Decoding Dark Matter at future e^+e^- colliders

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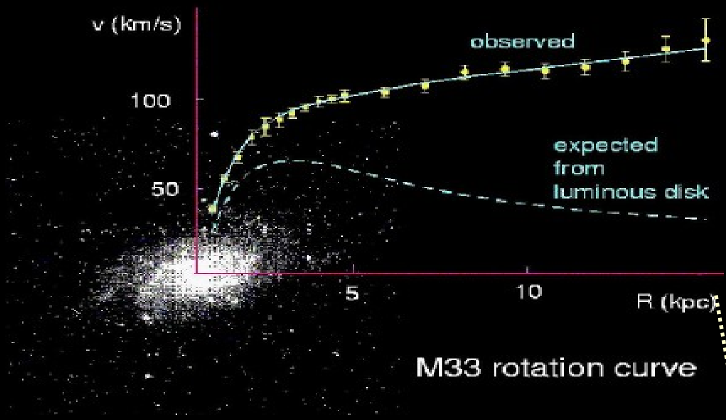
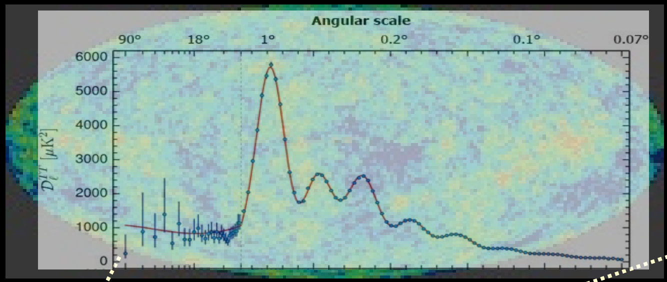
The existence of Dark Matter is confirmed by several independent observations at cosmological scale

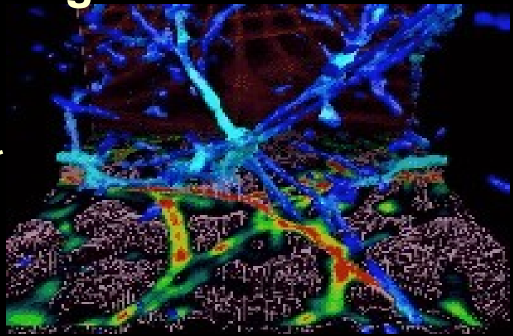
Galactic rotation curves

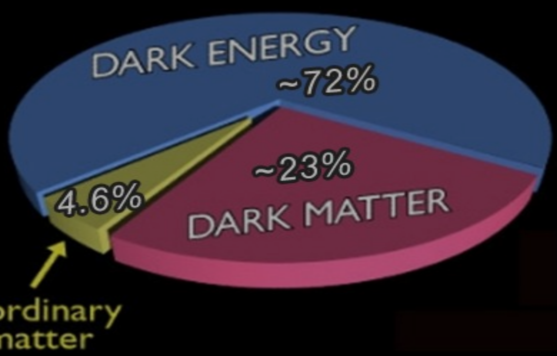


CMB: WMAP and PLANCK



Large Scale Structures





Gravitational lensing

Bullet cluster

DM is very appealing even though we know almost nothing about it!

Spin ?

Mass ?

Stable

Yes ? No ?

symmetry behind stability ?

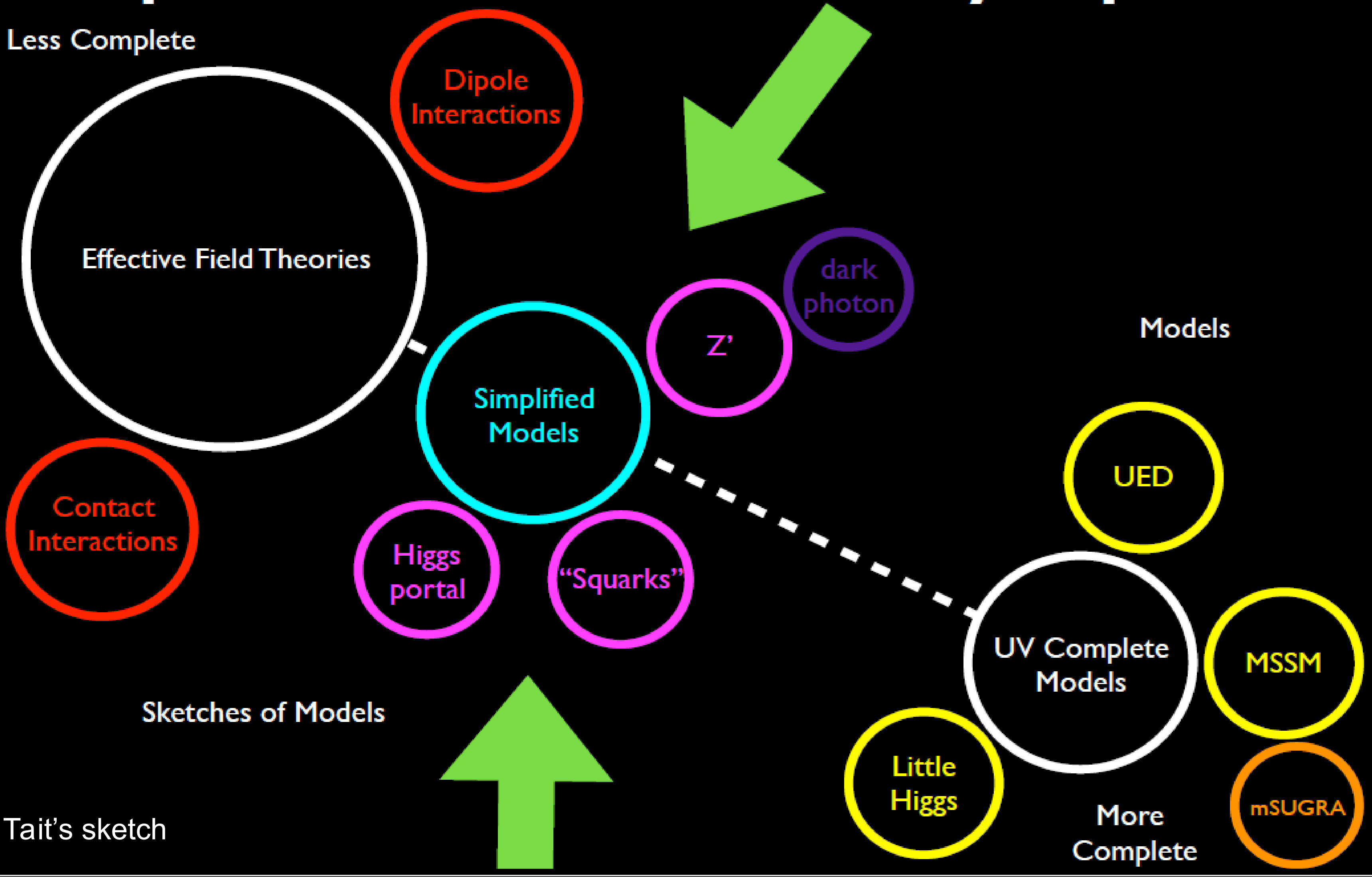
Couplings

- gravity V
- weak ?
- higgs ?
- quarks/gluons ?
- leptons ?
- New mediators ?

