol{H} + \nabla\psi + \dot{\boldsymbol{\Psi}} \qquad \text{and} \qquad \boldsymbol{D} \to \boldsymbol{D} + \nabla \times \boldsymbol{\Psi}$$

(b) Are $\boldsymbol{D}$ and $\boldsymbol{H}$ measurable quantities.

Question 4.

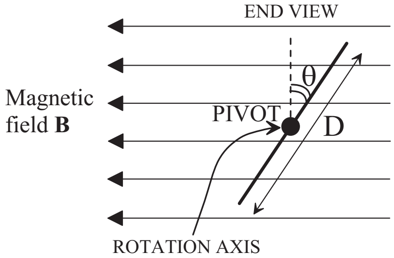
(a) (i) A straight wire of length L carries current I in a direction parallel to the unit vector $\hat{\mathbf{n}}$. Write down an equation for the force, $\mathbf{F}$, on the wire in a uniform magnetic field $\mathbf{B}$. Describe, using a sketch, the direction of $\mathbf{F}$ if the field, $\mathbf{B}$, is perpendicular to the direction of the wire, $\hat{\mathbf{n}}$.

(ii) Use the expression for the Lorentz force on a charge moving in a magnetic field to describe the voltage induced between the ends of a straight wire of length L when it is moved at velocity $\mathbf{v}$ through a uniform static field $\mathbf{B}$. How does the voltage depend on $|\mathbf{v}|$, L and $|\mathbf{B}|$, and on the direction of $\mathbf{B}$ relative to $\mathbf{v}$ and the orientation of the wire?

(b) The figure below shows the end view of a rectangular loop with its top and bottom sides each of length L perpendicular to the diagram and to the magnetic field, $\mathbf{B}$. The other sides are each of length D. The coil rotates with angular velocity $d\theta/dt$.

(i) Adapt your answer in (a)(ii) [based on the Lorentz force] to find an expression for the induced voltages in each of the four sides, and the net e.m.f. for the loop as a function of B, D, L, and angle θ.

(ii) Use Faraday's flux rule to calculate the net e.m.f. for the loop as a function of B, D, L, and angle θ.



(c) If the loop is now fixed at angle θ, and carries current I, adapt your answer in (a)(i) to find an expression for the magnitude of the torque on the loop in terms of B, I, D, L, and θ.

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