IOP HEPP & APP Annual Conference

MSHT Approximate N³LO Parton Distribution Functions

In the pursuit of theoretical uncertainties…

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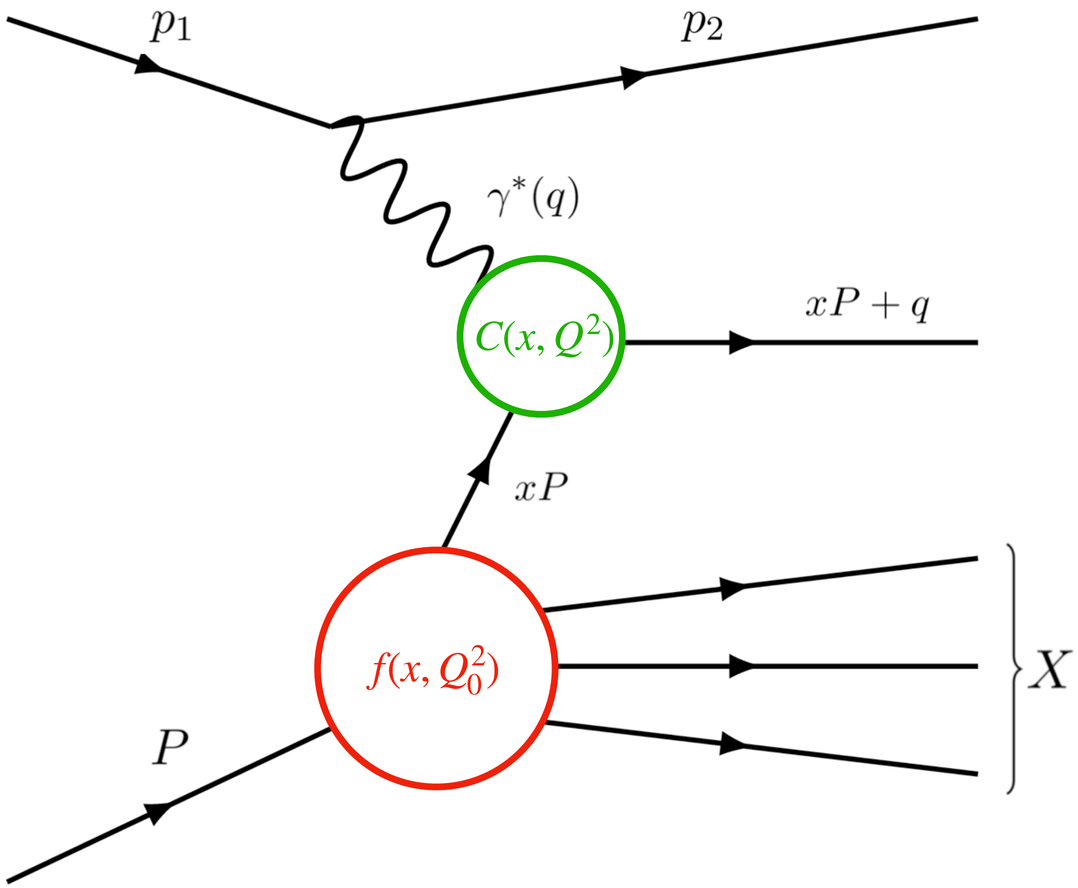
A bit of revision…

- PDFs - **probability** of a parton fluctuating out of proton.
- Coefficient function - **perturbatively** calculated.

$$C(x,Q^2) = C^{(0)}(x,Q^2) + \alpha_s C^{(1)}(x,Q^2) + \alpha_s^2 C^{(2)}(x,Q^2) + \alpha_s^3 C^{(3)}(x,Q^2) + \ldots$$

- PDFs are **determined from experiment** using complex parameterisations.



- ‘Global’ fit using many different data sets and **processes**.

A bit of revision…

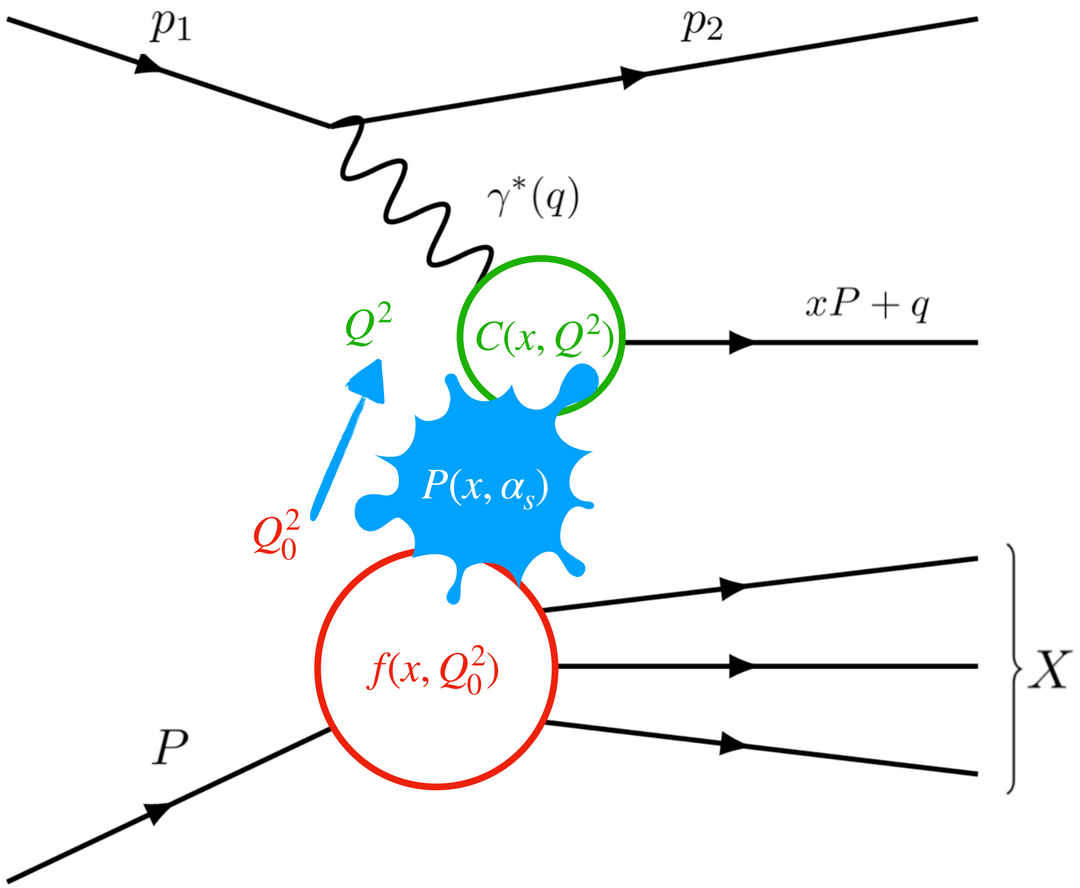
- Scale dependence of PDFs is **also calculable** in QCD perturbation theory!

