Mixing and CP Violation with $D^0 \to K_S^0 \pi^+ \pi^-$

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LHCb

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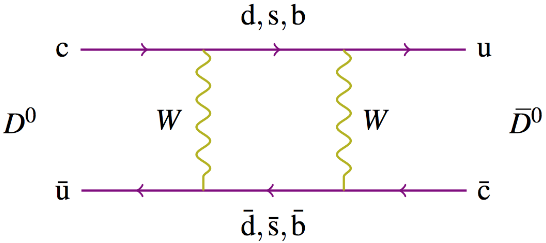
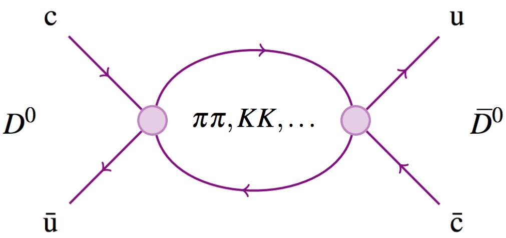
D^0 Mixing





Mass Eigenstates:

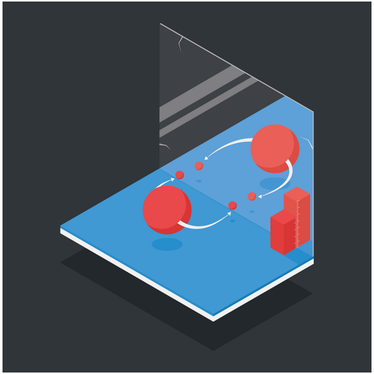
$$\left|D_{1,2}\right\rangle = p\left|D^0\right\rangle \pm q\left|\bar{D}^0\right\rangle$$

Mixing parameters:

$$x \equiv \frac{(m_1 - m_2)}{\Gamma} \qquad y \equiv \frac{(\Gamma_1 - \Gamma_2)}{2\Gamma}$$

CP Violation

- CP violation discovered in charm in 2019 at LHCb
- CPV in charm is predicted to be small in the Standard Model ($\sim \mathbf{10^{-4} - 10^{-3}}$)
- Theoretical prediction has large uncertainties due to strong interactions
- CPV searches in charm complementary to those in kaons and B mesons



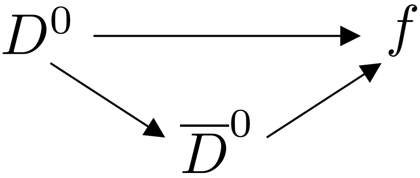
Types of CP Violation

Direct CP Violation:

$$\Gamma(D^0 \rightarrow f) \neq \Gamma(\bar{D}^0 \rightarrow \bar{f}) \text{ or } |A_f| \neq |\bar{A}_{\bar{f}}|$$

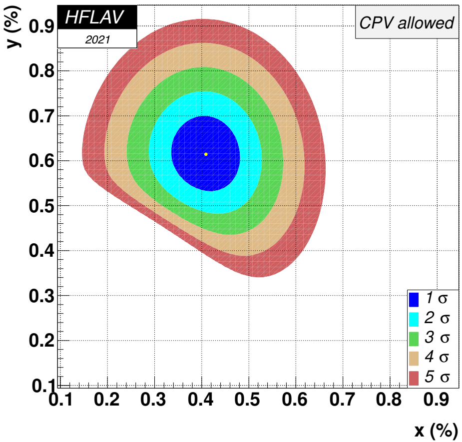
CP Violation in Mixing:

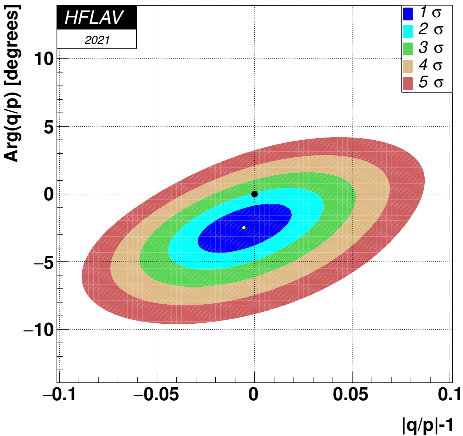
$$\Gamma(D^0 \rightarrow \bar{D}^0) \neq \Gamma(\bar{D}^0 \rightarrow D^0) \text{ or } |q| \neq |p|$$



CP Violation in interference in mixing and decay:

$$\Gamma(D^0 \rightarrow \bar{D}^0 \rightarrow f, t) \neq \Gamma(\bar{D}^0 \rightarrow D^0 \rightarrow f, t) \text{ or } \phi = arg\left(\frac{q\bar{A}_f}{pA_f}\right) \neq 0$$

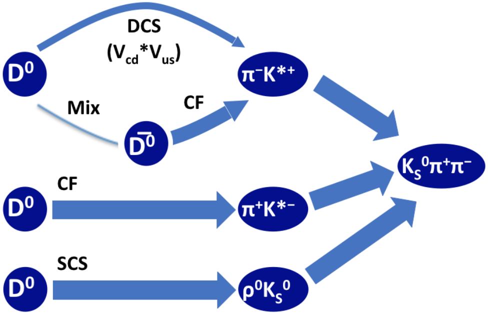


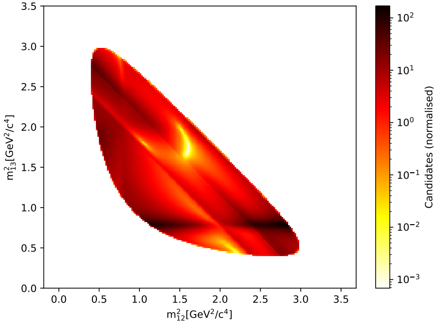


World averages of the mixing and CP-violation parameters 2021, from HFLAV.

