



Science and  
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# vFFA and novel optics for muon collider accelerator complex

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7 August 2020

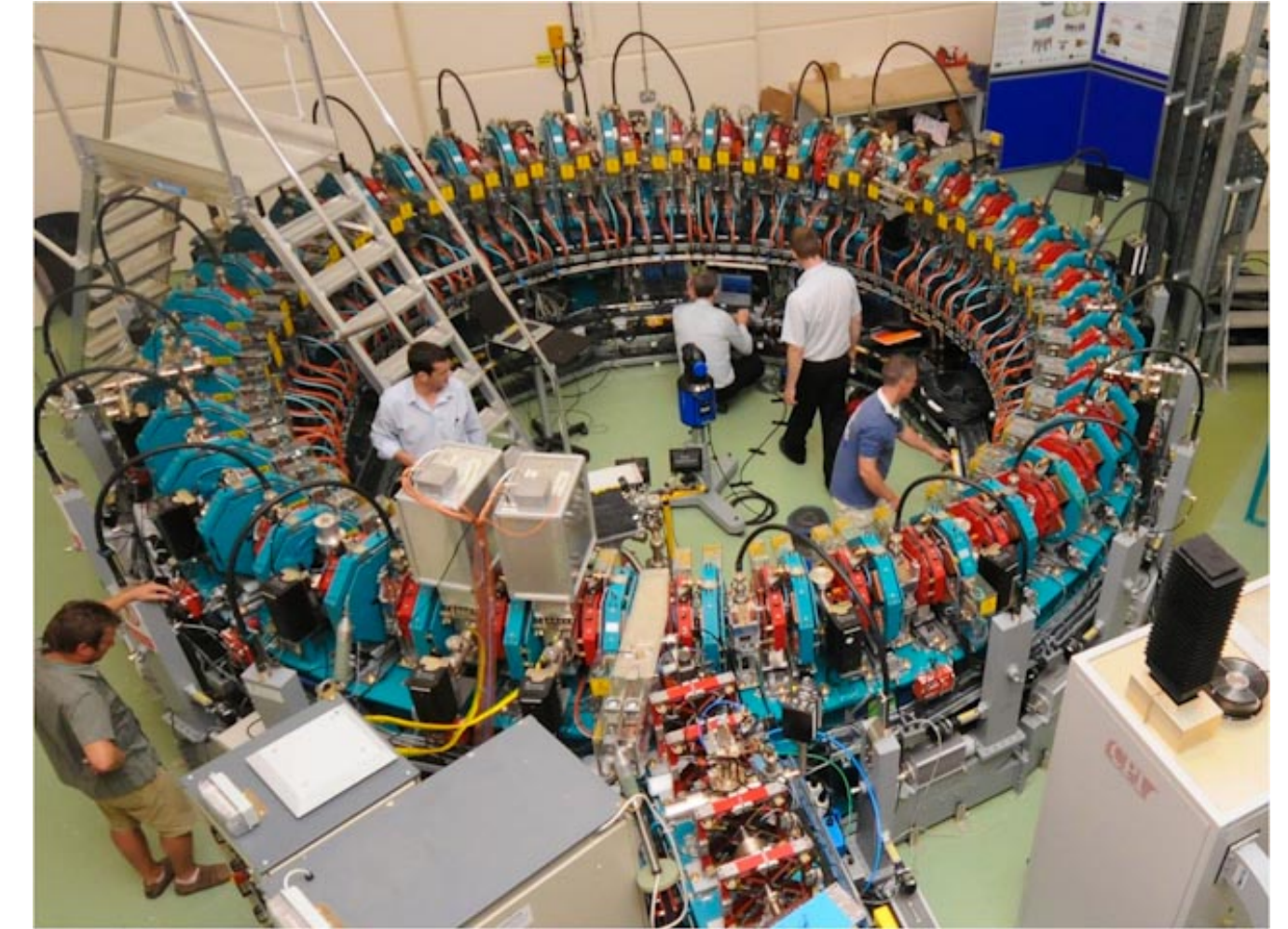
UK muon collider and nuStorm meeting

# Contents

- Muon accelerators and FFAs in the UK
- Vertical excursion FFA
- Muon collider accelerator complex
  - Muon accelerator
  - Muon collider
  - Muon accumulator (LEMMA)
  - Proton driver
  - (Positron accumulator (LEMMA))
- Summary

# Muon acceleration and FFA developments in the UK

- Neutrino factory design study
  - Accelerator complex with a FFA as the final muon accelerator.
- Linear nonscaling FFA
  - Electron Model of Many (or Muon) Applications: EMMA.
  - It was commissioned in 2012 at Daresbury Laboratory.
- PAMELA for proton therapy.
- FFA design for ISIS-II.
  - Prototype vertical excursion FFA.
  - Feasibility study is on going.



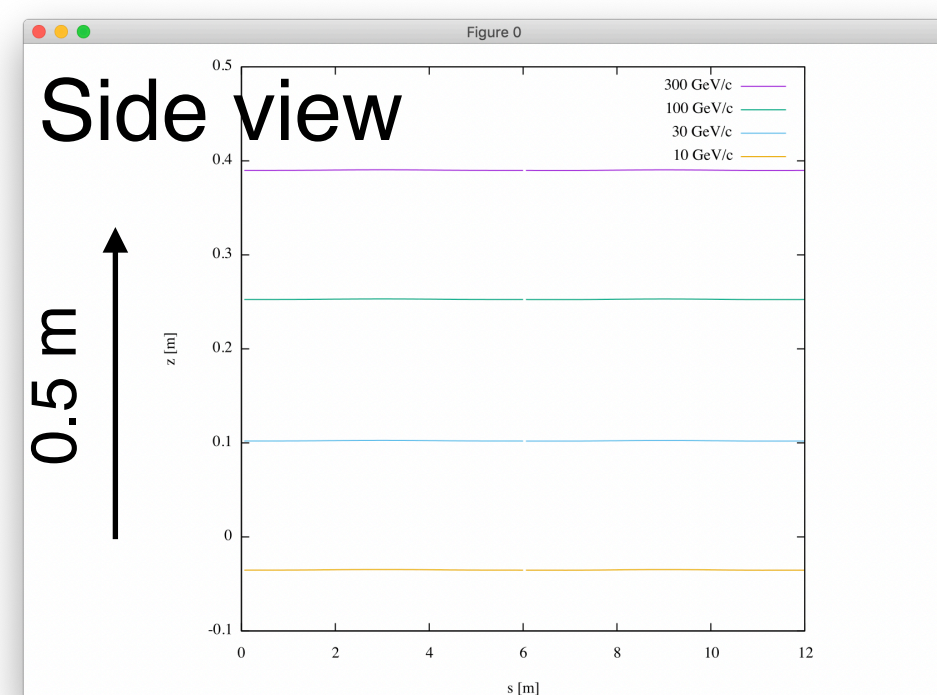
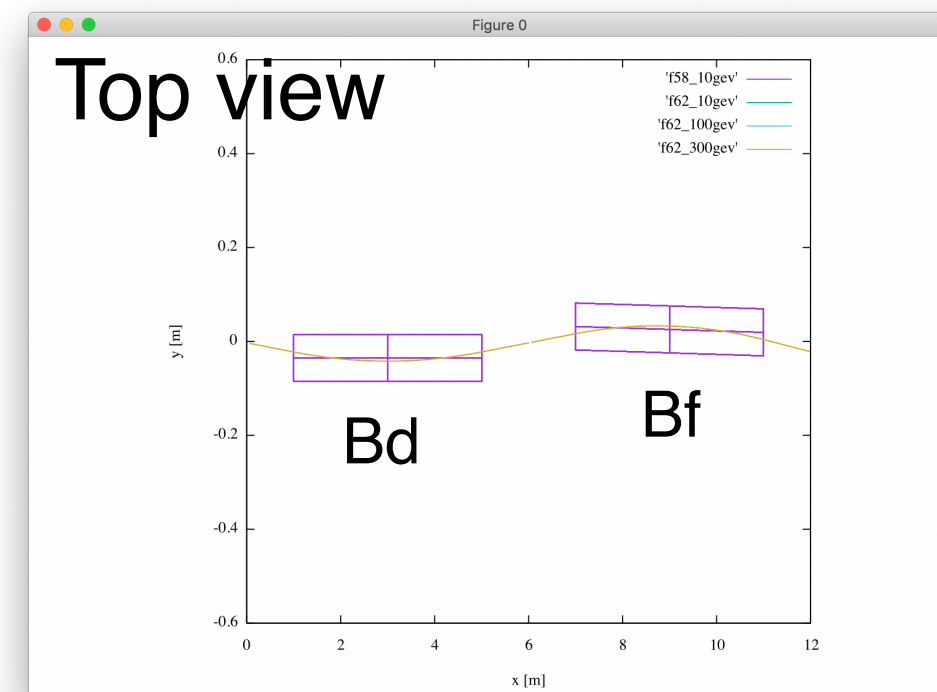
EMMA

# Vertical excursion FFA (vFFA)

- Invented in 1955 by Tihiro Ohkawa.
- Re-invented in 2013 by Stephen Brooks.
- Orbit moves vertically when the beams are accelerated.
- **Path length is constant for all the momenta.**  
**Momentum compaction factor is zero.**
- It was called electron cyclotron
  - Ultra-relativistic particles can be accelerated continuously with fixed field magnets.
- Ideal for muon acceleration.
  - No ramping of magnetic fields.
  - No RF frequency modulation.
  - Large momentum ratio from injection to extraction, e.g.  $\sim 30$ .
  - Wiggling orbits to spread out neutrino.
  - **Enough vertical aperture is needed.**

