A Quasi Model Independent method to measure the relative strong phase in $D^0 \to K_S^0 \pi^+ \pi^-$ decays

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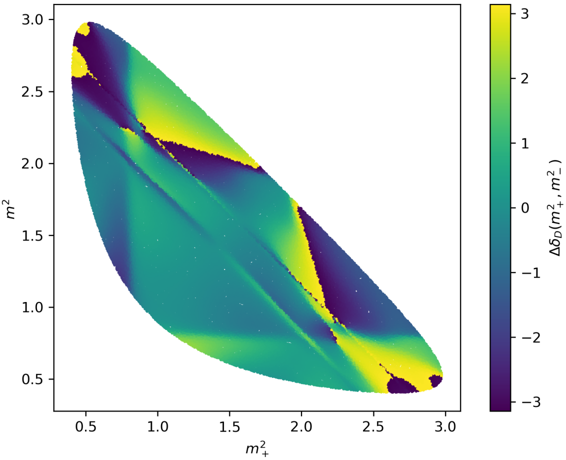
Model Dependent measurement of $\Delta\delta_D$

- $D^0(\bar{D}^0) \to K^0_S\pi^+\pi^-$ has 2 degrees of freedom - typically describe phasespace with the 'Dalitz variables' $m^2_\pm = (p_{K^0_S} + p_{\pi^\pm})^2$.
- We use the 'isobar' model to describe the complex valued amplitude of A_{D^0}:

$$A_{D^0}(\Phi) = \sum_r a_r A_r(\Phi) + \sum_{\mathrm{non-res}} A_{\mathrm{non-res}}(\Phi)$$

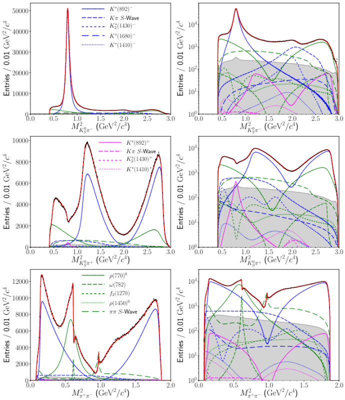
  with couplings a_r to every resonance.
- If we assume no CP violation then $A_{\bar{D}^0}(m^2_+, m^2_-) = A^*_{D^0}(m^2_-, m^2_+)$.

Strong Phase $\Delta\delta_D(\Phi)$ in $K^0_S\pi^+\pi^-$



Belle-BaBar 2018

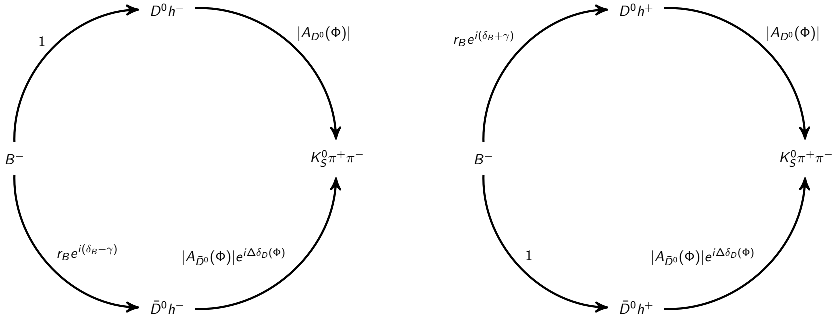
Belle and BaBar (https://arxiv.org/abs/1804.06153v1), used 1.2M signal $D^0 \to K_S^0\pi^+\pi^-$ events from $B\bar{B}$ pairs.