Mixing in $D^0 \to K^0_S \pi^+ \pi^-$

- 'Right-sign' (CF) and 'wrong-sign' (DCS or mixed) decay into **same final state**

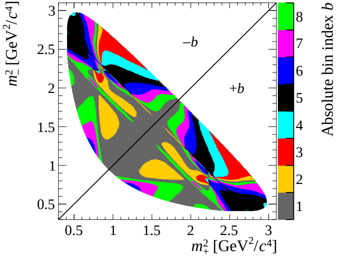




- Offers **direct** access to mixing and CPV parameters x, y, $|q/p|$, ϕ
- Requires **time and phase-space** dependent analysis

Bin-flip Model-independent analysis

- Data is binned in Dalitz coordinates where the binning scheme is chosen to have approximately constant strong-phase differences
- Measure the yield ratio $R^{\pm}_{bj}$ between $-b$ and b in bins of decay time

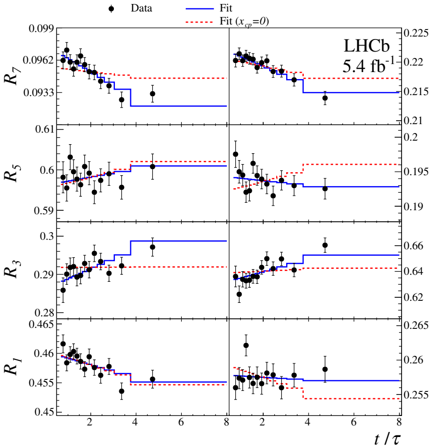


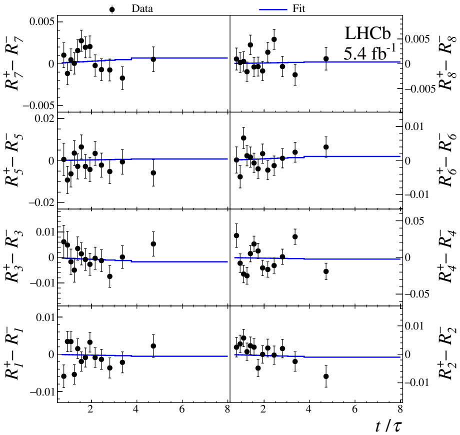
$$R^{\pm}_{bj} \approx \frac{r_b + \frac{1}{4} r_b \left\langle t^2 \right\rangle_j Re(z_{CP}^2 - \Delta z^2) + \frac{1}{4} \left\langle t^2 \right\rangle_j |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \left\langle t \right\rangle_j Re[X_b^* (z_{CP} \pm \Delta z)]}{1 + \frac{1}{4} \left\langle t^2 \right\rangle_j Re(z_{CP}^2 - \Delta z^2) + r_b \frac{1}{4} \left\langle t^2 \right\rangle_j |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \left\langle t \right\rangle_j Re[X_b (z_{CP} \pm \Delta z)]}$$

$$\boldsymbol{x_{CP} \equiv -\mathrm{Im}(z_{CP}) \qquad y_{CP} \equiv -\mathrm{Re}(z_{CP})}$$
$$\boldsymbol{\Delta x \equiv -\mathrm{Im}(\Delta z) \qquad \Delta y \equiv -\mathrm{Re}(\Delta z)}$$

and in the limit of CP symmetry $\boldsymbol{x_{CP} = x}$, $\boldsymbol{y_{CP} = y}$ and $\boldsymbol{\Delta x = \Delta y = 0}$

Bin-flip Results





Ratios R_b^+/R_b^-, the slope shows a sign of mixing, red line is $x = 0$ hypothesis.

$R_b^+ - R_b^-$ any slope shows sign of CP-violation (none seen).

Results

Parameter	Value $[10^{-3}]$	Stat. correlations			Syst. correlations		
		y_{CP}	Δx	Δy	y_{CP}	Δx	Δy
x_{CP}	$3.97 \pm 0.46 \pm 0.29$	0.11	−0.02	−0.01	0.13	0.01	0.01
y_{CP}	$4.59 \pm 1.20 \pm 0.85$		−0.01	−0.05		−0.02	0.01
Δx	$-0.27 \pm 0.18 \pm 0.01$			0.08			0.31
Δy	$0.20 \pm 0.36 \pm 0.13$						

Parameter	Value	95.5% CL interval
x $[10^{-3}]$	$3.98\,^{+0.56}_{-0.54}$	$[2.9, 5.0]$
y $[10^{-3}]$	$4.6\,^{+1.5}_{-1.4}$	$[2.0, 7.5]$
$\lvert q/p \rvert$	0.996 ± 0.052	$[\ \ 0.890, 1.110]$
ϕ	$-0.056\,^{+0.047}_{-0.051}$	$[-0.172, 0.040]$

First observation of a non-zero mass difference ($x > 0$) in neutral charm mesons.