$$\mu^2 \frac{d}{d\mu^2} f(x, \mu^2) = P(x, \alpha_s(\mu^2)) \otimes f(x, \mu^2)$$

$$P(x, \alpha_s) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x) + \alpha_s^4 P^{(3)}(x) + \ldots$$

where $P(x, \alpha_s)$ are the **splitting functions.**

- PDFs parameterised at a starting scale Q_0^2 and **evolved** to a desired scale Q^2.

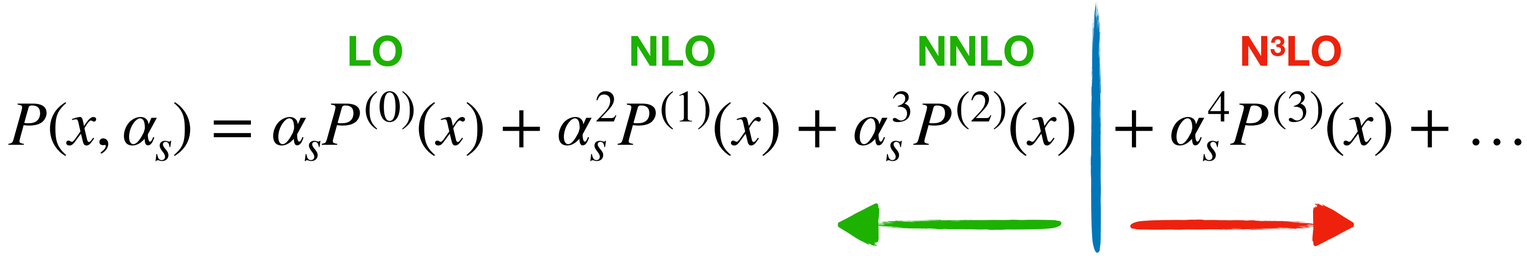


Takeaway: Perturbatively calculable quantities are essential **ingredients** for PDF **determination** (and making predictions using PDFs).

What is a theoretical uncertainty?

And also… why do we care?

- Leading source from **Missing Higher Orders** in perturbation theory.



- Current knowledge is up to NNLO, with **higher orders unknown**.
- Potentially **large corrections** hiding in higher orders beyond **theory truncation**.

What do we know?

…and what don't we know?

$$f(x \to 0) = \frac{C_A^3}{3\pi^4}\left(\frac{82}{81} + 2\zeta_3\right)\frac{1}{2}\frac{\ln^2 1/x}{x}$$

- Some knowledge of **leading terms** in the $x \to 0$ regime.
- Some **numerical constraints** (Low-integer Mellin moments).
- **Intuition** from lower orders/expectations from perturbation theory.

$$\mathscr{M}[f(x)](N) = \int_0^1 dx x^{N-1} f(x)$$

- Can attempt to **parameterise** the N³LO functions.

Splitting Functions up to N³LO

…approximately

- With N_m constraints, we employ:

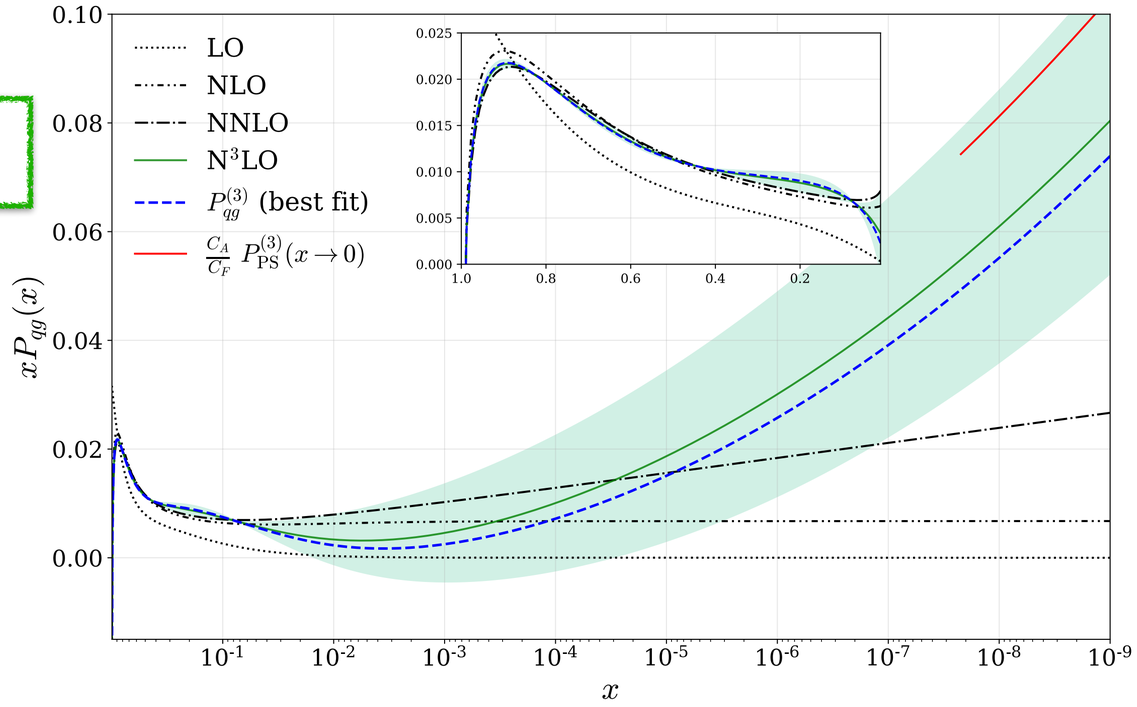
contains any known information.

$$P(x) = \sum_{i=1}^{N_m} A_i f_i(x) + f_e(x)$$

- Choose a set of **relevant functions** f_i and solve for A_i

- To allow control of this function, introduce a **degree of freedom** a.

$$f_e(x) \to f_e(x, a)$$



- a interpreted as a **nuisance parameter** allowed to vary in a PDF fit.