Resonance	Amplitude	Phase (deg)	Fit Fraction (%)
$K_S^0\rho(770)^0$	1 (fixed)	0 (fixed)	20.4
$K_S^0\omega(782)$	0.0388 ± 0.0005	120.7 ± 0.7	0.5
$K_S^0 f_2(1270)$	1.43 ± 0.03	-36.3 ± 1.1	0.8
$K_S^0\rho(1450)^0$	2.85 ± 0.10	102.1 ± 1.9	0.6
$K^*(892)^-\pi^+$	1.720 ± 0.006	136.8 ± 0.2	59.9
$K_2^*(1430)^-\pi^+$	1.27 ± 0.02	-44.1 ± 0.8	1.3
$K^*(1680)^-\pi^+$	3.31 ± 0.20	-118.2 ± 3.1	0.5
$K^*(1410)^-\pi^+$	0.29 ± 0.03	99.4 ± 5.5	0.1
$K^*(892)^+\pi^-$	0.164 ± 0.003	-42.2 ± 0.9	0.6
$K_2^*(1430)^+\pi^-$	0.10 ± 0.01	-89.6 ± 7.6	< 0.1
$K^*(1410)^+\pi^-$	0.21 ± 0.02	150.2 ± 5.3	< 0.1
$\pi^+\pi^-$ S-wave Parameters			10.0
β_1	8.5 ± 0.5	68.5 ± 3.4	
β_2	12.2 ± 0.3	24.0 ± 1.4	
β_3	29.2 ± 1.6	-0.1 ± 2.5	
β_4	10.8 ± 0.5	-51.9 ± 2.4	
f_{11}^{prod}	8.0 ± 0.4	-126.0 ± 2.5	
f_{12}^{prod}	26.3 ± 1.6	-152.3 ± 3.0	
f_{13}^{prod}	33.0 ± 1.8	-93.2 ± 3.1	
f_{14}^{prod}	26.2 ± 1.3	-121.4 ± 2.7	
s_0^{prod}	-0.07 (fixed)		
$K\pi$ S-wave Parameters			
$K_0^*(1430)^-\pi^+$	2.36 ± 0.06	99.4 ± 1.7	7.0
$K_0^*(1430)^+\pi^-$	0.11 ± 0.01	162.3 ± 6.6	< 0.1
$M_{K_0^*(1430)^\pm}$ (GeV$/c^2$)	1.441 ± 0.002		
$\Gamma_{K_0^*(1430)^\pm}$ (GeV)	0.193 ± 0.004		
F	$+0.96 \pm 0.07$		
R	1 (fixed)		
a	$+0.113 \pm 0.006$		
r	-33.8 ± 1.8		
ϕ_F (deg)	0.1 ± 0.3		
ϕ_R (deg)	-109.7 ± 2.6		
$K^*(892)^\pm$ Parameters			
$M_{K^*(892)^\pm}$ (GeV$/c^2$)	0.8937 ± 0.0001		
$\Gamma_{K^*(892)^\pm}$ (GeV)	0.0472 ± 0.0001		

Most accurate $D^0 \to K_S^0\pi^+\pi^-$ model to date.

CKM Measurements with $D^0(\bar{D}^0) \to K_S^0\pi^+\pi^-$



$$x_\pm + iy_\pm = r_{B^\pm}\exp(i(\delta_B \pm \gamma))$$

$D^0\bar{D}^0$ interfering terms lead to dependence on the relative phase between D^0 and $\bar{D}^0$.

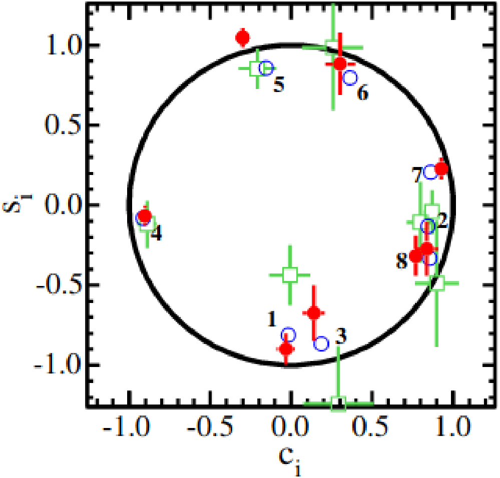
Model Dependent Measurement of CKM parameters

- If the model for $B^{\pm} \to D(\to K_S^0 \pi^+ \pi^-) h^{\pm}$ is completely true, then $\sigma^{\mathrm{syst}} = 0$ for the model.
- Problem is that the model for the decay might be wrong - particularly with the $\Delta\delta_D(\Phi)$ - even if the magnitudes for D^0 and $\bar{D}^0$ are correct.
- From LHCb (2014) (arxiv.org/abs/1407.6211) measurement with $1\mathrm{fb}^{-1}$, get $\gamma = (84^{+49}_{-42})^{\circ}$ including statistical and systematic uncertainties (would $\approx 16^{\circ}$ with the $9\mathrm{fb}^{-1}$ data).
- σ^{syst} in this measurement is between 2% and 20% (depending on the parameter) of the σ^{stat}, this systematic is dominated by the choice of model for $D^0 \to K_S^0 \pi^+ \pi^-$.