Amplitude formalism

The square of the time-dependent amplitude of the process $D^0 \to K_S^0\pi^+\pi^-$ is given by:

$$|\mathcal{A}_f(t)|^2 = \frac{1}{2}e^{-\Gamma t}\left[\left(|A|^2 - \left|\frac{q}{p}B\right|^2\right)\cos(x\Gamma t) - 2\,\mathrm{Im}\left(AB^*\left[\frac{q}{p}\right]^*\right)\sin(x\Gamma t)\right. \\ \left. + \left(|A|^2 + \left|\frac{q}{p}B\right|^2\right)\cosh(y\Gamma t) - 2\,\mathrm{Re}\left(AB^*\left[\frac{q}{p}\right]^*\right)\sinh(y\Gamma t)\right] \quad (1)$$

as well as a similar equation for $\left|\bar{\mathcal{A}}_f(t)\right|^2$ ($\bar{D}^0 \to K_S^0\pi^+\pi^-$).

$A = \mathcal{A}_f$ and $B = \bar{\mathcal{A}}_f$ are the amplitudes of $D^0(\bar{D}^0) \to f$ and depend on the phase-space defined by the Dalitz variables $m^2(K_s^0\pi^+)$ and $m^2(K_s^0\pi^-)$.

CP Violation

Include CP-violation in the fit by allowing x and y to be different for D^0 and $\bar{D}^0$:
$x = x_{CP} \pm \Delta x$ and $y = y_{CP} \pm \Delta y$ (Bin-flip paper)

$$x_{CP} = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\right] \tag{2}$$

$$\Delta x = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right] \tag{3}$$

$$y_{CP} = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\right] \tag{4}$$

$$\Delta y = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right] \tag{5}$$

The overall PDF is given by:

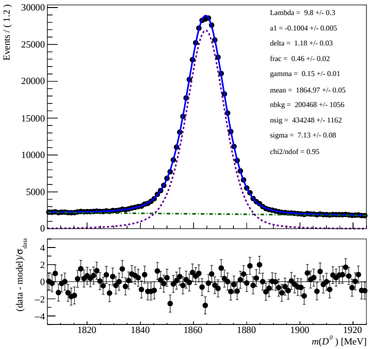
$$\begin{aligned}\mathcal{P}(t, m_+^2, m_-^2, p_{sig}, tag, \vec{\alpha}) = p_{sig} &\Big[(1-\omega_{muontag})\mathcal{P}_{sig}(t, m_+^2, m_-^2, tag, \vec{\alpha}) \\ &+ \omega_{muontag}\mathcal{P}_{mt}(t, m_+^2, m_-^2, tag, \vec{\alpha})\Big] \\ &+ (1-p_{sig})\mathcal{P}_{bkg}(t, m_+^2, m_-^2)\end{aligned} \quad (6)$$

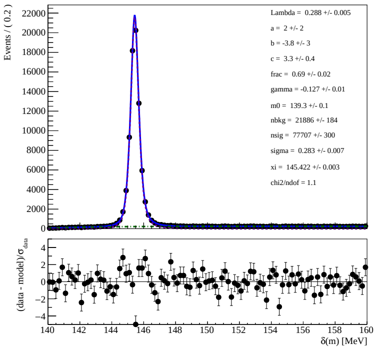
p_{sig} is the per-candidate signal probability, tag is the flavour tag, $\vec{\alpha}$ is a vector of the fit parameters, $\omega_{muontag}$ is the mistag fraction. The decay-time is given by t and the Dalitz variables are m_+^2 and m_-^2. The signal PDF is given by:

$$\mathcal{P}_{sig}(t, m_+^2, m_-^2, tag, \vec{\alpha}) = \left[\left(|\, \mathcal{A}_f(t', m_+^2, m_-^2)\,|^2\, \epsilon(t')\right) \otimes R(t, t', \mu_t, \sigma_t)\right] \epsilon(m_+^2, m_-^2) \quad (7)$$

where $\mathcal{A}_f(t', m_+^2, m_-^2)$ is the time-dependent amplitude model, $\epsilon(t')$ and $\epsilon(m_+^2, m_-^2)$ are the decay-time and phase-space efficiencies and $R(t, t', \mu_t, \sigma_t)$ is the decay-time resolution.

Data Selection





- Turbo trigger, offline pre-selection and data-trained MVA.
- 1.36M (2.80M) single-tagged LL (DD) signal candidates, and 0.22M (0.48M) double-tagged LL (DD) signal candidates, summing to $\sim$ 4.9M signal candidates in total.