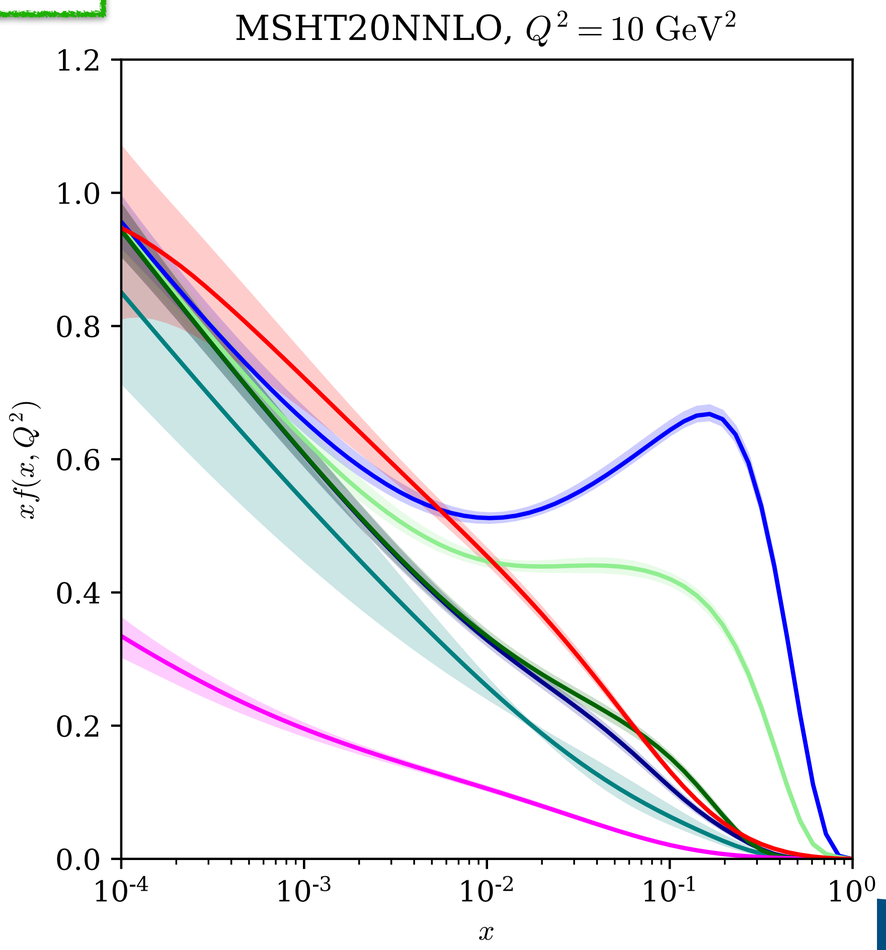
MSHT N³LO PDFs

NNLO: $\chi^2 \simeq 5121 \ / \ 4363$

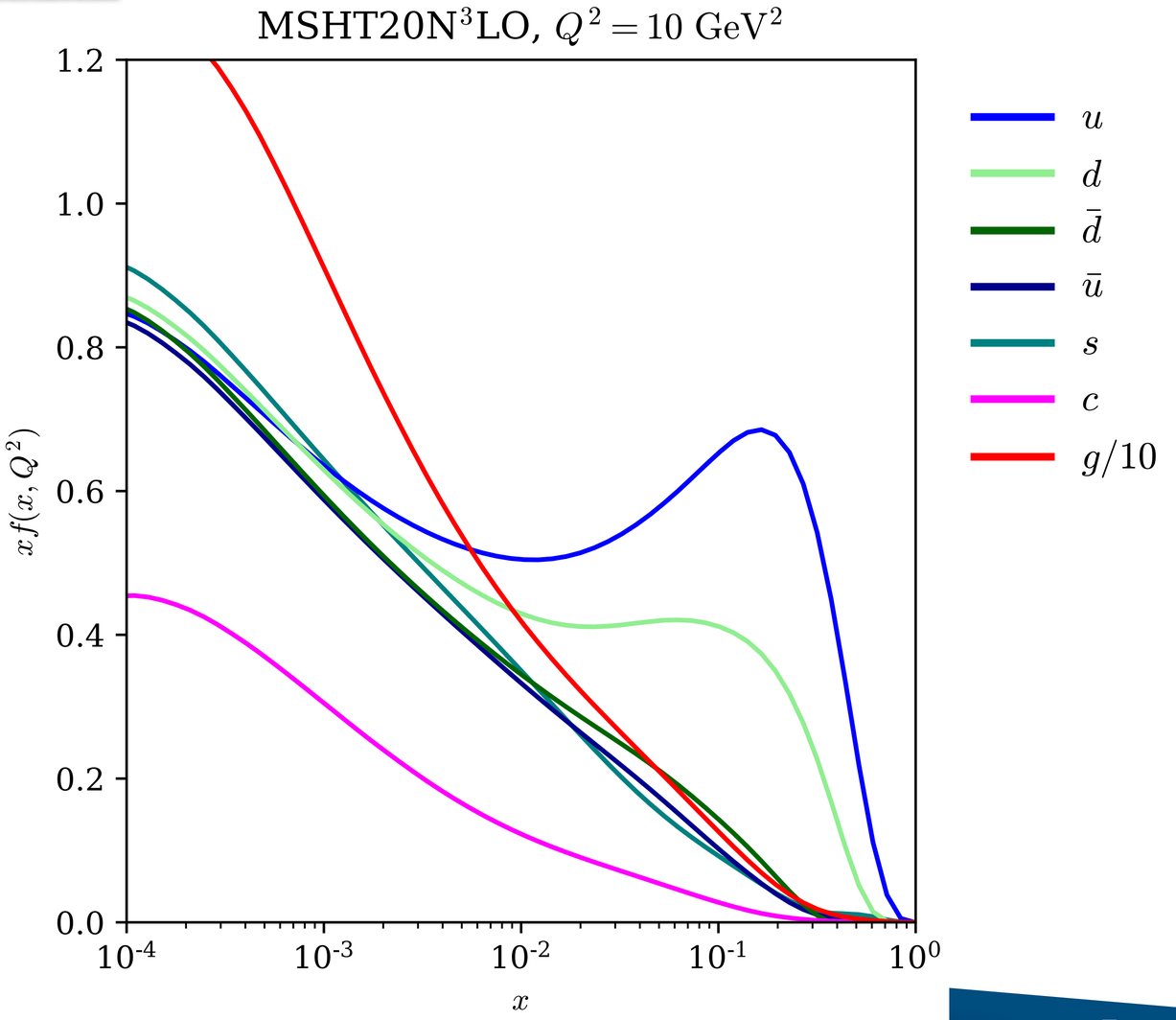
N³LO: $\chi^2 \simeq 4943 \ / \ 4363$

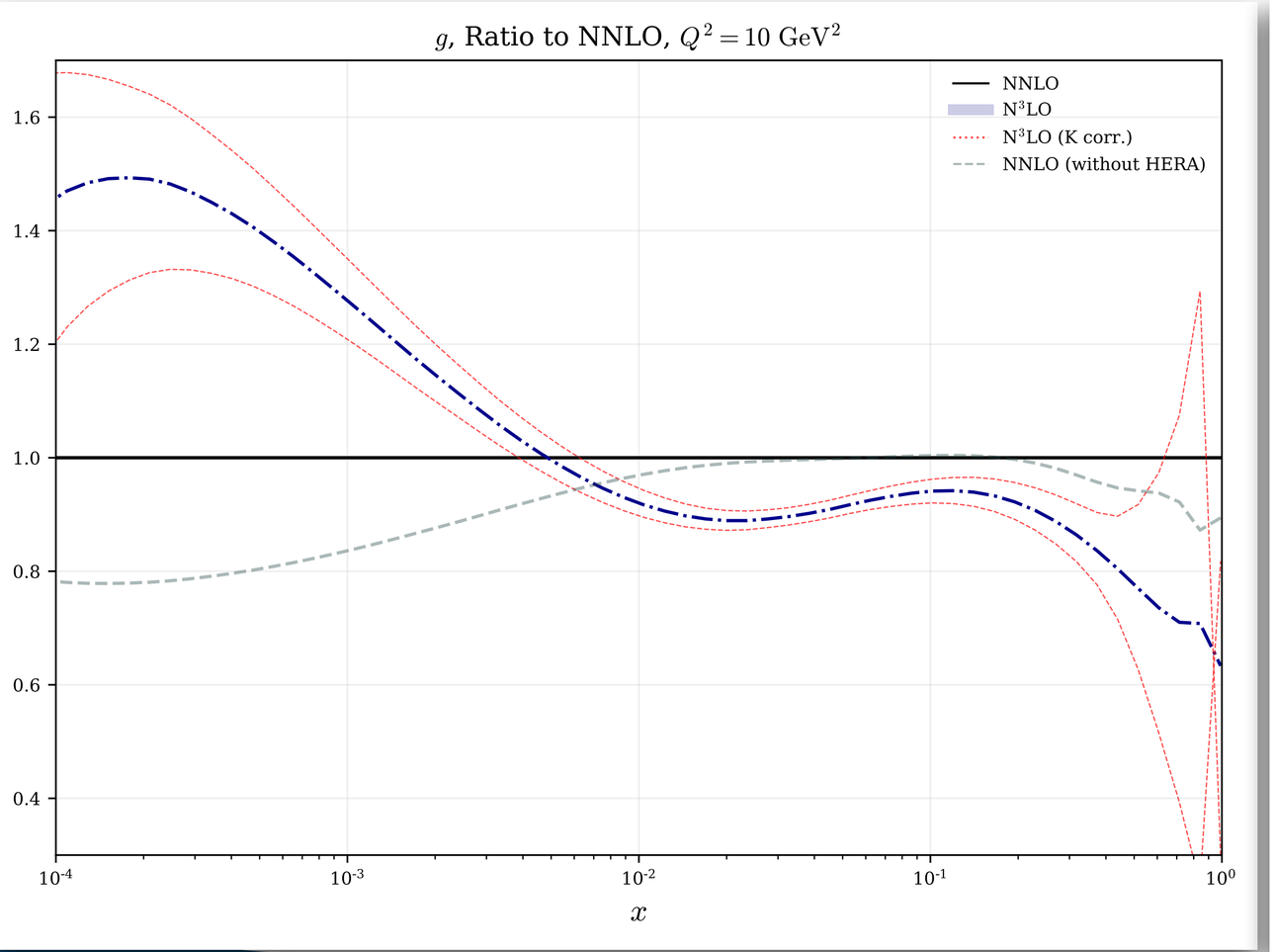
BREAKING NEWS

