



Useful things to know about accelerators – part I

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**Science and
Technology
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Accelerators - A Window on Nature

- Particle accelerators provide the source for most high energy physics experiments
 - Provide high luminosity, high energy beams for colliders
 - Provide high brightness beams for secondary particle production
 - Also key technology for life sciences, engineering, chemistry
- How do they work?
 - How can we get to high energy?
 - How can we keep the beam in the accelerator?
 - How can we get to high luminosity?
- What are the main HEP facilities in the world today?
- What might HEP facilities look like in the future?



Accelerator Components

- Most accelerators share similar components
- Main components of an accelerator
 - Bending – dipoles
 - Focussing – quadrupoles
 - Acceleration - RF cavities
- Also
 - Vacuum
 - Diagnostics
 - Targets for secondary particle production
- First Lecture: Derive basic theory of accelerator physics
- Second Lecture: Discuss accelerator equipment and techniques

Lorentz force law

- Fundamental equation for particles moving through fields

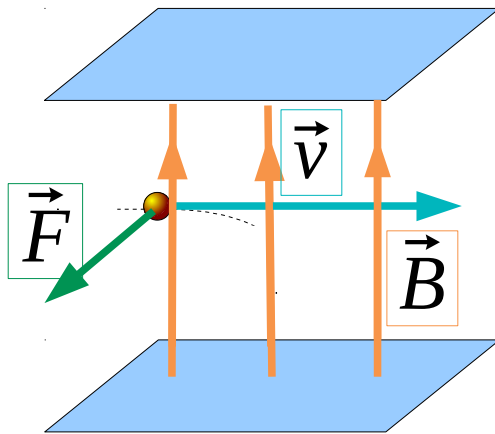
The diagram shows the Lorentz force law equation $\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$ with five color-coded labels above it: 'Force' (green), 'Charge' (cyan), 'Velocity' (red), 'Magnetic Field' (purple), and 'Electric Field' (orange). Arrows of corresponding colors point from each label to its respective term in the equation: Force to \vec{F} , Charge to q , Velocity to \vec{v} , Magnetic Field to \vec{B} , and Electric Field to \vec{E} . The equation is labeled '(eq. 1)' to the right.

$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E} \quad (\text{eq. 1})$$

- **Magnetic force** is perpendicular to velocity
 - Magnetic field conserves energy
- **Electric force** is weaker by factor velocity
 - Magnets are better for bending and focussing

Magnetic Rigidity and Bending

- Simplest magnet - “dipole”
 - Uniform magnetic field perpendicular to beam direction



Lorentz force (eq. 1) + centripetal motion:

$$qvB = \frac{pv}{\rho}$$

Radius

Rearranging:

$$B\rho = \frac{p}{q}$$

Magnetic Rigidity

- Constant force \rightarrow constant curvature \rightarrow circular motion
- Magnetic rigidity parameterises momentum
- Charge-to-mass ratio important when accelerating multiple particle species



Worked example – LHC

- If we wanted to accelerate, say, 7 TeV particles, what bending radius is required?
- Maximum dipole field around 8.3 T

