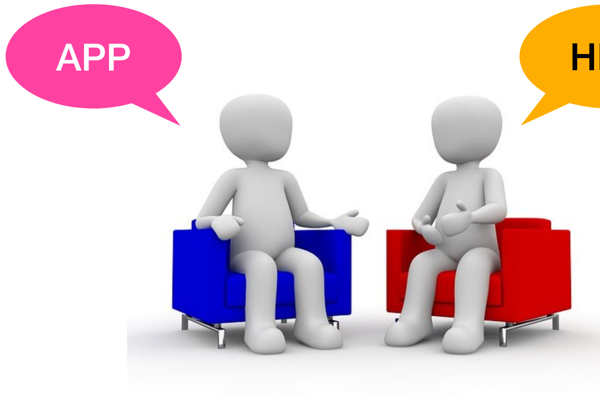
Institute of Physics



Developments in Formal Theory

IOP Joint APP / HEP Conference 2022

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Outline

- *Formal theory* is a broad term - nearly 4000 papers since 2020!
- There is also no hard boundary between what is “formal” and what is “applied”.
- At its best, theoretical physics identifies **common structures** in different physical theories…
- …and develops **new ways of thinking**.
- In this talk, I will focus on recent examples that I hope are interesting!

Gravity from gauge theory

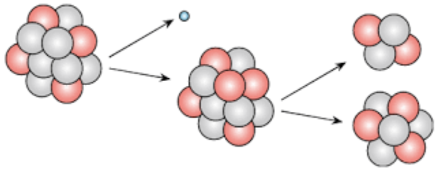
Celestial Holography

Machine Learning in Formal Theory

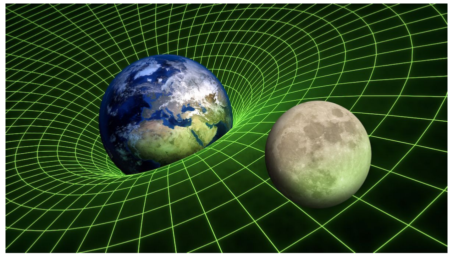
Fundamental Forces

- Our understanding of the universe is in terms of **matter** acted on by **fundamental forces.**

Electromagnetism



Weak / strong nuclear forces



Gravity

The Standard Model of Particle Physics

"Non-abelian gauge theory"

General Relativity

- Neither of these theories is complete.
- New calculational tools are constantly needed!

Scattering amplitudes

- One of our main ways of testing theories is through scattering experiments.
- Interactions described by **scattering amplitudes**, which are related to measurable quantities (e.g. cross-sections).
- Traditional methods ("Feynman rules") look VERY different in gauge theories / gravity.
- As an example, the rules for interacting gluons / gravitons are as follows:

$$\begin{aligned} g[&\eta^{\mu\nu}(q_1 - q_2)^\rho \\ +&\eta^{\mu\rho}(q_2 - q_3)^\nu \\ +&\eta^{\rho\nu}(q_3 - q_1)^\mu] \end{aligned}$$