Amplitude formalism

- The amplitude for $D \to abc$ through an intermediate resonance $r \to ab$ is given by:

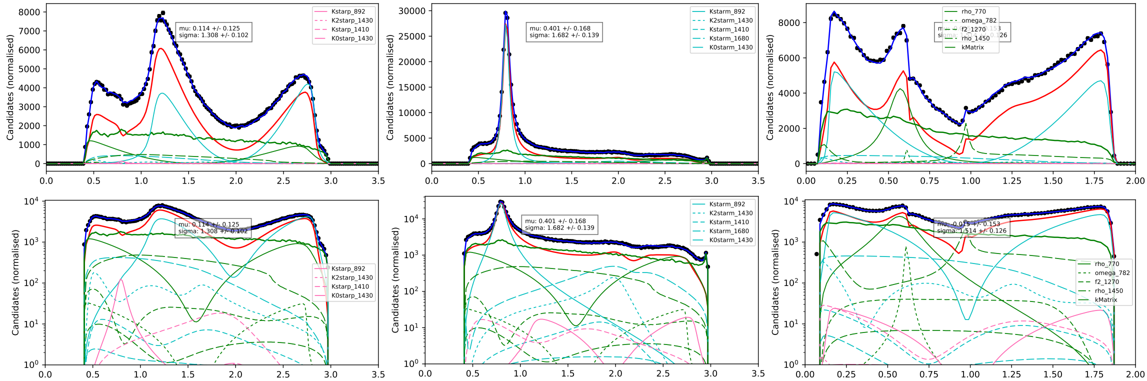
$$A_r(m_+^2, m_-^2) = F_D^{(L)}(q, q_0) \times F_r^{(L)}(p, p_0) \times Z_L(\Omega) \times \mathcal{T}_r(m) \tag{8}$$

- where the form factors $F_D^{(L)}$ and $F_r^{(L)}$ describe the decay $D \to rc$ and $r \to ab$,
- L is the orbital angular momentum between a and c,
- p and q are the momenta of c and a in the resonance rest frame,
- $Z_L(\Omega)$ describes the angular distribution of the final state particles,
- $\mathcal{T}_r$ is the dynamical function describing the resonance r

$$\mathcal{T}(D^0 \to K_S^0 \pi^+ \pi^-) = c_K \mathcal{T}_{\pi\pi} + c_L \mathcal{T}_{K\pi} + \sum_r c_r \mathcal{T}_r \tag{9}$$

- $\pi\pi$ S-wave described by K-matrix ($\mathcal{T}_{\pi\pi}$), $K\pi$ S-wave by LASS ($\mathcal{T}_{K\pi}$)

Time-integrated Fit



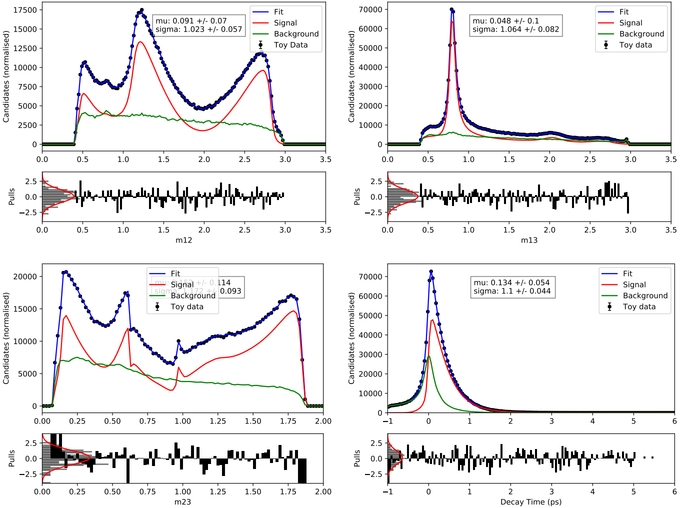
m^2_{12} [GeV] m^2_{13} [GeV] m^2_{23} [GeV]

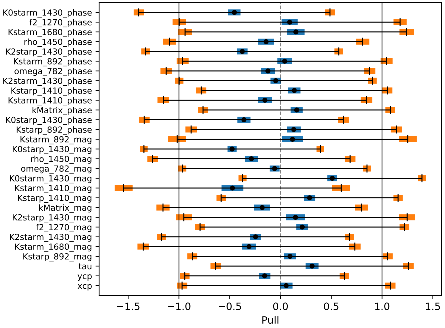
Fit projections. Single-tagged K^0_S (LL) 2016 sample. Fit $\chi^2/\text{ndof} = 1.18$ for 6763 ndof.