Binned Measurement of $\Delta\delta_D(\Phi)$

- Systematics are clearly a big problem for these measurements.
- Modelling $\Delta\delta_D(\Phi)$ is likely the problem.
- Can 'measure' model $\Delta\delta_D(\Phi)$ from BESIII $\psi(3770) \to D^0\bar{D}^0$ data, independent of the model https://arxiv.org/abs/2003.00091 by binning the strong phase into $\pm i = 1, 2, .., N$ bins.
- $c_i + is_i = \frac{\int_i |A_{D^0}(\Phi)||A_{Dzbar}(\Phi)| \exp(i\Delta\delta_D(\Phi))d\Phi}{\sqrt{\int_i |A_{D^0}(\Phi)|^2 d\Phi \int_i |A_{\bar{D}^0}(\Phi)|^2 d\Phi}}$

Binned Measurement of $\Delta\delta_D(\Phi)$



From https://arxiv.org/abs/2003.00091

Binned Measurement of CKM parameters

- Then use results c_i, s_i from $\psi(3770)$ data as inputs and perform 'model-independent' measurements of CKM parameters by splitting the Dalitz plane in to $i = \pm 1, \pm 2, \pm N$ bins of $\Delta\delta_D$ using some binning scheme (built from a model).
- No more model systematics, but lose statistical precision due to binning and introduce systematics from the BESIII input.
- $\langle N_i^{B^-} \rangle \propto F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(c_i x_- + s_i y_-)$
- $\langle N_i^{B^+} \rangle \propto \bar{F}_i + (x_+^2 + y_+^2)F_i + 2\sqrt{F_i\bar{F}_i}(c_i x_+ - s_i y_+)$
- The binned analysis at LHCb (https://arxiv.org/abs/1806.01202 with $9\mathrm{fb}^{-1}$ obtained $\gamma = (68.7^{+5.2}_{-5.1})^\circ$.
- Initially thought that binning $\Delta\delta_D(\Phi)$ equally would yield best precision (smallest changes in $\Delta\delta_D(\Phi)$ in each bin.
- Further optimisations for the binning scheme have been performed - can get approximately 85% of the precision of the MD method https://arxiv.org/abs/1010.2817.

Quasi Model Independent measurement of $\Delta\delta_D$

- If $|A_D^0(\Phi)|$ is close to the 'truth' (i.e. that we trust the results for $|A_D^0(\Phi)|$ from the B-Factories).
- Belle-BaBar have no access to $\Delta\delta_D(\Phi)$ since only $D^0 \to K_S^0\pi^+\pi^-$ is studied.
- We attempt to 'correct' the $\Delta\delta_D(\Phi)$ that the Belle-BaBar model produces
- Replace $\Delta\delta_D(\Phi)$ with $\Delta\delta_D(\Phi) + f(\Phi|C)$, correcting the strong phase from a given model
- This is different from the 'Binned' method - which calculates quantities that depend on $\Delta\delta_D(\Phi)$ in a model independent way
- We do not bin Φ at all - we attempt to do a point by point correction to the strong phase in the two-dimensional Dalitz phase-space using correlated $D^0\bar{D}^0$ prepared states (i.e. $\psi(3770) \to D^0\bar{D}^0$ with at least one deccaying to $K_S^0\pi^+\pi^-$.

Quasi Model Independent measurement of $\Delta\delta_D$

- $f(\Phi|C)$ is given as a two-dimensional polynomial in Dalitz space, $\Phi = (m_+^2, m_-^2)$:

$$f(\Phi|C) = \sum_{i=0}^{O} \sum_{j=0}^{O-i} C_{i,j} P_i(m_+^2) P_j(m_-^2)$$

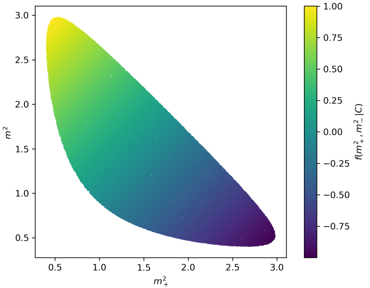
- To preserve antisymmetry of $\Delta\delta_D(\Phi)$, we require $f(\Phi) = -f(\Phi^T)$,
- Can achieve this with transformation $\Phi = (m_+^2, m_-^2) \to \Phi' = (w_+(m_+^2, m_-^2), w_-(m_+^2, m_-^2))$:

$$w_+(m_+^2, m_-^2) = m_+^2 + m_-^2$$
$$w_-(m_+^2, m_-^2) = m_-^2 - m_+^2$$

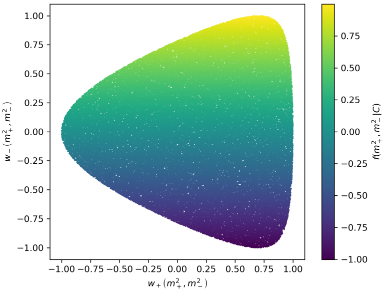
- Then these one-dimensional polynomials, $P_i(w_+)P_{2j+1}(w_-)$, $i, j = 0, 1, 2...O$ build up the two-dimensional polynomial $f(\Phi|C)$ with $C_{i,2j+1}$ as the free parameters.