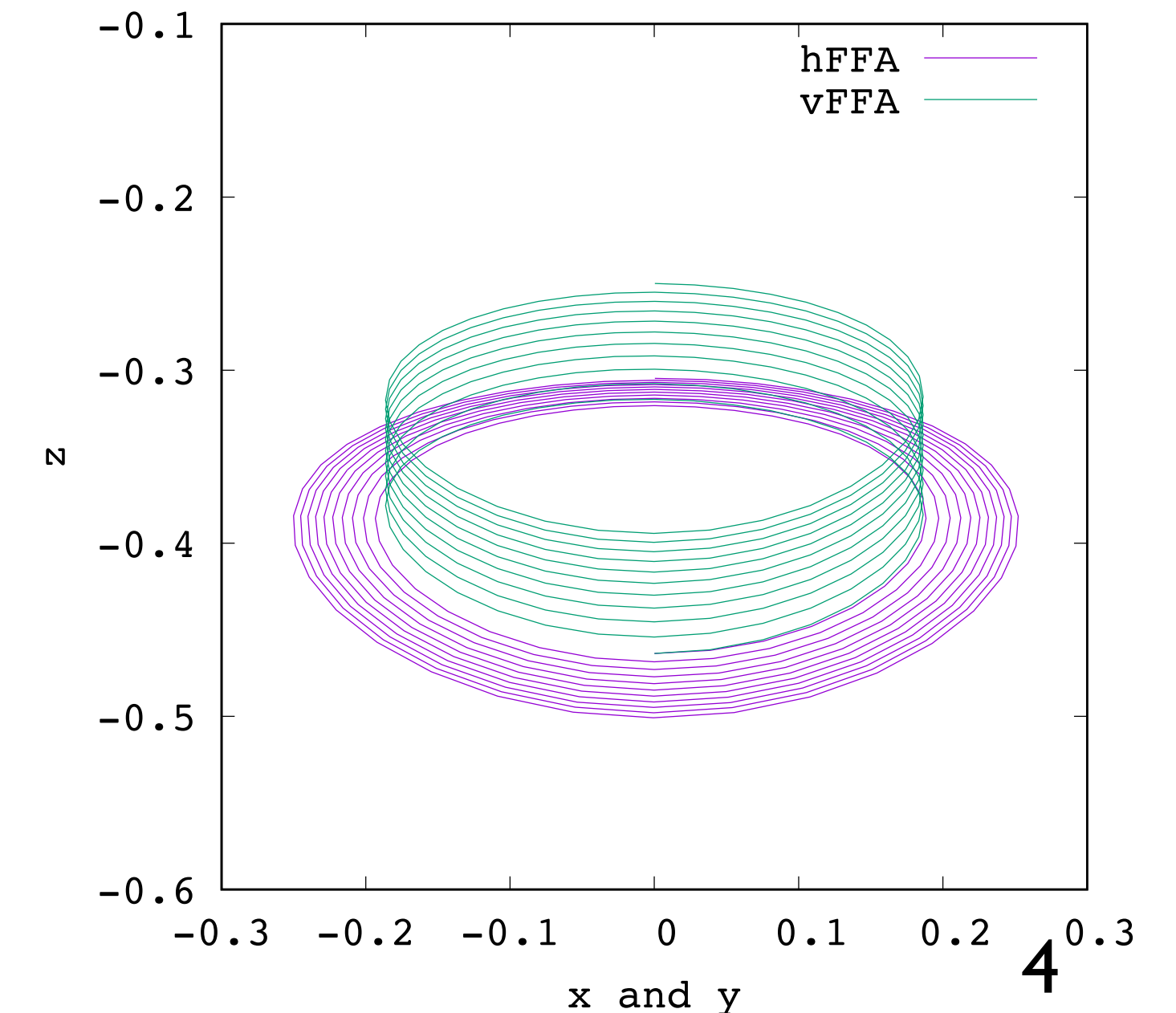
G8. FFAG Electron Cyclotron.\* TIHIRO OHKAWA, *University of Illinois* (introduced by D. W. Kerst).—New types of FFAG<sup>1</sup> accelerators having the same orbit length for all momenta are proposed. In these types electrons, injected with an energy of a few Mev, are accelerated by a fixed frequency electric field until the radiation loss becomes serious, probably

