Model Independent Measurement of CKM γ using $B^{\pm} \rightarrow D^{0}\,(\rightarrow 2\pi^{+}2\,\pi^{-})K^{\pm}$

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The CKM Matrix

- Quarks can change flavour

- CKM matrix quantifies transition amplitudes:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Must be unitary:

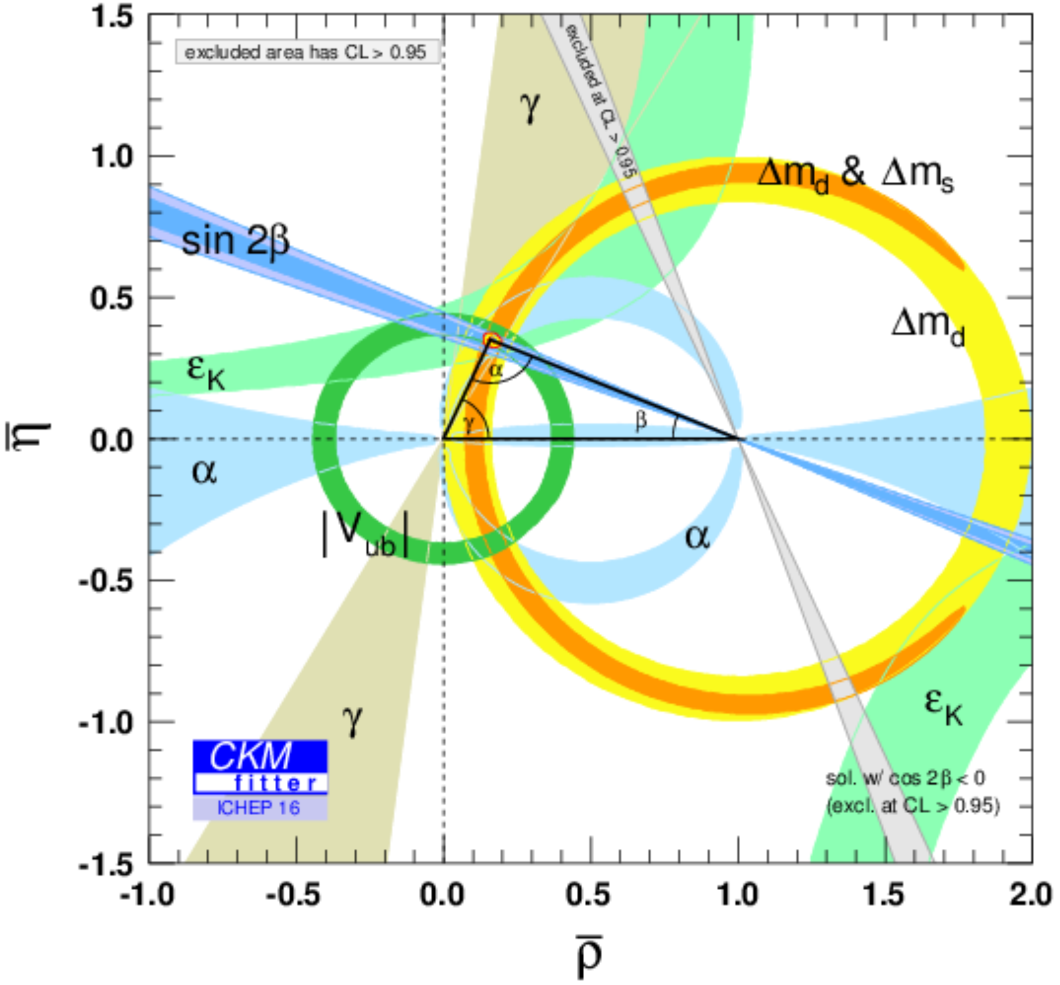
$$V_{CKM}^{\dagger} V_{CKM} = I$$

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

What is γ?

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

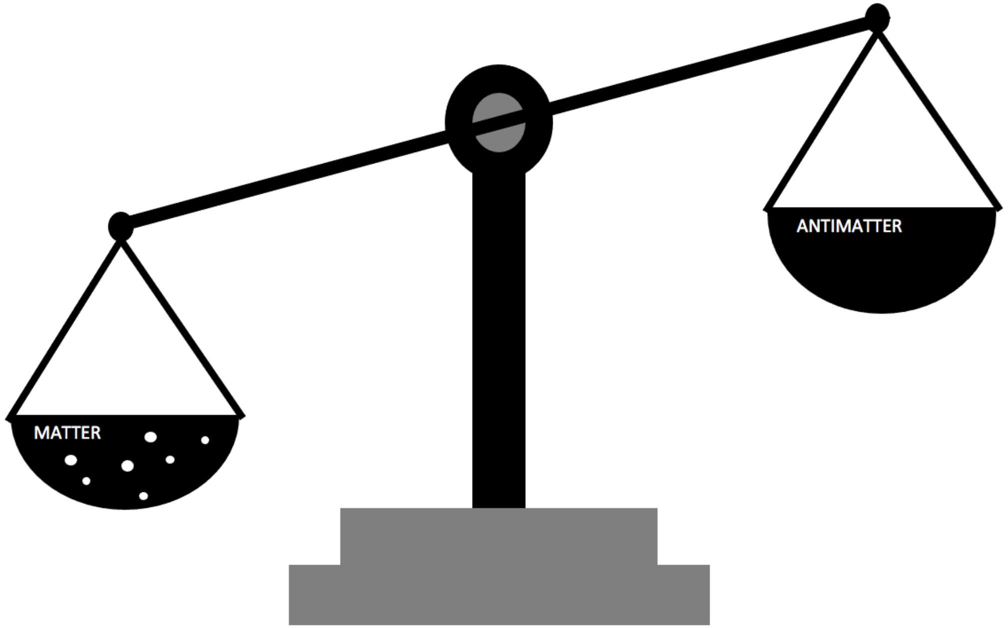
- The angle with one of the largest uncertainties in the unitary triangle. It accounts for the charge-parity violating phase between *b->u* and *b->c* quark transitions.
- $\boldsymbol{\gamma}$ is theoretically (and experimentally) very clean so sets a benchmark for the SM.
- Improving the measurement of the CKM angle $\boldsymbol{\gamma}$ is a key physics goal for LHCb.
- Current LHCb combination measurement is $(65.4^{+3.8}_{-4.2})^{\circ}$ (JHEP 12 (2021) 141)



Motivation

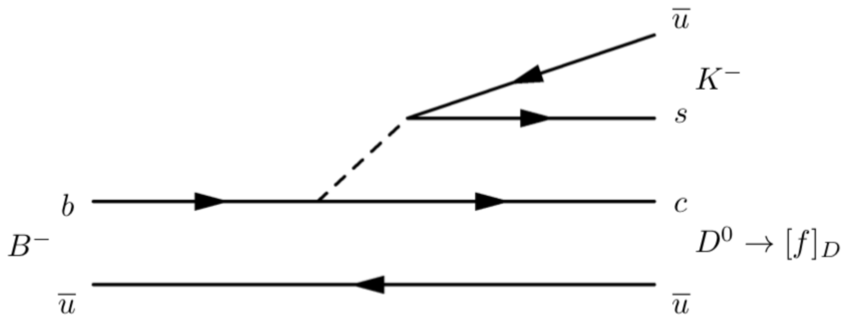
- The charge parity violation described by γ accounts for some of the matter and anti-matter difference we observe.

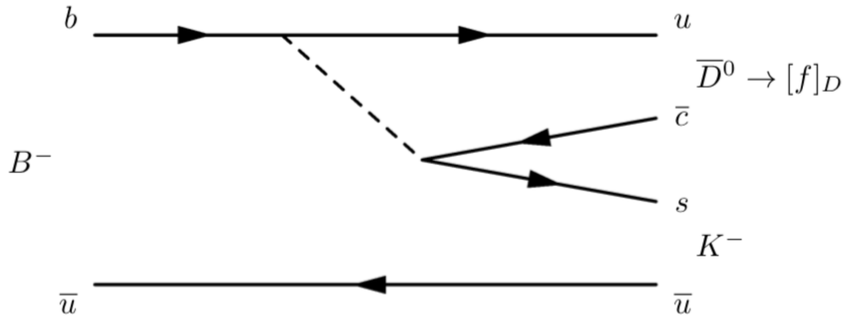
- The extent of the matter and anti-matter asymmetry we observe in the Universe is not accounted for in the standard model.