Toy studies

- Toy psuedoexperiments are used to validate the fitter, assess potential biases and estimate statistical precision.
- Amplitude model taken from initial time-integrated fit
- Realistic background and detector effects are included
- Decay-time is generated using the PDG value of D^0 lifetime
- Toys are generated with world average values of x and y $\pm 1\sigma$

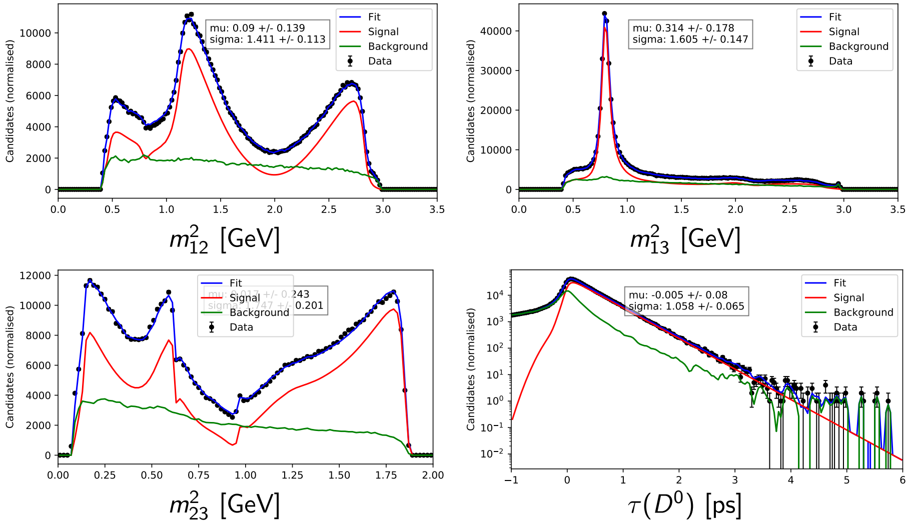
Toy Studies





Mean and width of pulls of fit parameters (above). Fit projections for one toy (left).

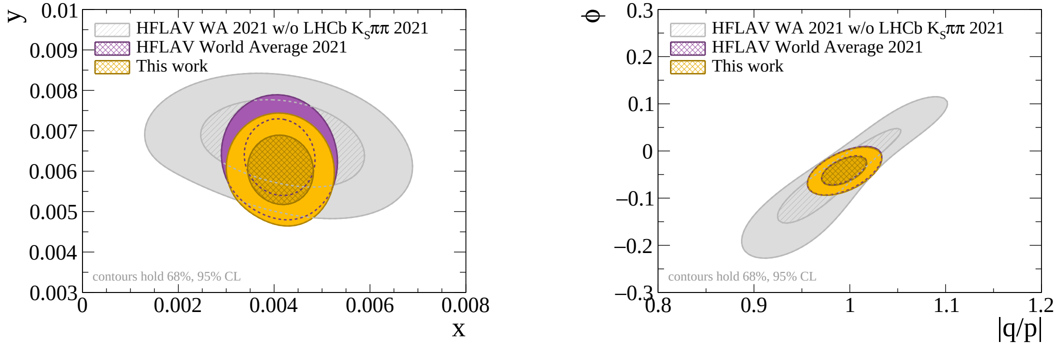
Time-dependent Fit (Single-tagged K_S^0 (LL) 2016)



Systematic Uncertainties

- Model-dependent systematic uncertainties are related to the choice of amplitude model
- Largest experimental systematic uncertainty is due to the background PDF
- Systematics are evaluated by resampling and rerunning fit to data or with toys
- Numbers are preliminary, some need to be updated with the simultaneous fit
- So far they are under control and comparable but below the statistical uncertainty

World Averages



This figure shows world average not including the bin-flip result (grey), current world averages on the mixing and CP-violation parameters (purple) including the bin-flip result, and this work (yellow)*.

* This work refers to the model-dependent amplitude analysis which is currently WIP and blind, here central values assumed are those of the bin-flip result.

Summary

- Time-dependent amplitude analysis of $D^0 \to K^0_S \pi^+ \pi^-$
- Mixing parameters x and y can be extracted from a time and phase-space dependent fit of the amplitude model to data
- Fit inputs include:
  - Signal model from Belle and BaBar
  - Decay-time acceptance and resolution from simulation
  - Phase-space acceptance from simulation
  - Background from data driven approach
  - Mistag from $D \to K\pi$ sample
- Blinded time-dependent fit to data
- Toy studies
- Systematic uncertainties and cross-checks