Bull. APS 30, 20 (1955)  
by Tihiro Ohkawa



$$B = B_0 \exp(my)$$

$m$  : field index  
 $y$  : vertical



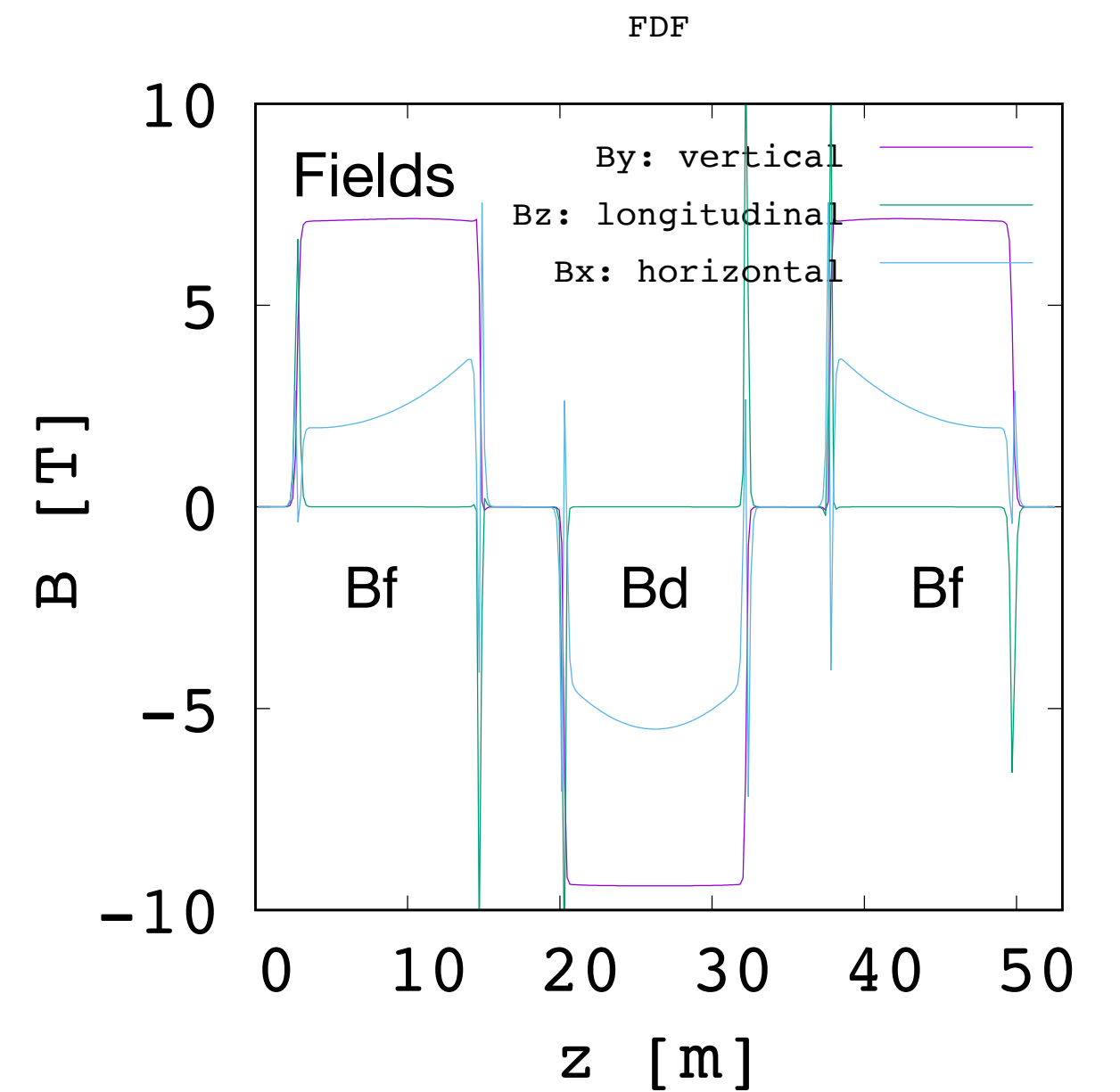
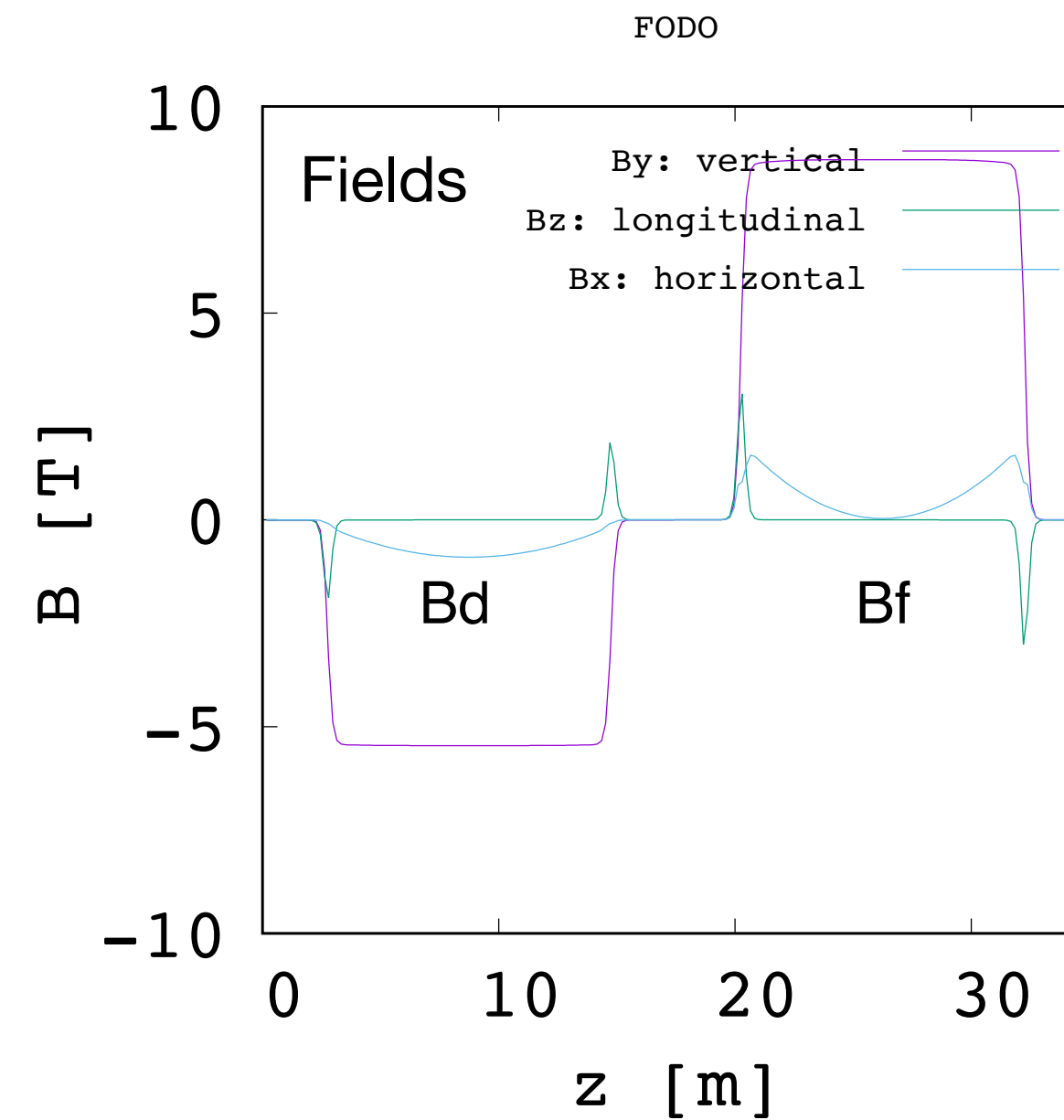
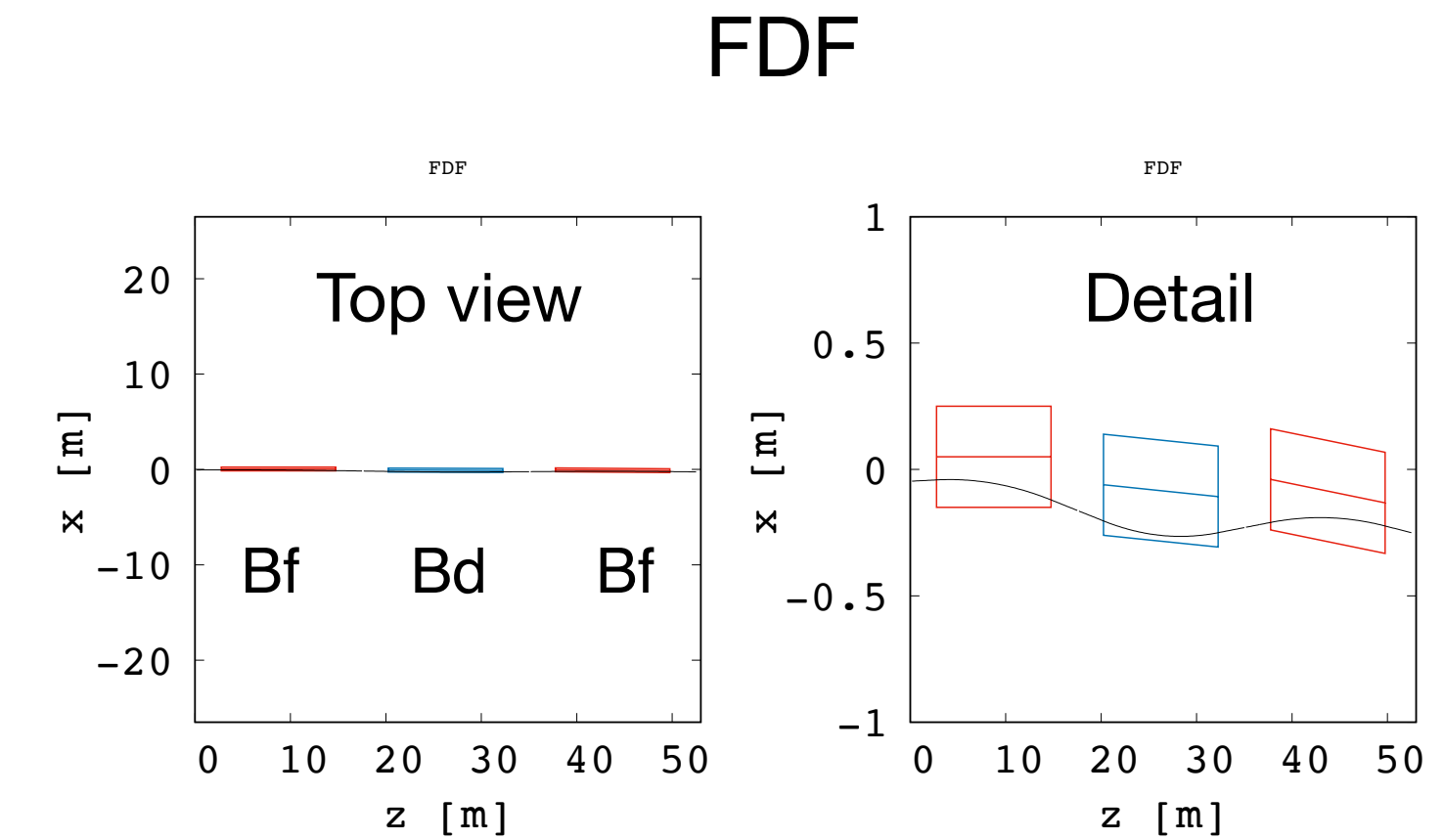
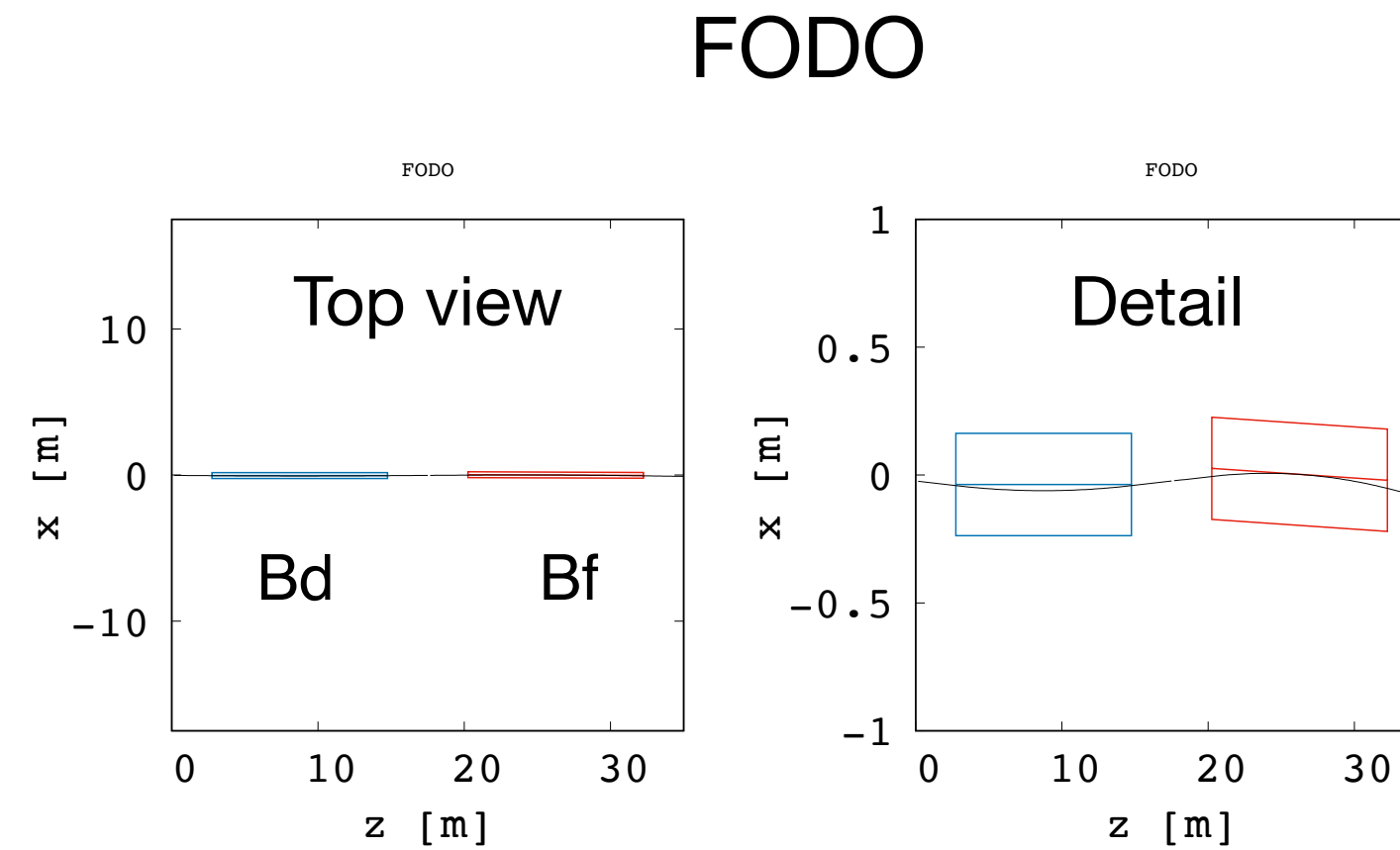
# Muon accelerator ring

# Design principle

- Circumference is about LHC.
- Top energy is 1.5 TeV.
- Momentum ratio from injection to top energy is about 30.
- Maximum field is no more than 10 T.
- Orbit excursion is less than 0.5 m.

# 1.5 TeV accelerator in LHC tunnel

	FODO	FDF
Energy	50 GeV to 1.5 TeV	50 GeV to 1.5 TeV
Cell length	35 m	52.5 m
Magnet length	2 x 15 m	3 x 15 m
# of cell	810	540
Maximum field	8.7 T	10.6 T
Field index m	6.8	3.0
Orbit excursion	0.50 m	1.13 m
Cell tune	0.3957 / 0.0861	0.3510 / 0.1515



# c.f. Acceleration with RCS

Neufer and Shiltsev, JINST **13** (2018) T10003

**Table 1.** Muon RCS Accelerator Parameters

<b>Scenario</b>	<b>“LHC-S”</b>	<b>“LHC-D”</b>	<b>“SPS”</b>	
$C$ , km	26.7	26.7	6.9	
$E_{\max}$ , TeV	7	7	0.45	
$E_{\text{inj}}$ , TeV	0.45	4	0.03	
$f_{\text{rep}}$ , Hz	5	4	20	
$\Delta E/\text{turn}$ , GeV	14.0	3.5	9.2	3.7
$B_{\text{SC}}$ , T	16	16	16	8
$L_{\text{SC}}$ , km	4.8	7.1	2.9	0.63
$B_{\text{pls}}$ , T	3.8	2.0	1.9	0.8
$\tau_{\text{ramp}}$ , ms	42	76	34	2.6
$dB_{\text{pls}}/dt$ , T/s	180	52	112	615