Motivation

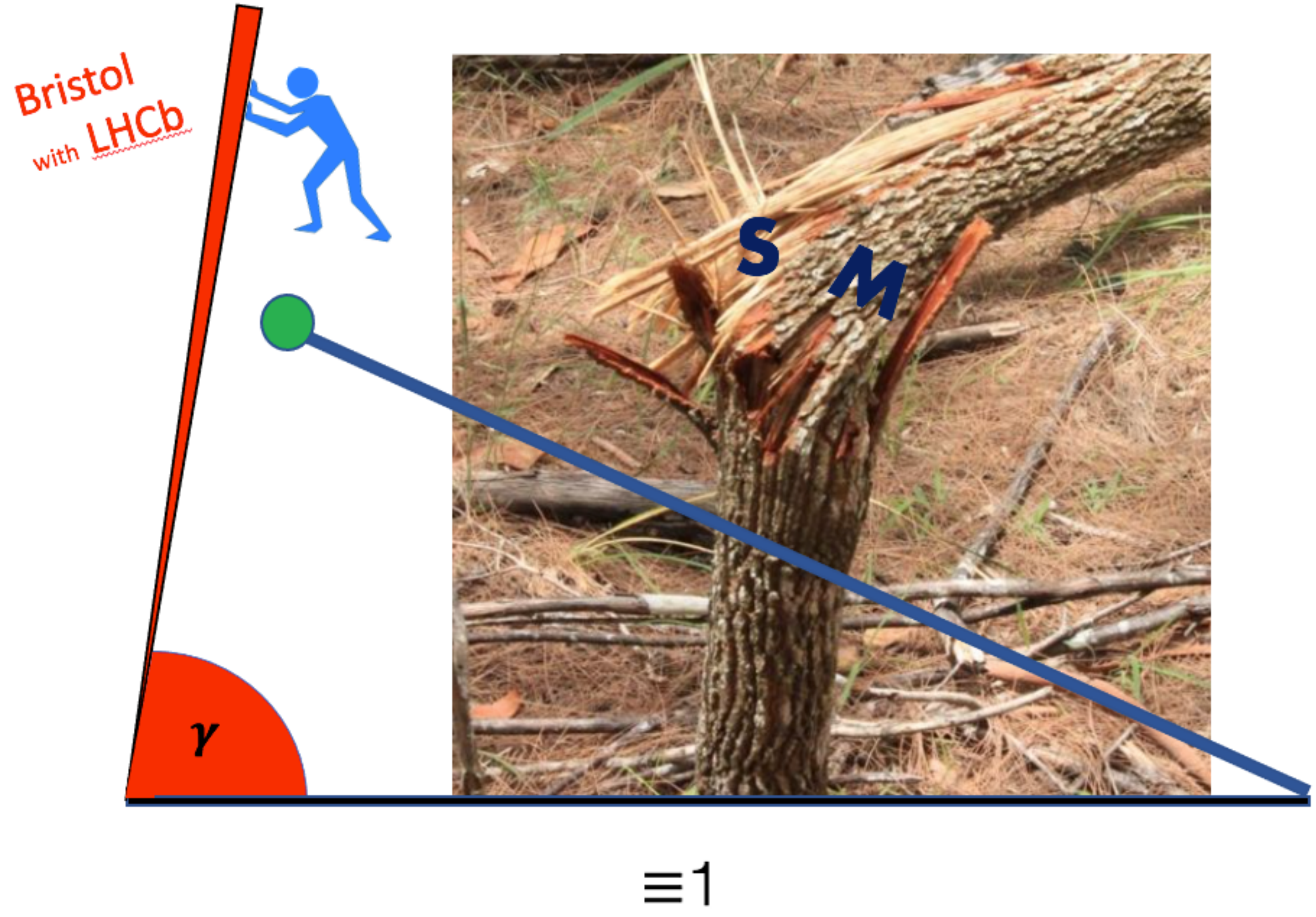
- $B^{\pm} \to D^0 K^{\pm}$ decays are predominantly mediated by tree-level processes.
- Physics beyond the standard model is expected to manifest itself as virtual particles within loop diagrams.
- Tree vs loop diagram measurements could point to New Physics (NP)