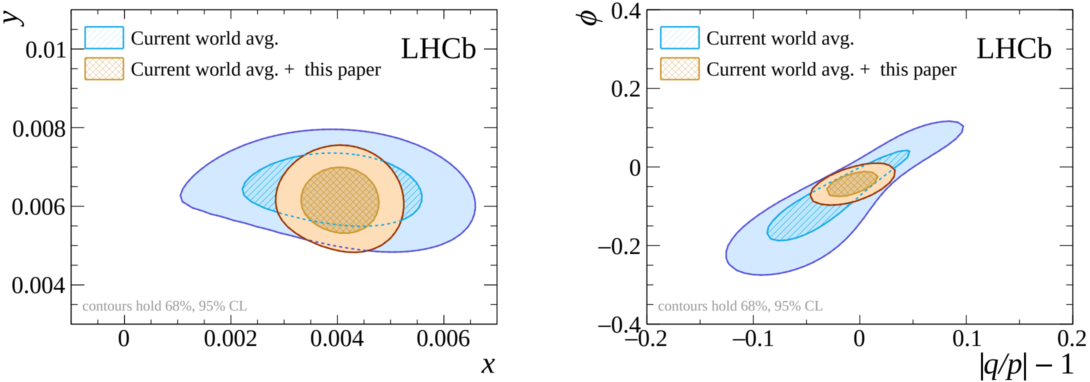
Back Up

Systematic uncertainties

Source	x_{CP}	y_{CP}	Δx	Δy
Reconstruction and selection	0.199	0.757	0.009	0.044
Secondary charm decays	0.208	0.154	0.001	0.002
Detection asymmetry	0.000	0.001	0.004	0.102
Mass-fit model	0.045	0.361	0.003	0.009
Total systematic uncertainty	0.291	0.852	0.010	0.110

Strong phase inputs	0.23	0.66	0.02	0.04
Detection asymmetry inputs	0.00	0.00	0.04	0.08
Statistical (w/o inputs)	0.40	1.00	0.18	0.35
Total statistical uncertainty	0.46	1.20	0.18	0.36

Uncertainties are in units of 10^{-3}.



World averages of the mixing and CPV parameters showing the impact of this result. Pre-2021 WA in blue and including this result in orange.

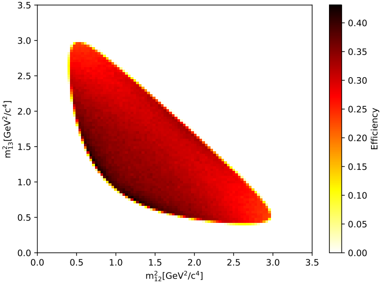
Paper: PhysRevLett.127.111801

Analysis overview

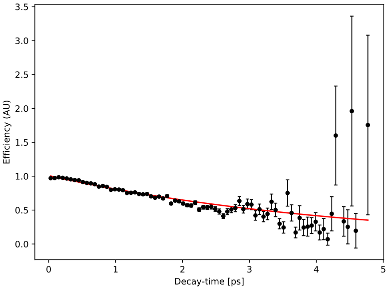
- Uses D mesons from semi-leptonic B meson decays:
  - $\overline{B} \to D^0(\to K_S^0\pi^+\pi^-)\mu^- X$ (single-tagged)
  - $\overline{B} \to D^{*-}(\to D^0(\to K_S^0\pi^+\pi^-)\mu^-)\pi^+ X$ (double-tagged)
- Model developed within GooFit and fits run on GPUs
- Initial amplitude model based on Belle and BaBar model (Paper)
- The inputs to the mixing fit consist of:
  - Signal model
  - Background from data-driven approach
  - Phase-space acceptance derived from Monte Carlo
  - Decay-time acceptance and resolution derived from Monte Carlo
  - Mistag fraction derived from two-body $D^0 \to K\pi$ sample

Simulation

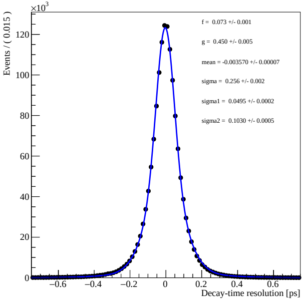
- Monte Carlo reweighted to match data using multi-dimensional kinematic reweighting
- MC is used for phase-space acceptance, decay-time acceptance and decay-time resolution



