Five-dimensional phase-space efficiency in $D \to K3\pi$ at LHCb for precision CP violation measurements

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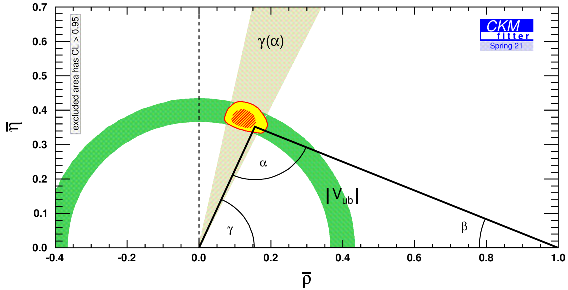
Introduction

- LHCb took $9fb^{-1}$ data between 2011 and 2018
- The detector is specialised for precision flavour physics measurements
- This work analyses $B \to (D \to K3\pi)K$ type events
- Measurement of the CKM angle γ - describes CP violation in $b \to u$ quark transition processes



[1]Figure from the CKMFitter Group, http://ckmfitter.in2p3.fr

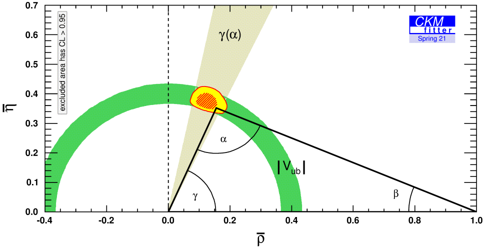
CKM Matrix

- The CKM matrix elements describe quark transition probabilities

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

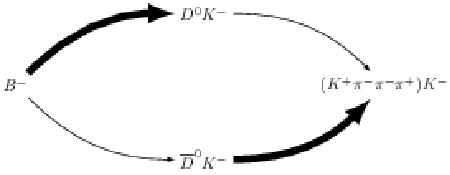
- Must be unitary - one unitarity condition is represented by the unitarity triangle

$$V_{ub}^{\star} V_{ud} + V_{cb}^{\star} V_{cd} + V_{tb}^{\star} V_{td} = 0$$



gamma

- $\gamma = arg\left(-\frac{V_{ub}^\star V_{ud}}{V_{cb}^\star V_{cd}}\right)$
- $\gamma = 67 \pm 4^{\circ}$[1]
- Expect similar precision from this analysis



- Rate for $B^\pm \to DK^\pm$; $D \to f$ for some final state f depends on γ
- Rate ratios (in a region of final state phase space Ω):

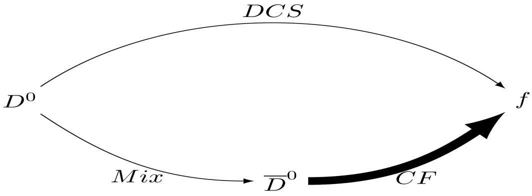
$$\frac{\Gamma(B^- \to DK^-, D \to f)_\Omega}{\Gamma(B^- \to DK^-, D \to \overline{f})_{\overline{\Omega}}} = r_{D,\Omega}^2 + r_B^2 + r_{D,\Omega} r_B |Z_\Omega^f| cos(\delta_B - \delta_D^f - \gamma)$$

$$\frac{\Gamma(B^+ \to DK^+, D \to \overline{f})_{\overline{\Omega}}}{\Gamma(B^+ \to DK^+, D \to f)_\Omega} = r_{D,\Omega}^2 + r_B^2 + r_{D,\Omega} r_B |Z_\Omega^f| cos(\delta_B - \delta_D^f + \gamma)$$

[1] "Updated LHCb combinedation of the CKM angle γ", LHCb Collaboration, 2020, http://cds.cern.ch/record/2743058

D Mixing

- The $K3\pi$ final state is accessible by both D^0 and $\overline{D}^0$
- These can decay directly or via mixing:

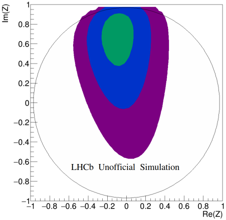


- Interference between these amplitudes is quantified by Z_Ω^f
- Z_Ω^f can be measured using LHCb data
- It has also been previously measured using quantum-correlated $c\overline{c}$ pairs, at e.g. CLEO and BES-III
- Best precision comes from combining these measurements

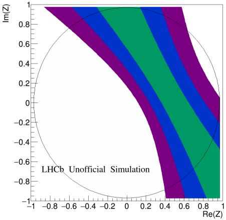
Measuring Z^f_Ω, γ

- We can measure Z^f_Ω from LHCb data by fitting to D^0 decay times
- Combine Z^f_Ω from fit with result from CLEO and BESIII to improve precision
- From simulation:

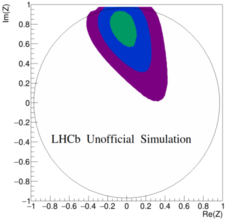
CLEO:



LHCb:



Combined:



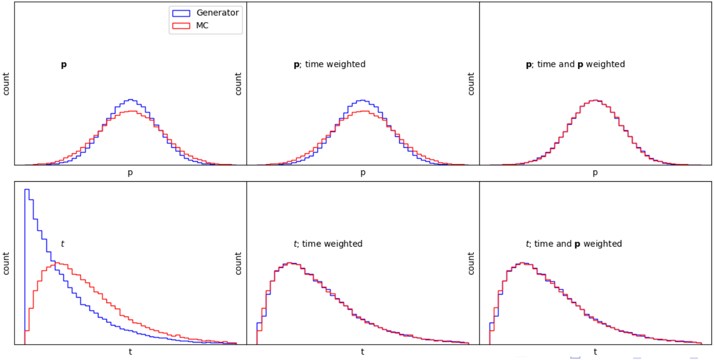
- We then can then use this value of Z^f_Ω to extract γ

Efficiency

- Correct for the detector acceptance - some events are missed in our reconstruction etc.
- Four body final state $\implies$ 5-dimensional phase space
- To correct for phase space and decay time efficiency, will therefore need a 6d efficiency model
- Want to fully describe this efficiency, including all correlations

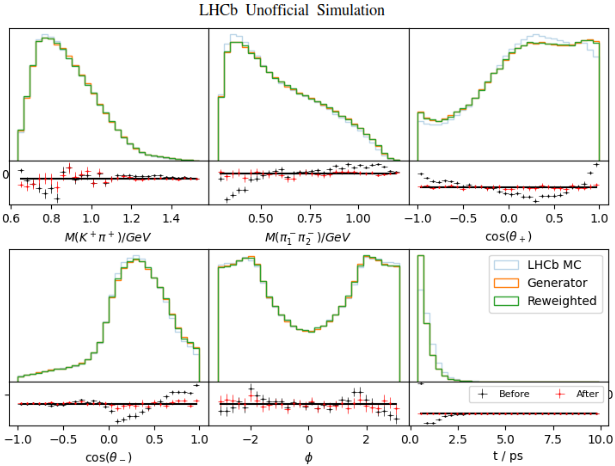
Reweighting Strategy

- Compare LHCb MC to generator level events to extract efficiency
- Perform a histogram division to find $\epsilon(t)$, correct for this then use a BDT to correct for the phase space efficiency (and keep correlations with time)
- Toy illustration of the method:



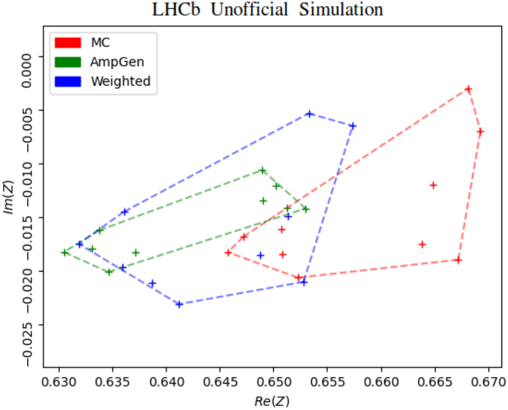
Validation

- We have several ways to validate the method
- Projections - necessary but insufficient