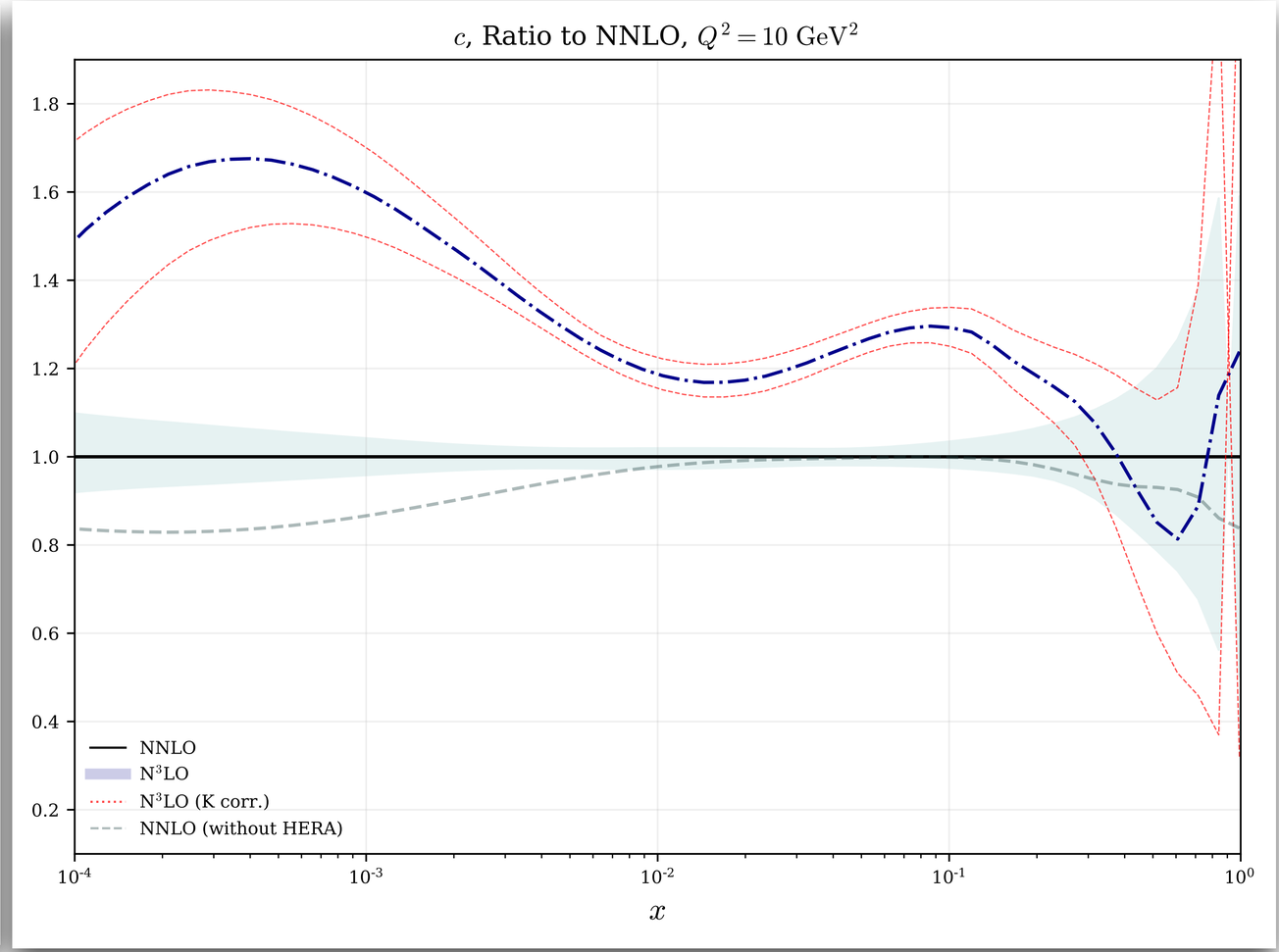
NNLO

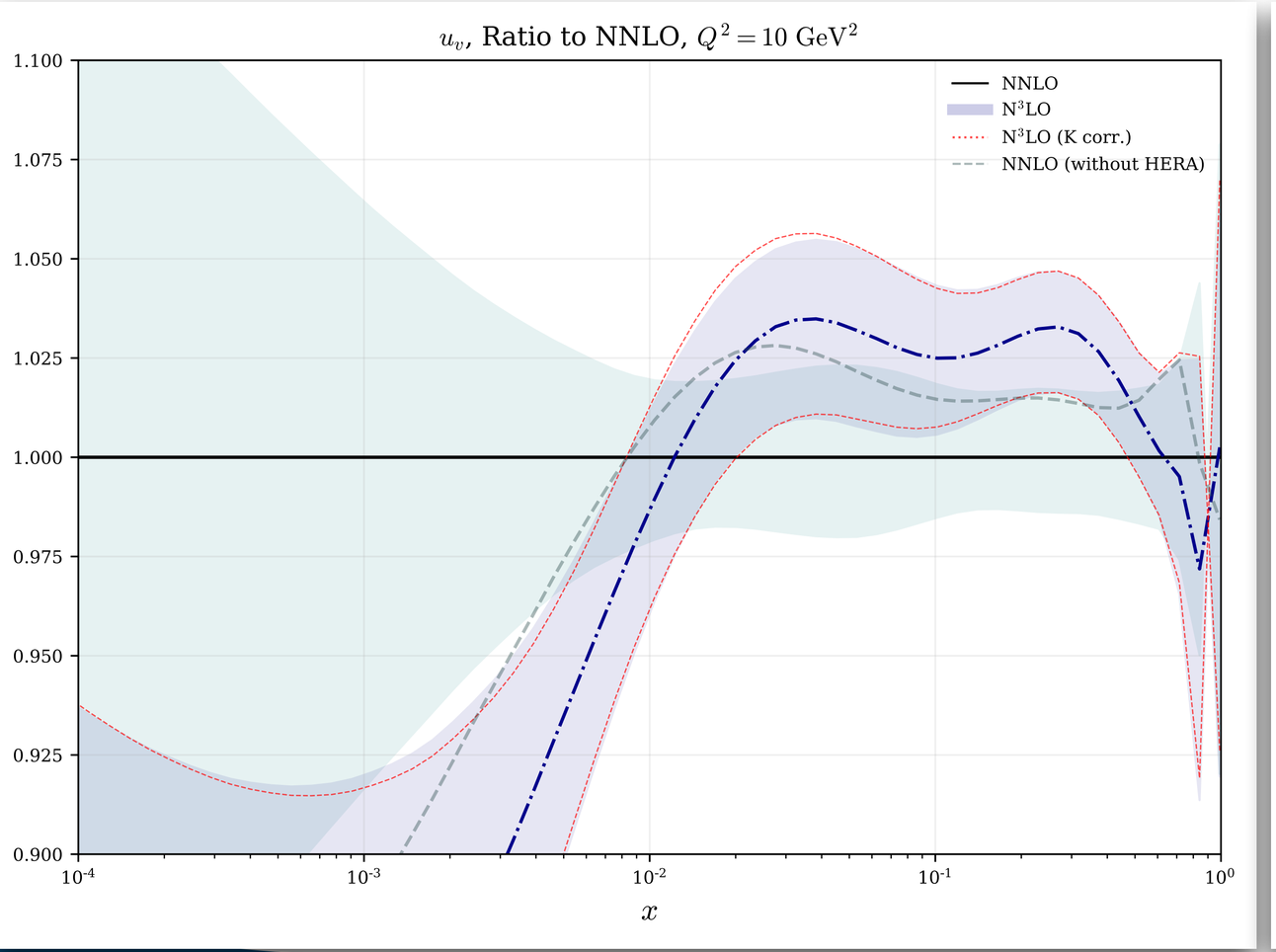


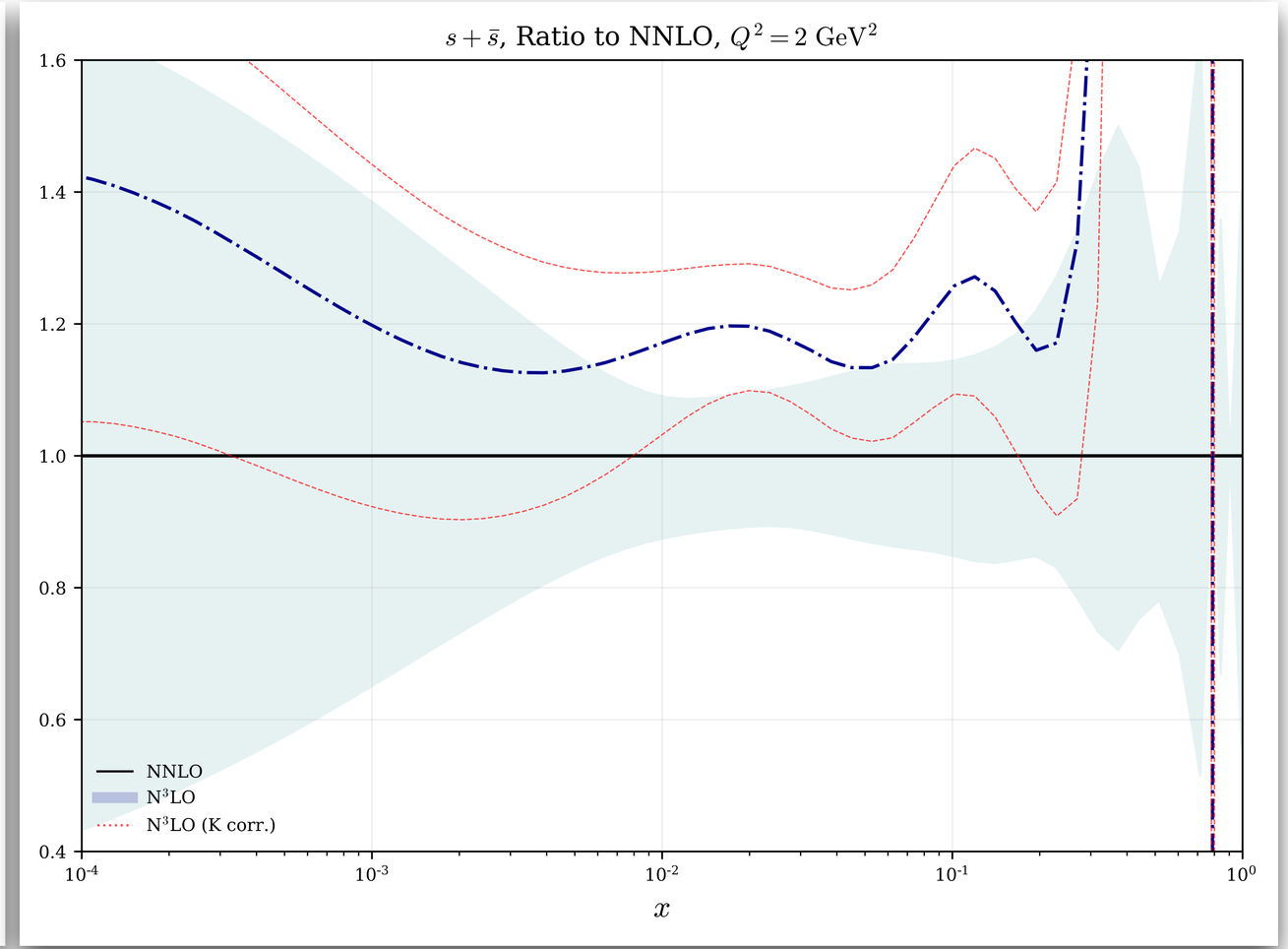
N³LO




g, Ratio to NNLO, $Q^2 = 10\ \mathrm{GeV}^2$
NNLO
N^{3}LO
N^{3}LO (K corr.)
NNLO (without