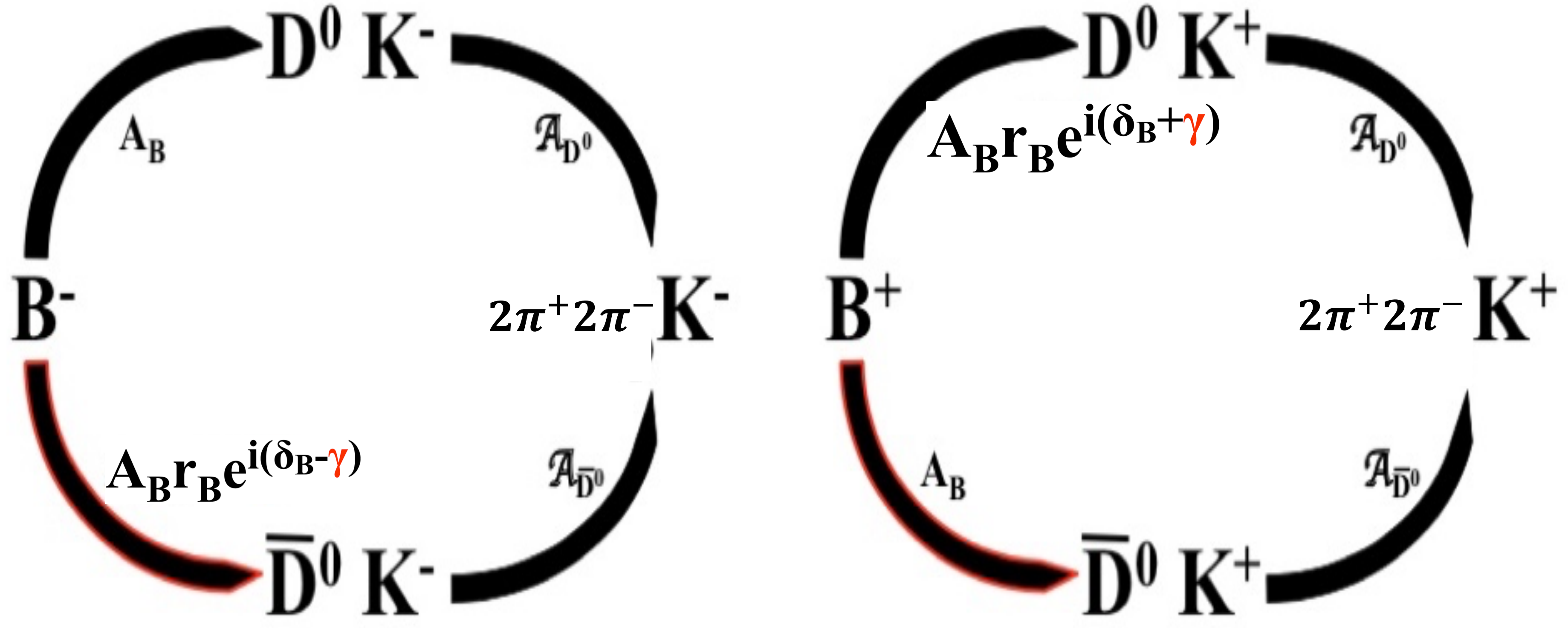
Motivation

What we're aiming for



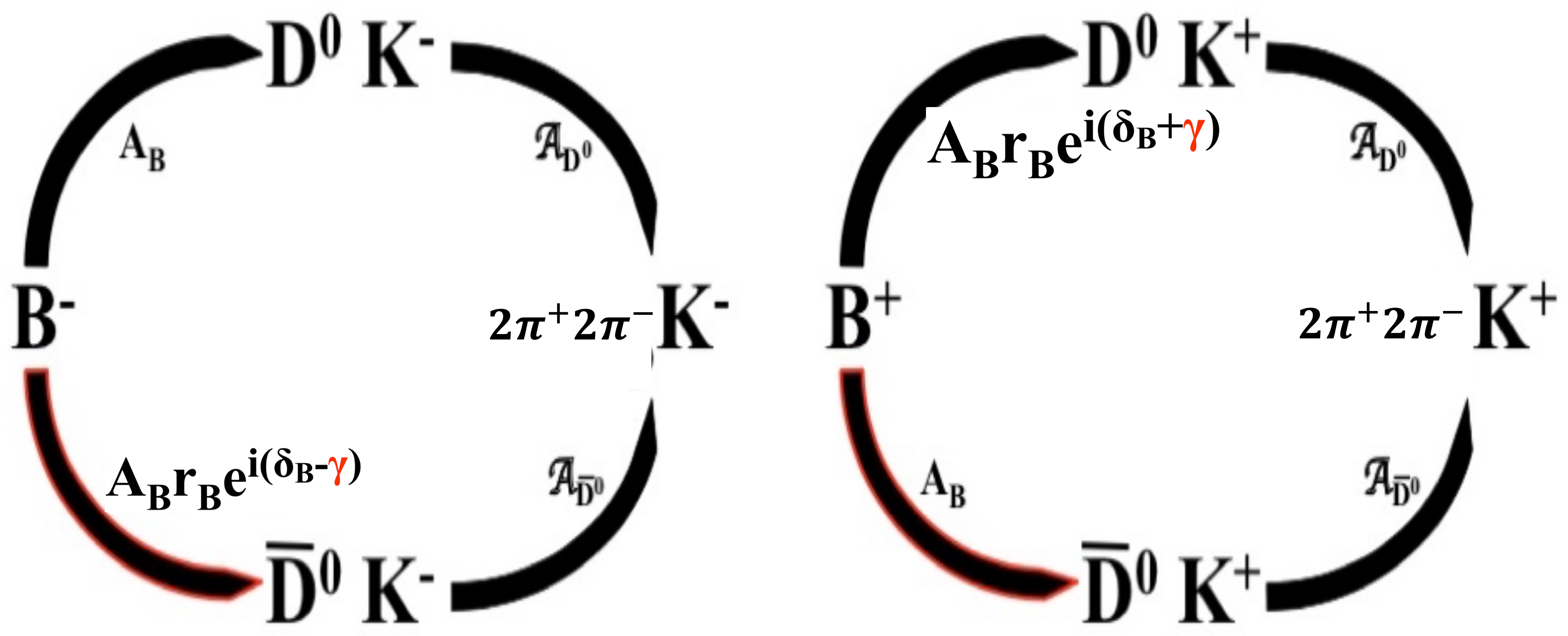
Sensitivity to γ

- $\boldsymbol{\gamma}$ **can be measured through interference** of $B^{\pm} \to D^0 K^{\pm}$ and $B^{\pm} \to \overline{D}^0 K^{\pm}$.



Sensitivity to γ

- $\boldsymbol{\gamma}$ **can be measured through interference** of $B^{\pm} \to D^0 K^{\pm}$ and $B^{\pm} \to \overline{D}^0 K^{\pm}$.
- Observation of the interference pattern over the 5-D phase space, since D is reconstructed as $4\pi^{\pm}$, increases the sensitivity to $\boldsymbol{\gamma}$.



Model Independent Method

- GGSZ method with $D^0 \to 2\pi^+ 2\,\pi^-$:

- This achieved by dividing the 5-D D decay phase space into bins according to strong phase difference and integrating over each bin.

- 10 bin pairs, the 5 positive bins are calculated using:

$$\Gamma(B^+ \to DK^+, D \to f_i) \propto T_i^f r_B^2 + \bar{T}_i^f + 2\sqrt{T_i^f \bar{T}_i^f}(c_i^f x_+ - s_i^f y_+)$$

$$\Gamma(B^- \to DK^-, D \to f_i) \propto \bar{T}_i^f r_B^2 + T_i^f + 2\sqrt{T_i^f \bar{T}_i^f}(c_i^f x_- + s_i^f y_-)$$

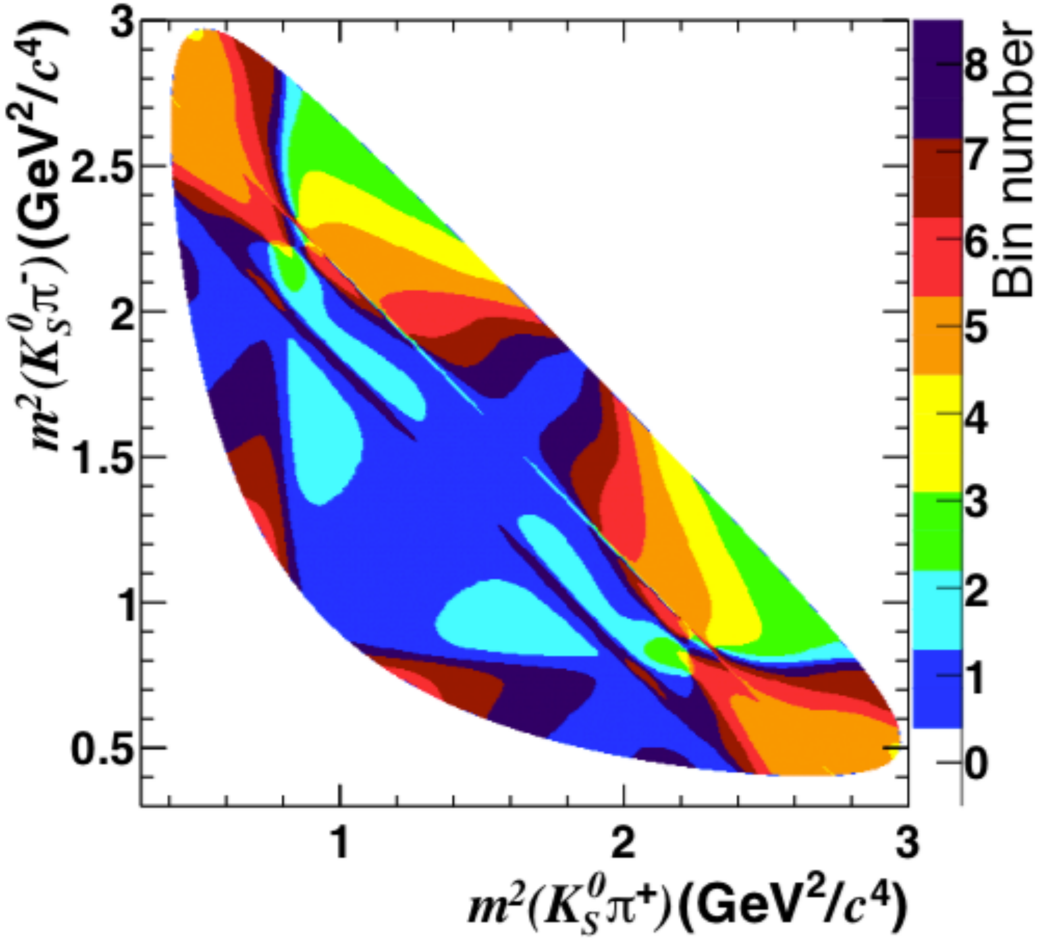
Where, $x_\pm = r_B \cos(\delta_B \pm \gamma)$ and $y_\pm = r_B \sin(\delta_B \pm \gamma)$

and $r_B^2 = x_\pm^2 + y_\pm^2$.

- Hadronic parameter inputs $(c_i, s_i, T_i, \bar{T}_i)$ from CLEO-c data in 5 different binning schemes and varying numbers of bins (JHEP 01 (2018) 144).

