

W-boson and Its Polarisation at NNLO

Andrei Popescu

in collaboration with

Rene Poncelet and Mathieu Pellen

based on 2102.13583, 2109.14336

Tuesday, 5 April 2022

Cavendish Laboratory



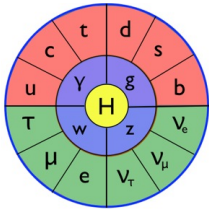
popescu@hep.phy.cam.ac.uk

Why study polarised W-boson at NNLO precision?

Why study polarised W-boson at NNLO precision?

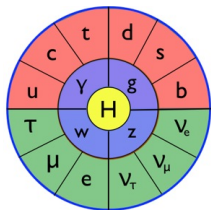
■ Probe the Electroweak mechanism:

- **Longitudinal polarisation** and massiveness of W^\pm , Z bosons is the direct consequence of the Electroweak symmetry breaking mechanism.
- Processes involving longitudinal boson **constrain** EFT operators.
- Any deviation from the Standard Model prediction will hint to **new physics**.



The Standard Model

Why study polarised W-boson at NNLO precision?



The Standard Model

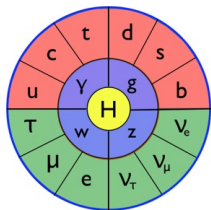
1 Probe the Electroweak mechanism:

- **Longitudinal polarisation** and massiveness of W^\pm , Z bosons is the direct consequence of the Electroweak symmetry breaking mechanism.
- Processes involving longitudinal boson **constrain** EFT operators.
- Any deviation from the Standard Model prediction will hint to **new physics**.

2 Use accumulated statistics

- Luminosities of Run 2 LHC (150 fb^{-1}) and beyond (Run 3: 300 fb^{-1} , High-lumi: 3000 fb^{-1}) will allow for **precise measurements** of processes involving weak bosons.

Why study polarised W-boson at NNLO precision?



The Standard Model

1 Probe the Electroweak mechanism:

- **Longitudinal polarisation** and massiveness of W^\pm , Z bosons is the direct consequence of the Electroweak symmetry breaking mechanism.
- Processes involving longitudinal boson **constrain** EFT operators.
- Any deviation from the Standard Model prediction will hint to **new physics**.

2 Use accumulated statistics

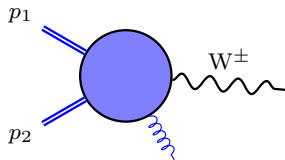
- Luminosities of Run 2 LHC (150 fb^{-1}) and beyond (Run 3: 300 fb^{-1} , High-lumi: 3000 fb^{-1}) will allow for **precise measurements** of processes involving weak bosons.

3 Squeeze out all what theory can offer

- Higher-order corrections are essential in reducing **theoretical uncertainty**:
LO $\sim 12\%$, NLO $\sim 7\%$, NNLO $\sim 2\%$ (process-dependent).

- As a massive spin-1 particle, the on-shell boson has a basis of 3 polarisation vectors ϵ_i^μ :

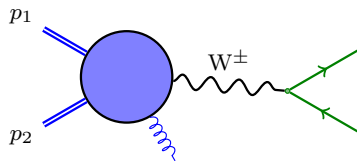
$$\epsilon_L = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad \epsilon_R = -\frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon_0 = \frac{1}{M}(p, 0, 0, E).$$



Boson Polarisation

- As a massive spin-1 particle, the on-shell boson has a basis of 3 polarisation vectors ϵ_i^μ :

$$\epsilon_L = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad \epsilon_R = -\frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon_0 = \frac{1}{M}(p, 0, 0, E).$$

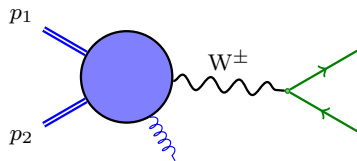


- Boson is short-lived ($\tau_W \approx 3 \cdot 10^{-25} s$), so we detect its **decay products**, which also prompt **interferences** between polarised amplitudes A_λ :

$$A_\lambda = \mathbf{P}_\mu \cdot \frac{\epsilon_\lambda^\mu \epsilon_\lambda^{\nu*}}{k^2 - M_V^2 + iM_V \Gamma_V} \cdot \mathbf{D}_\nu \quad \Rightarrow \quad A_{\text{unpol.}}^2 = \sum_\lambda |A_\lambda|^2 + \sum_{\lambda \neq \lambda'} A_\lambda^* \cdot A_{\lambda'}$$

- As a massive spin-1 particle, the on-shell boson has a basis of 3 polarisation vectors ϵ_i^μ :

$$\epsilon_L = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad \epsilon_R = -\frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon_0 = \frac{1}{M}(p, 0, 0, E).$$



- Boson is short-lived ($\tau_W \approx 3 \cdot 10^{-25} s$), so we detect its **decay products**, which also prompt **interferences** between polarised amplitudes A_λ :

$$A_\lambda = \mathbf{P}_\mu \cdot \frac{\epsilon_\lambda^\mu \epsilon_{\lambda'}^{\nu*}}{k^2 - M_V^2 + iM_V \Gamma_V} \cdot \mathbf{D}_\nu \quad \Rightarrow \quad A_{\text{unpol.}}^2 = \sum_\lambda |A_\lambda|^2 + \sum_{\lambda \neq \lambda'} A_\lambda^* \cdot A_{\lambda'}$$

- For polarisation interpretation one needs **on-shell** W-boson, which can be achieved via:
 - Narrow-width approximation:** generate on-shell phase space
 - On-shell projection:** map full phase space to on-shell phase space

These methods introduce *off-shell effects*, which are roughly of order $\Gamma_W/M_W \approx 0.025$.