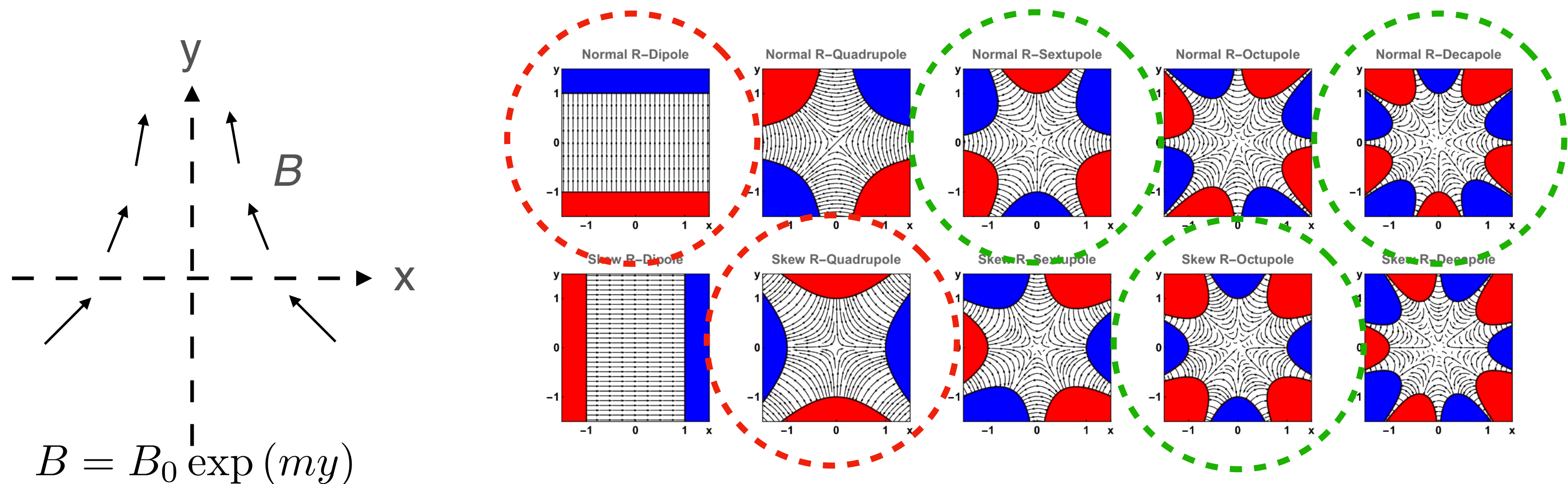
# Muon collider ring

# Design principle

- Circumference should be as small as possible..
- Energy is 1.5 TeV.
- Momentum compaction factor is zero if required.
- Large momentum acceptance.
- Maximum field is no more than 14 T (use the same criterion of LEMMA accumulator ring).
- Follow vFFA idea but use only up to skew quadrupole.

$$\frac{B_y}{B_0} = \exp(my) \cos(mx) \approx 1 + my + \frac{m^2}{2!} (y^2 - x^2) + \frac{m^3}{3!} (y^3 - 3x^2y) + \dots$$

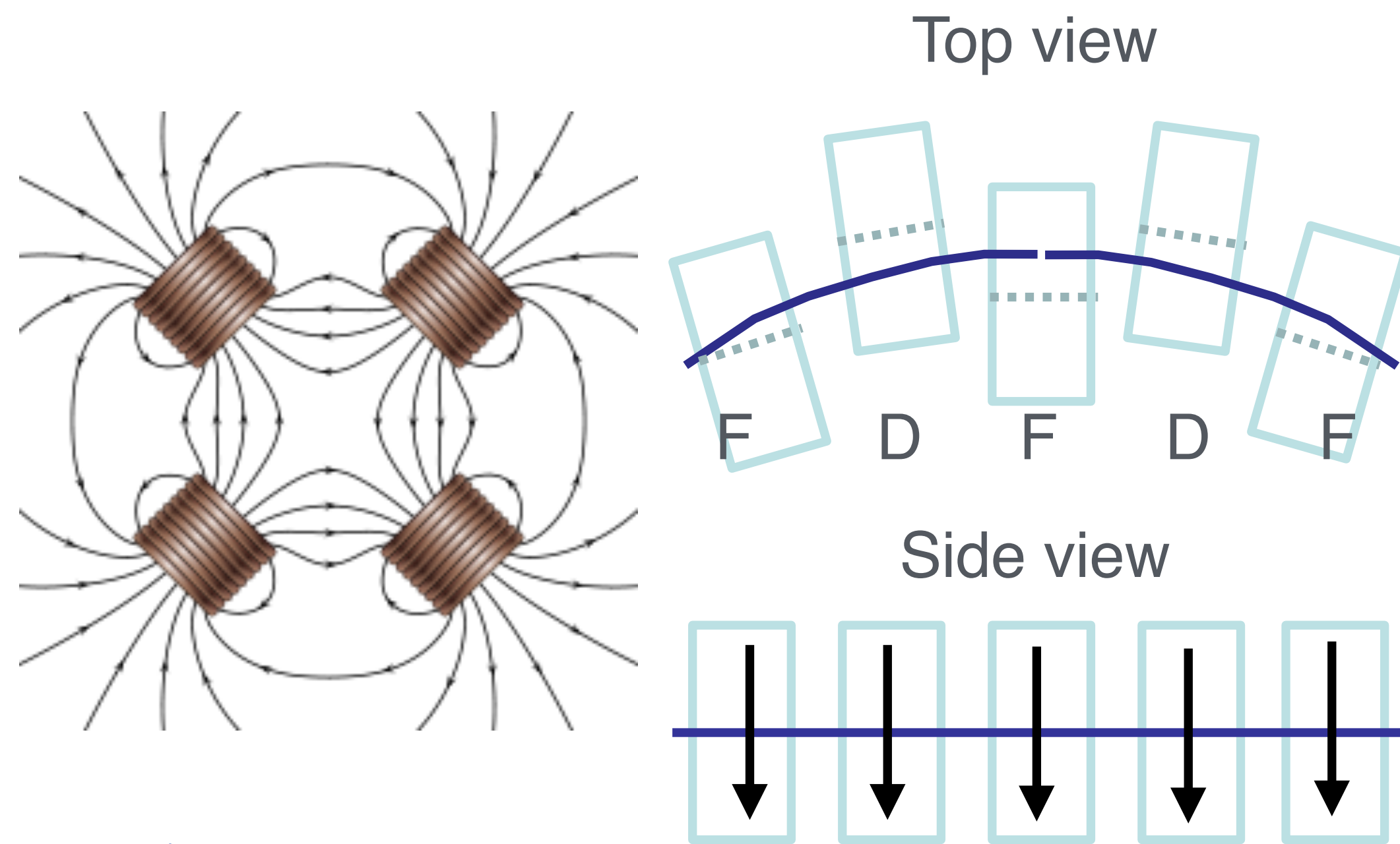
Normal Dipole
Skew Quad
Normal Sext
Skew Oct



# Combined function (CF) lattice

## Normal quadrupole

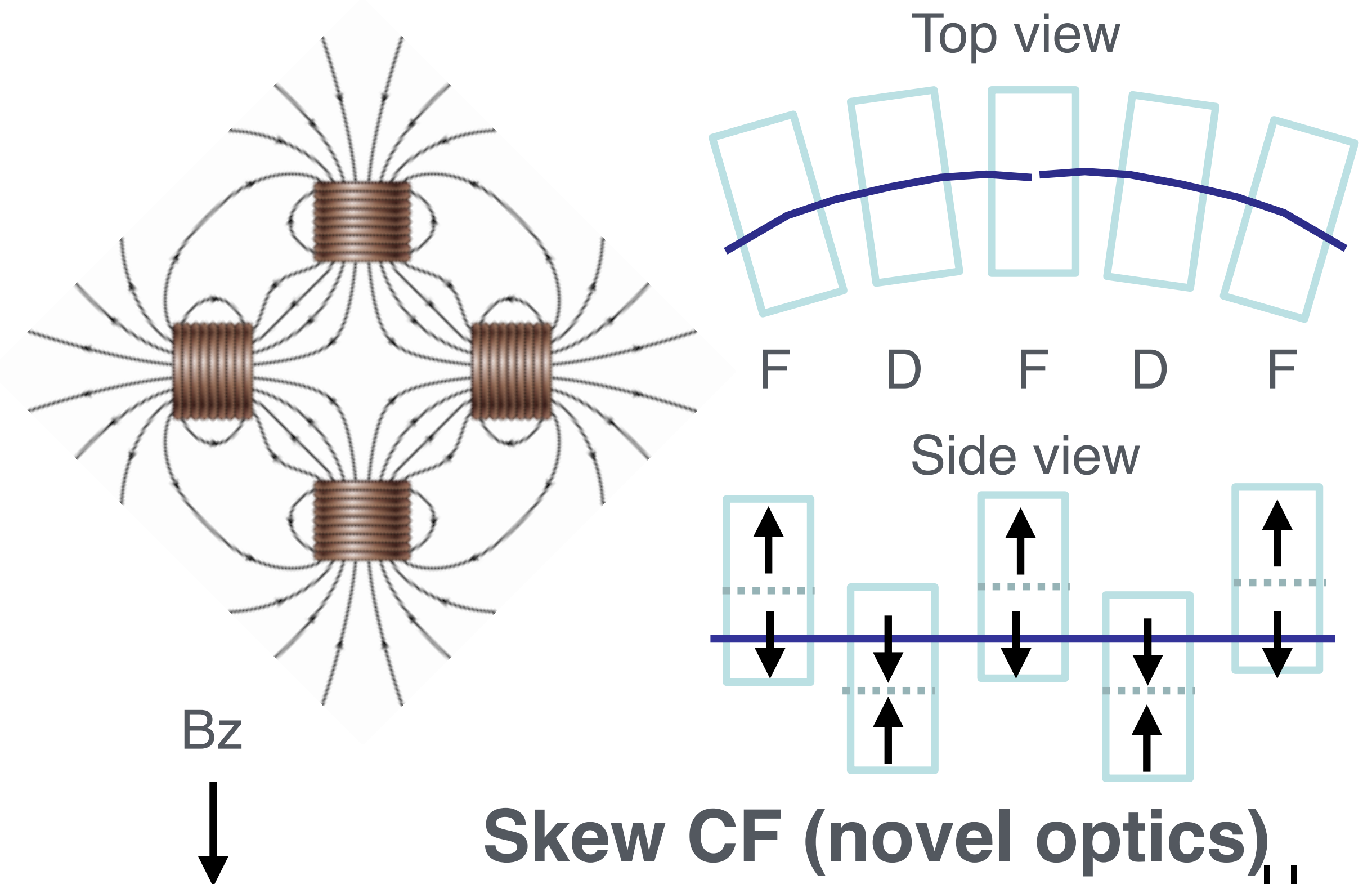
- F (normal bend) and D (normal bend) make the orbit circle.
- Vertical field get stronger **along radial direction**
- **Equilibrium orbit moves horizontally.**



Normal CF

## Skew quadrupole

- F (normal bend) and D (normal bend) make the orbit circle.
- Vertical field get stronger **along vertical direction**
- **Equilibrium orbit moves vertically.**