Phase-space acceptance

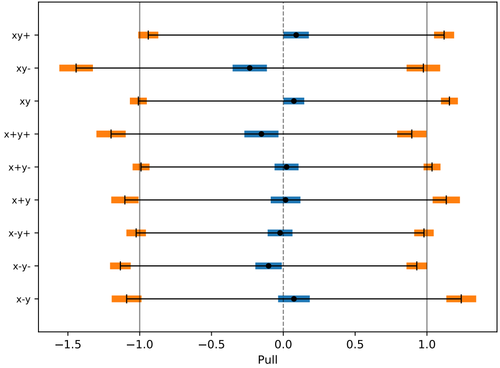


Decay-time acceptance

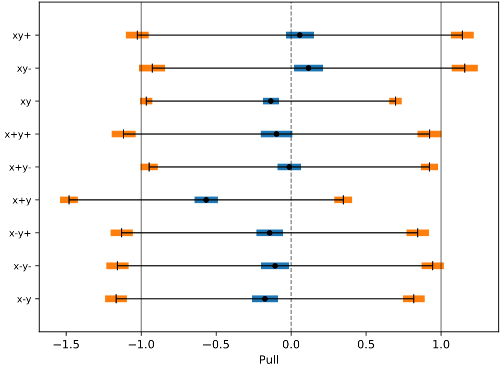


Decay-time resolution

Toy Studies

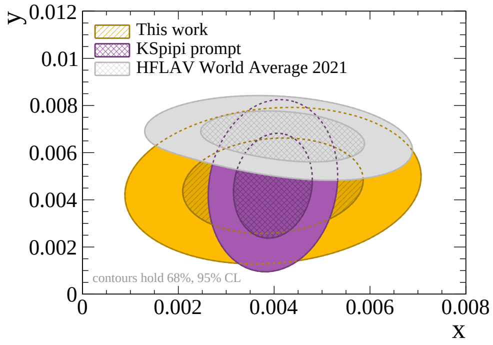


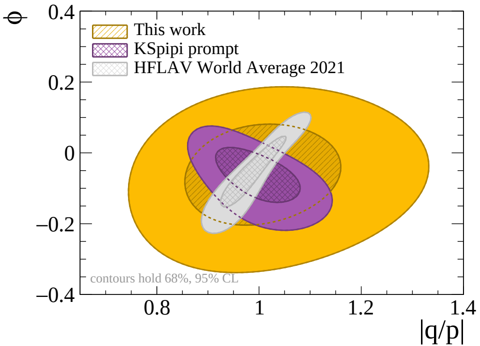
(a) Pull x



(b) Pull y

Summary of the results of running fits over ensembles of toy pseudo-experiments, generated at a range of different mixing parameter values covering the world average values $\pm 1\sigma$.



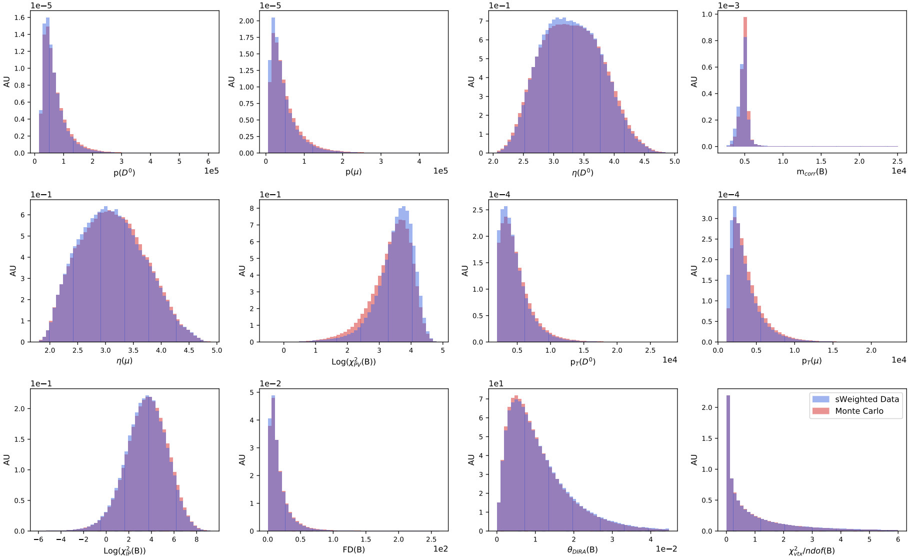


World averages on the mixing and CP-violation parameters without the bin-flip result (grey), the 2021 bin-flip result (purple) and this work (yellow)*.

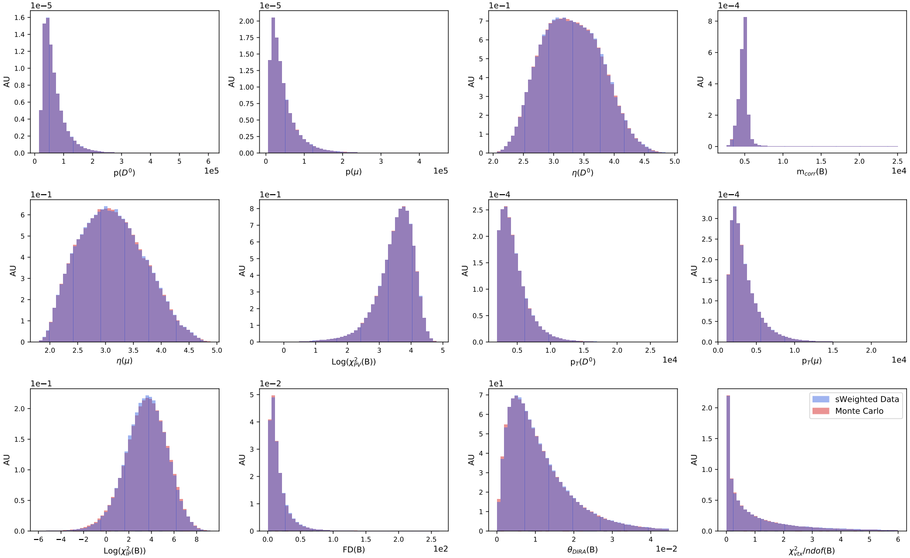
* Same assumptions as previous slide.

Monte Carlo Reweighting

- Inputs needed for the mixing fit model from simulation: Phase-space acceptance, decay-time resolution and acceptance.
- Monte Carlo samples: Event types 12875523 and 11876125 cocktail of neutral and charged B mesons (phase-space MC)
- MC reweighted using multidimensional kinematic reweighter GBReweighter from hep_ml

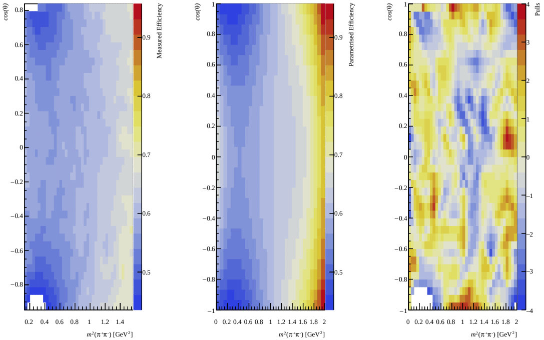


Original distributions. Single-tagged K_S^0 (LL) 2016 sample.