GLUONS

$$\begin{aligned} &U(q_1, q_2, q_3)_{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3} = \\ &-\frac{K}{2}\Big[q^2_{(\alpha_1}q^3_{\beta_1)}\left(2\eta_{\alpha_2(\alpha_3}\eta_{\beta_3)\beta_2} - \tfrac{2}{d-2}\eta_{\alpha_2\beta_2}\eta_{\alpha_3\beta_3}\right) \\ &\quad +q^1_{(\alpha_2}q^3_{\beta_2)}\left(2\eta_{\alpha_1(\alpha_3}\eta_{\beta_3)\beta_1} - \tfrac{2}{d-2}\eta_{\alpha_1\beta_1}\eta_{\alpha_3\beta_3}\right) \\ &\quad +q^1_{(\alpha_3}q^2_{\beta_3)}\left(2\eta_{\alpha_1(\alpha_2}\eta_{\beta_2)\beta_1} - \tfrac{2}{d-2}\eta_{\alpha_1\beta_1}\eta_{\alpha_2\beta_2}\right) \\ &\quad +2q^3_{(\alpha_2}\eta_{\beta_2)(\alpha_1}\eta_{\beta_1)(\alpha_3}q^2_{\beta_3)} + 2q^1_{(\alpha_3}\eta_{\beta_3)(\alpha_2}\eta_{\beta_2)(\alpha_1}q^3_{\beta_1)} + 2q^2_{(\alpha_1}\eta_{\beta_1)(\alpha_3}\eta_{\beta_3)(\alpha_2}q^1_{\beta_2)} \\ &\quad +q^2\cdot q^3\left(\tfrac{2}{d-2}\eta_{\alpha_1(\alpha_2}\eta_{\beta_2)\beta_1}\eta_{\alpha_3\beta_3} + \tfrac{2}{d-2}\eta_{\alpha_1(\alpha_3}\eta_{\beta_3)\beta_1}\eta_{\alpha_2\beta_2} - 2\eta_{\alpha_1(\alpha_2}\eta_{\beta_2)(\alpha_3}\eta_{\beta_3)\beta_1}\right) \\ &\quad +q^1\cdot q^3\left(\tfrac{2}{d-2}\eta_{\alpha_2(\alpha_1}\eta_{\beta_1)\beta_2}\eta_{\alpha_3\beta_3} + \tfrac{2}{d-2}\eta_{\alpha_2(\alpha_3}\eta_{\beta_3)\beta_2}\eta_{\alpha_1\beta_1} - 2\eta_{\alpha_2(\alpha_1}\eta_{\beta_1)(\alpha_3}\eta_{\beta_3)\beta_2}\right) \\ &\quad +q^1\cdot q^2\left(\tfrac{2}{d-2}\eta_{\alpha_3(\alpha_1}\eta_{\beta_1)\beta_3}\eta_{\alpha_2\beta_2} + \tfrac{2}{d-2}\eta_{\alpha_3(\alpha_2}\eta_{\beta_2)\beta_3}\eta_{\alpha_1\beta_1} - 2\eta_{\alpha_3(\alpha_1}\eta_{\beta_1)(\alpha_2}\eta_{\beta_2)\beta_3}\right)\Big] \end{aligned}$$

GRAVITONS

- Not at all clear that these theories are related!
- Remarkably, however, they are…

The double copy: gravity from gauge theory

- In 2010, it was noticed that scattering amplitudes in gauge / gravity theories look **almost identical** if phrased in the right way (Bern, Carrasco, Johansson):

$$\mathcal{A}_m^{(L)} \sim g^{m-2+2L} \sum_i \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{n_i c_i}{\prod_{\alpha_i} p_{\alpha_i}^2} \qquad \mathcal{M}_m^{(L)} \sim \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_i \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- Called the **double copy**, and suggests a new way of thinking about gravity.

- For Feynman diagrams with no loops, there is an explanation from string theory (Kawai, Lewellen, Tye).

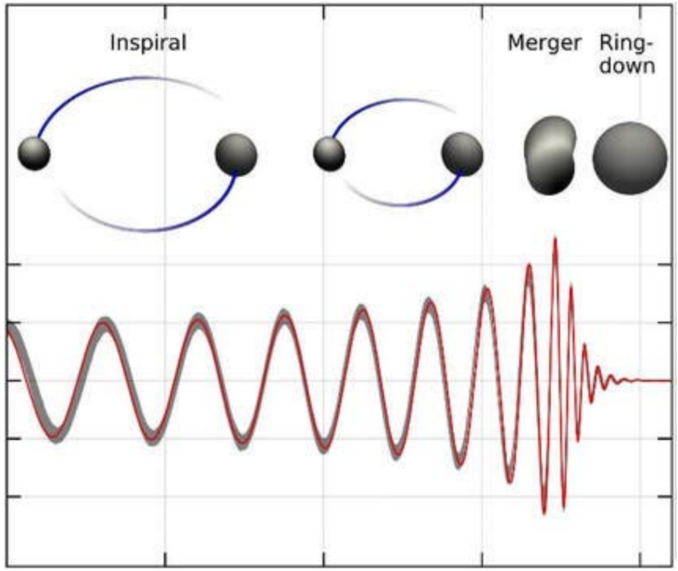
- Can be extended to **classical solutions** (Monteiro, O'Connell, White).

- Provides new calculational tools in GR!

Applications to gravitational waves

- There is a huge amount work at the moment applying QFT methods (e.g. scattering amplitudes) to **gravitational scattering**.

- Of direct relevance for gravitational wave experiments.



- New results at $\mathcal{O}(G_N^3)$ that were not previously known in the gravity literature (e.g. Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng).