Skew CF (novel optics)<sub>11</sub>

# 1.5 TeV collider ring

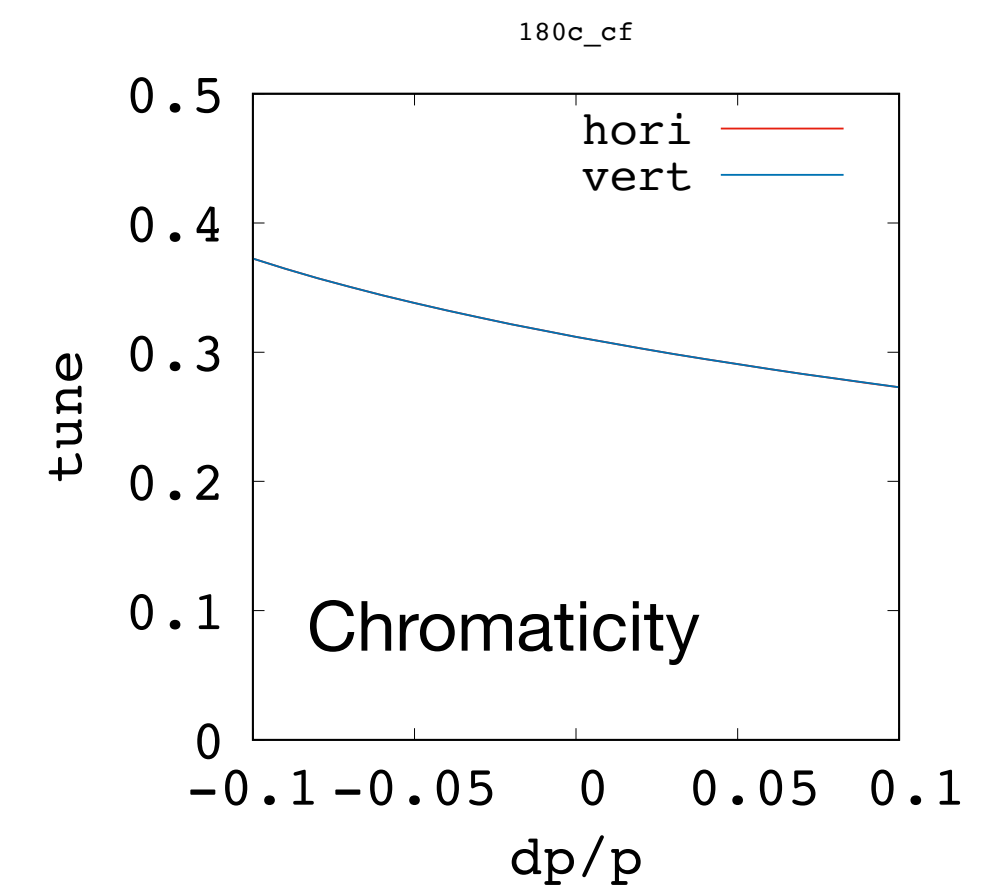
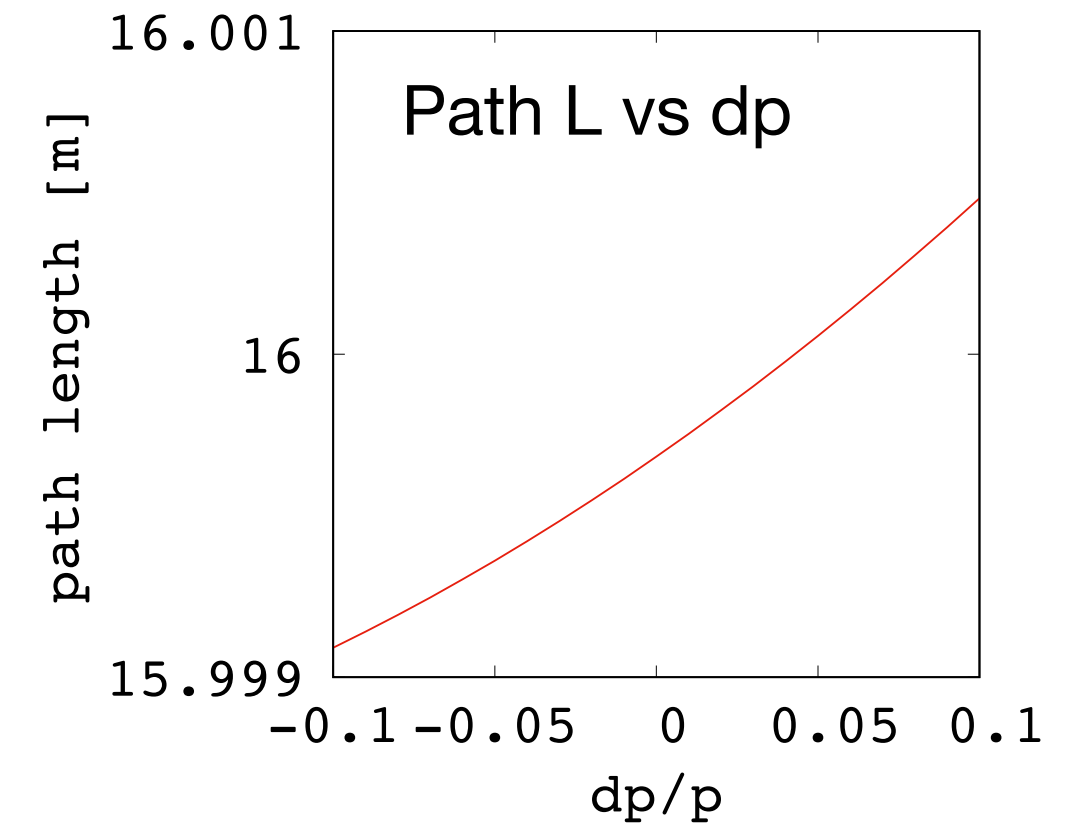
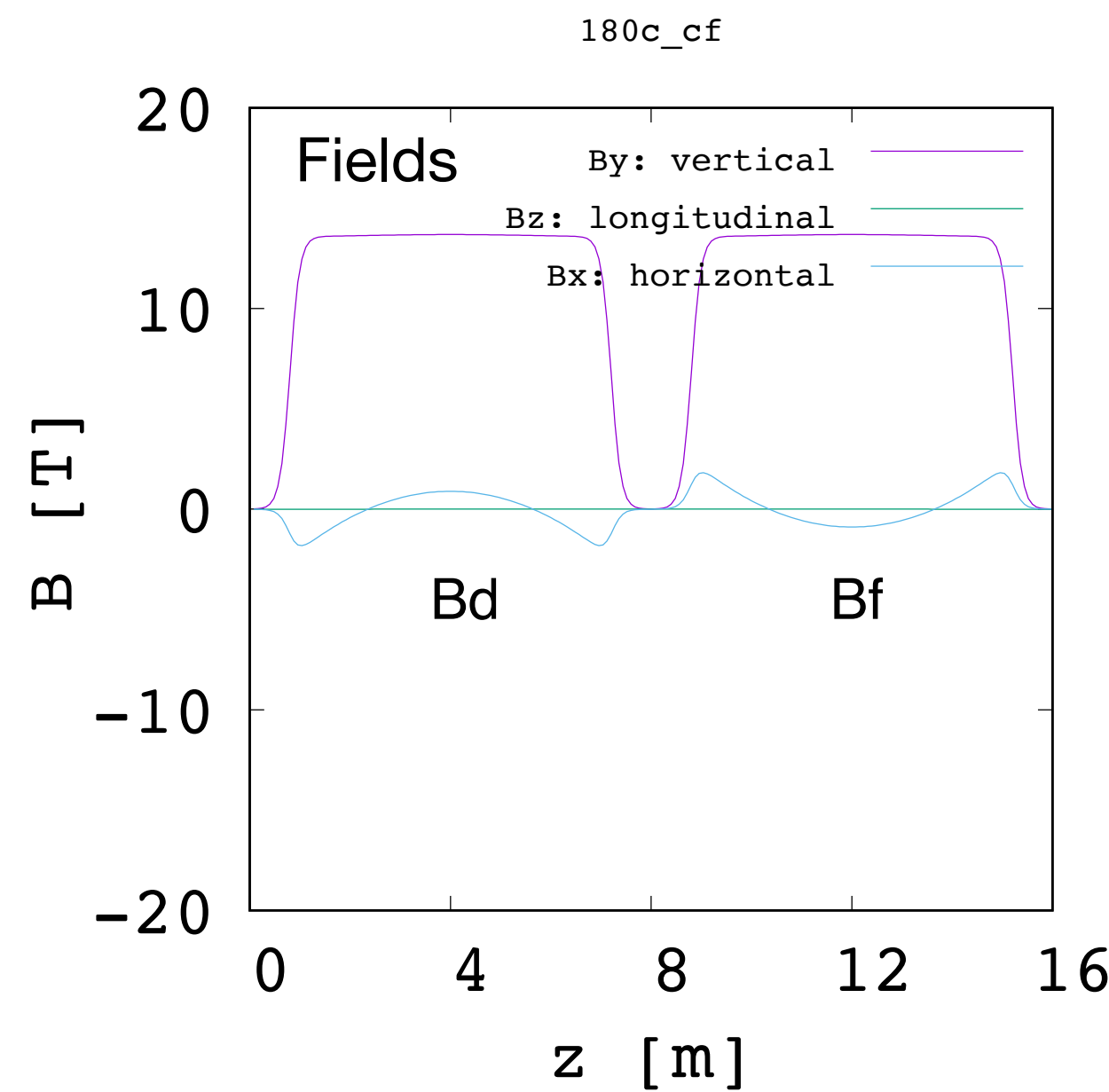
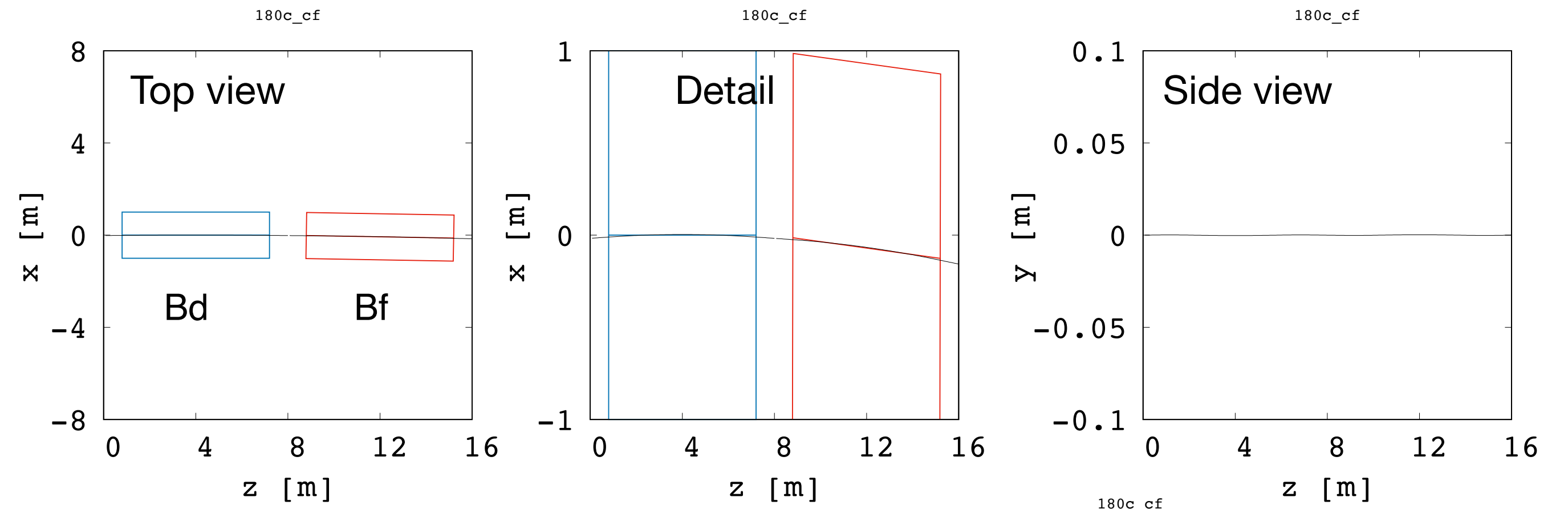
**Without** constraint of momentum compaction = 0.