Thermal relic

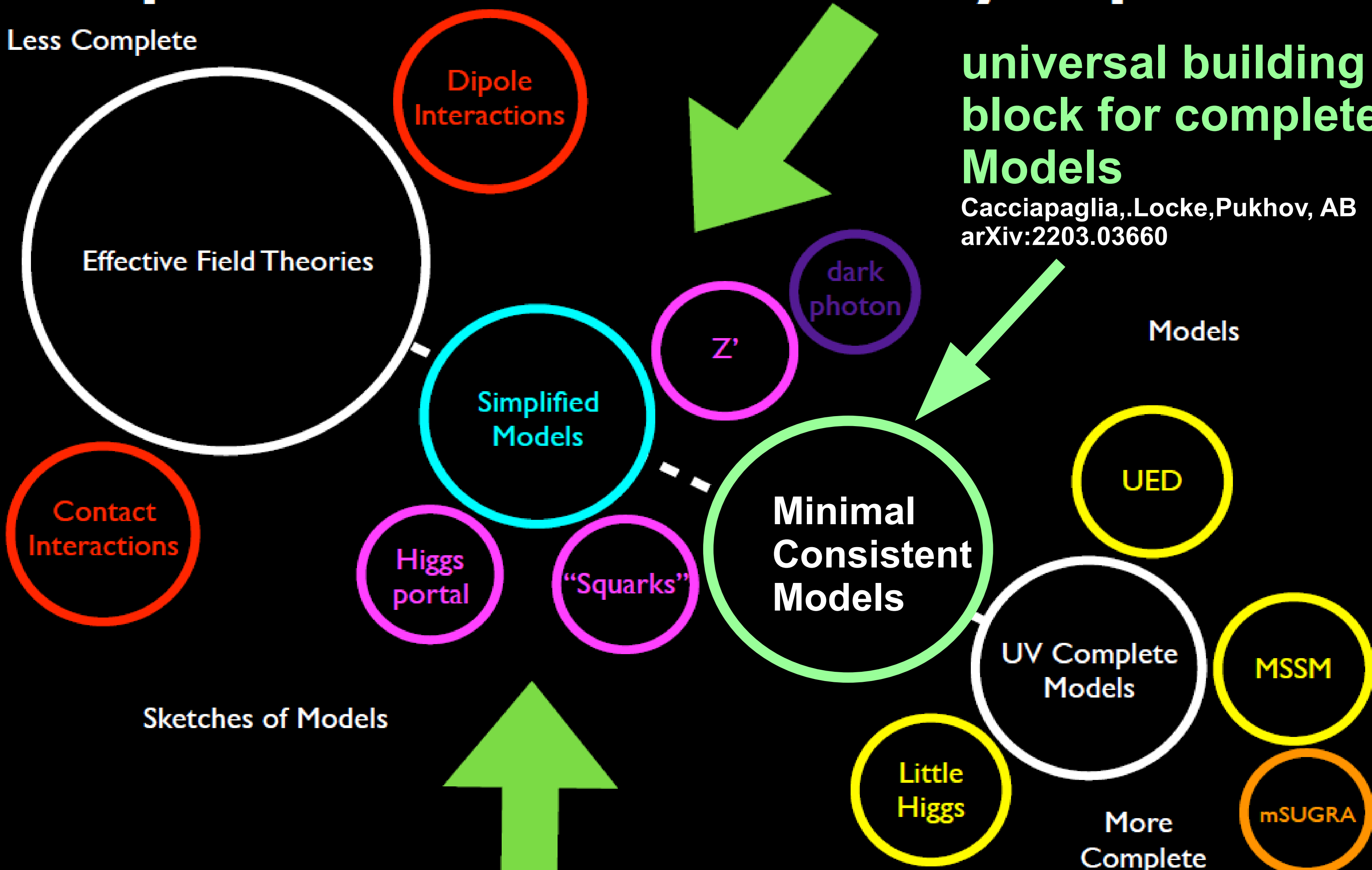
Yes ? No ?

Spectrum of Theory Space



Tim Tait's sketch

Spectrum of Theory Space

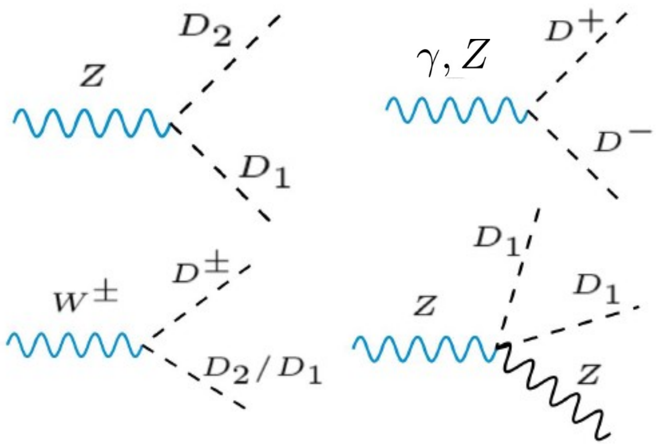


Inert 2 Higgs Doublet model (i2HDM)

$\tilde{S}^{1/2}_{1/2}$

$$\mathcal{L}_\phi = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - V(\phi_1, \phi_2)$$

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}, \quad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} D^+ \\ D_1 + i D_2 \end{pmatrix}$$



$$M_{D1}, \quad \Delta M^+ = M_{D+} - M_{D1}, \quad \Delta M^0 = M_{D2} - M_{D+}$$

Minimal fermion DM model (MFDM)

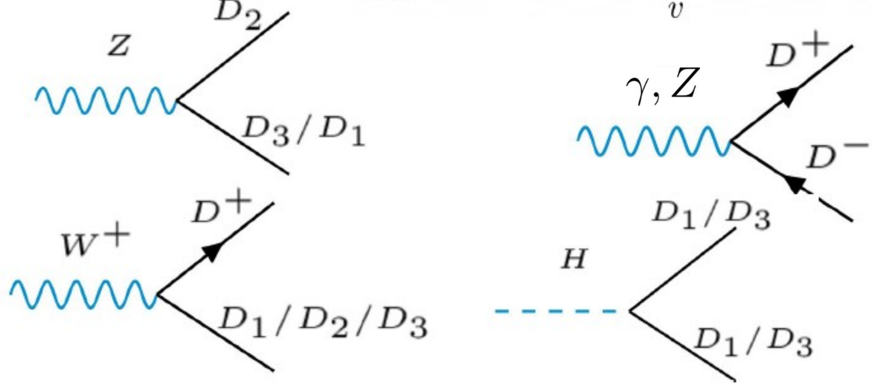
$\widetilde{F}^{1/2}_{1/2} \widetilde{M}^0_0$

$$\mathcal{L}_{FDM} = \mathcal{L}_{SM} + \bar{\psi}(i\not{D} - m_\psi)\psi + \frac{1}{2}\bar{\chi}^0_s(i\not{\partial} - m_s)\chi^0_s - (Y_{DM}(\bar{\psi}\Phi\chi^0_s) + h.c.)$$

$$\psi = \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}}(\chi^0_1 + i\chi^0_2) \end{pmatrix}$$

Majorana singlet χ^0_s

$$Y_{DM} = \frac{\sqrt{(m_{D3} - m_{D+})(m_{D+} - m_{D1})}}{v}$$



$$M_{D1} \quad \Delta M^+ = M_{D+} - M_{D1}, \quad \Delta M^0 = M_{D3} - M_{D+}$$

Benchmarks and tools

- CalcHEP+PYTHIA8+Delphes3
- ISR+Beamstrahlung (CalcHEP)

  ILC 500 Gev design (from ILC TDR)

Parameters \ Benchmarks		BP1	BP2
M_D		60	60
M_+		160	120
M_{D_2}		160.85	120.85
I2HDM parameters			
λ_{345}		6.5×10^{-4}	7.0×10^{-4}
λ_2		1.0	1.0
DM observables			
Ωh^2	SDM	0.111	0.112
	FDM	0.108	0.109
σ^p_{SI}[pb]	SDM	6.17×10^{-13}	6.17×10^{-13}
	FDM	1.67×10^{-11}	1.65×10^{-11}

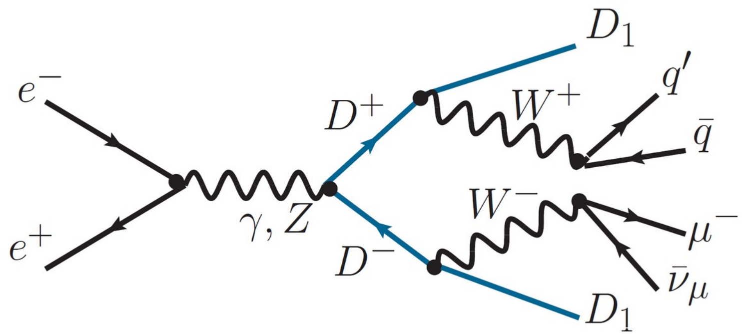
```
<
 ISR scale          = 1.00E+00*sqrtS
 Beamstralung             ON
 Bunch x+y sizes (nm)= 500.0
 Bunch lenght (mm)   = 0.300
 Number of particles = 2.0e+10
          *   N_gamma = 1.71
          *   Upsilon = 0.06
 Beamstrahlung F(x) plot
 Beamstrahlung F(x)*(1-x)^(2/3)
```