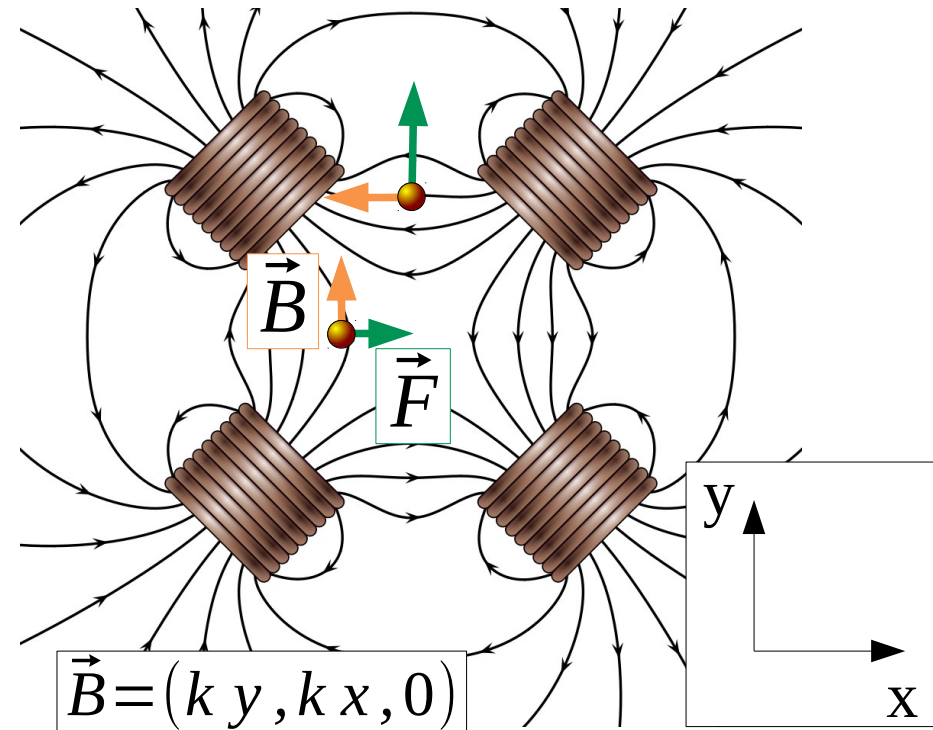
$$B\rho = \frac{p}{q}$$

$$\rho = \frac{p}{qB} = \frac{7}{0.3 \times 8.3} = 2.8 \text{ km}$$

- Nb: LHC radius ~ 4.1 km
 - Need space for detectors, etc

Quadrupole magnets

- If we only had bending magnets, particles would soon be lost from the accelerator
- Need to keep the particles in the accelerator using focussing elements
 - Usually use quadrupoles
- Field stronger away from beam centre
 - Like a spring or pendulum
 - Simple harmonic motion
- Overall focussing by alternating the gradient





Quadrupole field – horizontal (1)

- For a particle moving near to the z-axis

$$\vec{F} = q \vec{v} \times \vec{B} + q \vec{E}$$

$$\vec{B} = (b_0 y, b_0 x, 0)$$

- Considering only p_x for now

$$\frac{dp_x}{dt} = q \frac{dz}{dt} B_y$$

- Use the chain rule

$$\frac{dp_x}{dt} = \frac{dp_x}{dz} \frac{dz}{dt}$$

- Combining these equations:

$$\frac{dp_x}{dz} = q b_0 x$$

Quadrupole field – horizontal (2)

$$\frac{dp_x}{dz} = q b_0 x$$



- Definition of x-component of momentum

$$p_x = m \gamma v_x = m \gamma \frac{dz}{dt} \frac{dx}{dz} = p_z \frac{dx}{dz}$$

- Substitute this definition into  gives

$$p_z \frac{d^2 x}{dz^2} = q b_0 x$$

- Rearrange and wrap up constant terms in focussing strength k

$$\frac{d^2 x}{dz^2} - k x = 0$$

Quadrupole field – vertical

- Lorentz force law with quadrupole field definition

$$\frac{dp_y}{dt} = -q b_0 v_z y$$

- Use chain rule and eliminate v_z

$$p_z \frac{d^2 y}{dz^2} = -q b_0 y$$

- Rearrange and wrap up constant terms in defocussing strength k

$$\frac{d^2 y}{dz^2} + k y = 0$$

Solutions

- Motion is governed by

$$\frac{d^2 x}{dz^2} - k x = 0 \qquad \frac{d^2 y}{dz^2} + k y = 0$$

- This is simple harmonic motion – solutions are of form

$$x = x_0 \cos(\sqrt{k} z) + \frac{dx_0}{dz} \frac{1}{\sqrt{k}} \sin(\sqrt{k} z)$$

- Taking derivative

$$\frac{dx}{dz} = -x_0 \sqrt{k} \sin(\sqrt{k} z) + \frac{dx_0}{dz} \cos(\sqrt{k} z)$$

For y

$$y = y_0 \cosh(\sqrt{k} z) + \frac{dy_0}{dz} \frac{1}{\sqrt{k}} \sinh(\sqrt{k} z)$$

$$\frac{dy}{dz} = y_0 \sqrt{k} \sinh(\sqrt{k} z) + \frac{dy_0}{dz} \cosh(\sqrt{k} z)$$



Transfer Matrix

- Just thinking about x , the particles move according to

$$x = x_0 \cos(\sqrt{k} z) + \frac{dx_0}{dz} \sin(\sqrt{k} z)$$

$$\frac{dx}{dz} = -x_0 \sqrt{k} \sin(\sqrt{k} z) + \frac{dx_0}{dz} \sqrt{k} \cos(\sqrt{k} z)$$

- We can rewrite this as a matrix

$$\begin{pmatrix} x \\ \frac{dx}{dz} \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{k} z) & \frac{1}{\sqrt{k}} \sin(\sqrt{k} z) \\ -\sqrt{k} \sin(\sqrt{k} z) & \cos(\sqrt{k} z) \end{pmatrix} \begin{pmatrix} x_0 \\ \frac{dx_0}{dz} \end{pmatrix}$$

- This matrix is known as the quadrupole's **transfer matrix**

$$\underline{u}_1 = \underline{M}_{01} \underline{u}_0$$



Questions



Questions

- Exercise – what is the transfer matrix for a drift space, that is a region with no fields at all?
 - What is the force acting on the particle?
 - What is $x(z)$ in terms of dx_0/dz and x_0
 - What is dx/dz in terms of dx_0/dz
 - Now write that as a matrix

Questions

- Exercise – what is the transfer matrix for a drift space?