**Minimise circumference.**

# 1.5 TeV collider ring

*minimise circumference, arc only*

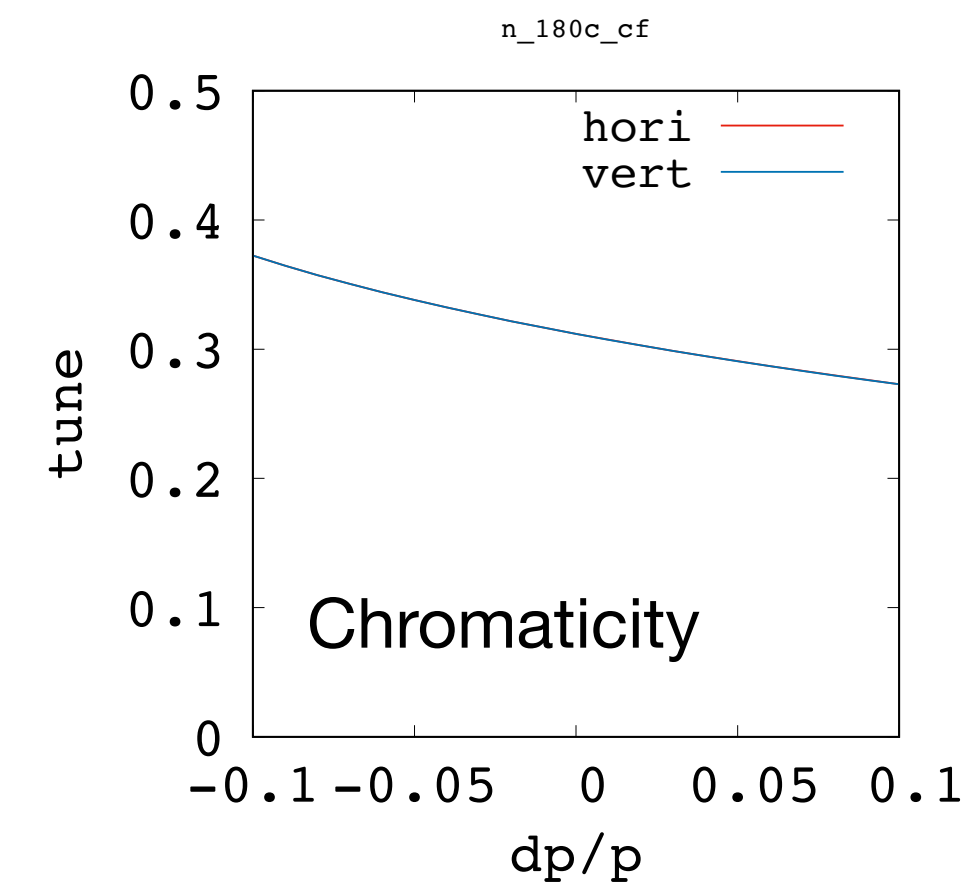
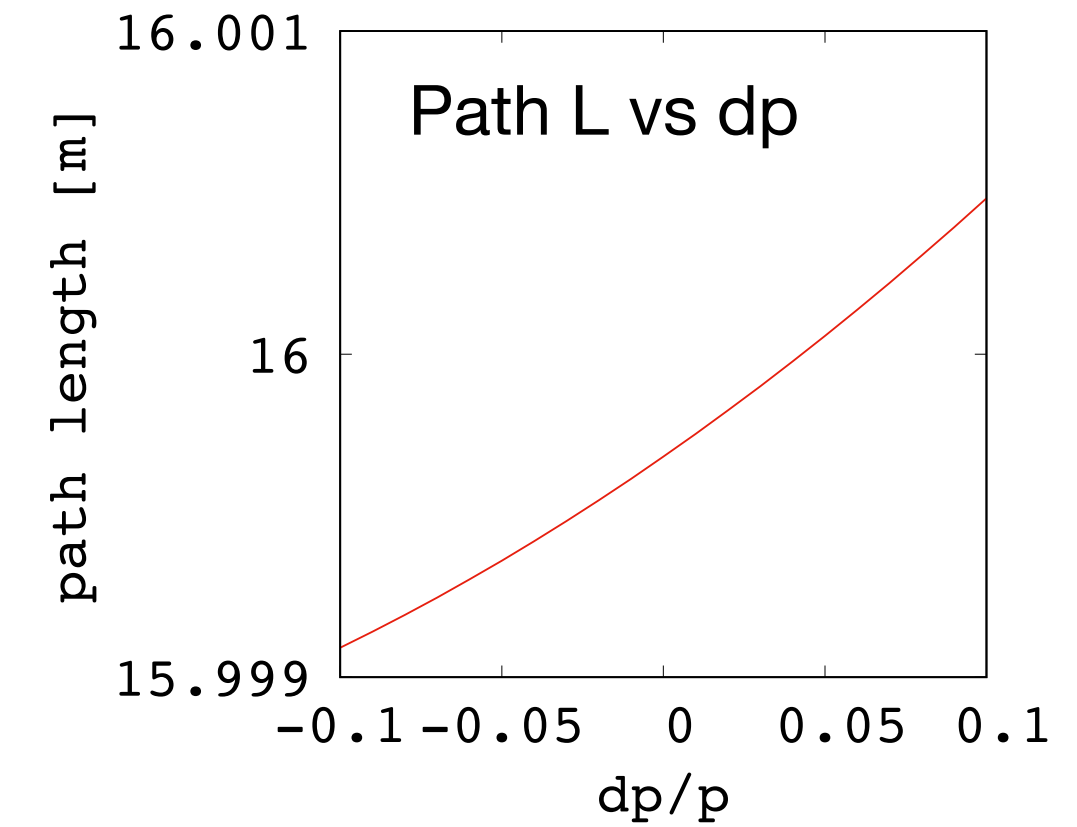
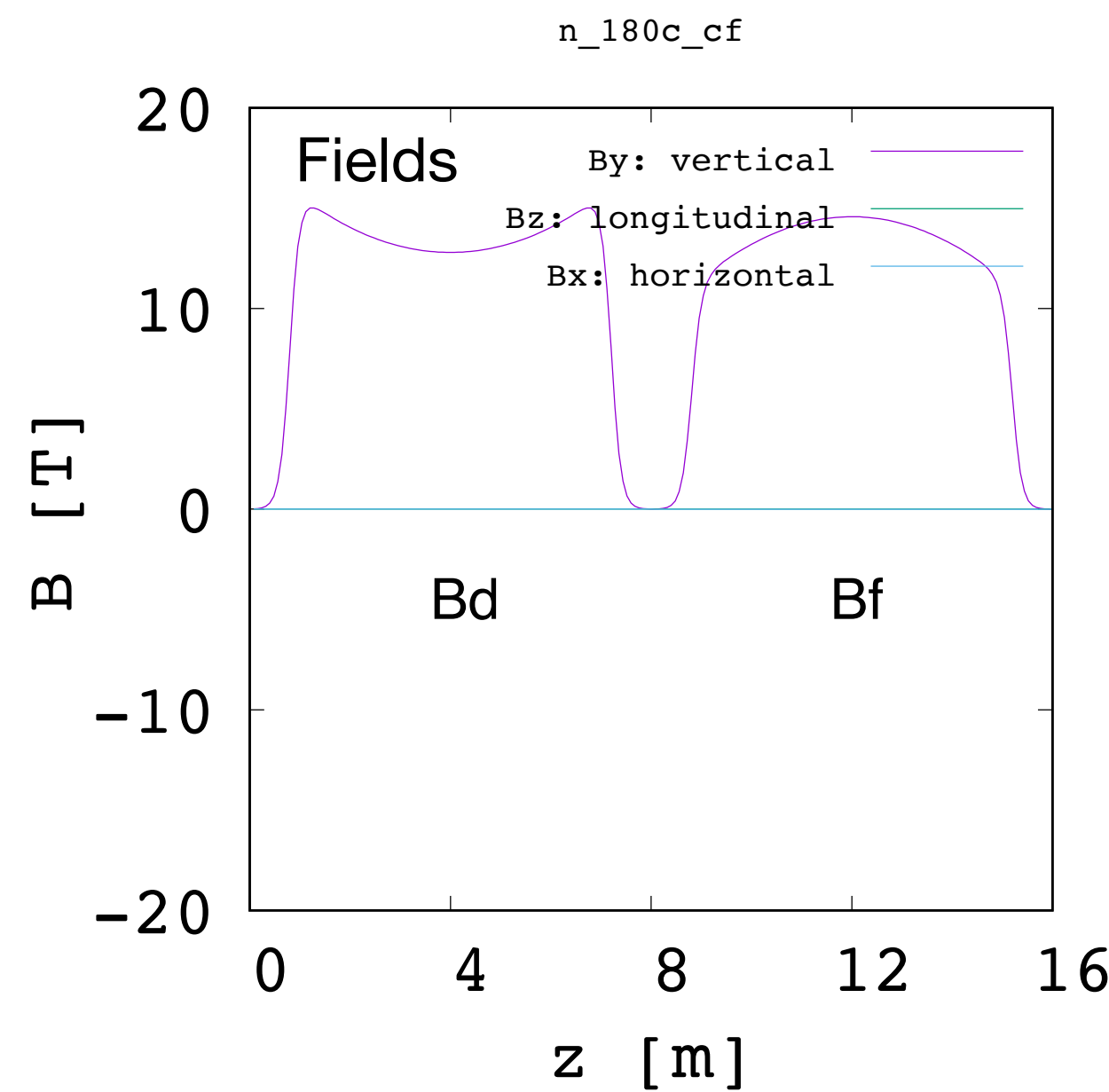
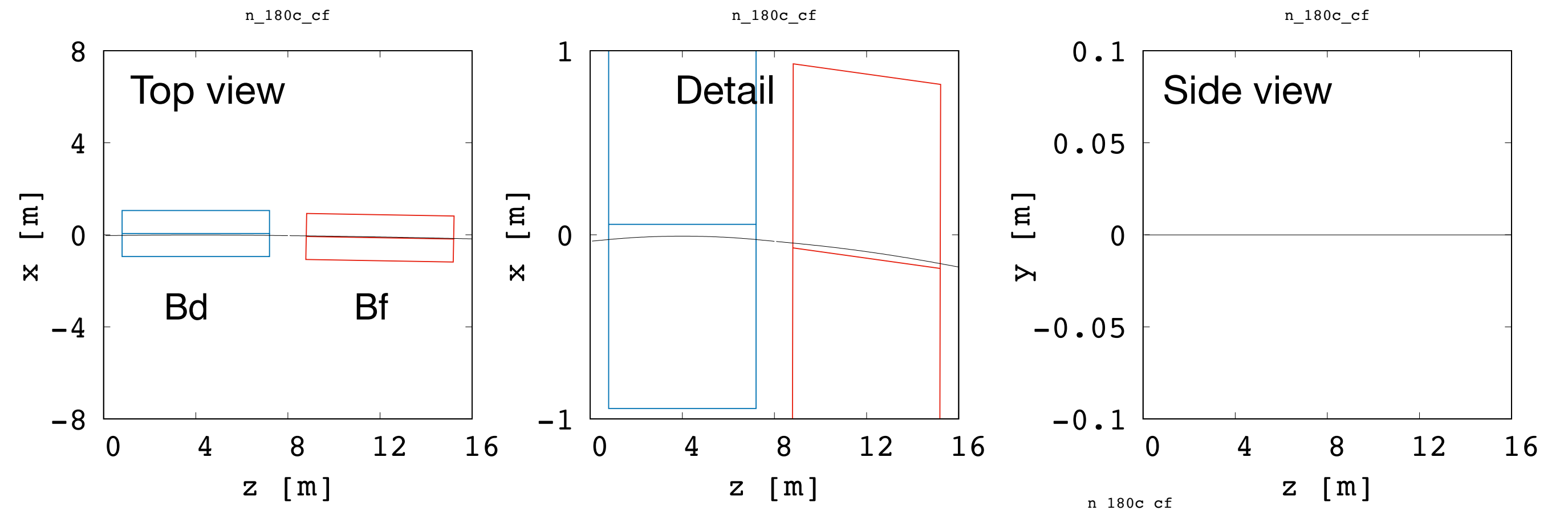
	<b>Skew FODO</b>
Energy	1.5 TeV
Momentum compaction	$4.32 \times 10^{-4}$
Circumference	2880 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	180
Maximum field	14 T
Field gradient	240 T/m
Cell tune	0.3119 / 0.3119