- MicrOMEGAs
  - relic density
  - DM DD and ID detection
  - Invisible Higgs decay

    (under control – the small value of $M_{D2} - M_+$ split)
- CheckMATE
  - test against LHC current limits

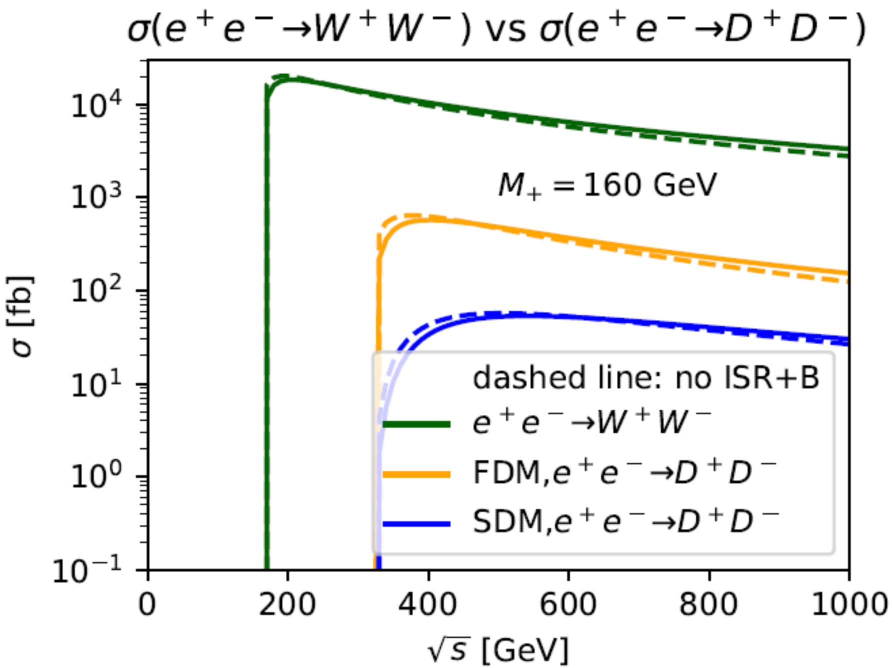
The process under study

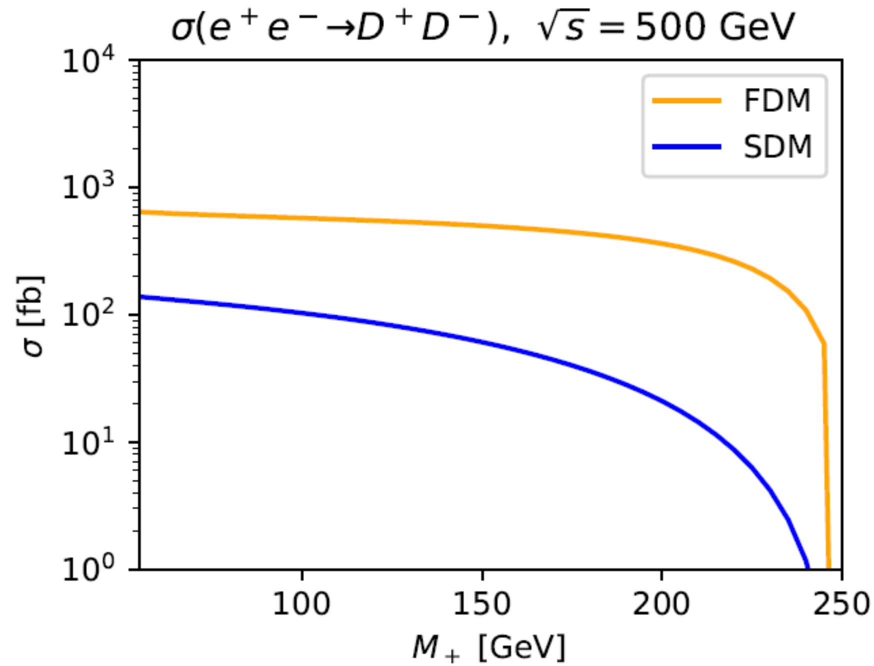
$$e^+e^- \to D^+D^- \to D_1 D_1 W^+W^- \to D_1 D_1 q'\bar{q}\mu\bar{\nu}$$



- Di-jet + muon + MET signature

$$\sigma_{\gamma\gamma} = \begin{cases} \sigma_0 \beta_+ \left[1 + \frac{2M_+^2}{s}\right] & \text{if } s_D = \frac{1}{2} \\ \sigma_0 \frac{\beta_+^3}{4} & \text{if } s_D = 0 \end{cases}$$