- Symmetry of self-conjugate D meson final state 2pi+2pi- can be exploited by defining bins in pairs which map onto each other.

Example binning scheme for $D^0 \to K_s\pi\pi$, CERN-THESIS-2017-334 (Babar 2008 binning scheme)



$$K_{-i}^f = \bar{K}_i^f \qquad \bar{K}_{-i}^f = K_i^f$$

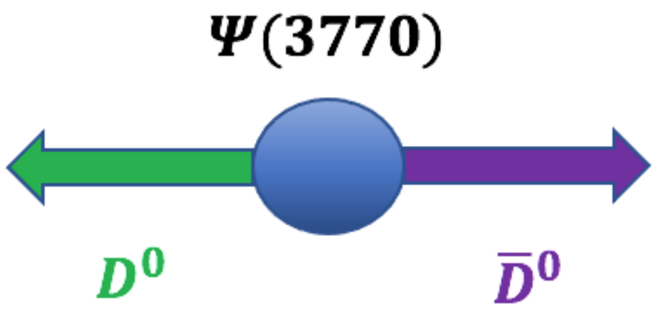
$$T_{-i}^f = \bar{T}_i^f \qquad \bar{T}_{-i}^f = T_i^f$$

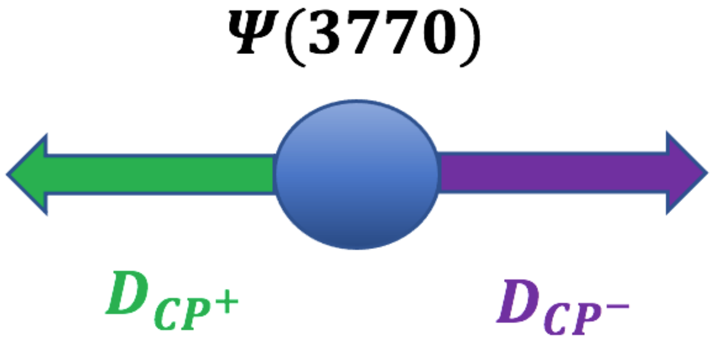
$$c_{-i}^f = c_i^f \qquad s_{-i}^f = -s_i^f\,.$$

Hadronic Parameters

- S. Harnew and C. Prouve ([JHEP 01 (2018) 144]) measured the hadronic parameters model independently with Cleo-c data for 5 binnings using quantum correlated $\bar{D}D$ decays.

- The hadronic parameters are related only to the D decays (not the B decay.)





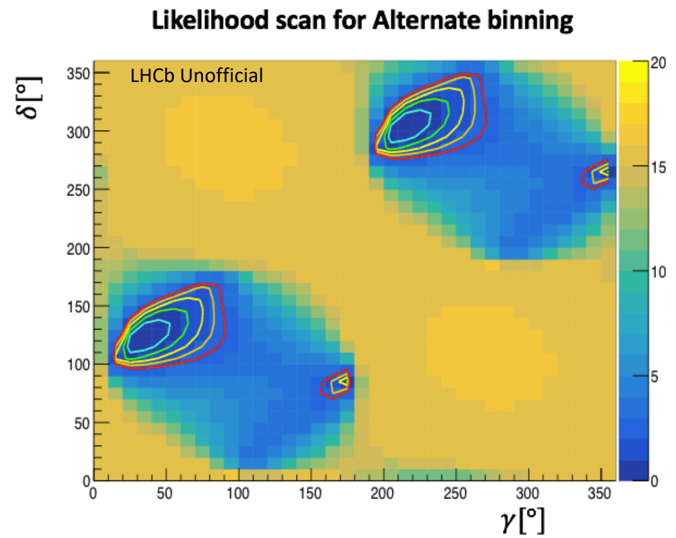
Sensitivity Study

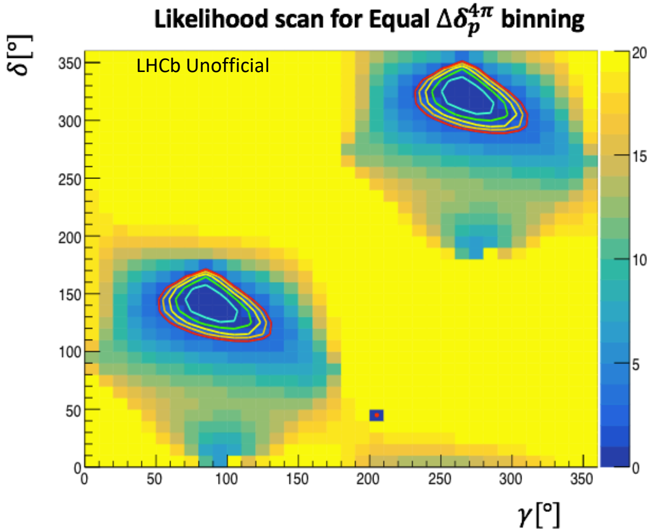
- Run 1 + Run 2 event yield - 9000 events
- Generated gaussian $B^{\pm}$ mass distributions of $B^{\pm} \rightarrow \overline{D}^0 K^{\pm}, D \rightarrow 4\pi^{\pm}$ decays
- Hadronic parameters $(c_i, s_i, T_i, \overline{T}_i)$ constrained according to uncertainties and correlation. 5 different binning schemes all with 1-5bin options.
- Fit for 2 sets of parameters:
  - γ , r_B , δ_B.
  - x_+ , x_- , y_+, y_-.

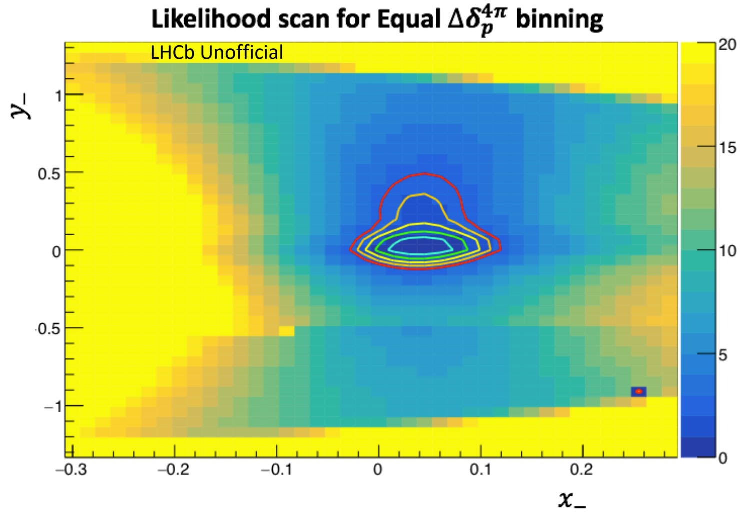
$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

- Sensitivity to $\boldsymbol{\gamma}$ ~ 15° for run 1 + run 2 data
- Sensitivity to $\boldsymbol{x_{\pm}}$, $\boldsymbol{y_{\pm}}$ ~ 0.031, 0.035, 0.065, 0.076 for run 1 + run 2 data