Quasi Model Independent measurement of $\Delta\delta_D$

- Begin with Dalitz Coordinates m_+^2, m_-^2.
- Right shows a polynomial with $C_{0,1} = 1$.



- Rotation of (m_+^2, m_-^2) to $(w_+(m_+^2, m_-^2), w_-(m_+^2, m_-^2))$ is one way to build polynomials that obey $f(m_+^2, m_-^2) = -f(m_-^2, m_+^2)$.

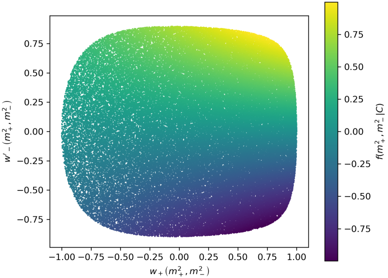


Stretching w_+, w_-

Can stretch the input parameters into a square with:

$$w'_- = \frac{\alpha w_-}{b w_+ + 1 - \varepsilon}$$

on right: $\alpha = 2, b = 1, \varepsilon = 0.01$



Form of correction

So we have the final form of our polynomial, $f(\Phi|C) = f(m_+^2, m_-^2|C)$:

$$f(m_+^2, m_-^2|C) = \sum_{i=0}^{O(C)} \sum_{j=0}^{O(C)-i} C_{i,2j+1} \times P_i^{\mathrm{legendre}}(w_+(m_+^2, m_-^2)) \times P_{2j+1}^{\mathrm{legendre}}(w'_-(m_+^2, m_-^2))$$

with $O(C)$ as our 'order' , we chose Legendre polynomials for $P_i(x)$ since they gave smaller correlations of the polynomials we considered ('simple' ($P_i(x) = x^i$) and 'Chebyshev').

Toy Monte Carlo

- We used AmpGenhttps://github.com/GooFit/AmpGen to generate toy $\psi(3770) \to D^0 \bar{D}^0$ and $B^\pm \to DK^\pm$ samples.

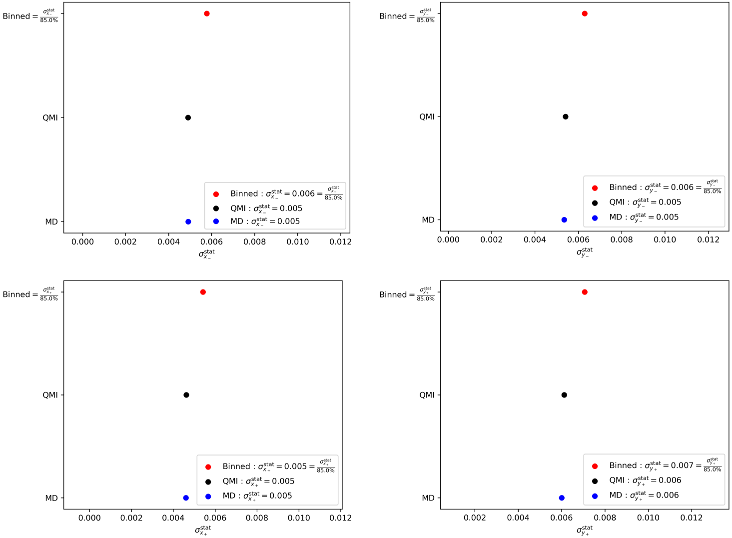
Tag Type	$N_{\text{Generated}}$	$N_{\mathrm{BESIII2020}}$
CP Odd (K^+K^-)	500	443 ± 22
CP Even $(K_S^0\pi^0)$	500	643 ± 26
D^0 flavour $(K^+\pi^-)$	5000	4740 ± 71
$\bar{D}^0$ flavour $(K^-\pi^+)$	5000	4740 ± 71
Double Tag $K_S^0\pi^+\pi^-$	1000	899

https://arxiv.org/abs/2002.12791

Tag Type	$N_{\text{Generated}}$	$N_{\mathrm{LHCb-2021}}$
$B^- \to DK^-$	20000	$(3798 \pm 41) + (8735 \pm 89)$
$B^+ \to DK^+$	20000	$(3798 \pm 41) + (8735 \pm 89)$

https://arxiv.org/abs/2010.08483

Comparison of CKM precision between unbinned, QMI and binned measurements of CKM parameters