- What is the force acting on the particle?

- No force

- What is $x(z)$ in terms of dx_0/dz and x_0

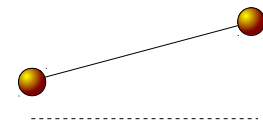
$$x = x_0 + \frac{dx_0}{dz} z$$

- What is dx/dz in terms of dx_0/dz

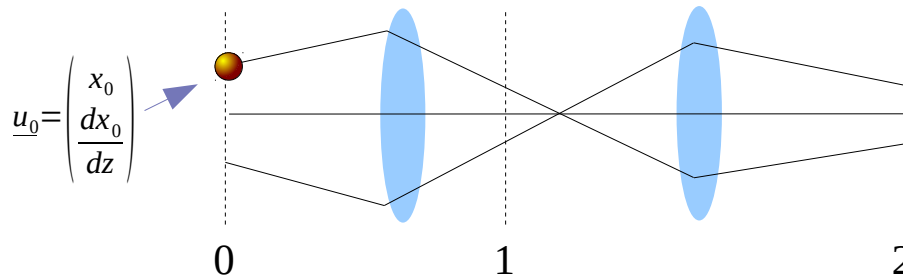
$$\frac{dx}{dz} = \frac{dx_0}{dz}$$

- Now write that as a matrix

$$\begin{pmatrix} x \\ \frac{dx}{dz} \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ \frac{dx_0}{dz} \end{pmatrix}$$



Transfer Lines



- Transfer matrix defines transport through a region
- Transfer matrices can be combined by multiplication
- Say we have transfer matrices like:

$$\underline{u}_1 = \mathbf{M}_{01} \underline{u}_0$$

$$\underline{u}_2 = \mathbf{M}_{12} \underline{u}_1$$

- Then

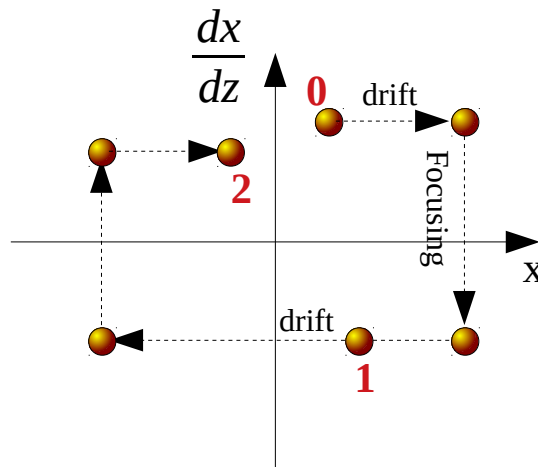
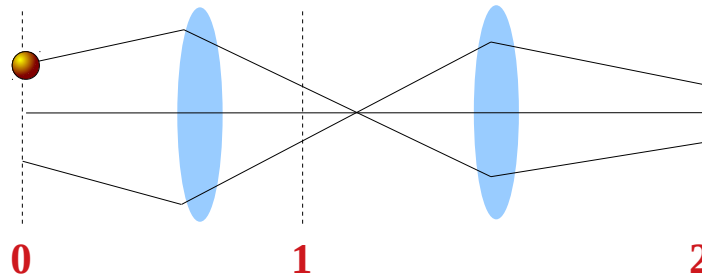
$$\underline{u}_2 = \mathbf{M}_{12} \mathbf{M}_{01} \underline{u}_0$$

- i.e. we can define a combined transfer matrix like

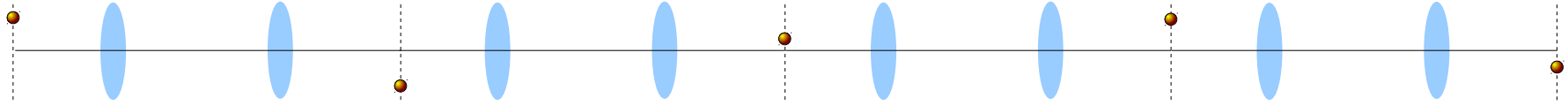
$$\mathbf{M}_{02} = \mathbf{M}_{12} \mathbf{M}_{01}$$

Phase space

- Another instructive way to look at beam optics is by considering the phase space



Periodic Lattices



- Following n identical cells or turns in a ring with one-turn matrix \mathbf{M}

$$\underline{u}_n = \mathbf{M}^n \underline{u}_0$$

- Rewrite

$$\mathbf{M} = \mathbf{I} \cos \mu + \mathbf{J} \sin \mu$$

$$\mathbf{J} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \quad \text{with} \quad \gamma\beta - \alpha^2 = 1 \quad \text{and} \quad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