HERA)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x


c, Ratio to NNLO, $Q^2 = 10\ \mathrm{GeV}^2$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
NNLO
N^{3}LO
N^{3}LO (K corr.)
NNLO (without HERA)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x

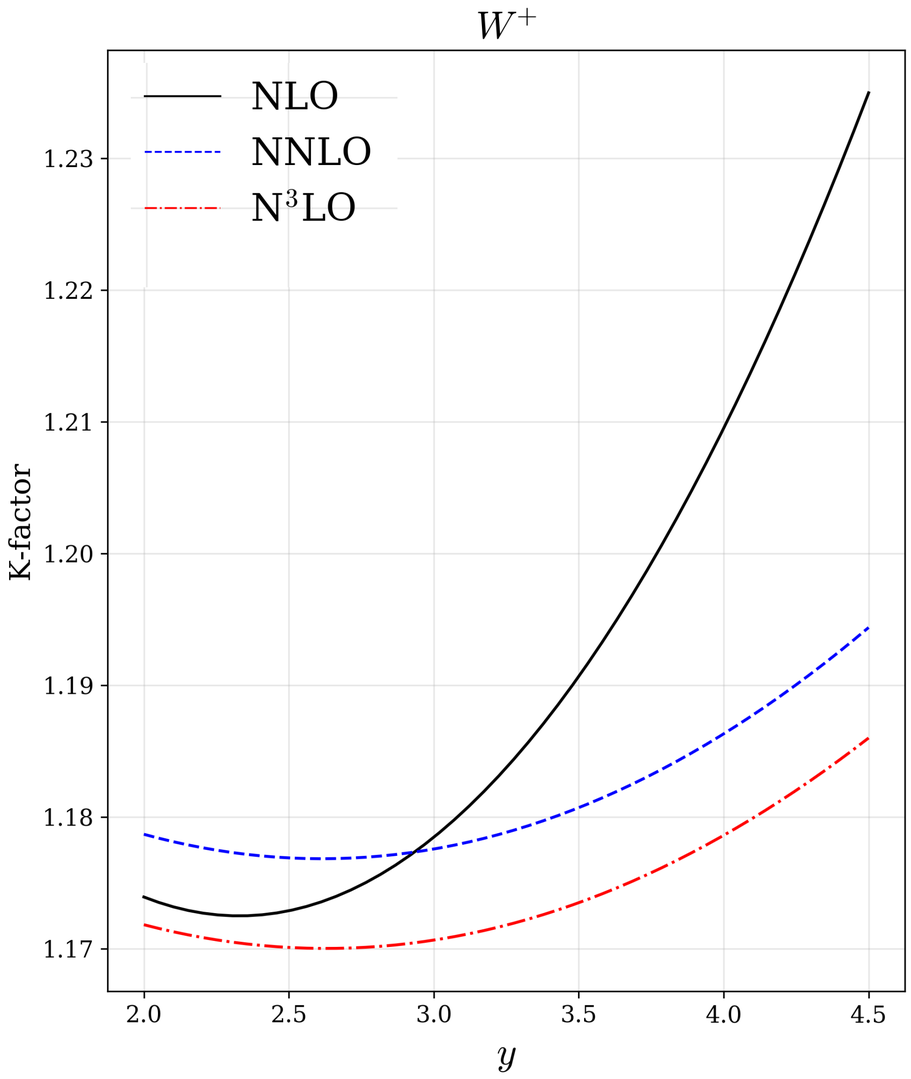

u_v, Ratio to NNLO, $Q^2 = 10$ GeV^2
NNLO
N^{3}LO
N^{3}LO (K corr.)
NNLO (without HERA)
1.100
1.075
1.050
1.025
1.000
0.975
0.950
0.925
0.900
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x