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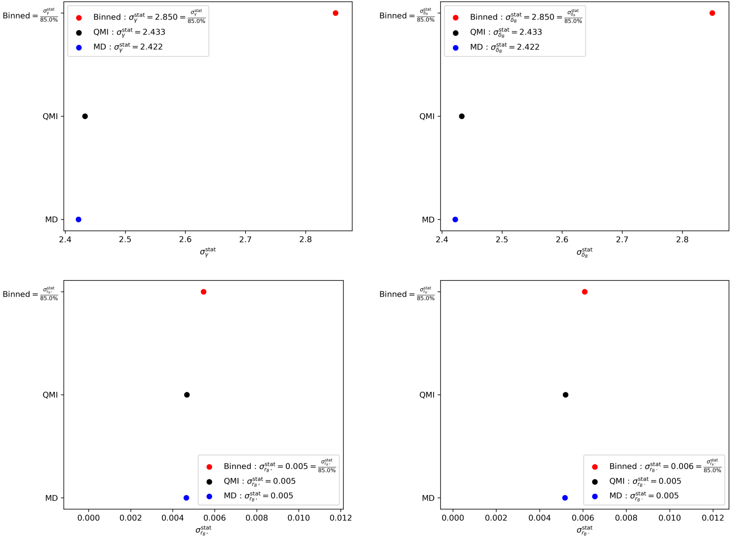
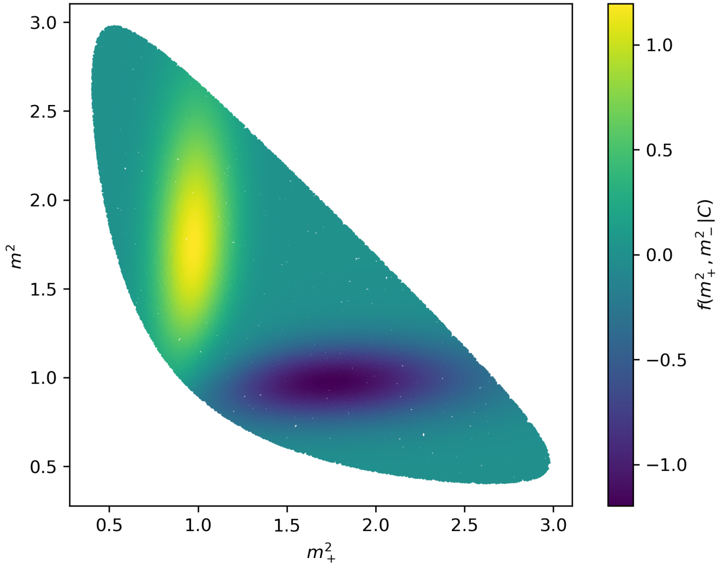


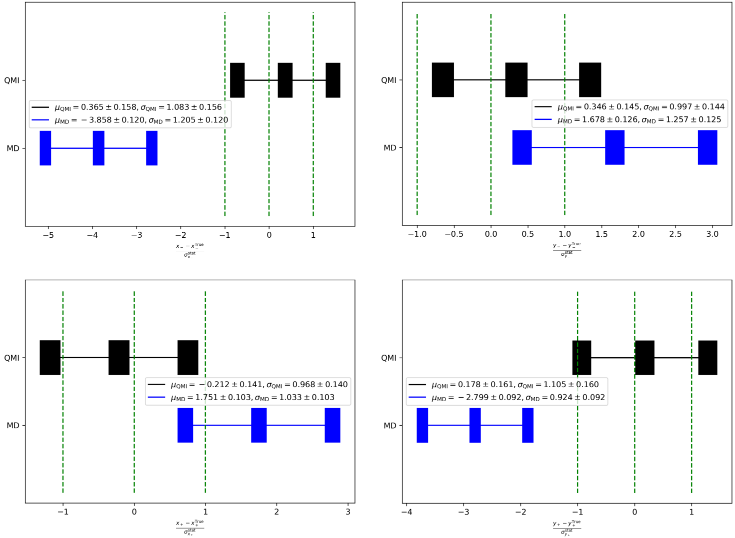
Impact of a bias in $\Delta\delta_D$ on the CKM measurement