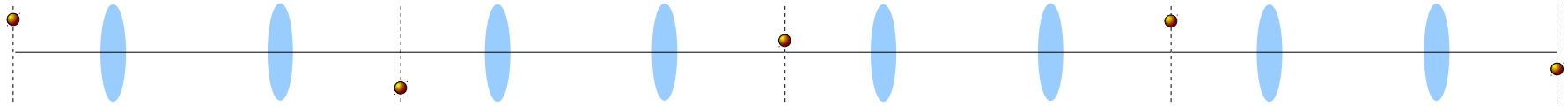
- So

$$\mathbf{J}^2 = -\mathbf{I}$$

- And

$$\mathbf{M}^n = \mathbf{I} \cos(n\mu) + \mathbf{J} \sin(n\mu)$$

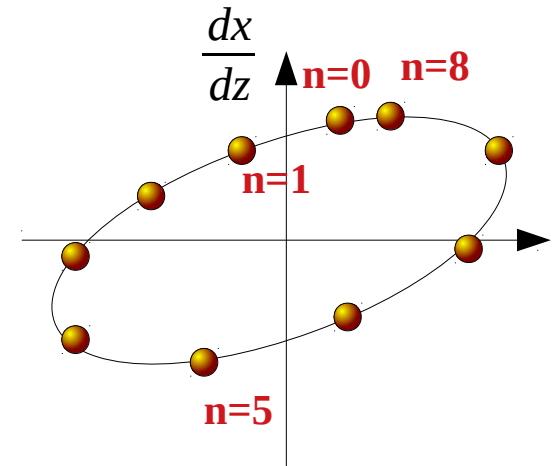
Periodic Lattices



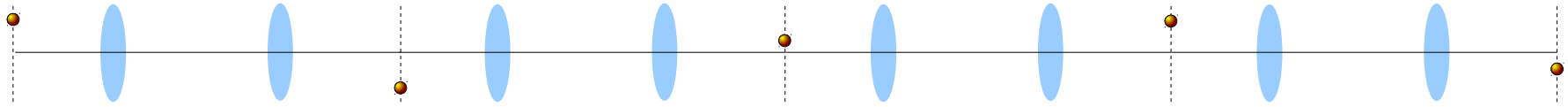
- What does this mean?

$$\mathbf{M}^n = \mathbf{I} \cos(n\mu) + \mathbf{J} \sin(n\mu)$$

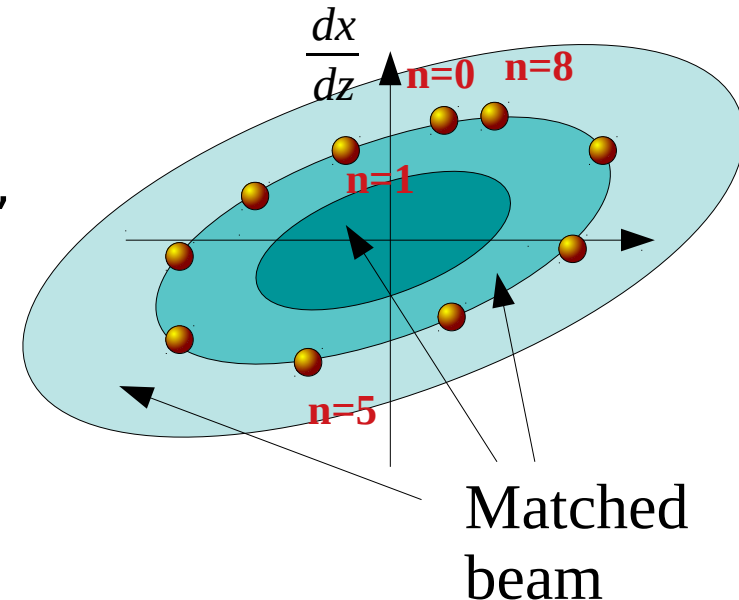
- Particles move around an ellipse in phase space if $\text{Trace}(\mathbf{M}) < 2$
- μ is the “phase advance”
- α , β and γ are “Twiss parameters”
 - Tell us the alignment of the ellipse
- Each particle sits on ellipse area ε - the particle’s amplitude