Z Numerical Integral

- Perform a Monte Carlo integral to numerically estimate Z_Ω^f for our MC
- Split the datasets into 10 equal chunks - evaluate Z_Ω^f for each
- Expect reweighted Z_Ω^f to match up with generator Z_Ω^f ("AmpGen")



Conclusion

- $B \to (D \to K3\pi)K$ events at LHCb give a powerful way to constrain γ
- Correcting for detector efficiency smoothly across the phase space is an important step in the analysis

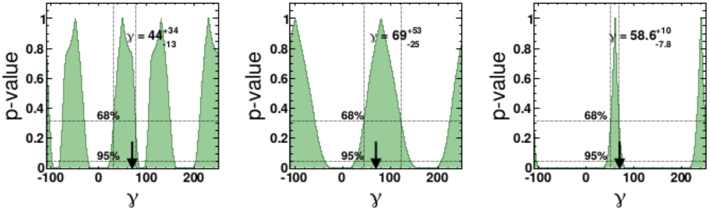
- We have a BDT based reweighting method to correct for the detector efficiency
- Initial validation seems to show it largely works

Backup

Prior Art

- This project follows on from previous work in this group; notably Sam Harnew's thesis
- Previously predicted that we could measure γ to within 4°
- Recent amplitude models have the potential to improve our phase space binning + give even better precision (1-2°?)

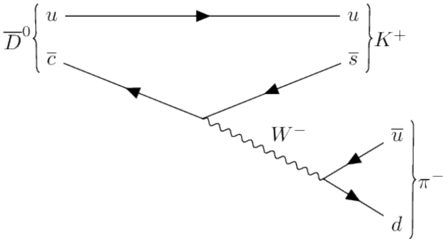
D-mixing (binned) alone | BES III (binned) alone | D-mixing (binned) with BES III binned



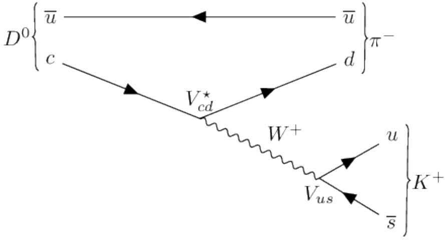
(From CERN-THESIS-2015-317)

RS/CF and WS/DCS Events

- Two types of decay for the analysis - "right sign" / "Cabibbo favoured" (RS/CF), "wrong sign" / "doubly Cabibbo suppressed" (WS/DCS)
- $K\pi$ digram shown in this slide but it's the same for $K3\pi$
- i.e. in this case $\overline{D}^0$ decays rapidly to f; D^0 slowly



RS/CF



WS/DCS

Analysis Strategy

- Time-dependent ratio of DCS to CF decay rates:

$$\frac{\Gamma(D^0(t)\to f)}{\Gamma(\overline{D}^0(t)\to f)} \approx r_{D,\Omega}^2 + r_{D,\Omega}\left(yRe(Z_\Omega^f) + xIm(Z_\Omega^f)\right)(\Gamma_D t) + \frac{y^2+x^2}{4}(\Gamma_D t)^2$$

with

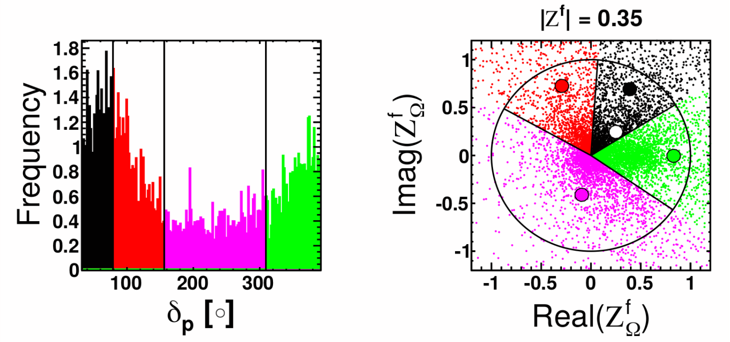
$$x = \frac{M_2 - M_1}{\Gamma_D} \qquad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