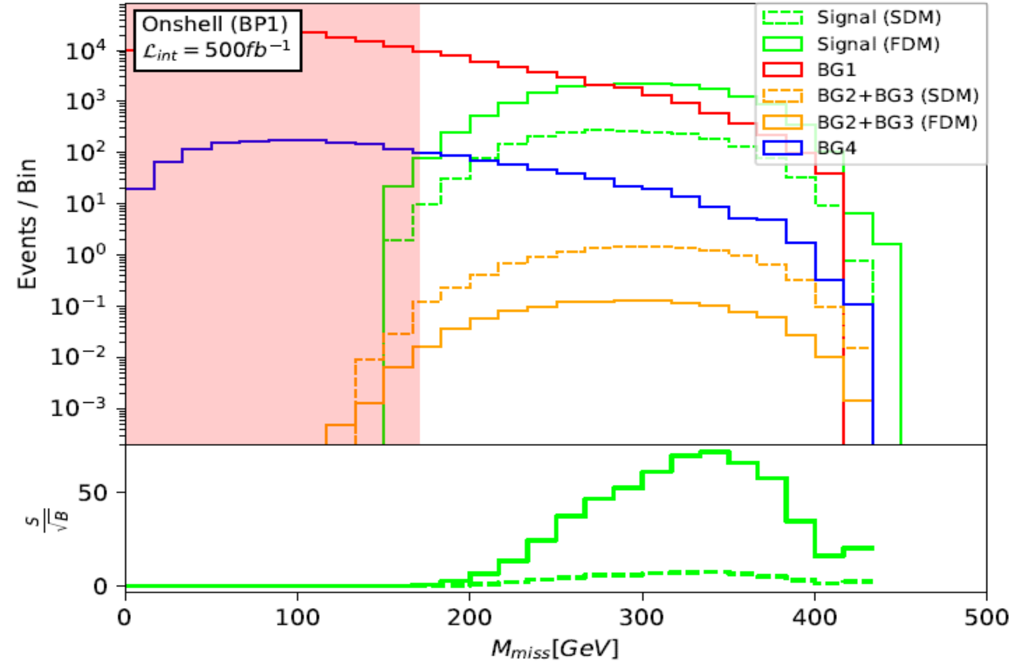




Observables

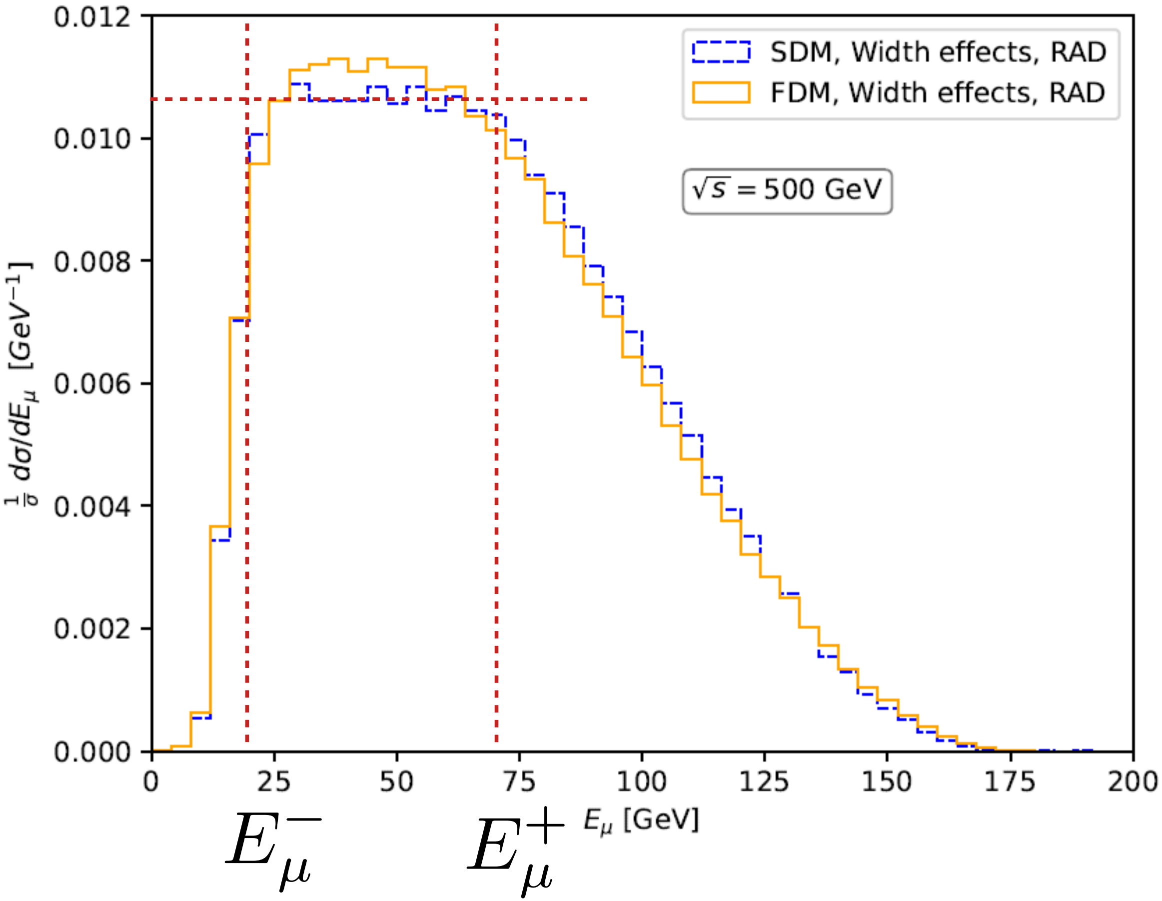
$$e^+e^- \to D^+D^- \to D_1D_1W^+W^- \to D_1D_1q'\bar{q}\mu\bar{\nu}$$

- Di-jet + muon + MET signature
  - $\sqrt{s}$ Is fixed (up to ISR+BRM effects)
  - M_{miss} can be reconstructed: $M_{miss} = \sqrt{\left((\sqrt{s},0,0,0) - \sum_{vis} P_{vis}\right)^2}$
  - Missing transverse momentum, $\not{E}_T$
  - charged lepton energy (muon), E_μ
  - angle of reconstructed W-boson in the LAB system, $\cos\theta_W$
  - the energy of W-boson reconstructed from the di-jet pair, E_{jj}
  - The cross section itself, which includes spin factors



W-boson and charged lepton energy distributions

$$e^+e^- \to D^+D^- \to D_1D_1W^+W^- \to D_1D_1q'\bar{q}\mu\bar{\nu}$$



- W energy distribution (from D^+ decay) have edges

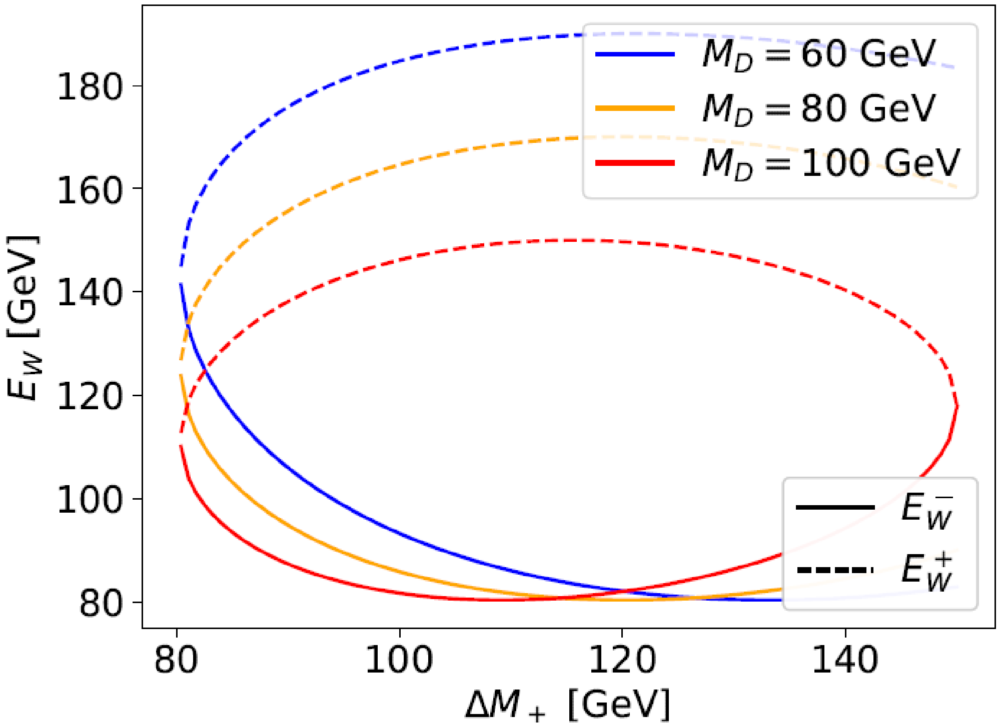
$$E_W^{(\pm)}(M_W^*) = \gamma_D(E_W^D \pm \beta_D p_W^D)$$

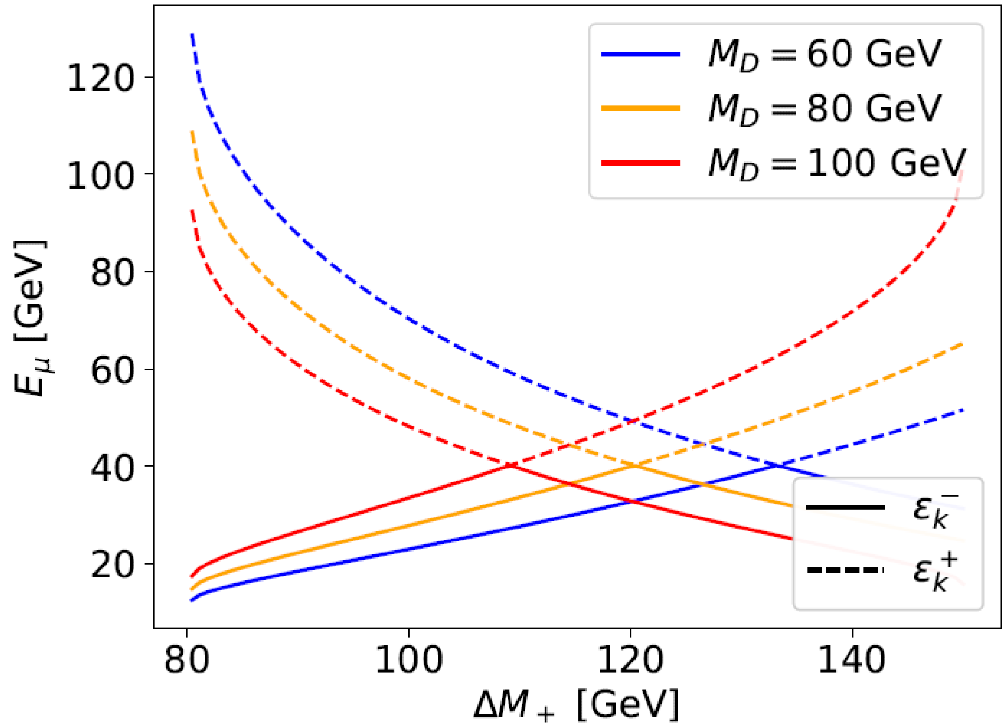
which lead to kinks in muon energy distributions

$$E_\mu^{(\pm)} = \frac{E_W^{(-)} \pm \sqrt{(E_W^{(-)})^2 - M_W^2}}{2}$$

- between these kinks distribution is approximately flat
- the positions of the upper edge of the di-jet (W) energy distribution and the lower kink in the muon energy distribution give two equations to determine M_D and M+