Boson Polarisation (2)

- The goal is to break down differential distributions into **polarised** components:

$$\frac{d\sigma}{dX} \rightarrow \frac{d\sigma_L}{dX} + \frac{d\sigma_R}{dX} + \frac{d\sigma_0}{dX}$$

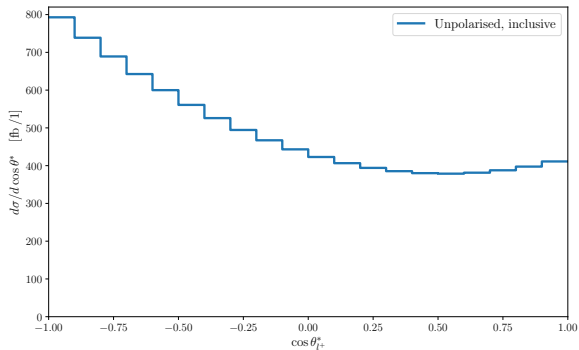


Figure: Angle of lepton emission for W^+ at the LHC

Boson Polarisation (2)

- The goal is to break down differential distributions into **polarised** components:

$$\frac{d\sigma}{dX} \rightarrow \frac{d\sigma_L}{dX} + \frac{d\sigma_R}{dX} + \frac{d\sigma_0}{dX}$$

- Can be done at amplitude level:

$$\left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) \rightarrow \epsilon_\lambda \epsilon_\lambda$$

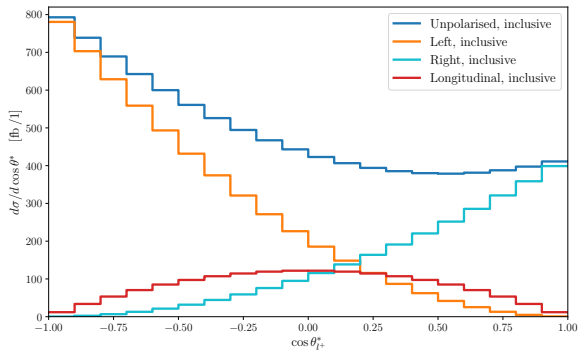


Figure: Angle of lepton emission for W^+ at the LHC

Boson Polarisation (2)

- The goal is to break down differential distributions into **polarised** components:

$$\frac{d\sigma}{dX} \rightarrow \frac{d\sigma_L}{dX} + \frac{d\sigma_R}{dX} + \frac{d\sigma_0}{dX}$$

- Can be done at amplitude level:

$$\left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right) \rightarrow \epsilon_\lambda \epsilon_\lambda$$

- With **no cuts**, angular emission distribution is described analytically:

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta} = \frac{3}{4} \sin^2\theta \cdot f_0 + \frac{3}{8} (1 \mp \cos\theta)^2 \cdot f_L + \frac{3}{8} (1 \pm \cos\theta)^2 \cdot f_R$$

where f_i are coefficients that depend on production part of the process.

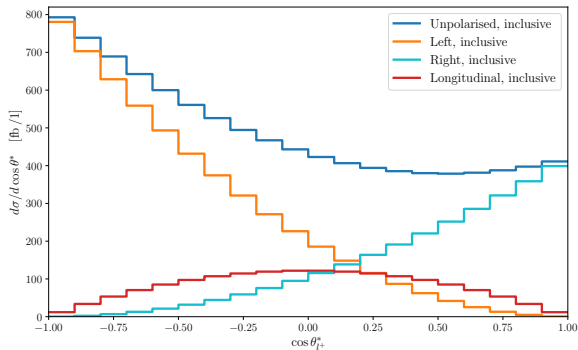


Figure: Angle of lepton emission for W^+ at the LHC

Boson Polarisation (3)

- We focus on the longitudinal polarisation, so we use transverse polarisation as correlated sum of left and right pol.:

$$\frac{d\sigma_T}{dX} \equiv \frac{d\sigma_L}{dX} + \frac{d\sigma_R}{dX} + \frac{d\sigma_{\text{interf.}}}{dX}$$

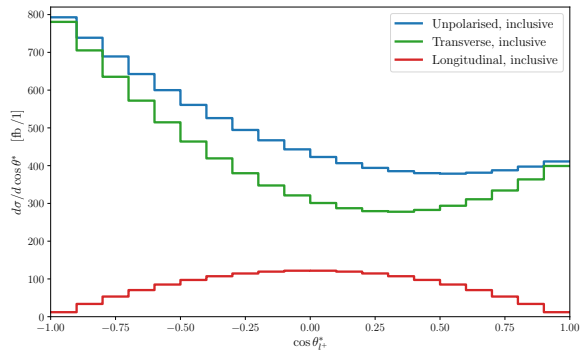


Figure: Angle of lepton emission for W^+ at the LHC

Boson Polarisation (3)

- We focus on the longitudinal polarisation, so we use transverse polarisation as correlated sum of left and right pol.:

$$\frac{d\sigma_T}{dX} \equiv \frac{d\sigma_L}{dX} + \frac{d\sigma_R}{dX} + \frac{d\sigma_{\text{interf.}}}{dX}$$

- Ratios to polarisation sum define **polarisation shapes** which can be used in experimental fits to **unpolarised** data.