$s + \bar{s}$, Ratio to NNLO, $Q^2 = 2$ GeV^2
1.6
1.4
1.2
1.0
0.8
0.6
0.4
NNLO
N^{3}LO
N^{3}LO (K corr.)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x

N³LO Drell-Yan

(K-factors up to N³LO)

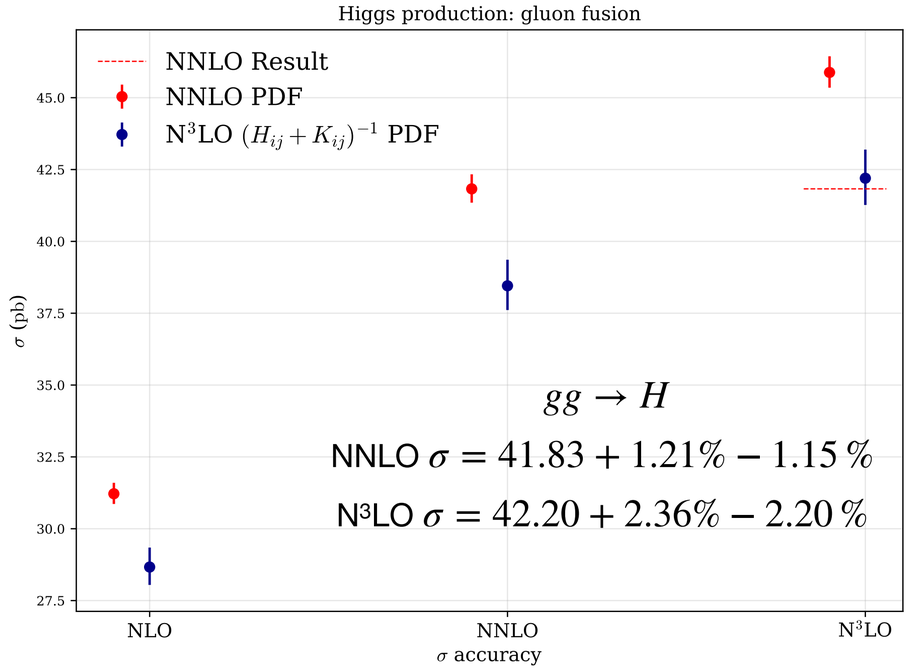
- K-factors transform the hard cross section between orders.
- Predict a **2% decrease** in the DY K-factors from NNLO. arXiv: 2107.09085
- In agreement with **recent results** found using NNLO PDFs with N³LO cross section.

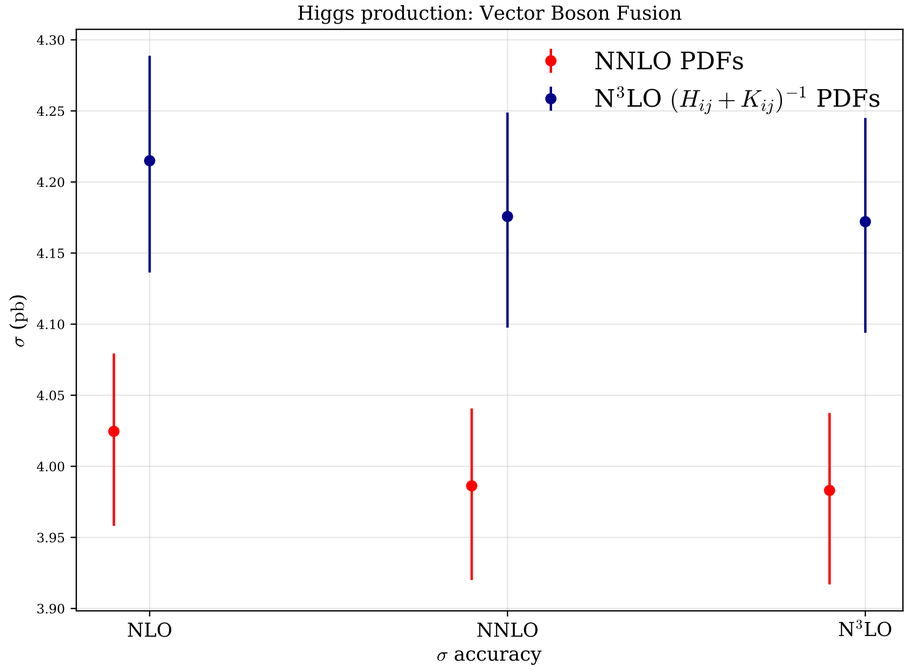


Higgs Predictions

For gluon fusion and Vector Boson Fusion (VBF)

- Good agreement between NNLO and N³LO for **gluon fusion** (top).
- Cancellation between N³LO cross section and PDFs **not guaranteed**.
- **Less cancellation** for VBF (bottom).
- However **variation between orders** is smaller for VBF σ.





VBF

NNLO $\sigma = 3.99 + 1.37\% - 1.66\,\%$

N³LO $\sigma = 4.17 + 1.75\% - 1.87\,\%$

Summary

- **Approximate N^{3}LO PDFs** are on their way.
- Provide an intuitive and controllable way to **include theoretical uncertainties** into PDFs.
- Preliminary results show **good agreement** with current N^{3}LO results.
- Paper **near to completion** (and hopefully thesis soon afterwards).

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