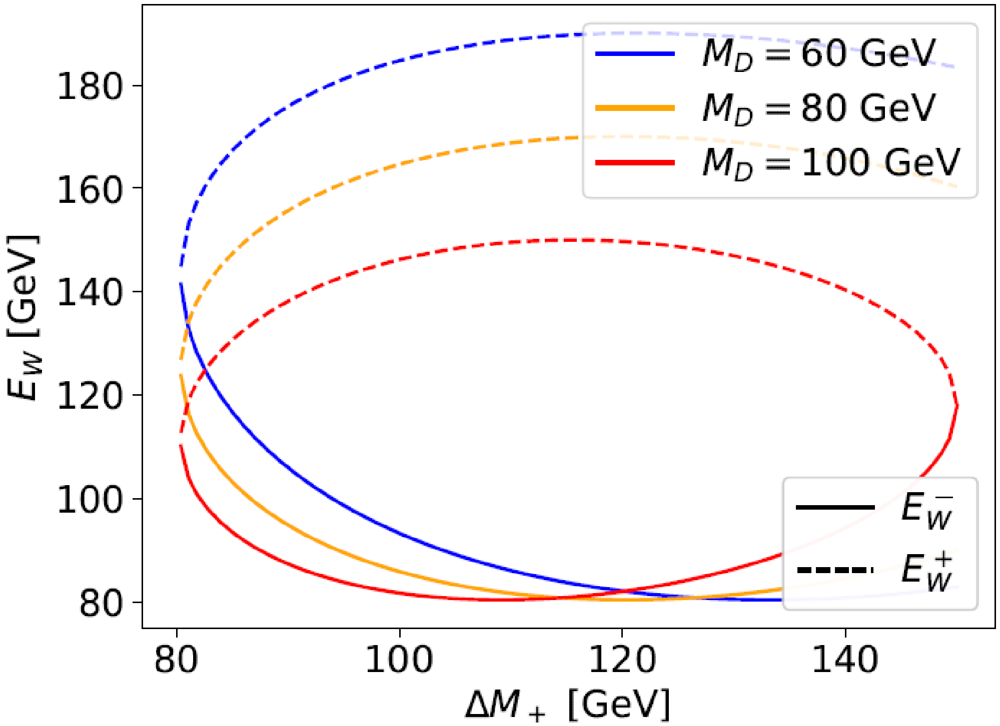
Kinks and M_D and M_+ determination

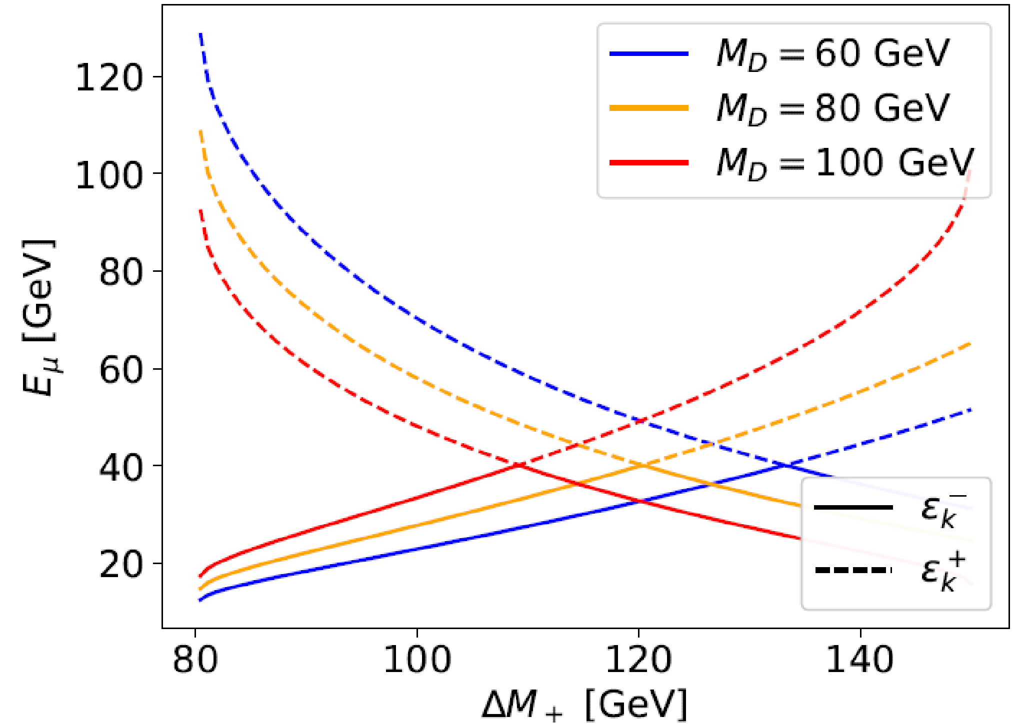




- Either of two edges in E(W) or in E(muon) distributions can be used to determine M_D and M+
- For certain D^+ and DM masses, edges either in E(W) or in E(mu) can overlap

Kinks and M_D and M_+ determination



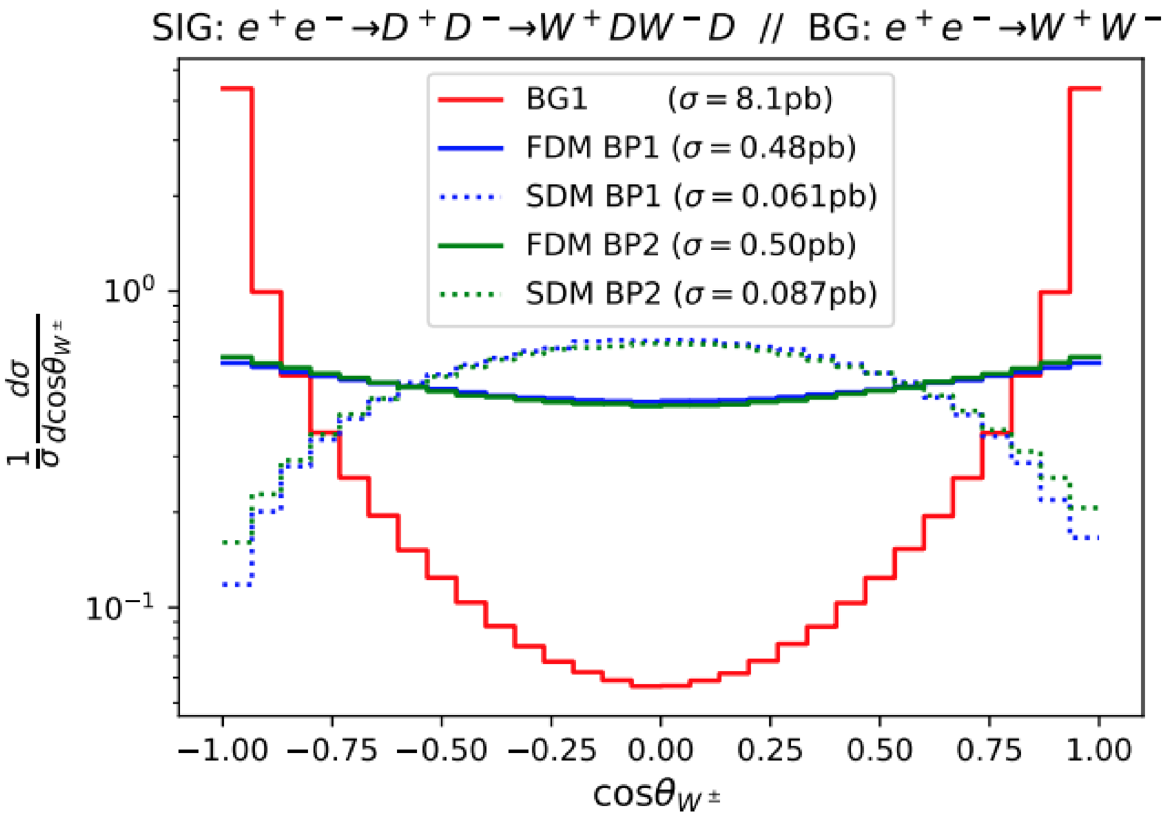


- Either of two edges in E(W) or in E(muon) distributions can be used to determine M_D and M+
- For certain D^+ and DM masses, edges either in E(W) or in E(mu) can overlap

- **But the edges in E(W) and E(muon) never overlap simultaneously:**
if distance between edges in E(W) distribution is small, the distance between edges in E(mu) is maximal and vice versa – so the **M_D and M+ can always be determined**