- Fruitful interplay between HEP and astrophysics communities.

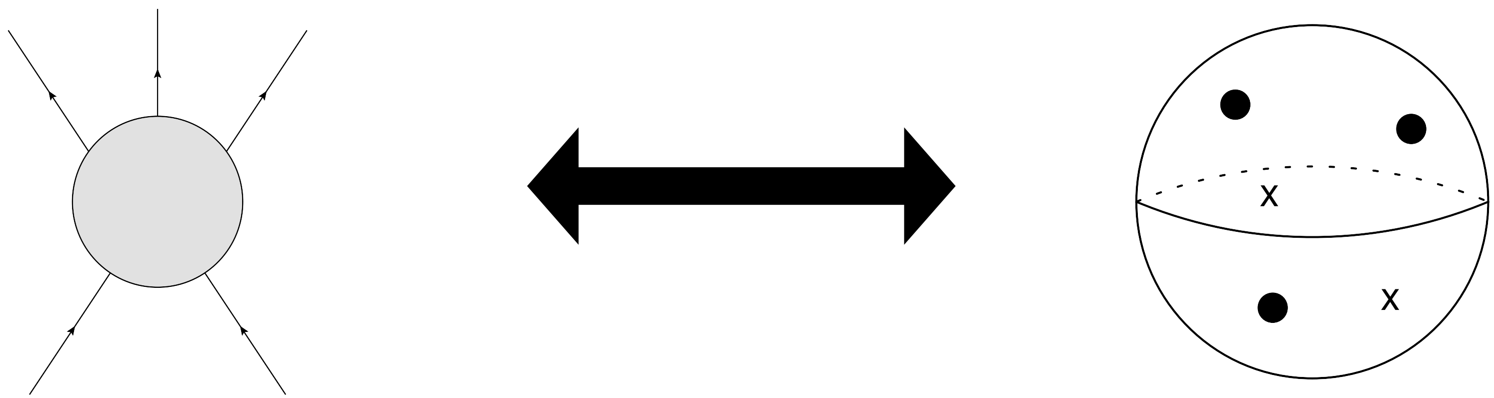
- Annual conference (*QCD Meets Gravity*) devoted to this topic.

Celestial Holography

- *Holography* is the idea that physics in four (or more!) dimensions can be reproduced by a lower-dimensional "dual theory".

- Not a new idea (e.g. 't Hooft; Susskind), and the most well-known incarnation is the AdS / CFT correspondence (Maldacena).

- Recently, a new holography proposal has been made for **flat space**.



SCATTERING IN 4D FLAT SPACE

FIELD THEORY ON
2D "CELESTIAL SPHERE"

Celestial Holography

- Lorentz transformations in 4D are equivalent to 2D conformal (angle-preserving) transformations on the celestial sphere.

- A dictionary has been proposed that translates 4D scattering amplitudes into correlators in a *conformal field theory (CFT)* on the celestial sphere (Pasterski, Shao, Strominger):

Correlation function of operators →

$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}^{\pm}_{\Delta_n}(z_n,\bar{z}_n)\rangle = \prod_{i=1}^{n}\int_0^{\infty} d\omega_i \omega_i^{\Delta_i-1}\langle out|\mathcal{S}|in\rangle$$

← 4D amplitude in flat space

Operator at a point on the celestial sphere

Mellin transform in particle energies

Celestial Holography

- The reason this is useful is that a HUGE amount is known about 2D CFTs!
- The hope is that we can fully identify a self-consistent *Celestial Conformal Field Theory* (CCFT), that is dual to 4D (quantum) gravity / gauge theory.
- Many recent developments.

New gravitational memory effects

Black hole information

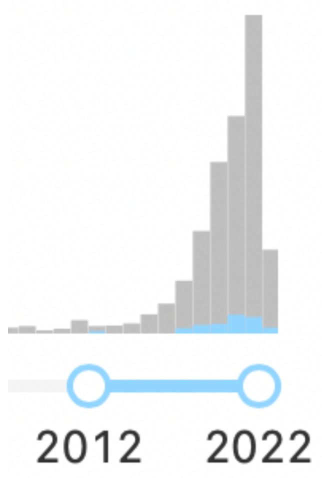
New results in QCD?

Relations with string theory

New algebraic structures in amplitudes

Machine learning

- We are living in a golden age of computational science.
- Artificial Intelligence and Machine Learning are ubiquitous in applied physics.