The cross section breaks down into polarisation shapes and fractions:

$$\frac{d\sigma}{dX} \rightarrow f_0 \cdot \left[\frac{d\sigma_0}{dX} \right]_{\text{norm.}} + f_T \cdot \left[\frac{d\sigma_T}{dX} \right]_{\text{norm.}}$$

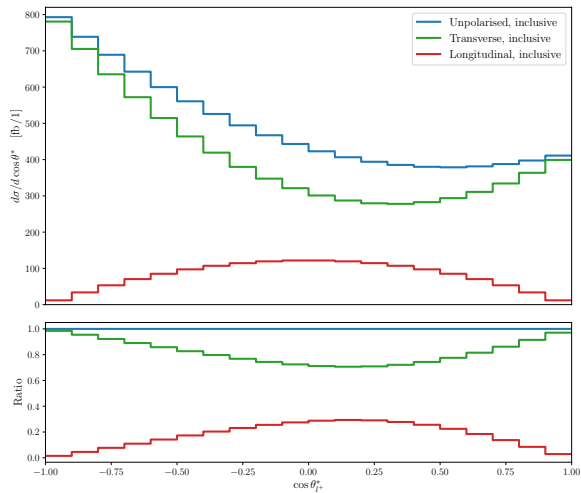


Figure: Angle of lepton emission for W^+ at the LHC

Boson Polarisation (3)

- We focus on the longitudinal polarisation, so we use transverse polarisation as correlated sum of left and right pol.:

$$\frac{d\sigma_T}{dX} \equiv \frac{d\sigma_L}{dX} + \frac{d\sigma_R}{dX} + \frac{d\sigma_{\text{interf.}}}{dX}$$

- Ratios to polarisation sum define **polarisation shapes** which can be used in experimental fits to **unpolarised** data.

The cross section breaks down into polarisation shapes and fractions:

$$\frac{d\sigma}{dX} \rightarrow f_0 \cdot \left[\frac{d\sigma_0}{dX} \right]_{\text{norm.}} + f_T \cdot \left[\frac{d\sigma_T}{dX} \right]_{\text{norm.}}$$

- Experimental cuts modify polarisation shapes and introduce interferences.

→ We need to pick **phase space regions** where interference and off-shell effects are small.

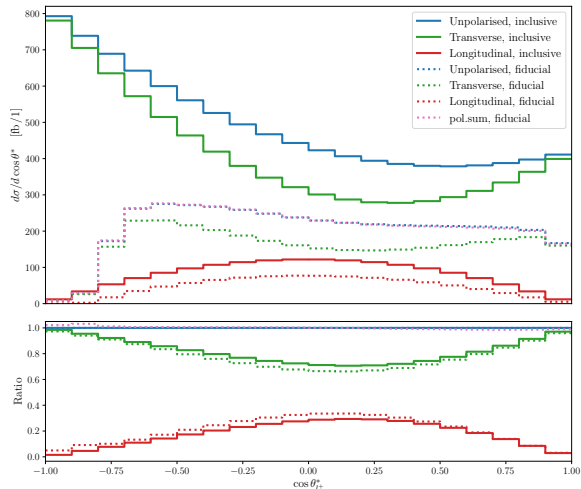
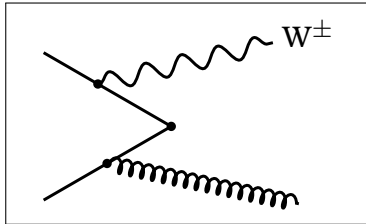


Figure: Angle of lepton emission for W^+ at the LHC

W in Association With One Jet



Literature

- Experimental
 - Polarised W+j
[ATLAS 1203.2165, CMS 1104.3829]
- Theoretical
 - Polarised W+j up to NLO QCD
[Bern et al. 1103.5445]
[Stirling et al. 1204.6427]
[Belyaev et al. 1303.3297]

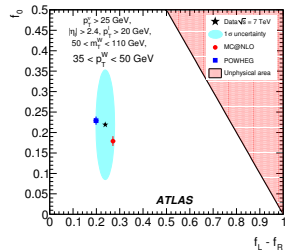
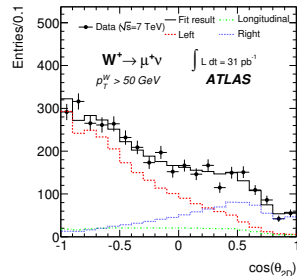


Figure: ATLAS measurements (2012)

Literature

- Experimental
 - Polarised W+j
[ATLAS 1203.2165, CMS 1104.3829]
- Theoretical
 - Polarised W+j up to NLO QCD
[Bern et al. 1103.5445]
[Stirling et al. 1204.6427]
[Belyaev et al. 1303.3297]
 - Unpolarised W+j up to NNLO QCD + NLO EW
[Denner et al. 0906.1656]
[Boughezal et al. 1602.06965, 1504.02131]
[Gehrmann et al. 1901.11041]

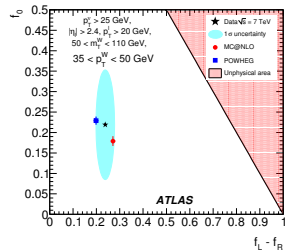
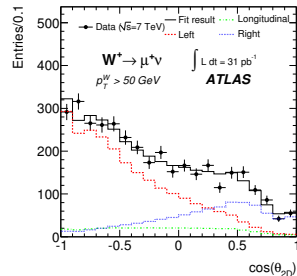