# 1.5 TeV collider ring

*minimise circumference, arc only*

	Normal FODO
Energy	1.5 TeV
Momentum compaction	$4.32 \times 10^{-4}$
Circumference	2880 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	180
Maximum field	14 T
Field gradient	240 T/m
Cell tune	0.3119 / 0.3119



# 1.5 TeV collider ring

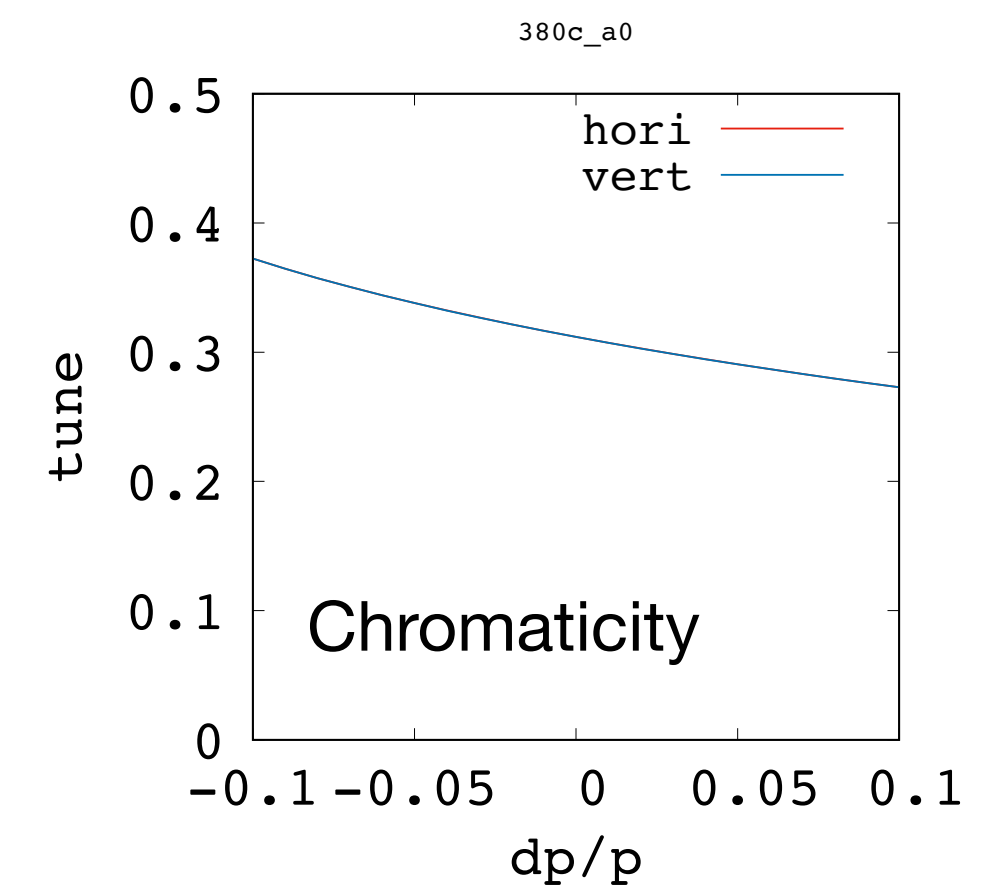
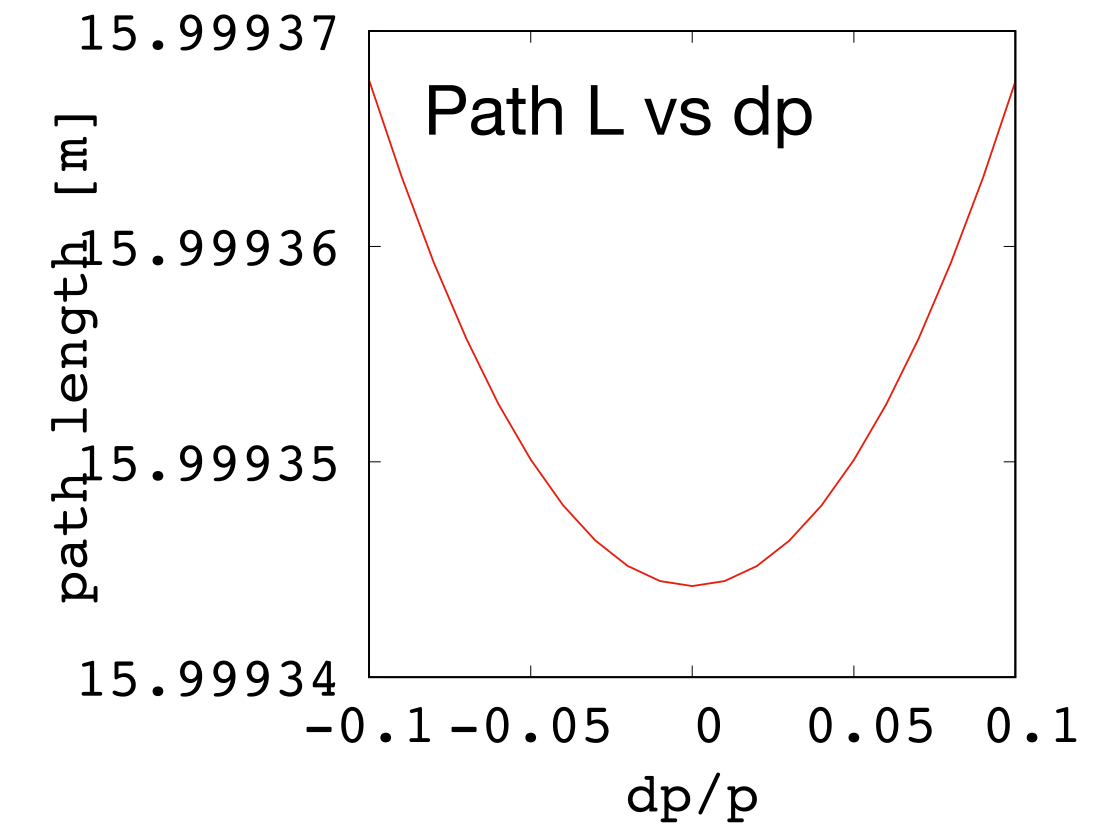
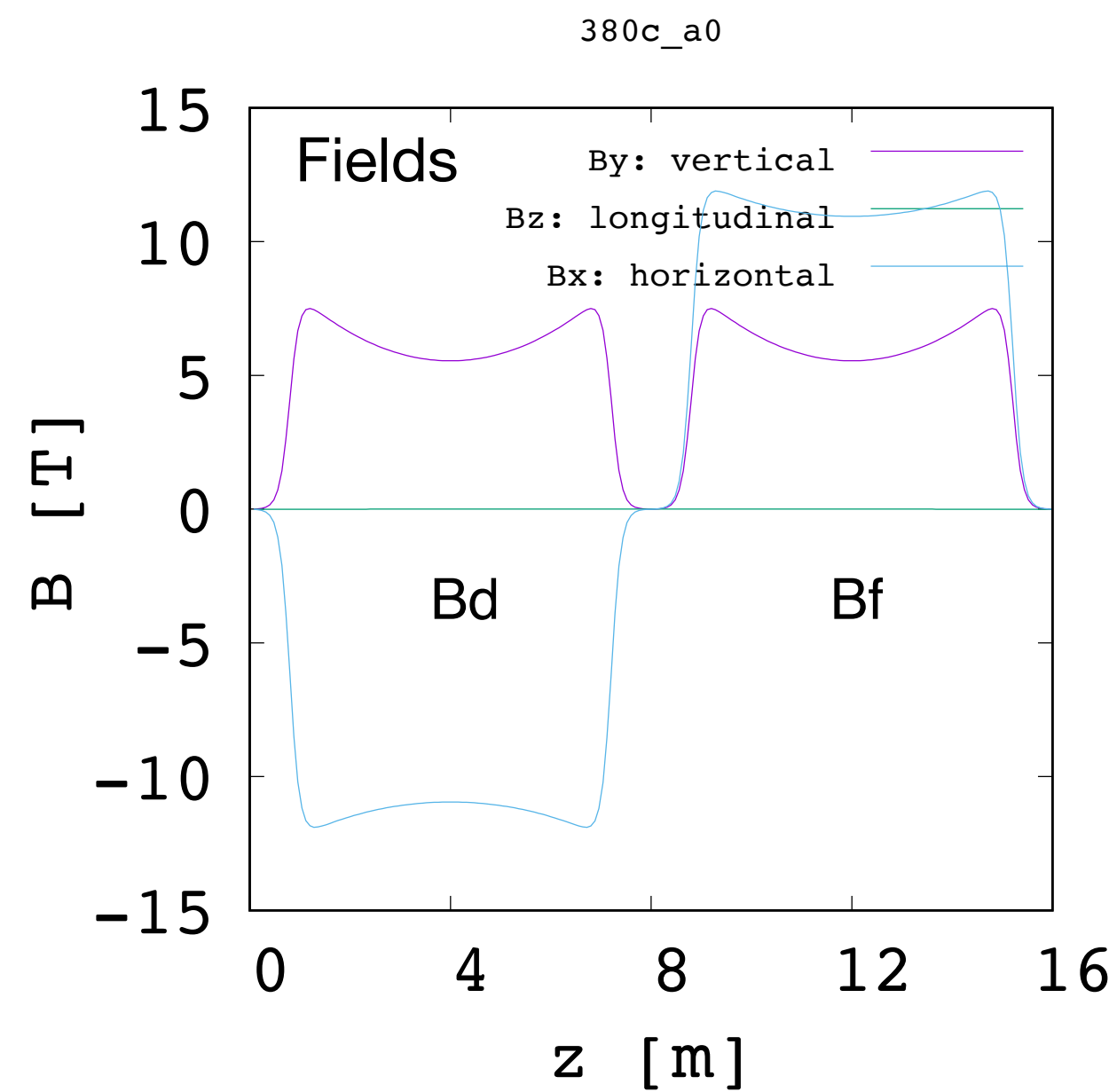
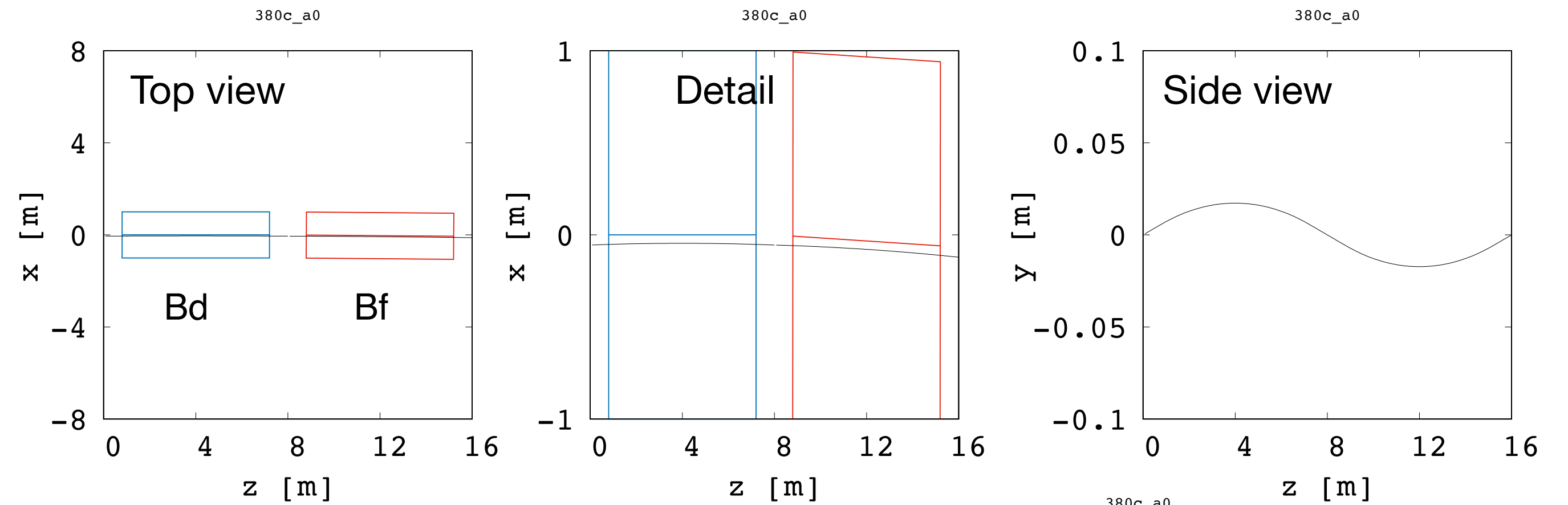
**Impose** constraint of momentum compaction = 0.