- The HEP INSPIRE database shows a rapid rise of papers with “learning” in the title!
- There are applications of the same techniques in more formal theory…
- Different types of machine learning are tailored to different tasks.

Supervised learning

- A classic example of **supervised** learning is *image recognition*. Given a large number of pictures of cats and dogs, can we teach a computer to recognise each type?

- To do this, we split our data set into a *training set*, where we tell the computer which type of image is which, and a *validation set*, on which we test the code.

- Some theoretical physics problems can be mapped precisely to image recognition!

Application to string theory

- String theory naturally lives in 10 spacetime dimensions.
- Four of these would be the ones we see, and the other six are curled up ("compactification").
- These six-dimensional spaces are called *Calabi-Yau manifolds*, and they are classified by various numbers that label their topology.
- These numbers are related to the spectrum of particles at low energy.
- But calculating them is traditionally very slow…

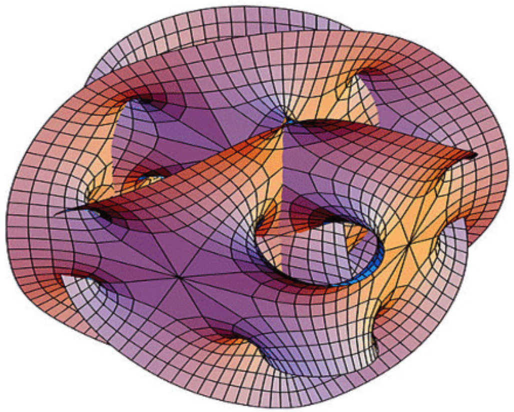


Image recognition in string theory!

- Large databases of Calabi-Yau manifolds have been assembled, together with their topological numbers (e.g. *Betti*, *Hodge*, and *Euler numbers*).
- One can train a machine learning algorithm on this dataset, to recognise the topological structure of an arbitrary CY manifold (He)!
- As an example, a certain family of Calabi-Yaus are defined by the intersection of 8 polynomials in a larger space.
- The coefficients form a matrix, which can then be converted to an image.

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

- Calculating topological data is now precisely an image recognition problem!
- Looking for interesting string theory solutions is much quicker!

Reinforcement learning

- Another technique is **reinforcement learning**, in which an algorithm learns a by increasing a given "reward".
- This has been recently used in the study of conformal field theories (CFTs).
- As well as the celestial sphere encountered earlier, CFTs arise all over physics, (e.g. condensed matter, special limits of high energy physics theories).
- A CFT contains certain operators $\{\mathcal{O}_i\}$ of given spin, with *scaling dimensions* $\{\Delta_i\}$:

$$\mathcal{O}_i(\lambda x) = \lambda^{-\Delta_i}\mathcal{O}(x)$$

The conformal bootstrap

- Products of operators obey a formula known as the *operator product expansion*:

$$\mathcal{O}_i(x_1)\mathcal{O}(x_2) = \sum_k C_{ij}^k \, \hat{f}_{ij}^k(x_1, x_2, \partial_{x_2}) \, \mathcal{O}_k(x_2)$$

Known differential operator

- A given CFT is *completely fixed* by the data $\{\Delta_i, C_{ij}^k\}$, as expressed by an infinite number of consistency equations.
- We can thus “solve” certain CFTs exactly, and non-perturbatively.
- By making assumptions about the CFT data, can we tell if an allowed solution is possible? This is called the *conformal bootstrap*.

Machine learning CFTs

- Assume we know how many operators of a given spin we have.
- Then, one use reinforcement learning to efficiently find allowed CFTs.
- The “reward” in this case corresponds to closely satisfying the CFT consistency requirements.

- This idea has been tested on known CFTs, which can be “learnt” by machines very efficiently (Kántor, Papageorgakis, Niarchos).
- Clear scope for using similar methods in other areas of theoretical physics!

Conclusions

- Theoretical physics has been invigorated in recent years by **new ways of thinking** about our theories (QFT, strings…).
- This has led to surprising new connections between "real" physical theories, that are **conceptually interesting**, and **practically useful**!
- Formal theory questions can be answered with **cutting-edge computational tools.**
- This is a whole new way of doing mathematics / theoretical physics!

hep-ph gr-qc astro-ph

hep-th Plenty of scope for collaboration and discussion! cond-mat

hep-ex hep-lat math-ph