Figure: ATLAS measurements (2012)

Literature

- Experimental
 - Polarised W+j
[ATLAS 1203.2165, CMS 1104.3829]
- Theoretical
 - Polarised W+j up to NLO QCD
[Bern et al. 1103.5445]
[Stirling et al. 1204.6427]
[Belyaev et al. 1303.3297]
 - Unpolarised W+j up to NNLO QCD + NLO EW
[Denner et al. 0906.1656]
[Boughezal et al. 1602.06965, 1504.02131]
[Gehrmann et al. 1901.11041]
- Our contribution
 - Polarised W+j at NNLO
[Pellen, Poncelet, AP 2109.14336]

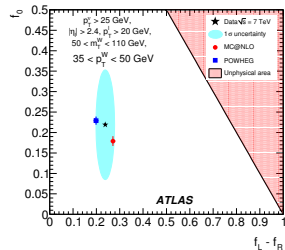
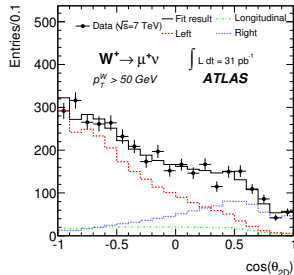
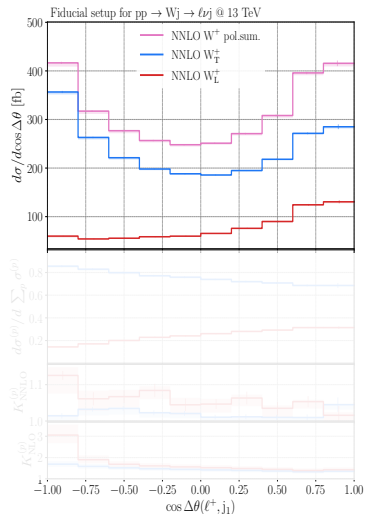
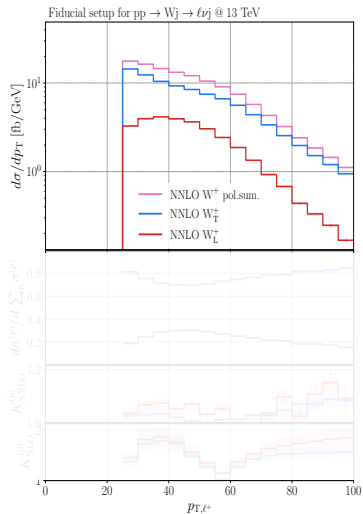


Figure: ATLAS measurements (2012)

W in Association With One Jet ::: Sensitive observables

- Looking for distinctive polarisation shapes (i.e not flat)

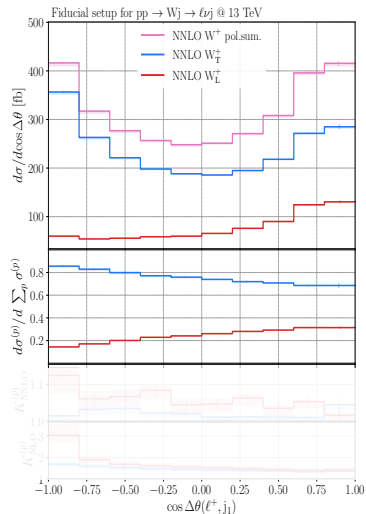
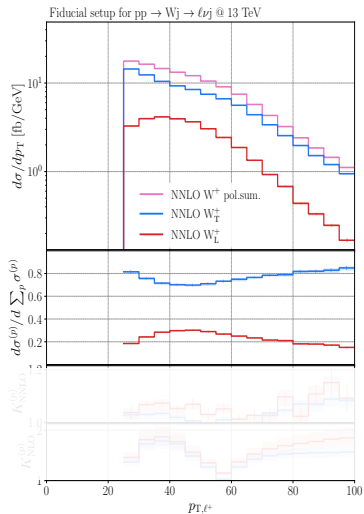


W in Association With One Jet ::: Sensitive observables

- Looking for distinctive polarisation shapes (i.e not flat)

e.g:

- lepton p_T
- polar angle between ℓ^\pm , jet



W in Association With One Jet ::: Sensitive observables

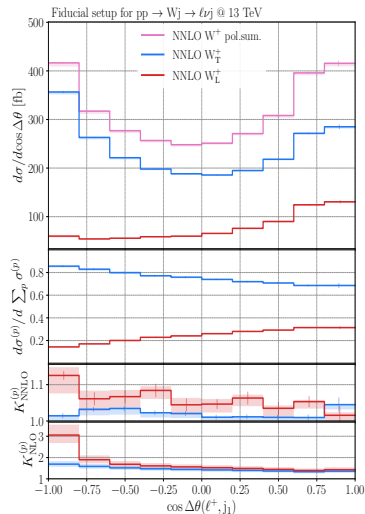
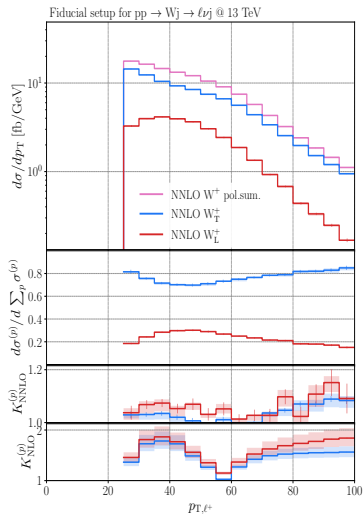
- Looking for distinctive polarisation shapes (i.e not flat)

e.g:

- lepton p_T
- polar angle between ℓ^\pm , jet

- QCD corrections:

- NLO: up to 200-1000% due to new kinematic freedom
- NNLO: 5-10%



W in Association With One Jet ::: Sensitive observables

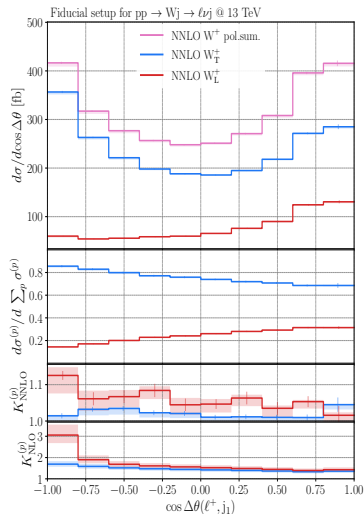
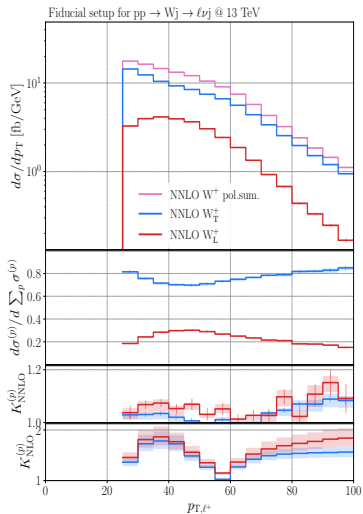
- Looking for distinctive polarisation shapes (i.e not flat)

e.g:

- lepton p_T
- polar angle between ℓ^\pm , jet

- QCD corrections:

- NLO: up to 200-1000% due to new kinematic freedom
- NNLO: 5-10%
- Longitudinal W is affected more than transverse by a factor of 3 at NNLO



W in Association With One Jet ::: Sensitive observables

- Looking for distinctive polarisation shapes (i.e not flat)

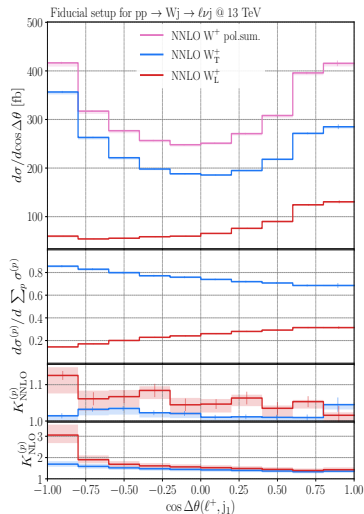
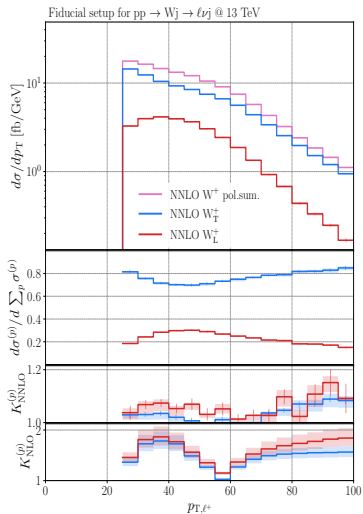
e.g:

- lepton p_T
- polar angle between ℓ^\pm , jet

- QCD corrections:

- NLO: up to 200-1000% due to new kinematic freedom
- NNLO: 5-10%
- Longitudinal W is affected more than transverse by a factor of 3 at NNLO
- Theoretical uncertainty drops from $\sim 7\%$ at NLO to $\sim 2\%$ at NNLO.

Importantly, theoretical uncertainty on polarised shapes is reduced at NNLO.



- Due to **large systematic errors**, currently published data do not allow for precise shape fits.

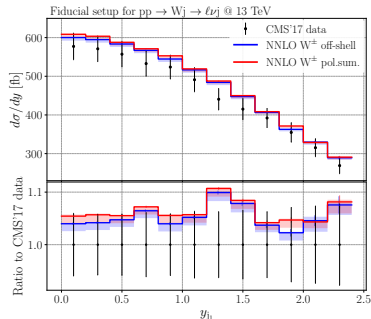


Figure: CMS'17 data shows **6-9% errors** on distribution of rapidity of the hardest jet.

W in Association With One Jet :: Fit to extrapolated data

- However, we have determined that at **projected luminosity** of 250 fb^{-1} (mid-Run 3), NNLO corrections shrink overlapping fit uncertainty of the fit by a factor of 2 across several observables.
 - distribution of fits (\mathbf{x}) for each scale is much denser for NNLO shapes.
 - fit uncertainty depends on data errors

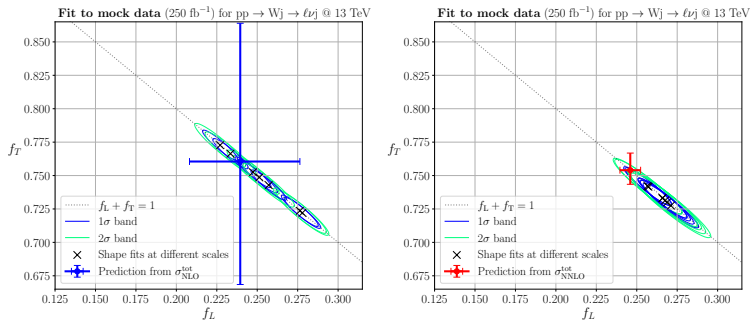
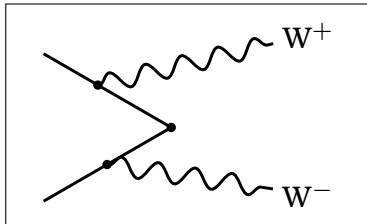


Figure: Fit of polarised shapes at NLO (left) and NNLO (right) to mock data (off-shell distribution) for the distribution of angle between the lepton and the hardest jet.