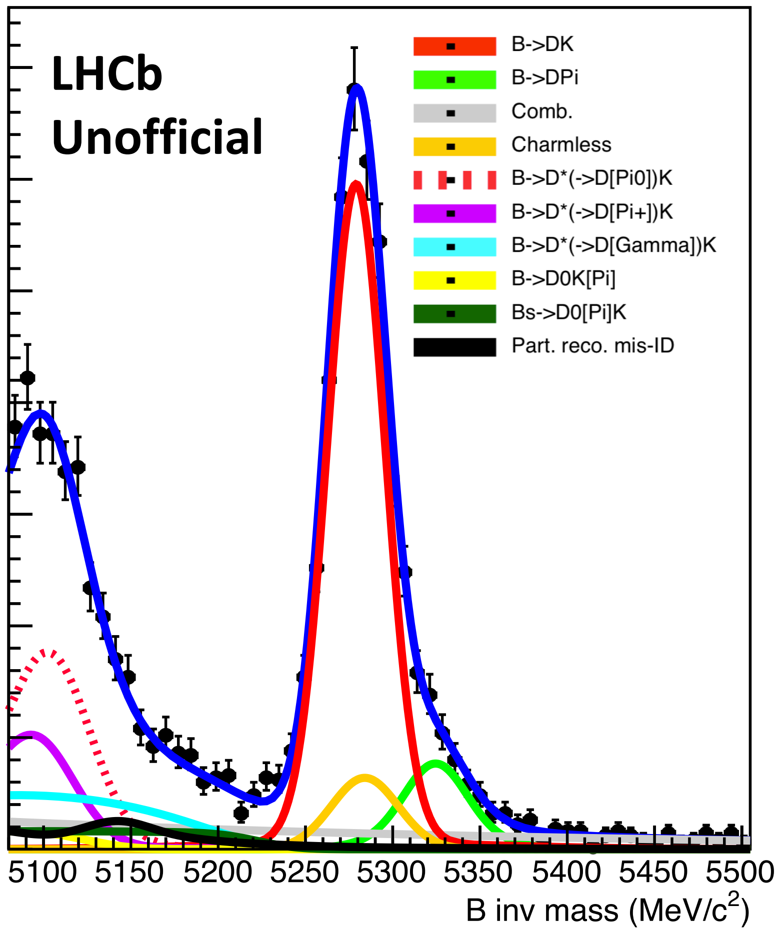




Simultaneous Global Fit (Unofficial, Preliminary)

- Pre-selections
- Boosted Decision Tree
- $D \to K_S^0 \pi^+ \pi^-$ veto
  $480\,\mathrm{MeV} < m(\pi^+\pi^-) < 505\,\mathrm{MeV}$
- Cut to reduce charmless bkg
- Particle ID Cuts
- Preliminary Fit for DK
- DPi used as a reference channel and constrain partially reconstructed background yields.

B->DK, D->2Pi+2Pi- (LHCb Unofficial)



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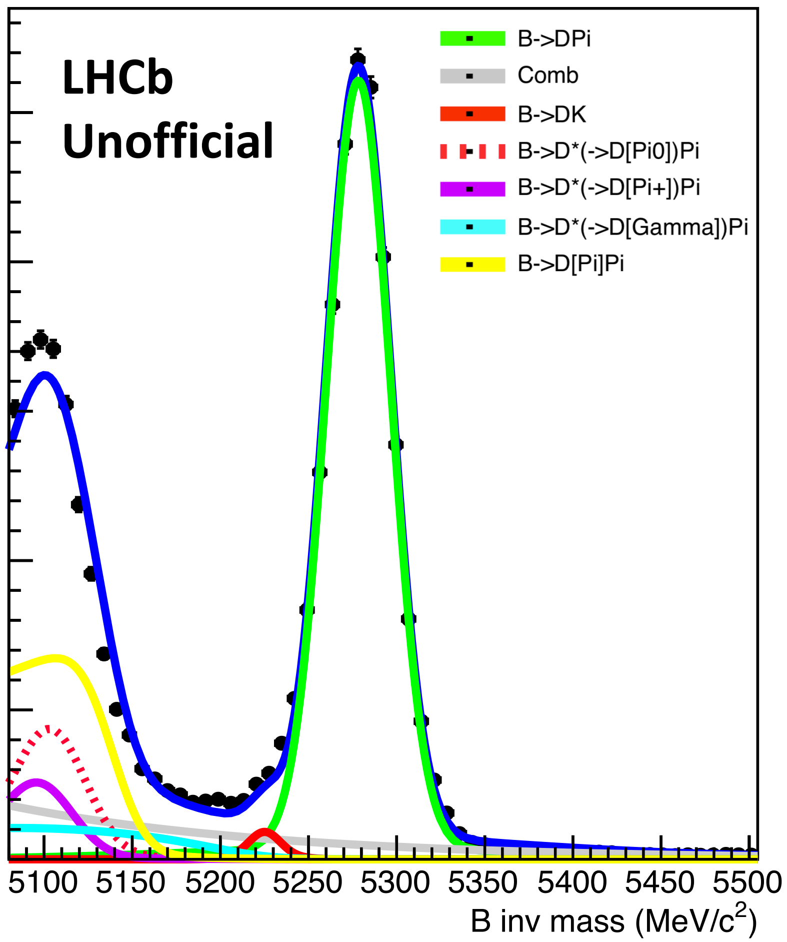
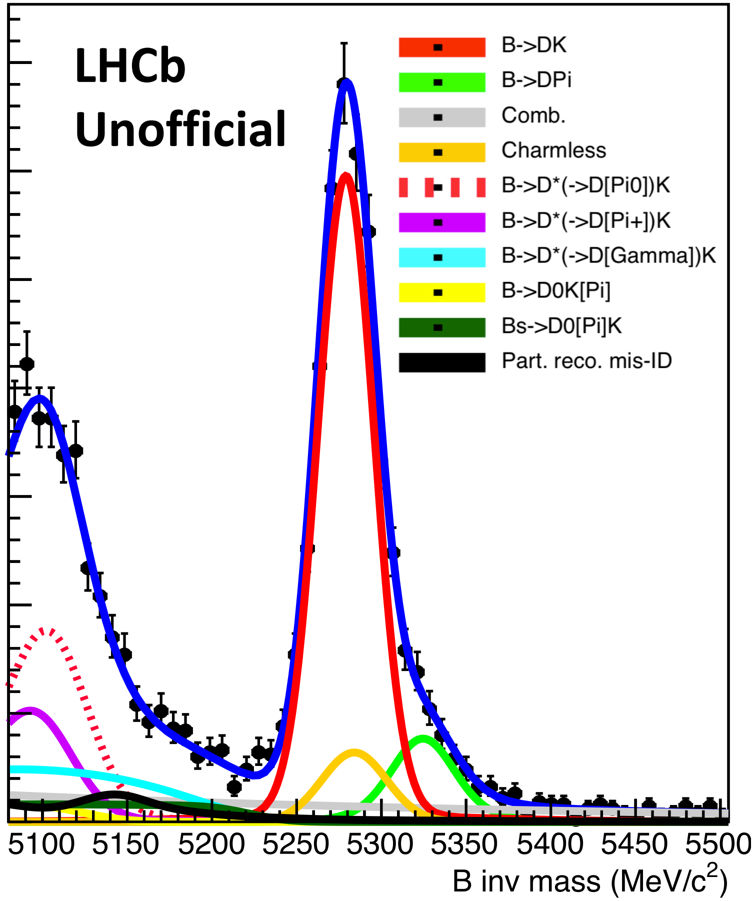


Next Steps

- Optimise cuts.
- Simultaneous fit combining all bins to determine shapes.
- Simultaneous fit to each bin, split by charge to determine x_+ , x_- , y_+, y_- .
- Systematics.

B->DK, D->2Pi+2Pi- (LHCb Unofficial)



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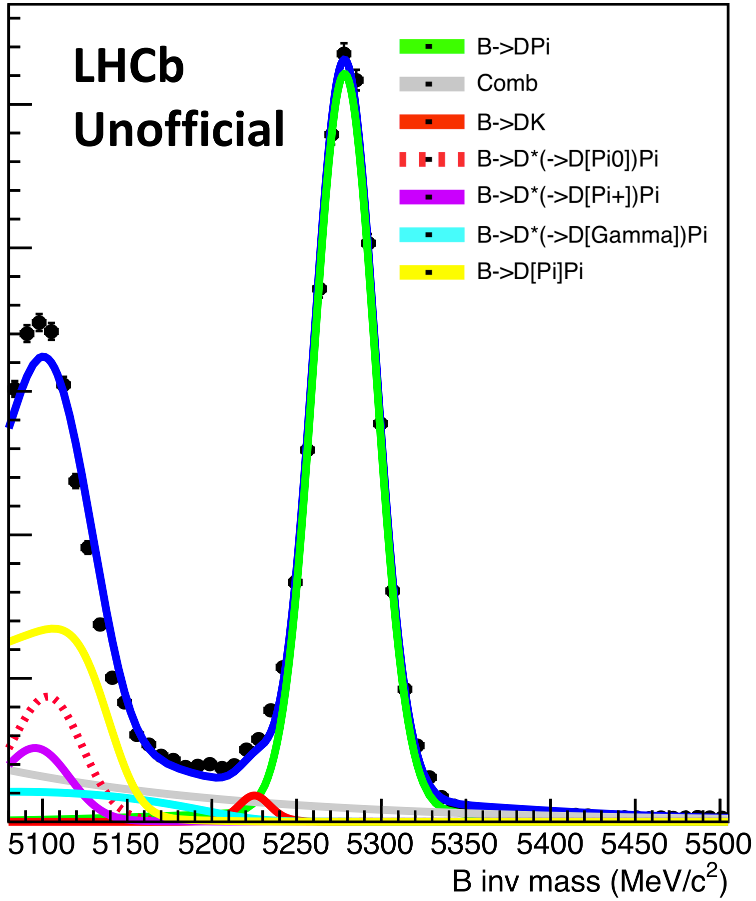