- Define a 'Gaussian' bias to $\Delta\delta_D(\Phi)$, $f_G(\Phi|\mu_+, \mu_-, \sigma_+, \sigma_-, A, w^0_-)$
- $$f(\Phi) = A \times \mathrm{erf}\left(\frac{w_-(m^2_+, m^2_-)}{w^0_-}\right) \times \begin{cases} G(m^2_+, \mu_+, \sigma_+)G(m^2_-, \mu_-, \sigma_-) & m^2_+ > m^2_- \\ G(m^2_-, \mu_+, \sigma_+)G(m^2_+, \mu_-, \sigma_-) & m^2_+ > m^2_- \end{cases}$$
- $G(x, \mu, \sigma) = \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$
- $\mathrm{erf}(x)$ is the 'error' function used to smoothly vary from -1 to $+1$ over $w_- < 0$ to $w_- > 0$.

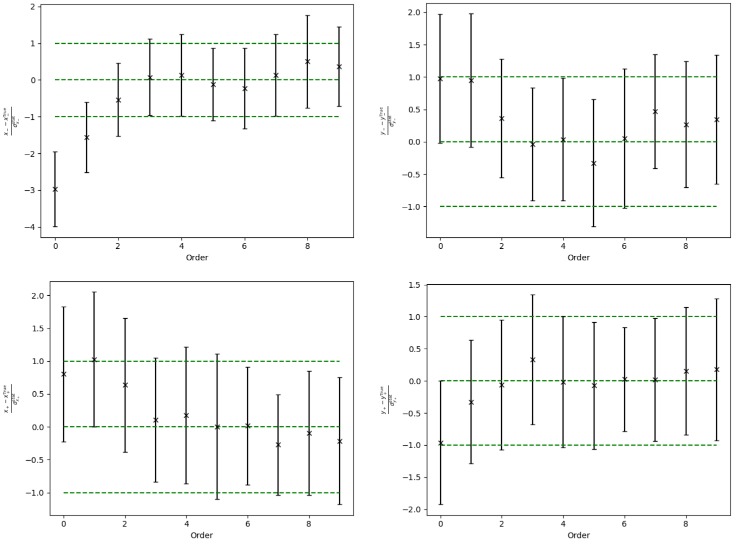
An extreme example of a bias to $\Delta\delta_D(\Phi)$



Recovery of $\Delta\delta_D(\Phi)$ with our method



Deciding on the order of the correcting polynomial in the QMI method



Summary

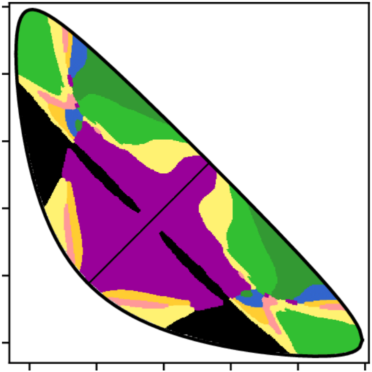
- Introduced a new method to measure $\Delta\delta_D(m_+^2, m_-^2)$.
- Method has much greater precision than the binned method.
- Method recovers from bias to $\Delta\delta_D(m_+^2, m_-^2)$ and therefore avoids large shifts in CKM parameters .
- $\sigma_{\mathrm{MD}}^{\mathrm{stat}} < \sigma_{\mathrm{QMI}}^{\mathrm{stat}} < \sigma_{\mathrm{Binned}}^{\mathrm{stat}}$.

Next...

- Write the method paper (In progress).
- Optimize implementation of method (lots of thanks to Tim Evans for creating and supporting AmpGen).
- Actually use the method in a measurement with data.