Diboson production



Literature

- Theoretical
 - EFT operators and polarised measurements
[Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD
[Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA)
[Billoni et al. 1310.1564]
[Ballestrero et al. 1710.09339, 1907.04722]

Literature

- Theoretical
 - EFT operators and polarised measurements
[Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD
[Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA)
[Billoni et al. 1310.1564]
[Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW
[Caola et al. 1511.08617]
[Grazzini et al. 1605.02716, 1912.00068]
[Lombardi et al. 2103.12077]

Literature

- Theoretical
 - EFT operators and polarised measurements
[Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD
[Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA)
[Billoni et al. 1310.1564]
[Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW
[Caola et al. 1511.08617]
[Grazzini et al. 1605.02716, 1912.00068]
[Lombardi et al. 2103.12077]
- Experimental
 - ATLAS $W^\pm Z$ measurement (13 TeV) [1902.05759]

Literature

- Theoretical
 - EFT operators and polarised measurements
[Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD
[Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA)
[Billoni et al. 1310.1564]
[Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW
[Caola et al. 1511.08617]
[Grazzini et al. 1605.02716, 1912.00068]
[Lombardi et al. 2103.12077]
- Experimental
 - ATLAS $W^\pm Z$ measurement (13 TeV) [1902.05759]
- Our contribution
 - Polarised W^+W^- at NNLO (fully-leptonic)
[Poncelet, AP 2102.13583]
First polarised prediction at NNLO!

Literature

- Theoretical
 - EFT operators and polarised measurements
[Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD
[Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA)
[Billoni et al. 1310.1564]
[Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW
[Caola et al. 1511.08617]
[Grazzini et al. 1605.02716, 1912.00068]
[Lombardi et al. 2103.12077]
- Experimental
 - ATLAS $W^\pm Z$ measurement (13 TeV) [1902.05759]
- Our contribution
 - Polarised W^+W^- at NNLO (fully-leptonic)
[Poncelet, AP 2102.13583]
First polarised prediction at NNLO!

Features of W^+W^- (fully-leptonic)

Literature

- Theoretical
 - EFT operators and polarised measurements [Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD [Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA) [Billoni et al. 1310.1564] [Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW [Caola et al. 1511.08617] [Grazzini et al. 1605.02716, 1912.00068] [Lombardi et al. 2103.12077]
- Experimental
 - ATLAS $W^\pm Z$ measurement (13 TeV) [1902.05759]
- Our contribution
 - Polarised W^+W^- at NNLO (fully-leptonic) [Poncelet, AP 2102.13583]
First polarised prediction at NNLO!

Features of W^+W^- (fully-leptonic)

- Largest cross-section among other diboson processes (setup of [1912.00068])

$$\sigma_{WW} \approx 415 \text{ fb} \quad \sigma_{ZZ} \approx 40 \text{ fb} \quad \sigma_{WZ} \approx 28 \text{ fb}$$

Literature

- Theoretical
 - EFT operators and polarised measurements [Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD [Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA) [Billoni et al. 1310.1564] [Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW [Caola et al. 1511.08617] [Grazzini et al. 1605.02716, 1912.00068] [Lombardi et al. 2103.12077]
- Experimental
 - ATLAS $W^\pm Z$ measurement (13 TeV) [1902.05759]
- Our contribution
 - Polarised W^+W^- at NNLO (fully-leptonic) [Poncelet, AP 2102.13583]
 - First polarised prediction at NNLO!*

Features of W^+W^- (fully-leptonic)

- Largest cross-section among other diboson processes (setup of [1912.00068])

$$\sigma_{WW} \approx 415 \text{ fb} \quad \sigma_{ZZ} \approx 40 \text{ fb} \quad \sigma_{WZ} \approx 28 \text{ fb}$$

- Polarised W^+W^- channels probe EFT operators

Literature

- Theoretical
 - EFT operators and polarised measurements [Liu, Wang 1804.08688]
 - Polarised diboson studies at NLO QCD [Denner et al. 2006.14867, 2010.07149]
 - Double-pole approximation (DPA) [Billoni et al. 1310.1564] [Ballestrero et al. 1710.09339, 1907.04722]
 - Off-shell production up to NNLO QCD + NLO EW [Caola et al. 1511.08617] [Grazzini et al. 1605.02716, 1912.00068] [Lombardi et al. 2103.12077]
- Experimental
 - ATLAS $W^\pm Z$ measurement (13 TeV) [1902.05759]
- Our contribution
 - Polarised W^+W^- at NNLO (fully-leptonic) [Poncelet, AP 2102.13583]

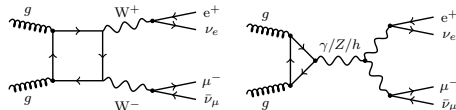
First polarised prediction at NNLO!