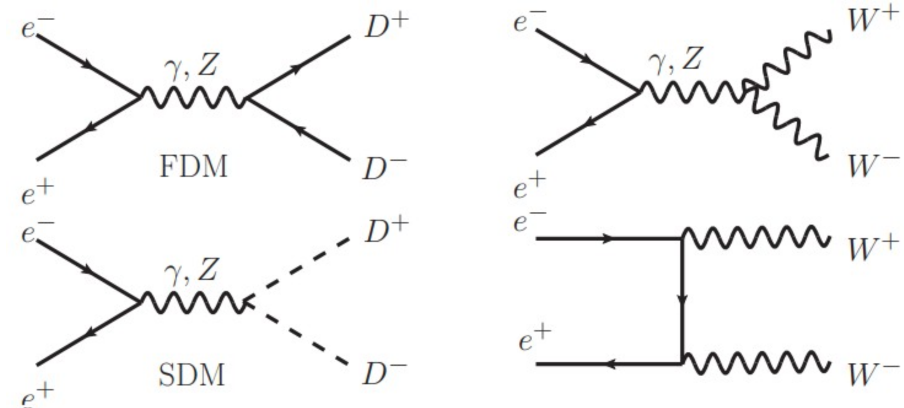
The role of the ILC in decoding the spin of DM

e+e- → D+ D- → DM DM W+ W- → DM DM jj μ ν



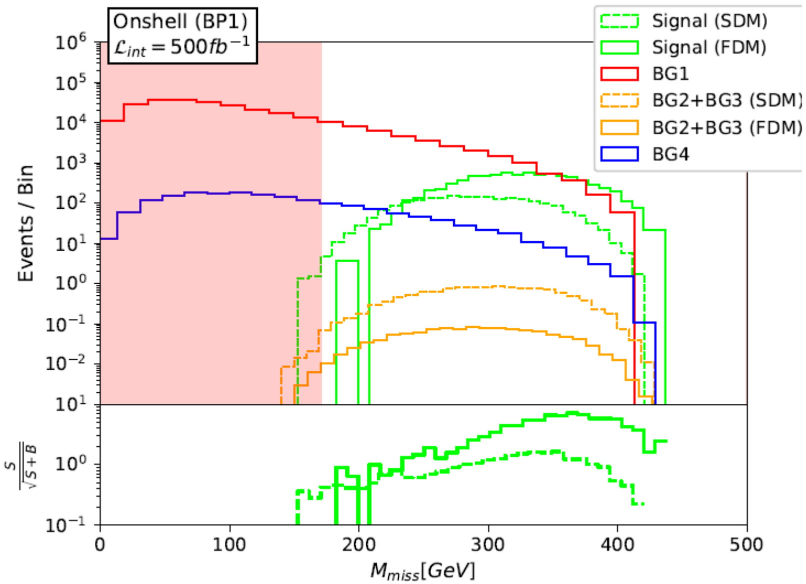
AB, Ginzburg, Locke, Freegard, Pukhov arXiv:**2112.15090**

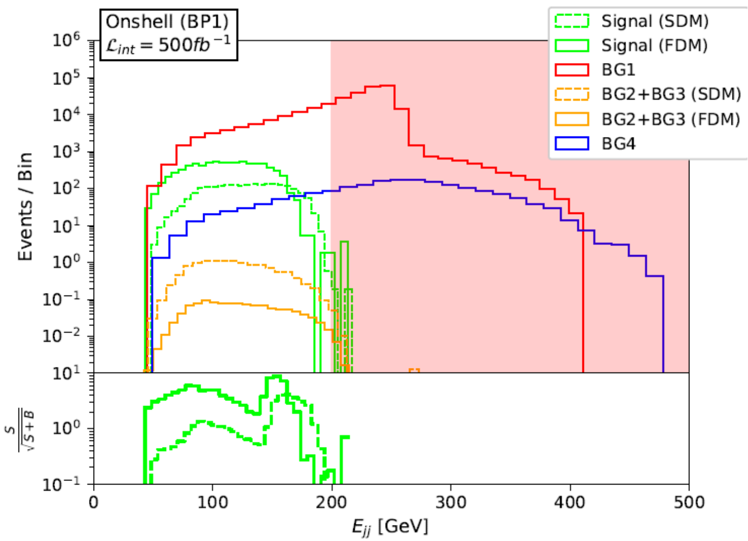
$$\frac{d\sigma}{d\cos\theta_{D\pm}} \propto \begin{cases} 1-\cos^2\theta_{D\pm}, & \text{for SDM} \\ 1+\dfrac{s-4M_+^2}{s+4M_+^2}\cos^2\theta_{D\pm}, & \text{for FDM} \end{cases}$$



- The angular W-boson distribution (either for real or virtual W) is found to be very important discriminator between DM spin as well as the main BG
- The shape of angular W-boson distribution is the same for two benchmarks for DM of the same spin

Signal vs BG analysis





SM BG cut flow

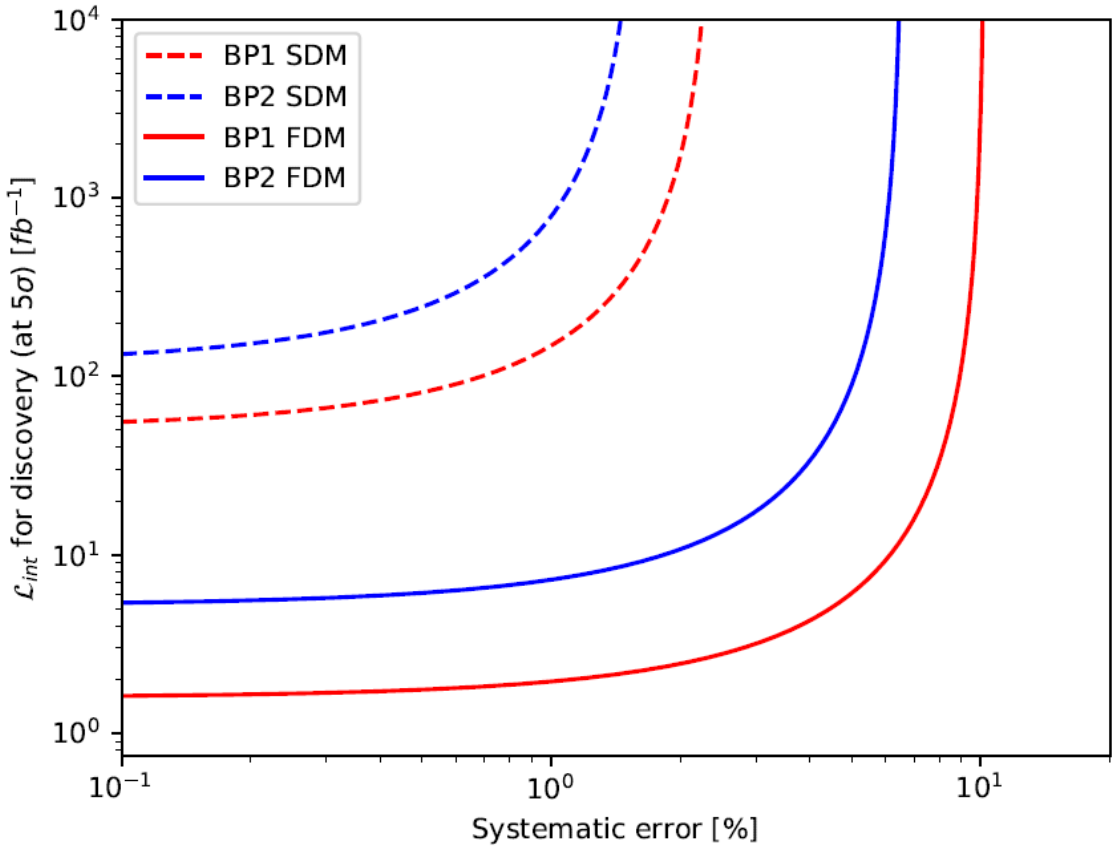
Cut	BG1	BG4	B_I	ε_{B_I}
Parton Level	6.600×10^5	1.947×10^4	6.795×10^5	—
Reco Level	2.921×10^5	1.842×10^3	2.939×10^5	0.433
$M_{miss} > 170$	4.053×10^4	4.881×10^2	4.101×10^4	0.140
$E_{jj} < 200$	3.718×10^4	2.993×10^2	3.748×10^4	0.914
$\lvert\cos\theta_{jj}\rvert < 0.9$	1.902×10^4	2.332×10^2	1.925×10^4	0.514
$\lvert\cos\theta_{\mu}\rvert < 0.9$	1.456×10^4	1.981×10^2	1.476×10^4	0.767

Cutflow for the SM BG (BG1 and BG4), which are BP independent.