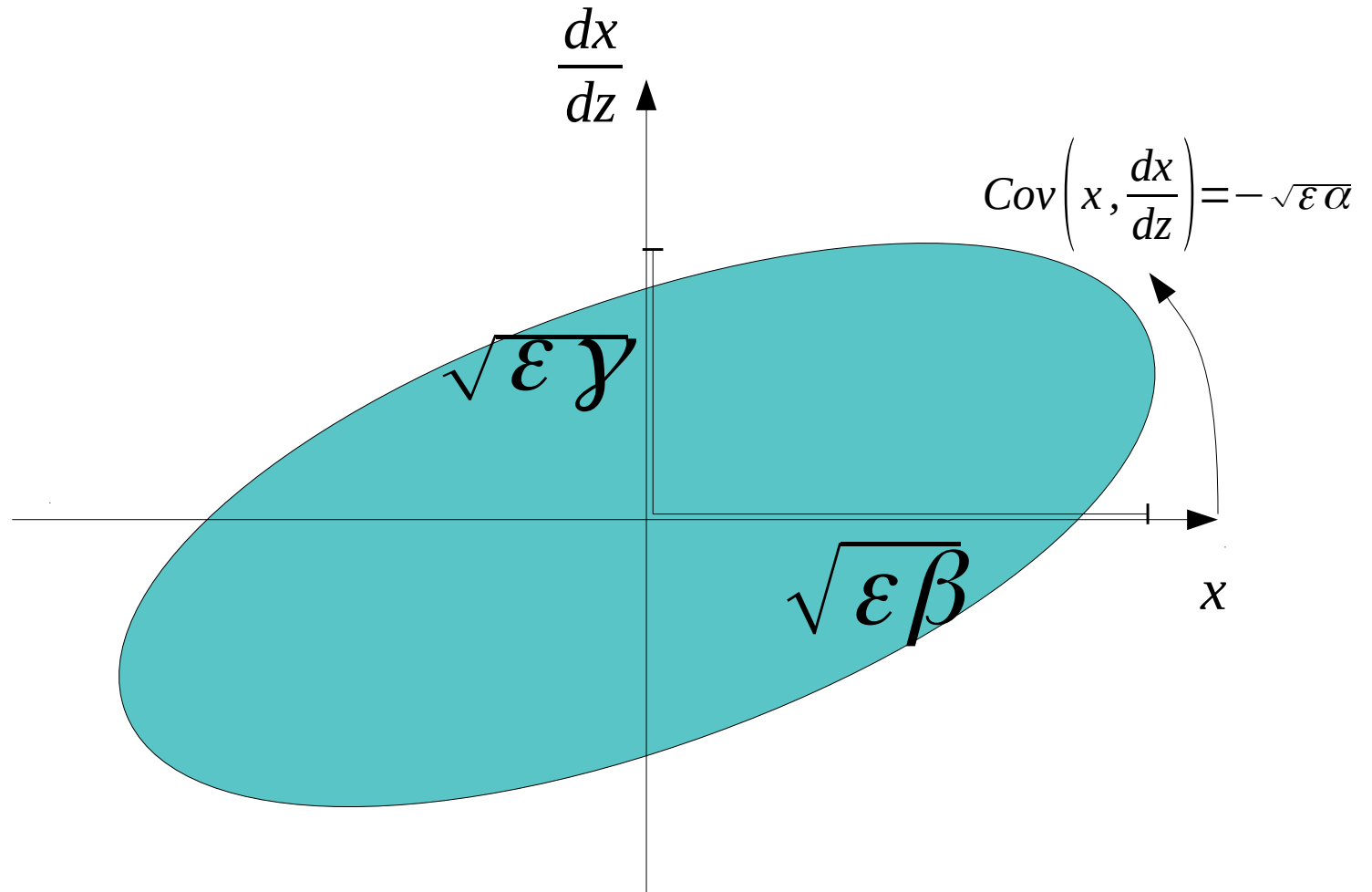
Periodic Lattices and beams



- Beam is composed of many particles
 - Particles occupy a region in phase space
- “Emittance” is area occupied by the entire beam
- Sometimes classify “RMS emittance”
 - Area occupied by ellipse 1 RMS distance from beam centre
- Low emittance is crucial for
 - High luminosity
 - Low losses