Features of W^+W^- (fully-leptonic)

- Largest cross-section among other diboson processes (setup of [1912.00068])

$$\sigma_{WW} \approx 415 \text{ fb} \quad \sigma_{ZZ} \approx 40 \text{ fb} \quad \sigma_{WZ} \approx 28 \text{ fb}$$

- Polarised W^+W^- channels probe EFT operators
- Loop-induced (LI) channel opens up at $\mathcal{O}(\alpha_s^2)$



→ expect a large $\mathcal{O}(\alpha_s^2)$ contribution

Diboson Production ::: Polarised distributions

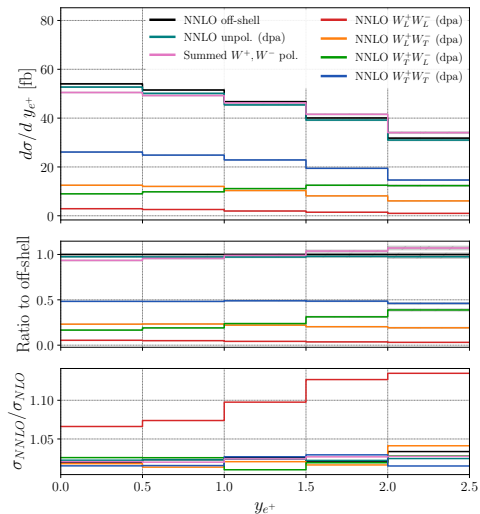


Figure: Positive lepton rapidity at NNLO

Diboson Production ::: Polarised distributions

- Polarisation fractions

TT	TL	LT	LL
48.5%	24.9%	22.1%	4.4%

→ Production is dominated by $W_T^+ W_T^-$, especially at high energies

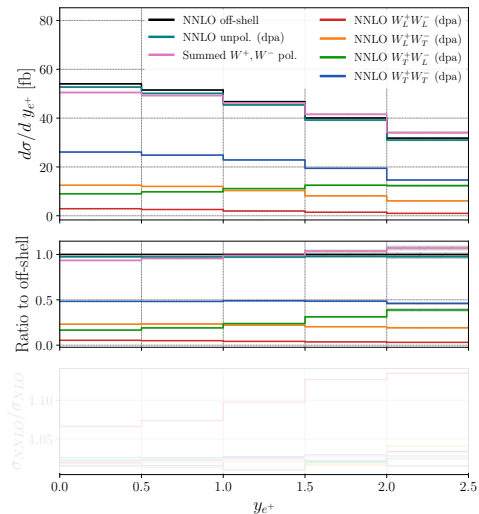


Figure: Positive lepton rapidity at NNLO

Diboson Production ::: Polarised distributions

- Polarisation fractions

TT	TL	LT	LL
48.5%	24.9%	22.1%	4.4%

→ Production is dominated by $W_T^+ W_T^-$, especially at high energies

- QCD corrections

order	TT	TL	LT	LL
NLO	7%	8%	9%	36%
NNLO	2%	2%	2%	9%

→ small except for LL

→ distinct corrections profile for LL

→ theoretical uncertainty reduced by 3

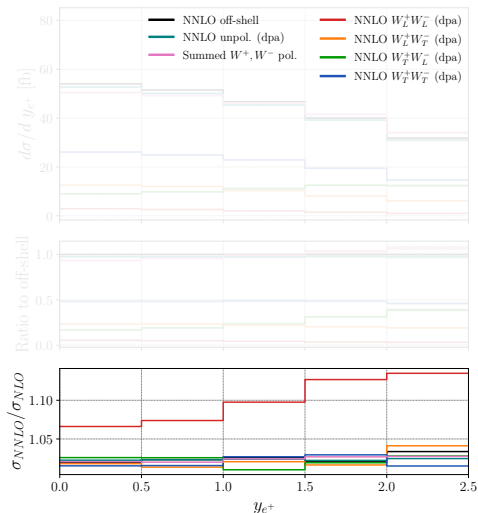


Figure: Positive lepton rapidity at NNLO

- QCD corrections

order	TT	TL	LT	LL
NNLO	2%	2%	2%	9%
LI	9%	3%	4%	8%

- distinct corrections profile for LL
- theoretical uncertainty increased by 2

⇒ this calls for $\mathcal{O}(\alpha_s^3)$ corrections.

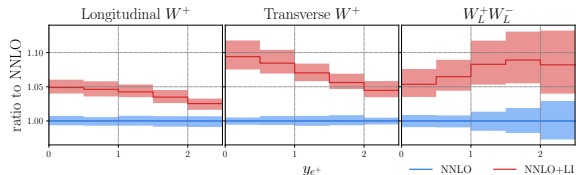


Figure: Loop-induced channel corrections in lepton rapidity.