$\alpha(\delta_{sys})$ for the 500 fb^{-1} **BP1 cut flow**

Cut	S	ε_S	B_{II}	$\varepsilon_{B_{II}}$	$\frac{(S+B_{II})}{B_I}$	$\alpha(\delta_{sys})$ $\alpha(0)$	$\alpha(0.01)$
SDM							
Parton Level	4.519×10^3	—	16.55	—	0.007	5.464	0.589
Reco Level	2.185×10^3	0.484	12.56	0.759	0.007	4.016	0.623
$M_{miss} > 170$	2.182×10^3	0.999	12.52	0.996	0.054	10.50	3.411
$E_{jj} < 200$	2.182×10^3	1.000	12.49	0.998	0.059	10.96	3.663
$\lvert\cos\theta_{jj}\rvert < 0.9$	2.132×10^3	0.977	10.64	0.852	0.111	14.58	5.921
$\lvert\cos\theta_{\mu}\rvert < 0.9$	2.027×10^3	0.951	9.587	0.901	0.138	15.65	6.816
FDM							
Parton Level	3.556×10^4	—	1.540	—	0.052	42.06	4.448
Reco Level	1.848×10^4	0.520	1.185	0.769	0.063	33.06	5.017
$M_{miss} > 170$	1.845×10^4	0.999	1.174	0.991	0.450	75.67	22.01
$E_{jj} < 200$	1.844×10^4	1.000	1.168	0.994	0.492	78.00	23.18
$\lvert\cos\theta_{jj}\rvert < 0.9$	1.651×10^4	0.895	0.946	0.810	0.858	87.30	30.20
$\lvert\cos\theta_{\mu}\rvert < 0.9$	1.542×10^4	0.934	0.851	0.899	1.045	88.77	32.43

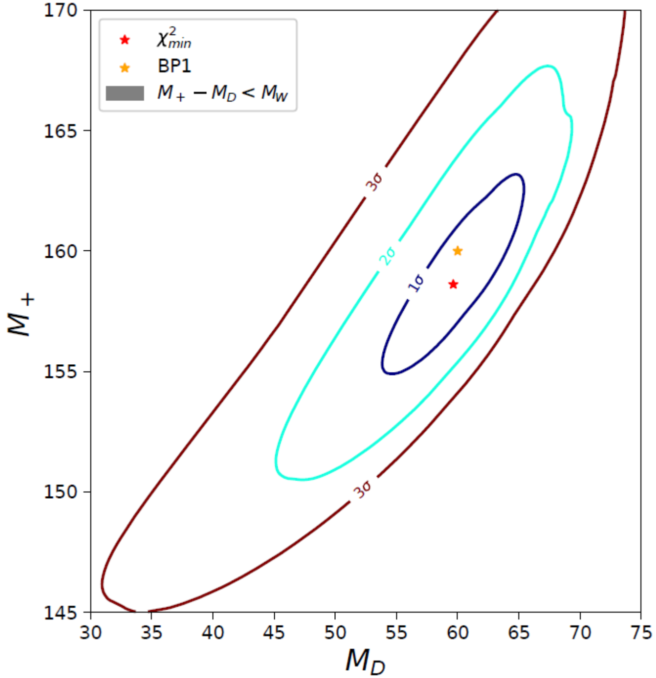
Signal vs BG analysis

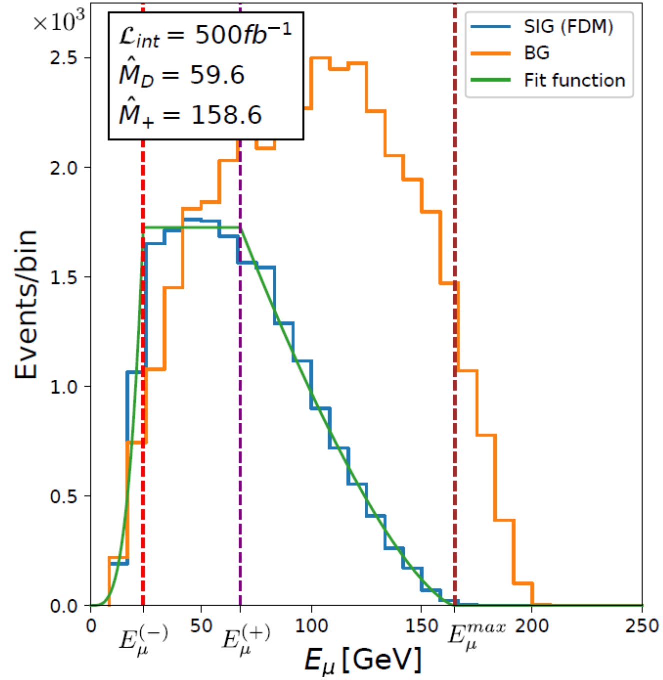


$$\alpha(\delta_{sys}) = \frac{S}{\sqrt{S+B} + \delta_{sys}(S+B)}$$

		Luminosity required for discovery (at 5σ)/fb^{-1}	
		$\alpha(0)$	$\alpha(0.01)$
SDM	BP1	51.1	149.
	BP2	117.	789.
FDM	BP1	1.59	1.95
	BP2	5.21	7.25

Mass determination





		$500\,fb^{-1}$	$20ab^{-1}$
FDM	M_D	$58.4^{+5.7}_{-6.0}$	$57.6^{+1.9}_{-2.2}$
	M_+	$158.1^{+4.0}_{-3.7}$	$157.4^{+2.7}_{-2.4}$
SDM	M_D	$66.0^{+19.2}_{-64.3}$	$64.3^{+3.2}_{-6.1}$
	M_+	$161.3^{+14.7}_{-52.8}$	$161.0^{+3.3}_{-3.9}$

		$500\,fb^{-1}$	$20ab^{-1}$
FDM	M_D	$60.0^{+0.7}_{-0.8}$	$60.0^{+0.1}_{-0.1}$
	M_+	$120.0^{+1.5}_{-1.7}$	$120.0^{+0.2}_{-0.3}$
SDM	M_D	$60.0^{+24.1}_{-19.7}$	$60.0^{+4.4}_{-1.3}$
	M_+	$120.0^{+22.3}_{-45.9}$	$120.0^{+2.3}_{-2.7}$

$$f(E_\mu) = \begin{cases} b\left(\dfrac{E_\mu}{E_\mu^{(-)}}\right)^a & \text{if } E_\mu \le E_\mu^{(-)} \\ b & \text{if } E_\mu^{(-)} < E_\mu < E_\mu^{(+)} \\ b\left(1 - \dfrac{E_\mu - E_\mu^{(+)}}{E_\mu^{max} - E_\mu^{(+)}}\right)^c & \text{if } E_\mu^{(+)} \le E_\mu < E_\mu^{max} \\ 0 & \text{if } E_\mu \ge E_\mu^{max} \end{cases},$$

The profile χ^2 is calculated by minimising over nuisance parameters a, b, c.
The minimum of this profiled χ^2 corresponds to the global minimum for the fit, when M_D, M_+ are also allowed to vary.

Spin discrimination

	$\mathcal{L}_{int}$ to differentiate at 95% CL $/ fb^{-1}$			
	Shape only		Shape and cross-section	
Assumed nature	SDM	FDM	SDM	FDM
BP1	9.8×10^2	30	1.9	3.4
BP2	2.3×10^3	1.2×10^2	9.6	13.

We assume that the mass of the DM is precisely known: a more complete treatment would involve a simultaneous fit of mass and spin.

Events are generated with the model assigned to 'Assumed nature', before statistical comparison with the alternative model is conducted.

We perform the analysis for two cases:
1) using only the shape: signal strength becomes a nuisance parameter μ
2) using the signal strength predicted by the specific model realisations.