Where $M_{1,2}$ and $\Gamma_{1,2}$ are the mass + width of the D mass eigenstates

- Fit to this equation to constrain Z_Ω^f
- Get x and y from external input
- Combine Z_Ω^f from fit with result from CLEO and BESIII to improve precision
- Given the $B \to (D \to K3\pi)K$ rates, we can then extract γ
- Perform the analysis in phase space bins - different value of Z_Ω^f in each

Phase Space Binning

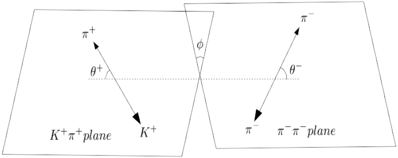
- Z_{Ω}^{f} describes average interference over a region of phase space Ω
- If we split phsp into regions we get an expression for γ in each region
- This has the potential to give us increased sensitivity

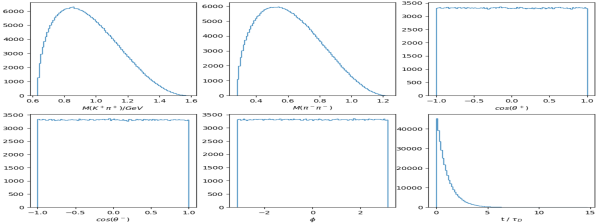


The distribution of Z_{Ω}^{f} for simulated DCS events. From Sam Harnew's Thesis CERN-THESIS-2015-317

Dimensionality and Efficiency Parameterisation

- Four body decay $\implies$ 5 free parameters; 6 including time
- Using Cabibbo-Maksymowicz parameterisation[2]:
- Toy phsp $D^0 \to K^+\pi^-\pi^+\pi^-$ evts:





[2] Cabibbo, N., & Maksymowicz, A. (1965)

Parametrising mixing

- Define the following amplitudes for suppressed and favoured decays to a final state f:

$$A(\mathbf{p}) \equiv \left\langle f_{\mathbf{p}} \middle| \hat{H} \middle| D^0 \right\rangle$$
$$B(\mathbf{p}) \equiv \left\langle f_{\mathbf{p}} \middle| \hat{H} \middle| \overline{D}^0 \right\rangle$$

- If we're looking at all decays in a region of phase space Ω, we are interested in:

$$\mathcal{A}_\Omega^2 \equiv \int_\Omega A(\mathbf{p}) A^\star(\mathbf{p}) \frac{d\Phi}{d\mathbf{p}} d\mathbf{p}$$
$$\mathcal{B}_\Omega^2 \equiv \int_\Omega B(\mathbf{p}) B^\star(\mathbf{p}) \frac{d\Phi}{d\mathbf{p}} d\mathbf{p}$$

- Interference described by the cross term:

$$\mathcal{Z}_\Omega^f \equiv \frac{\int_\Omega A(\mathbf{p}) B^\star(\mathbf{p}) \frac{d\Phi}{d\mathbf{p}} d\mathbf{p}}{\mathcal{A}_\Omega \mathcal{B}_\Omega}$$

Z Numerical Integral

- Charm interference parameter $R_{K3\pi}e^{-i\delta_{K3\pi}} \equiv Z$ is a key parameter in this analysis
- We can estimate it using a numerical integral:

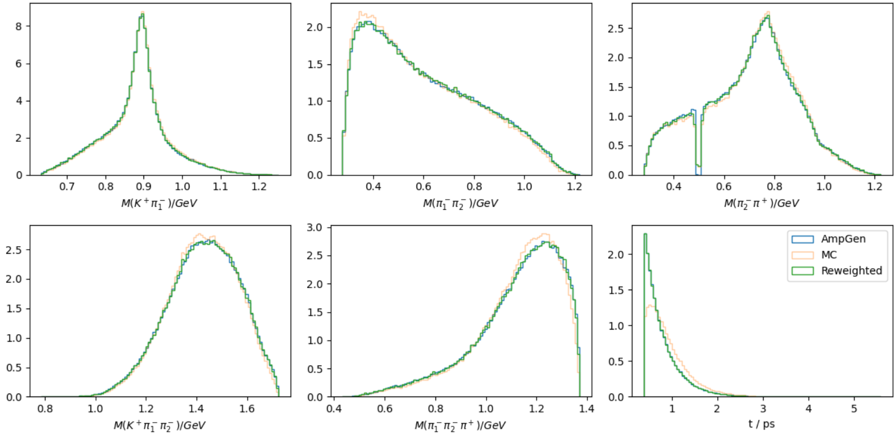
$$Z = \sum_i \frac{A_{CF}(\mathbf{p}_i)A_{DCS}(\mathbf{p}_i)w_i}{N} \quad (1)$$

where w_i is each event's weight and N is normalisation:

$$N = \sqrt{\sum_i A_{CF}(\mathbf{p}_i)A^*_{CF}(\mathbf{p}_i)w_i}\sqrt{\sum_i A_{DCS}(\mathbf{p}_i)A^*_{DCS}(\mathbf{p}_i)w_i} \quad (2)$$

- We can evaluate the amplitudes using AmpGen
- Ideally, we should get the same value of Z from a dataset generated with AmpGen and our reweighted LHCb MC

Combined Reweighter: Alternate parameterisation



Combining distributions I

- $\epsilon(\mathbf{p}, t)$ should be the same for suppressed $D^0 \to K^+ 3\pi$ (DCS) and favoured $\overline{D}^0 \to K^+ 3\pi$ (CF) events
- Our reweighter should be trained on Generator and MC datasets
- MC should be the combined CF + DCS LHCb MC datasets
- Generator should be a combination of CF + DCS generator-level events
- Need to combine CF + DCS generator-level MC in the "right" proportion
- We don't know what this is a priori, but we can calculate it...

Combining distributions II

- We generated $\alpha A^{CF} + \beta A^{DCS}$; we dont know α or β
- We reconstruct $\epsilon\alpha A^{CF} + \epsilon\beta A^{DCS}$

- Introduce some factors $C^{CF} = \alpha \int \epsilon(x)dx$, $C^{DCS} = \beta \int \epsilon(x)dx$
- These can be evaluated with e.g.

$$\alpha \int \epsilon(x)dx = \alpha \int \epsilon(x)\frac{A^{CF}(x)}{A^{CF}(x)}dx = \int \left(\alpha\epsilon(x)A^{CF}(x)\right)\frac{1}{A^{CF}(x)}dx = \sum_{CF\ MC} \frac{1}{A^{CF}(x)} \tag{3}$$

Combining distributions III

- Reminder: factors $C^{CF} = \alpha \int \epsilon(x)dx$, $C^{DCS} = \beta \int \epsilon(x)dx$

- We can write $\epsilon(x) = \frac{\epsilon\alpha A^{CF} + \epsilon\beta A^{DCS}}{\alpha A^{CF} + \beta A^{DCS}} = \frac{\epsilon\alpha A^{CF} + \epsilon\beta A^{DCS}}{C^{CF} A^{CF} + C^{DCS} A^{DCS}} \times \int \epsilon(x)dx$
- $\int \epsilon(x)$ is a constant factor that we can ignore

- i.e. we can find the right numbers (C^{CF}, C^{DCS}) of evts to generate with AmpGen by using the amplitude models A^{CF}, A^{DCS}
- Generate CF and DCS events in the proportion $C^{CF}:C^{DCS}$

- This is really just a general way to add together two LHCb MC samples

$\frac{WS}{RS}$ Ratio

- MC was generated with no D mixing - expect $\frac{WS}{RS}$ counts to be constant with time
- This ratio is what we fit to to measure Z^f_Ω, so getting the right behaviour here is important