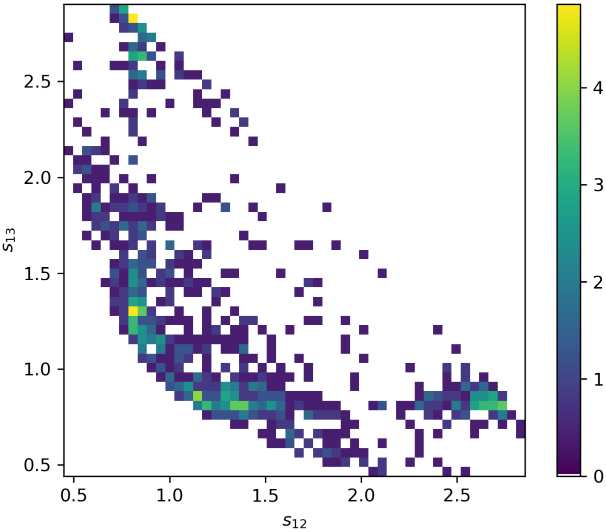
Backup Slides

Optimal Binning scheme

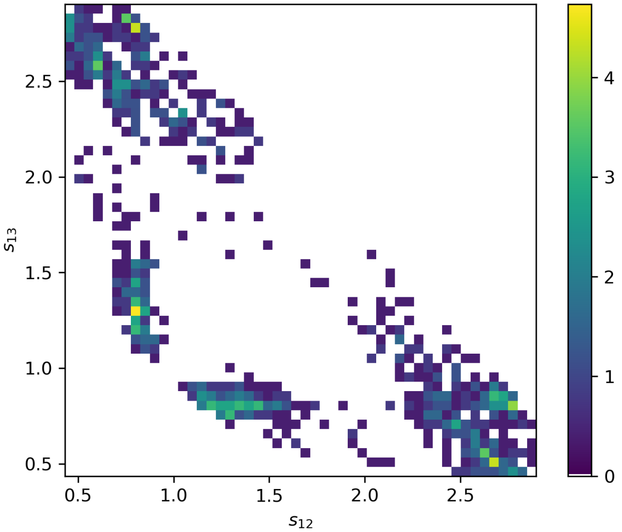


https://arxiv.org/abs/1904.01129 https://arxiv.org/abs/1010.2817

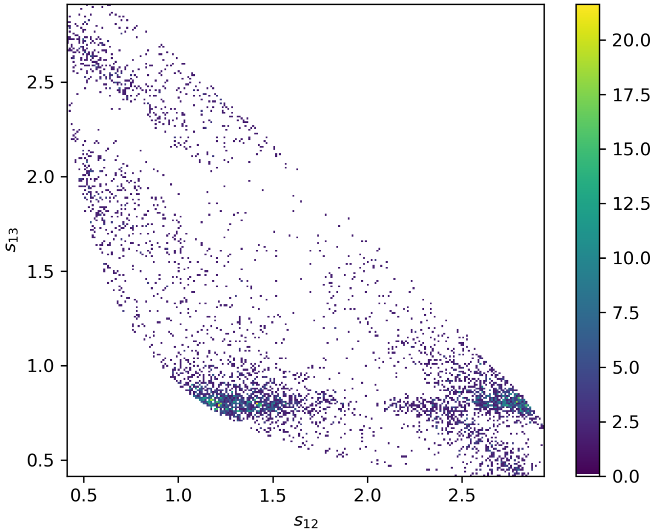
Toy K^+K^- v.s. $K_S^0\pi^+\pi^-$



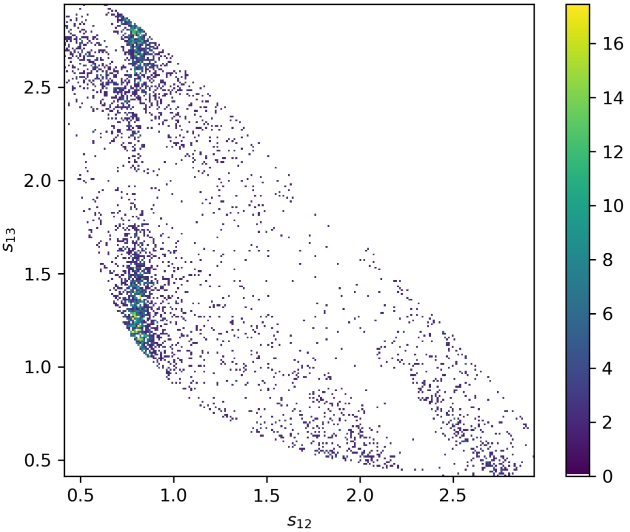
Toy $K_S^0\pi^0$ v.s. $K_S^0\pi^+\pi^-$



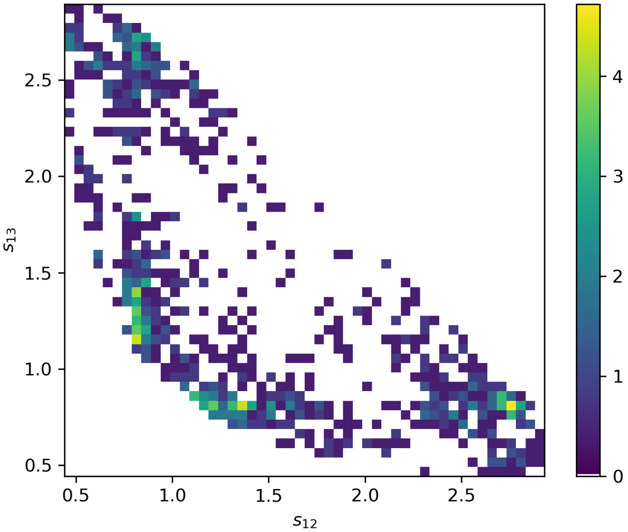
Toy $K^+\pi^-$ v.s. $K^0_S\pi^+\pi^-$