Conclusion

- γ being measured using $B^{\pm} \rightarrow D^{0}\ (\rightarrow 2\pi^{+}2\ \pi^{-})K^{\pm}$ decays

- Model independent approach implemented.

- γ is evaluated at different points over the 5-dimensional phase space of the D meson decay, increasing the sensitivity to Gamma.

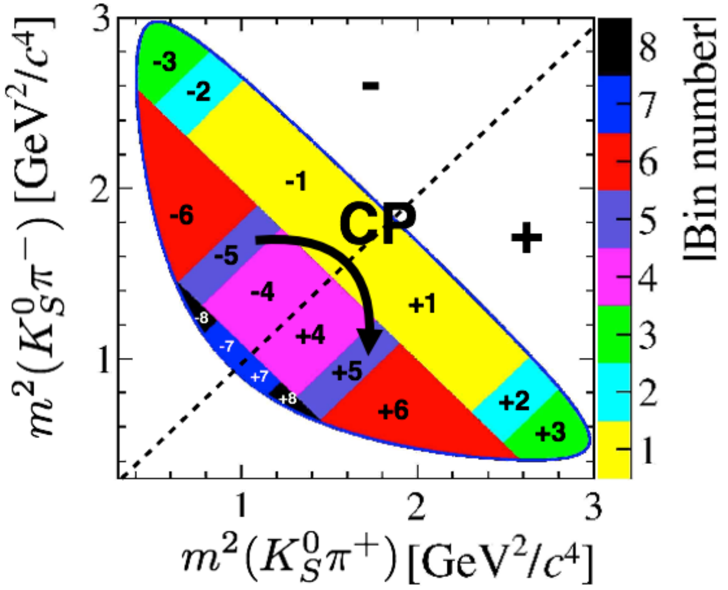
- This is the first time γ will be measured in the $D^{0} \rightarrow 2\pi^{+}2\ \pi^{-}$ decay mode with a model independent approach using phase space bins.

Any Questions

Back Up

Model Independent Method

Example binning scheme from analysis
$D^0 \to K_s\pi\pi$ analysis JHEP 10 (2014) 097



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$$\Gamma(B^- \to DK^-, D \to f_i) \propto \bar{T}_i^f r_B^2 + T_i^f + 2\sqrt{T_i^f \bar{T}_i^f}(c_i^f x_- + s_i^f y_-)$$

Where, $x_\pm = r_B \cos(\delta_B \pm \gamma)$ and $y_\pm = r_B \sin(\delta_B \pm \gamma)$ and $r_B^2 = x_\pm^2 + y_\pm^2$.

- Hadronic parameter inputs $(c_i, s_i, T_i, \bar{T}_i)$ from CLEO-c data in 5 different binning schemes and varying numbers of bins (JHEP 01 (2018) 144).

- Symmetry of self-conjugate D meson final state 2pi+2pi- can be exploited by defining bins in pairs which map onto each other.

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Hadronic Parameters

- Sam Harnew and Claire Prouve (JHEP 01 (2018) 144) measured the hadronic parameters with Cleo-c data for 5 binnings using quantum correlated $\bar{D}D$ decays.
- The hadronic parameters are related only to the D decays (not the B decay)

$$c_i^f = \frac{1}{\sqrt{K_i^f \bar{K}_i^f}} \int_i |A_{\mathbf{p}}^f||\bar{A}_{\mathbf{p}}^f| \cos\left(\Delta\delta_{\mathbf{p}}^f\right) \phi(\mathbf{p})\mathrm{d}\mathbf{p}$$

$$K_i^f = \int_i |A_{\mathbf{p}}^f|^2 \phi(\mathbf{p})\mathrm{d}\mathbf{p} \qquad \bar{K}_i^f = \int_i |\bar{A}_{\mathbf{p}}^f|^2 \phi(\mathbf{p})\mathrm{d}\mathbf{p}$$

$$s_i^f = \frac{1}{\sqrt{K_i^f \bar{K}_i^f}} \int_i |A_{\mathbf{p}}^f||\bar{A}_{\mathbf{p}}^f| \sin\left(\Delta\delta_{\mathbf{p}}^f\right) \phi(\mathbf{p})\mathrm{d}\mathbf{p}$$

$$T_i^f = \frac{K_i^f}{\sum_i K_i^f} \qquad \bar{T}_i^f = \frac{\bar{K}_i^f}{\sum_i \bar{K}_i^f}$$

Equal $\Delta\delta_{\mathbf{p}}^{4\pi}$ binning

i	c_i	s_i	T_i	$\overline{T}_i$
1	$0.881 \pm 0.053 \pm 0.044$	$0.303 \pm 0.149 \pm 0.046$	$0.237 \pm 0.008 \pm 0.004$	$0.217 \pm 0.008 \pm 0.003$
2	$0.501 \pm 0.084 \pm 0.046$	$-0.032 \pm 0.201 \pm 0.025$	$0.122 \pm 0.006 \pm 0.002$	$0.127 \pm 0.006 \pm 0.003$
3	$0.450 \pm 0.113 \pm 0.064$	$0.441 \pm 0.228 \pm 0.072$	$0.059 \pm 0.004 \pm 0.002$	$0.075 \pm 0.005 \pm 0.002$
4	$-0.201 \pm 0.167 \pm 0.068$	$0.132 \pm 0.304 \pm 0.039$	$0.039 \pm 0.004 \pm 0.002$	$0.045 \pm 0.004 \pm 0.001$
5	$-0.397 \pm 0.152 \pm 0.036$	$-0.446 \pm 0.381 \pm 0.132$	$0.040 \pm 0.004 \pm 0.001$	$0.039 \pm 0.004 \pm 0.002$
$\tilde{F}_+^{4\pi}$	$0.768 \pm 0.021 \pm 0.013$			
Q	$0.733 \pm 0.052 \pm 0.035$			

Hadronic Parameters

- Bin average sine and cosine of strong phase difference (describes interference of $D^0 \to f$ and $\bar{D}^0 \to f$ amplitudes over the region i)

$$c_i^f = \frac{1}{\sqrt{K_i^f \bar{K}_i^f}} \int_i |A_{\mathbf{p}}^f||\bar{A}_{\mathbf{p}}^f| \cos\left(\Delta\delta_{\mathbf{p}}^f\right) \phi(\mathbf{p}) d\mathbf{p}$$

$$s_i^f = \frac{1}{\sqrt{K_i^f \bar{K}_i^f}} \int_i |A_{\mathbf{p}}^f||\bar{A}_{\mathbf{p}}^f| \sin\left(\Delta\delta_{\mathbf{p}}^f\right) \phi(\mathbf{p}) d\mathbf{p}$$

- The branching fraction for $D^0 \to fi$ and $\bar{D}^0 \to fi$ decays that populate phase space bin i

$$K_i^f = \int_i |A_{\mathbf{p}}^f|^2 \phi(\mathbf{p}) d\mathbf{p} \qquad \bar{K}_i^f = \int_i |\bar{A}_{\mathbf{p}}^f|^2 \phi(\mathbf{p}) d\mathbf{p}$$