Summary

- Polarised bosons are a focused test of the Standard Model
- We provided the first two NNLO-precise studies of polarised W production
- Higher order QCD corrections are:
 - stable
 - particularly strong for the longitudinal mode
 - crucial for precision studies
- Looking forward to:
 - theory improvements, e.g higher-order corrections to the loop-induced channel in W^+W^-
 - measurements in diboson processes and an update on $W+j$

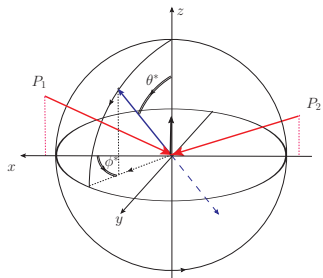
- Polarised bosons are a focused test of the Standard Model
- We provided the first two NNLO-precise studies of polarised W production
- Higher order QCD corrections are:
 - stable
 - particularly strong for the longitudinal mode
 - crucial for precision studies
- Looking forward to:
 - theory improvements, e.g higher-order corrections to the loop-induced channel in W^+W^-
 - measurements in diboson processes and an update on $W+j$

Thank you for your attention!

Appendix

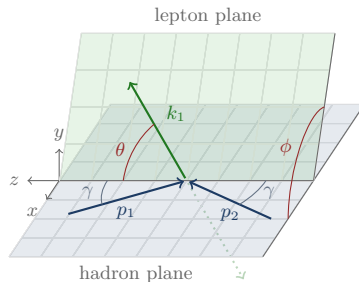
Reference frames

- *Helicity frame* is a rest frame of the W-boson, where z-axis is directed along the boson momentum in lab frame, and x-axis belongs to the plane containing boosted parton momenta \vec{p}'_1, \vec{p}'_2 .



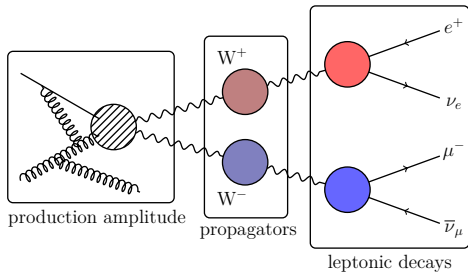
Helicity frame [1103.5445]

- *Collins-Soper (CS) frame* is a rest frame of the W-boson in which the z-axis is taken to bisect the opening angle between incoming proton momenta \vec{p}_1, \vec{p}_2 , and the y-axis is along the direction of $\vec{p}_1 \times \vec{p}_2$. The azimuthal angle is measured from the x-axis which lies in the scattering plane [hep-ph/0604208].



Collins-Soper frame [1708.00008]

Method: Double-pole approximation



Full amplitude is approximated by **on-shell sub-amplitudes** and the **off-shell propagators**:

$$A \approx \sum_{\alpha, \beta} A_{\alpha\beta, \mu\nu}^{pp \rightarrow W^+ W^-} \frac{A_{\alpha}^{\mu}(W^+ \rightarrow l^+ \nu) \cdot A_{\beta}^{\nu}(W^- \rightarrow l^- \bar{\nu})}{(p_+^2 - M^2) \cdot (p_-^2 - M^2)},$$

boson polarisations

- A selected **on-shell projection** defined on-shell sub-amplitudes
(we choose to preserve leptonic angles in the decay frames, and boson angles in the diboson frame).
- Cross-term amplitude contributions coming from $A_{\alpha} A_{\bar{\alpha}}$ terms create **interferences** for cross sections.

W in Association With One Jet: Off-shell and interference effects

- Off-shell effects mean that on-shell approximation/projection does not agree with off-shell calculation:

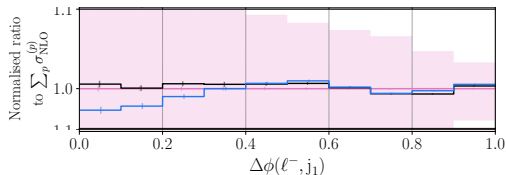
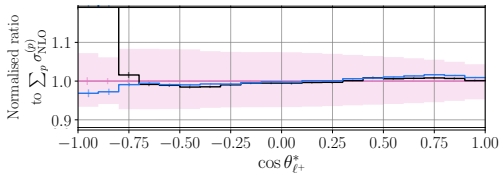
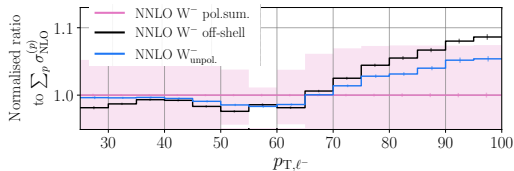
$$\frac{d\sigma_{\text{off-shell}}}{dX} \neq \frac{d\sigma_{\text{unpol}}}{dX}$$

- Interference effects cause polarised sum to break down:

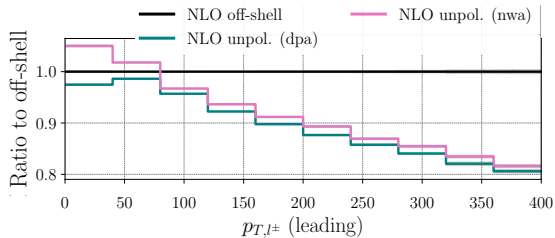
$$\frac{d\sigma_{\text{unpol}}}{dX} \neq \frac{d\sigma_0}{dX} + \frac{d\sigma_T}{dX} \equiv \frac{d\sigma_{\text{pol.sum}}}{dX}$$

- Regions suitable for polarisation fits:

- $\rightarrow \Delta\phi(\ell^-, j_1) > 0.3$
- $\rightarrow 25 < p_{T,\ell} < 70 \text{ GeV}$
- $\rightarrow \cos\theta_{\ell^+}^* > -0.75$
- $\rightarrow |y_j| < 2 \text{ GeV}$



Diboson Production: Approximations



Unpolarised narrow-width (NWA), double-pole (DPA) approximations, and off-shell.