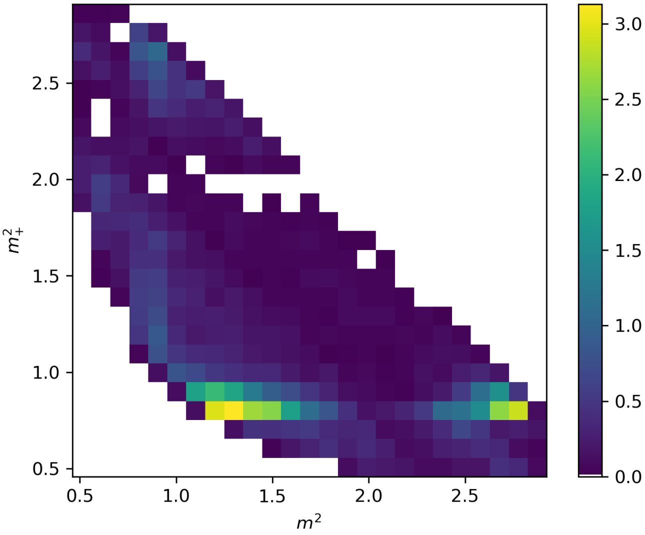
Toy $K^-\pi^+$ v.s. $K_S^0\pi^+\pi^-$



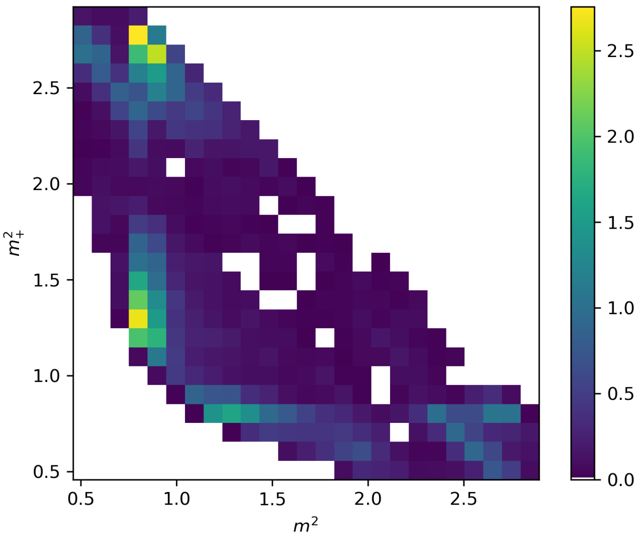
Toy $K_S^0\pi^+\pi^-$ v.s. $K_S^0\pi^+\pi^-$



Toy $B^- \to Dh^-$ $D \to K_S^0\pi^+\pi^-$



Toy $B^+ \to Dh^+$ $D \to K_S^0\pi^+\pi^-$



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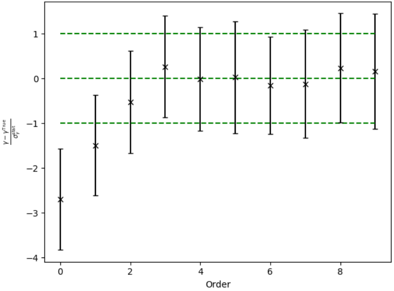

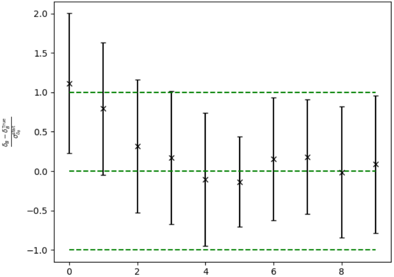

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