Beam ellipse





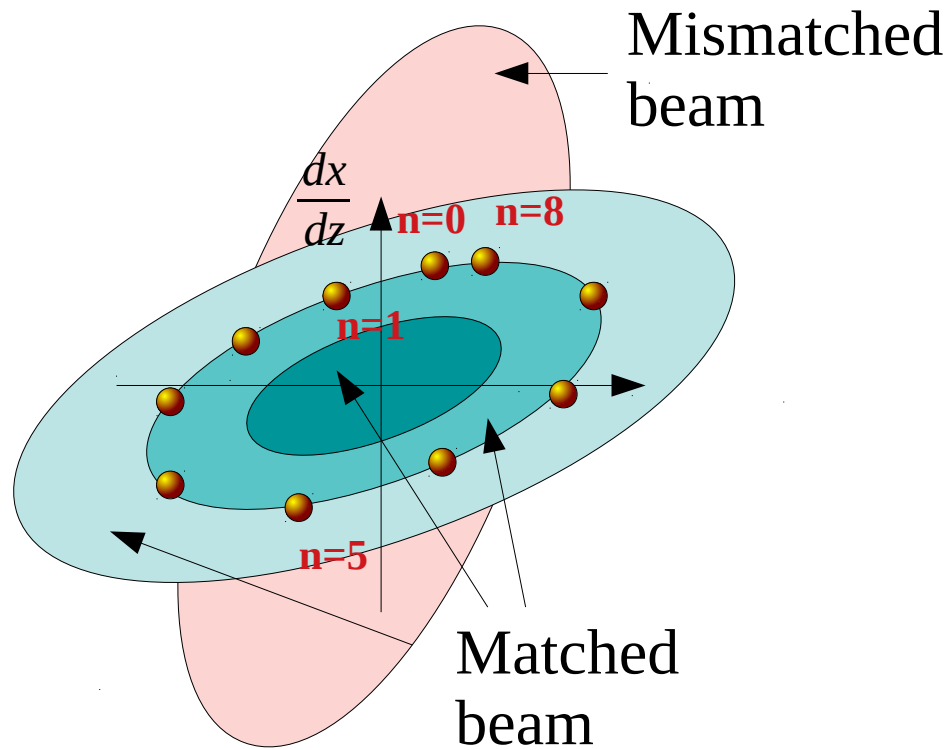
Emittance Growth

- Ideally emittance is conserved, but this is not always the case
- Long list of effects that can cause emittance growth
 - Beam mismatch
 - Scattering off residual gas
 - Scattering off particles in the same beam
 - Scattering off particles in other beams (e.g. in collider)
 - Space charge
 - Resonances:

$$\nu_x = \frac{\mu_x}{2\pi}$$

$$j \nu_x + k \nu_y = N$$

Mismatch





Emittance Reduction (Cooling)

- Several techniques to reduce emittance
 - Synchrotron radiation cooling
 - Stochastic cooling
 - Laser cooling
 - Electron cooling
 - Ionisation cooling
- Fundamental principle is to remove “heat” from the beam using a neighbouring heat sink
 - Comoving electron beam → electron cooling
 - Comoving laser → laser cooling
 - Emission of synchrotron radiation
 - Photon emission caused by (principally) electrons bending in magnetic field



Questions



Questions

- What is behaviour of particles in phase space if
 - $\text{Trace}(M) < 2$
 - $\text{Trace}(M) = 2$
 - $\text{Trace}(M) > 2$



Questions

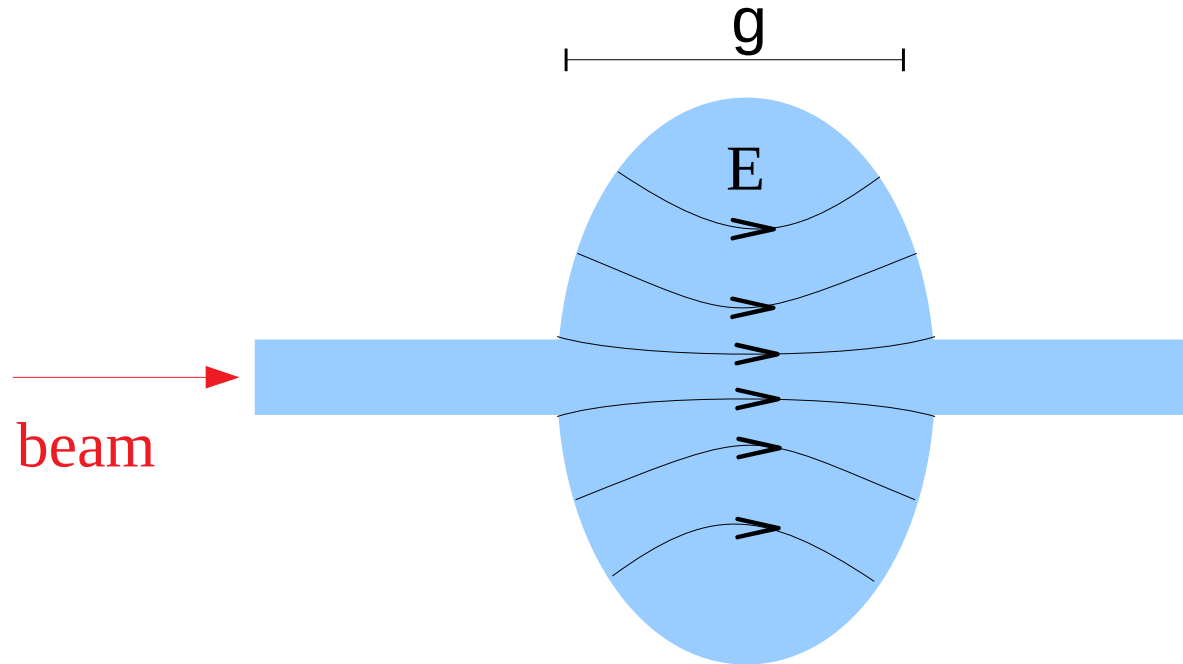
- What is behaviour of particles in phase space if
 - $\text{Trace}(M) < 2$
 - Motion is an ellipse
 - $\text{Trace}(M) = 2$
 - $x \rightarrow +/- x$
 - $\text{Trace}(M) > 2$
 - Motion is a hyperbola