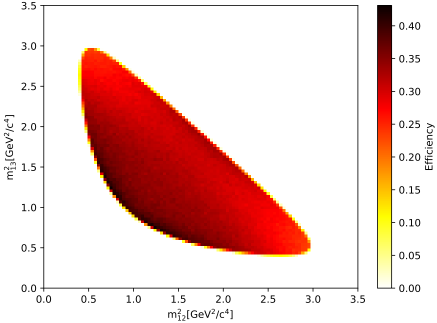


After reweighting. Single-tagged K_S^0 (LL) 2016 sample.

Phase-space Acceptance



Phase-space acceptance. Single-tag K_S^0 (LL) 2016. MC (left), model (middle), pulls (right).

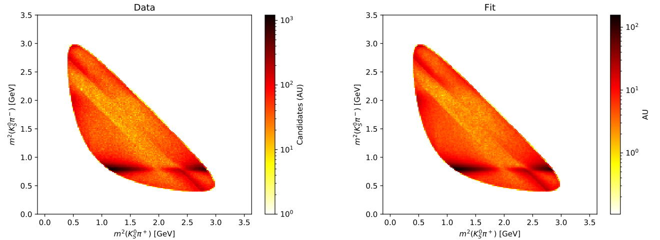


Regular Dalitz coordinates.

Time-integrated fit

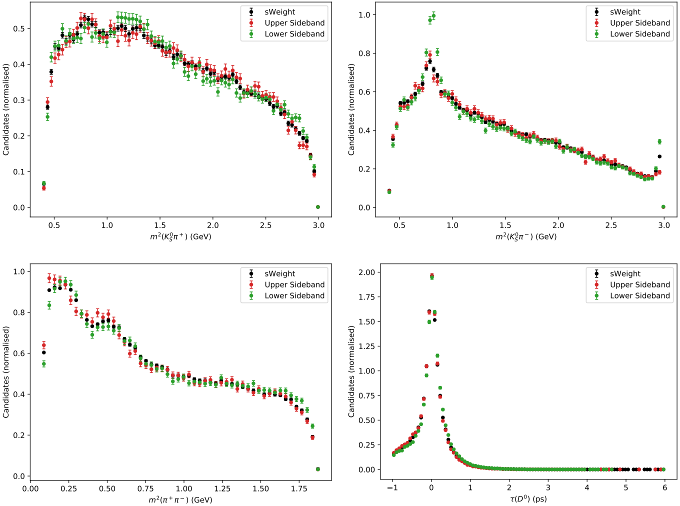
Resonance	Fit Fraction (%)
$\rho(770)$	18.40
$\omega(782)$	0.44
$f_2(1270)$	0.87
$\rho(1450)$	0.15
$K^*(892)^-$	56.05
$K_2^*(1430)^-$	2.01
$K^*(1410)^-$	0.19
$K^*(1680)^-$	$<$0.1
$K^*(892)^+$	0.26
$K_2^*(1430)^+$	$<$0.1
$K^*(1410)^+$	0.26
$\pi^+\pi^-$ S-wave	11.91
$K_0^*(1430)^-$	8.21
$K_0^*(1430)^+$	$<$0.1

Fit fractions. Single-tag K_S^0 (LL) 2016.

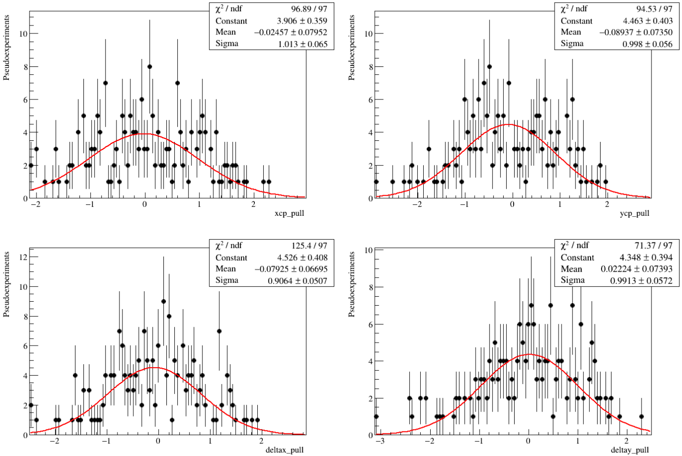


Dalitz plots: Data (left), fit model (right). Single-tagged K_S^0 (LL) 2016 sample.

Combinatorial background model



Toy studies with CP violation



Pulls from the ensemble of pseudoexperiments including CP violation allowing Δx and Δy to float.