- Fraction of $D^0 \to f$ and $\bar{D}^0 \to f$ decays that populate phase space bin i.

$$T_i^f = \frac{K_i^f}{\sum_i K_i^f} \qquad \bar{T}_i^f = \frac{\bar{K}_i^f}{\sum_i \bar{K}_i^f}$$

Binning Schemes

- To best exploit the symmetries of the self-conjugate $4\pi^{\pm}$ final state, phase space bins are defined in pairs to map to each other under the CP operation s.t:

$$K^f_{-i} \equiv \bar{K}^f_i,\ \bar{K}^f_{-i} \equiv K^f_i,\ c^f_{-i} \equiv c^f_i \text{ and } s^f_{-i} \equiv -s^f_i$$

- Equal and Variable $\Delta\delta_p^{4\pi}$:
  - Based on an equal/variable division of $\Delta\delta_p^{4\pi} = \arg(A_p^{4\pi}) - \arg(\bar{A}_p^{4\pi})$
- Alternate binning
  - Also uses the relative magnitude of $D^0 \to 4\pi^{\pm}$ to $\bar{D}^0 \to 4\pi^{\pm}$ amplitudes but also considers the variation across each bin.
- Optimal and Alternate Optimal
  - Defined to optimise the expected sensitivity to $\boldsymbol{\gamma}$ in $\boldsymbol{B^{\pm} \to DK^{\pm}}$decays.
- Note: Although an amplitude model is used to inspire the binning schemes the results are model-unbiased.

Equal $\Delta\delta_{\mathbf{p}}^{4\pi}$ binning

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$\tilde{F}_+^{4\pi}$	$0.768 \pm 0.021 \pm 0.013$			
Q	$0.733 \pm 0.052 \pm 0.035$			

Variable $\Delta\delta_{\mathbf{p}}^{4\pi}$ binning

i	c_i	s_i	T_i	$\overline{T}_i$
1	$0.966 \pm 0.101 \pm 0.052$	$0.086 \pm 0.316 \pm 0.068$	$0.069 \pm 0.005 \pm 0.001$	$0.062 \pm 0.004 \pm 0.003$
2	$0.810 \pm 0.070 \pm 0.051$	$-0.136 \pm 0.229 \pm 0.051$	$0.123 \pm 0.006 \pm 0.003$	$0.112 \pm 0.006 \pm 0.002$
3	$0.910 \pm 0.080 \pm 0.059$	$0.225 \pm 0.259 \pm 0.107$	$0.078 \pm 0.005 \pm 0.001$	$0.078 \pm 0.005 \pm 0.002$
4	$0.405 \pm 0.083 \pm 0.046$	$0.215 \pm 0.188 \pm 0.041$	$0.133 \pm 0.006 \pm 0.003$	$0.152 \pm 0.006 \pm 0.003$
5	$-0.154 \pm 0.105 \pm 0.047$	$0.213 \pm 0.207 \pm 0.031$	$0.090 \pm 0.005 \pm 0.003$	$0.103 \pm 0.006 \pm 0.002$
$\tilde{F}_+^{4\pi}$	$0.772 \pm 0.021 \pm 0.010$			
Q	$0.698 \pm 0.049 \pm 0.020$			

Alternative binning

i	c_i	s_i	T_i	$\overline{T}_i$
1	$-0.205 \pm 0.189 \pm 0.094$	$-0.057 \pm 0.384 \pm 0.127$	$0.057 \pm 0.004 \pm 0.001$	$0.019 \pm 0.003 \pm 0.003$
2	$0.445 \pm 0.105 \pm 0.066$	$-0.041 \pm 0.259 \pm 0.073$	$0.129 \pm 0.006 \pm 0.004$	$0.060 \pm 0.005 \pm 0.004$
3	$0.888 \pm 0.053 \pm 0.045$	$-0.150 \pm 0.159 \pm 0.027$	$0.263 \pm 0.008 \pm 0.007$	$0.192 \pm 0.007 \pm 0.004$
4	$0.530 \pm 0.097 \pm 0.044$	$0.239 \pm 0.209 \pm 0.084$	$0.121 \pm 0.006 \pm 0.004$	$0.073 \pm 0.005 \pm 0.003$
5	$-0.451 \pm 0.162 \pm 0.053$	$-0.238 \pm 0.416 \pm 0.157$	$0.059 \pm 0.004 \pm 0.002$	$0.027 \pm 0.003 \pm 0.002$
$\tilde{F}_+^{4\pi}$	$0.764 \pm 0.022 \pm 0.011$			
Q	$0.702 \pm 0.051 \pm 0.027$			

Optimal binning

i	c_i	s_i	T_i	$\overline{T}_i$
1	$0.949 \pm 0.057 \pm 0.039$	$-0.041 \pm 0.171 \pm 0.041$	$0.193 \pm 0.007 \pm 0.004$	$0.173 \pm 0.007 \pm 0.003$
2	$0.641 \pm 0.110 \pm 0.073$	$0.331 \pm 0.257 \pm 0.087$	$0.045 \pm 0.004 \pm 0.004$	$0.123 \pm 0.006 \pm 0.005$
3	$0.542 \pm 0.094 \pm 0.059$	$0.034 \pm 0.224 \pm 0.063$	$0.135 \pm 0.006 \pm 0.005$	$0.070 \pm 0.005 \pm 0.004$
4	$0.309 \pm 0.123 \pm 0.073$	$0.294 \pm 0.236 \pm 0.058$	$0.054 \pm 0.005 \pm 0.003$	$0.092 \pm 0.005 \pm 0.002$
5	$-0.492 \pm 0.130 \pm 0.041$	$0.665 \pm 0.256 \pm 0.100$	$0.069 \pm 0.004 \pm 0.002$	$0.045 \pm 0.004 \pm 0.002$
$\tilde{F}_+^{4\pi}$	$0.768 \pm 0.021 \pm 0.012$			
Q	$0.757 \pm 0.052 \pm 0.026$			