Longitudinal Dynamics

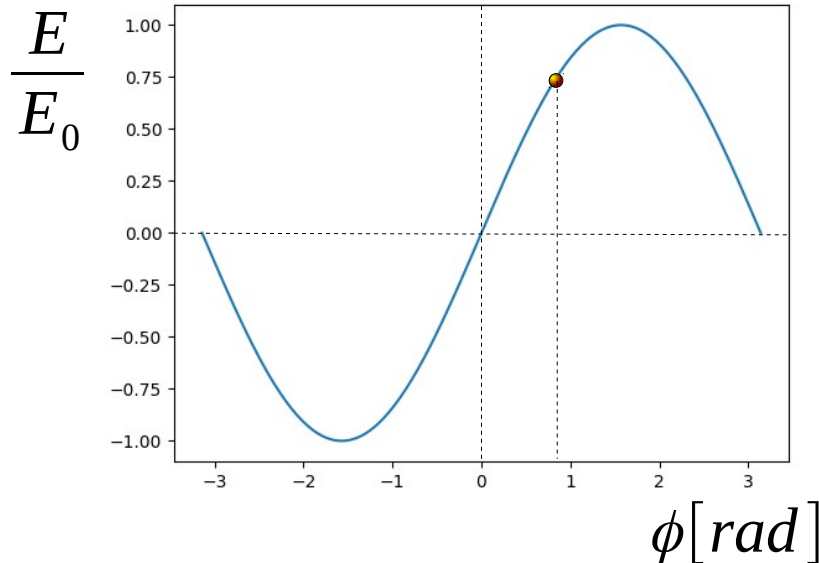
- So much for transverse motion (i.e. x and y planes)
- What about energy and acceleration?
- Electrostatic acceleration limited by breakdown potential
 - Change in energy is given by voltage differential
 - High voltage differentials cause breakdown (sparks)
 - Practically limits electrostatic acceleration to few MeV
- To accelerate beyond MeV require oscillating electric field
- RF Cavities

RF cavity field



- RF cavity holds a resonating EM wave
- Recall Lorentz force law
$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$$
- Force is in direction of motion - energy changes!

RF cavity field



- In RF cavity

$$\vec{E} = E_0 \sin(\omega t + \phi)$$

- Energy change of synchronous particle crossing at ϕ_s

$$\delta W = qTg E_0 \sin(\phi_s)$$

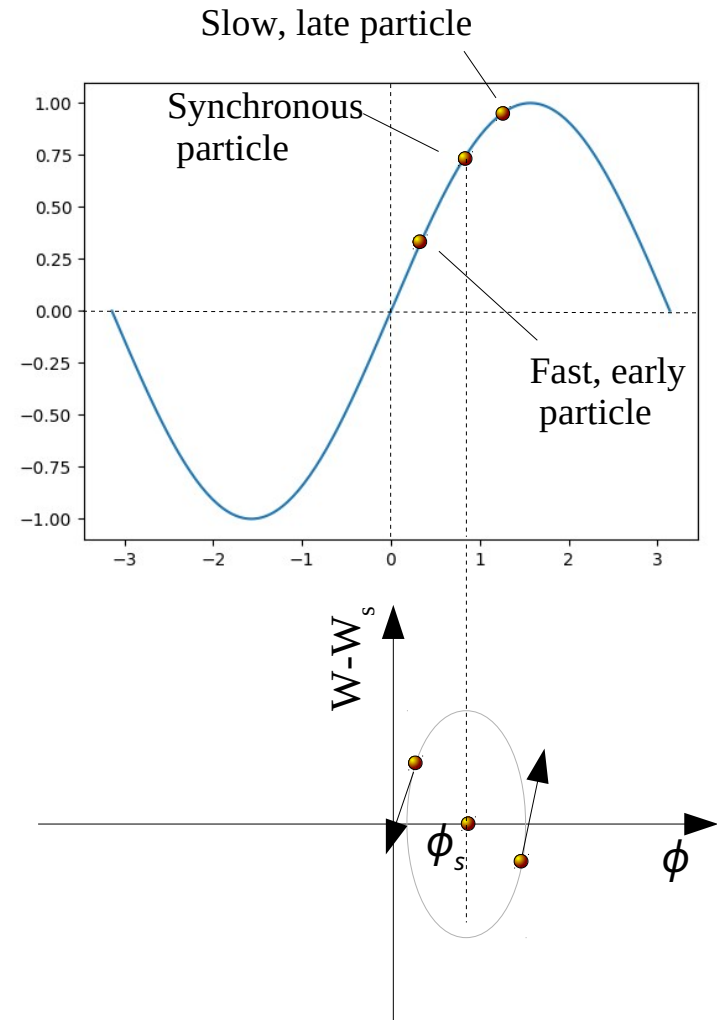
- T is factor to allow for phase to vary a bit during crossing
- g is the gap length