Result: the luminosity required to exclude a given hypothesis at the expected 95% CL

Conclusions and Outlook

- **Future e^+e^- colliders have unique power to determine the properties of DM, including its spin!**
  - Two minimal models with DM spin ½ and 0 as an example of the case study
- **New results:** the power of **E(mu), E(W), Cos(Θ_W)** and **missing mass** to
  - discover 100 GeV FDM (SDM) with the few (hundred) inverse fb integrated luminosity
  - determine mass of DM with up to a percent accuracy
  - discriminate DM spin (especially Cos(Θ_W))
  - the edges of **E(mu), E(W)** distributions are very complementary: they never overlap simultaneously, so the **M_D and M+ can always be determined**
- Next step: **vector DM case**

Thank you!

Backup slides

It is convenient to use the cross section for SM process

$$\sigma_0 \equiv \sigma(e^+e^- \to \gamma \to \mu^+\mu^-) = 4\pi\alpha^2/3s$$

as a normalizer for the cross sections of the $e^+e^- \to D^+D^-$ processes under study. For γ-factors and velocities of D^+,

$$\gamma_+ = \frac{\sqrt{s}}{2M_+}, \qquad \beta_+ = \sqrt{1 - 4M_+^2/s}$$

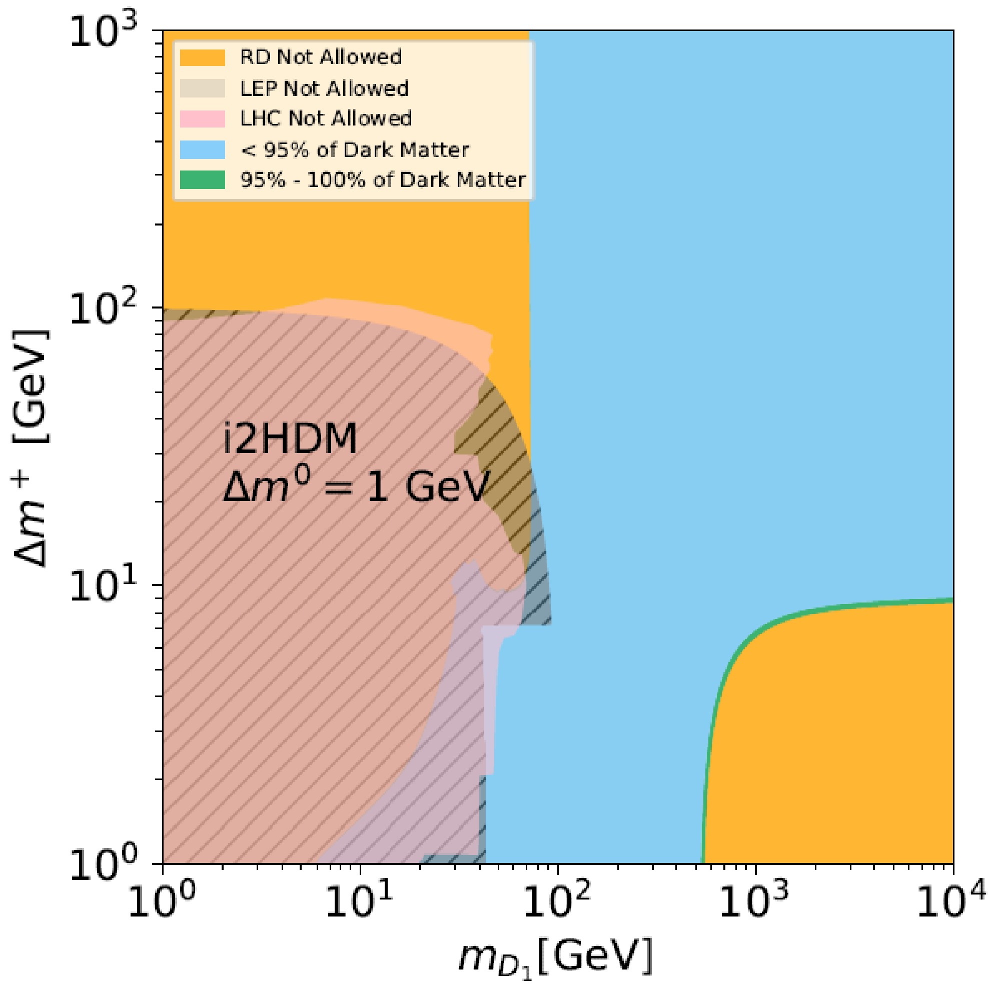
the QED cross section of $e^+e^- \to D^+D^-$ process from the squared amplitude with the photon exchange only is given by

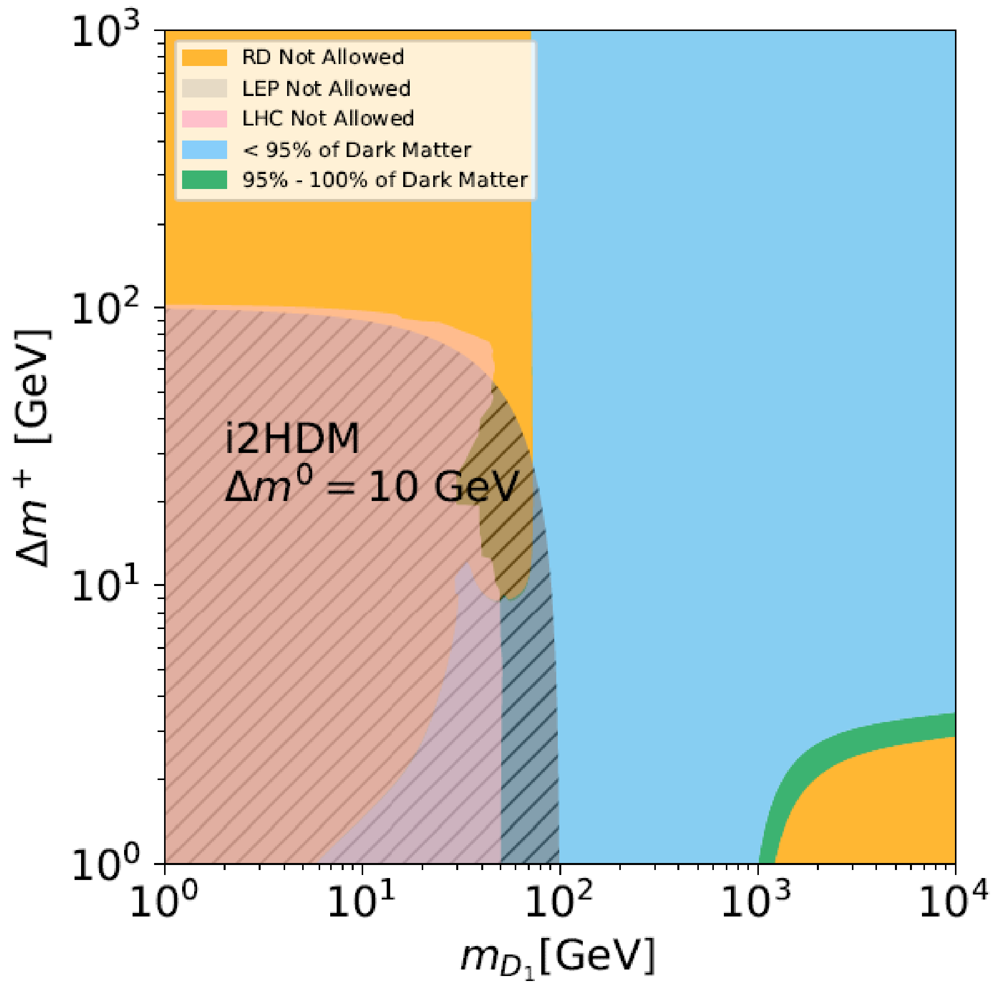
$$\sigma_{\gamma\gamma} = \begin{cases} \sigma_0\beta_+ \left[1 + \frac{2M_+^2}{s}\right] & \text{if } s_D = \frac{1}{2} \\ \sigma_0\frac{\beta_+^3}{4} & \text{if } s_D = 0 \end{cases},$$

while the total cross section is given by

$$\sigma = \sigma_{\gamma\gamma} + \sigma_{\gamma Z} + \sigma_{ZZ} = \sigma_{\gamma\gamma}\left[1 + \frac{\kappa_{\gamma Z}}{1 - \frac{M_Z^2}{s}} + \frac{\kappa_{ZZ}}{\left(1 - \frac{M_Z^2}{s}\right)^2}\right],$$

i2HDM parameter space: the current status





MFDM parameter space: the current status