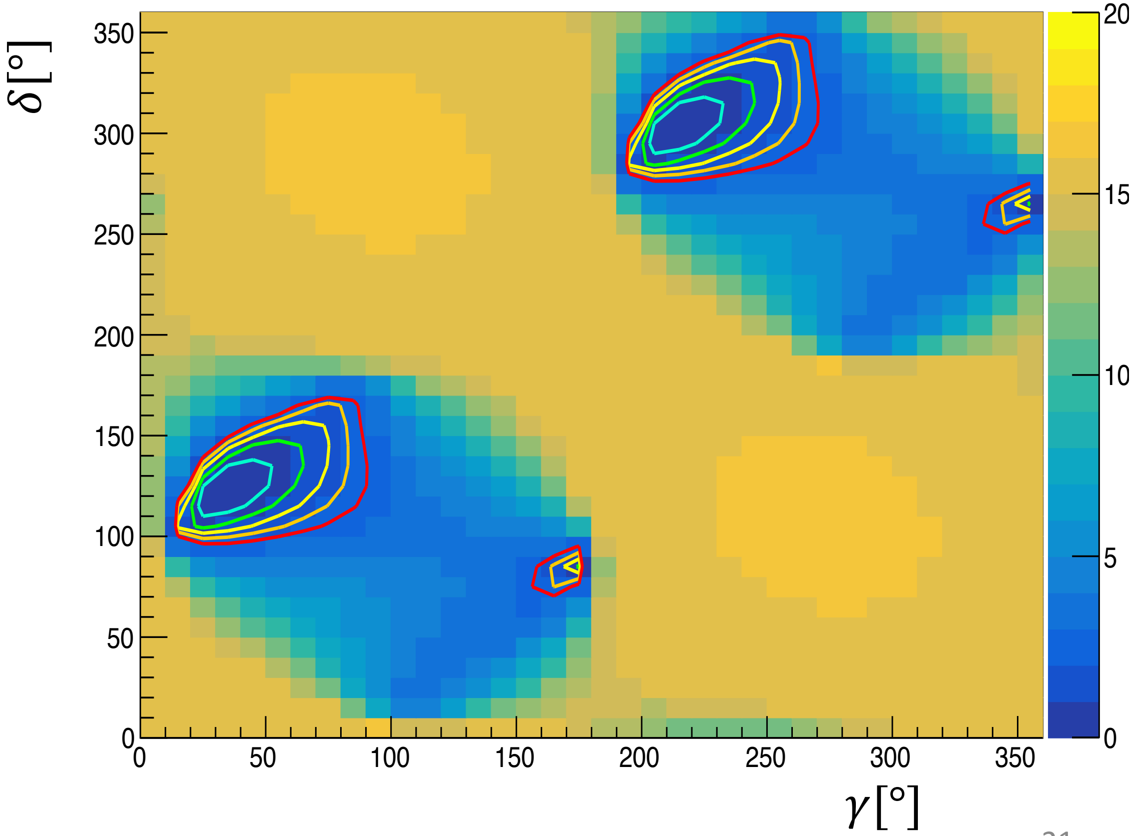
Optimal-alternative binning

i	c_i	s_i	T_i	$\overline{T}_i$
1	$0.279 \pm 0.143 \pm 0.101$	$-0.379 \pm 0.291 \pm 0.104$	$0.096 \pm 0.005 \pm 0.002$	$0.032 \pm 0.004 \pm 0.005$
2	$0.622 \pm 0.095 \pm 0.059$	$-0.486 \pm 0.237 \pm 0.072$	$0.123 \pm 0.006 \pm 0.004$	$0.055 \pm 0.004 \pm 0.003$
3	$0.969 \pm 0.057 \pm 0.038$	$-0.089 \pm 0.162 \pm 0.038$	$0.202 \pm 0.007 \pm 0.005$	$0.164 \pm 0.007 \pm 0.003$
4	$0.463 \pm 0.099 \pm 0.046$	$0.245 \pm 0.215 \pm 0.067$	$0.134 \pm 0.006 \pm 0.005$	$0.077 \pm 0.005 \pm 0.004$
5	$-0.332 \pm 0.138 \pm 0.041$	$0.484 \pm 0.263 \pm 0.132$	$0.074 \pm 0.005 \pm 0.002$	$0.043 \pm 0.004 \pm 0.003$
$\tilde{F}_+^{4\pi}$	$0.771 \pm 0.021 \pm 0.010$			
Q	$0.760 \pm 0.057 \pm 0.017$			

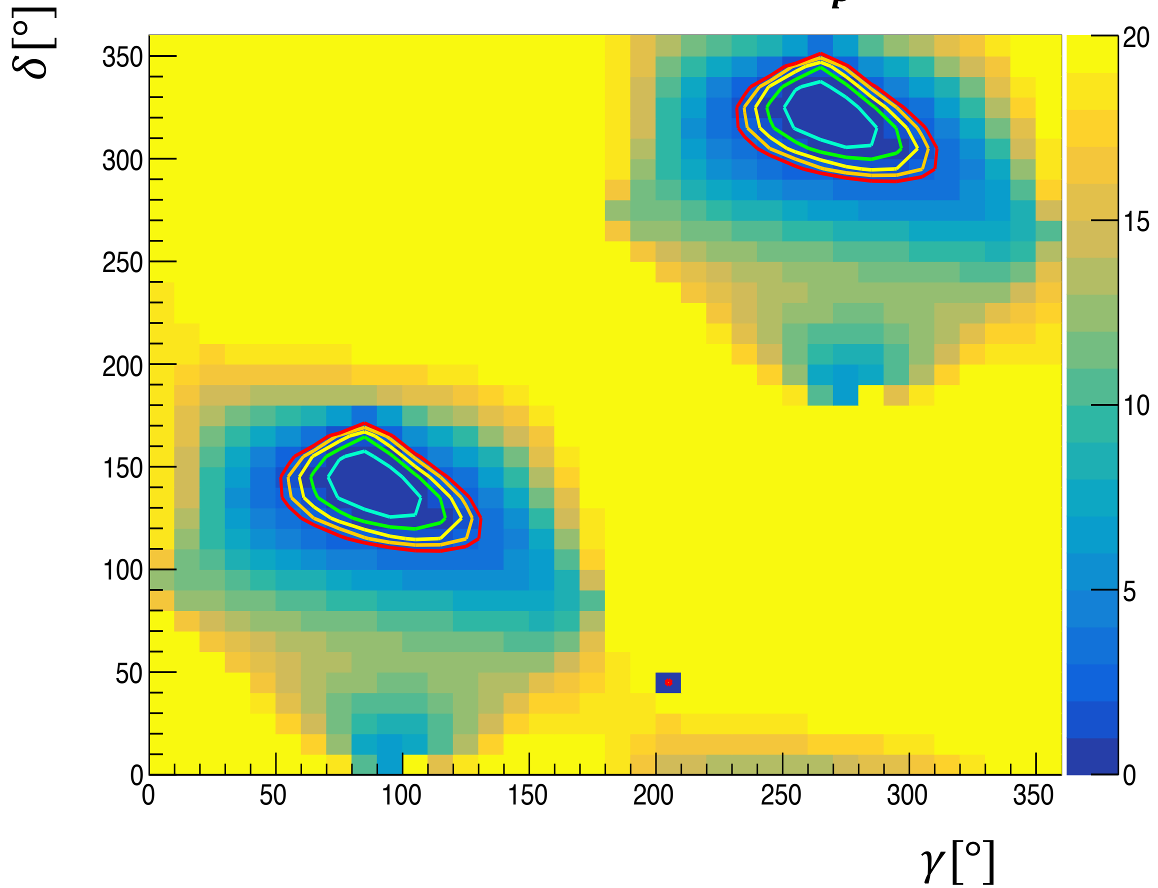
Likelihood scans for $\boldsymbol{\gamma}$, $\boldsymbol{r_B}$, $\boldsymbol{\delta_B}$ fitter

- Ambiguities arising from the $\cos(\delta_B \pm \gamma)$ and $\sin(\delta_B \pm \gamma)$ terms.
- Two primary maxima in both plots as expected from the cos and sin relation
- Two secondary maxima in Alternate binning are more prevalent than Equal $\Delta\delta_p^{4\pi}$ binning explains fitters reduced performance in the Alternate binning case.

Likelihood scan for Alternate binning



Likelihood scan for Equal $\Delta\delta_p^{4\pi}$ binning



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Full PDFs

$$PDF^{D\pi channel} = N_{D\pi sig}PDF^{D\pi sig} + N_{D\pi comb}PDF^{D\pi comb} + N_{DK\to D\pi}PDF^{DK\to D\pi} + N_{D\pi partial}PDF^{D\pi partial}$$

$where,$

$$N_{D\pi partial}PDF^{D\pi partial} = N_{partial1}PDF^{partial1} + N_{partial2}PDF^{partial2} + N_{partial3}PDF^{partial3} + N_{partial4}PDF^{partial4}$$

$$PDF^{DK\ channel} = N_{DKsig}PDF^{DKsig} + N_{DKcomb}PDF^{DKcomb} + N_{charmless}PDF^{charmless} + N_{D\pi\to DK}PDF^{D\pi\to DK} + N_{DKpartial}PDF^{DKpartial} + N_{D\pi\to DKpartial}PDF^{D\pi\to DKpartial}$$

$where,$

$$N_{DKpartial}PDF^{DKpartial} = N_{partial5}PDF^{partial5} + N_{partial6}PDF^{partial6} + N_{partial7}PDF^{partial7} + N_{partial8}PDF^{partial8} + N_{partial9}PDF^{partial9}$$

$$N_{D\pi\to DKpartial}PDF^{D\pi\to DKpartial} = N_{partia1\to 5}PDF^{partial1\to 5} + N_{partial2\to 6}PDF^{partial2\to 6} + N_{partial3\to 7}PDF^{partial3\to 7} + N_{partial4\to 8}PDF^{partial4\to 8}$$