Phase stability

- Particle crossing at phase ϕ relative to synchronous particle

$$\delta W = q T g E_0 \sin(\phi + \phi_s)$$

- Particle arriving early
 - Fast
 - t negative
 - Gets smaller energy kick
 - Ends up relatively slower
- Particle arriving late
 - Slow
 - t positive
 - Gets bigger energy kick
 - Ends up relatively faster
- Phase stability!

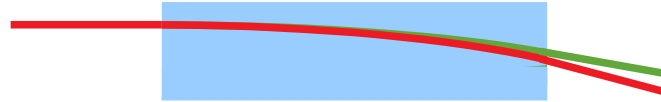




Dealing with momentum spread

- Momentum spread introduces a few effects
 - Dispersion
 - Chromaticity
 - Momentum compaction
- Dispersion:
 - Off-momentum particles follow a different trajectory
- Momentum compaction (rings):
 - Different path length yields different time of flight
- Chromaticity:
 - Off-momentum particles get a different focussing strength

Dispersion



- Recall the definition of magnetic rigidity

$$B\rho = \frac{p}{q}$$

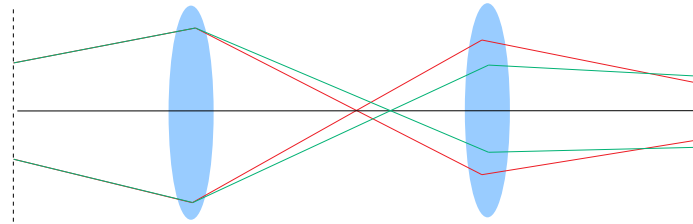
- Particles having different momentum (p) get different radius of curvature
 - Introduce dispersion D

$$D = p \frac{dx}{dp}$$

- Which is another optical function that we must make periodic

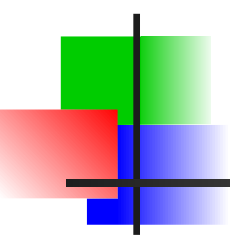
Chromaticity

- Chromaticity arises because quadrupoles focus differently for different momenta



$$k = q \frac{b_0}{p}$$

- This often limits the degree of focussing at a collision point
 - Limits luminosity
- Can deliberately enhance/reduce chromaticity by
 - Introduce a dispersion
 - Using a magnet with variable focussing strength across the aperture - “sextupole”



Questions



Review

- Dipoles are used to bend a beam – rigidity is $B\rho = \frac{p}{q}$
- Quadrupoles are used to focus a beam: $k = q \frac{b_0}{p}$
- Beam in each of x and y can be characterised by 3 Twiss parameters and an emittance
- Lattices can be characterised by a phase advance
- RF cavities are used to accelerate the beam
- Introducing momentum spread, one can also define a dispersion (and its derivative with respect to z)

Finally... luminosity

- Luminosity defines the number of interactions in a collider per unit time for a given cross section
- Luminosity will increase if
 - Beam is narrower
 - Current is higher

The diagram illustrates the formula for luminosity, $\tilde{L} = \frac{N_1 N_2 f N_b}{4 \pi \sigma_x \sigma_y}$, with callouts for each variable:

- Number of particles in each bunch** (green box) points to N_1 and N_2 .
- Revolution frequency** (blue box) points to f .
- Number of bunches** (red box) points to N_b .
- Width of Each bunch** (orange box) points to σ_x and σ_y .

$$\tilde{L} = \frac{N_1 N_2 f N_b}{4 \pi \sigma_x \sigma_y}$$



What dictates luminosity?

$$\tilde{L} = \frac{N_1 N_2 f N_b}{4 \pi \sigma_x \sigma_y}$$

- Typically
 - Number of particles → space charge
 - Revolution frequency → ring circumference
 - Number of bunches → RF frequency
 - Beam width → $\sqrt{\varepsilon \beta}$
 - Emittance (cooling?)
 - Twiss beta (final focus and chromaticity)



Next lecture...

- Accelerator equipment
- Types of accelerator
- Current facilities
- Future facilities