Allow increase of circumference.

# 1.5 TeV collider ring

*momentum comp=0, arc only*

	<b>Skew FODO</b>
Energy	1.5 TeV
Momentum compaction	0
Circumference	6080 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	380
Maximum field	14 T
Field gradient	240 T/m
Cell tune	0.3131 / 0.3131

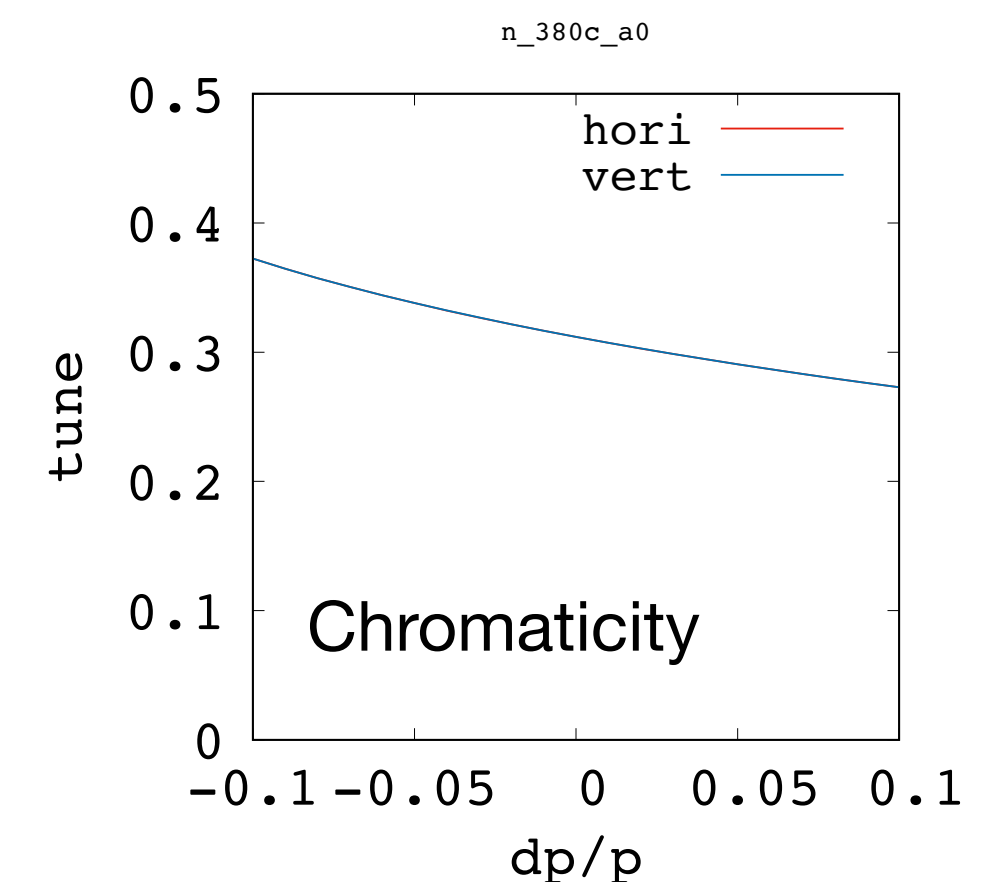
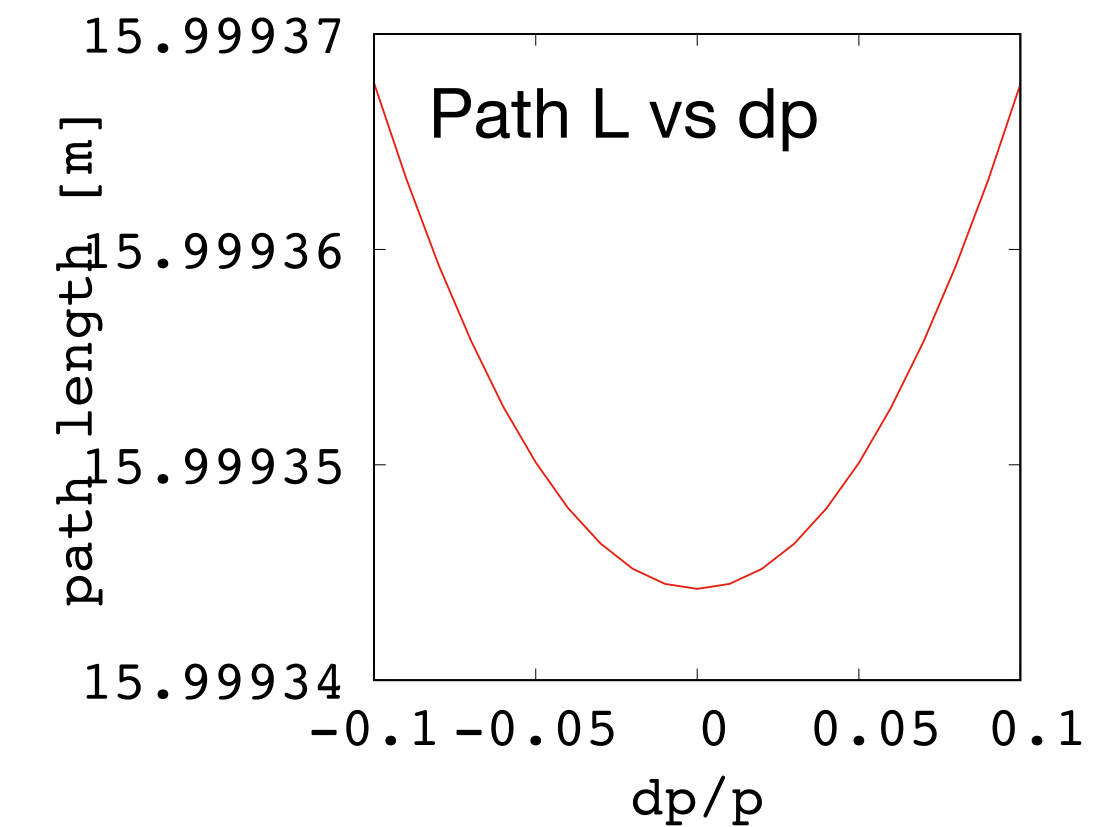
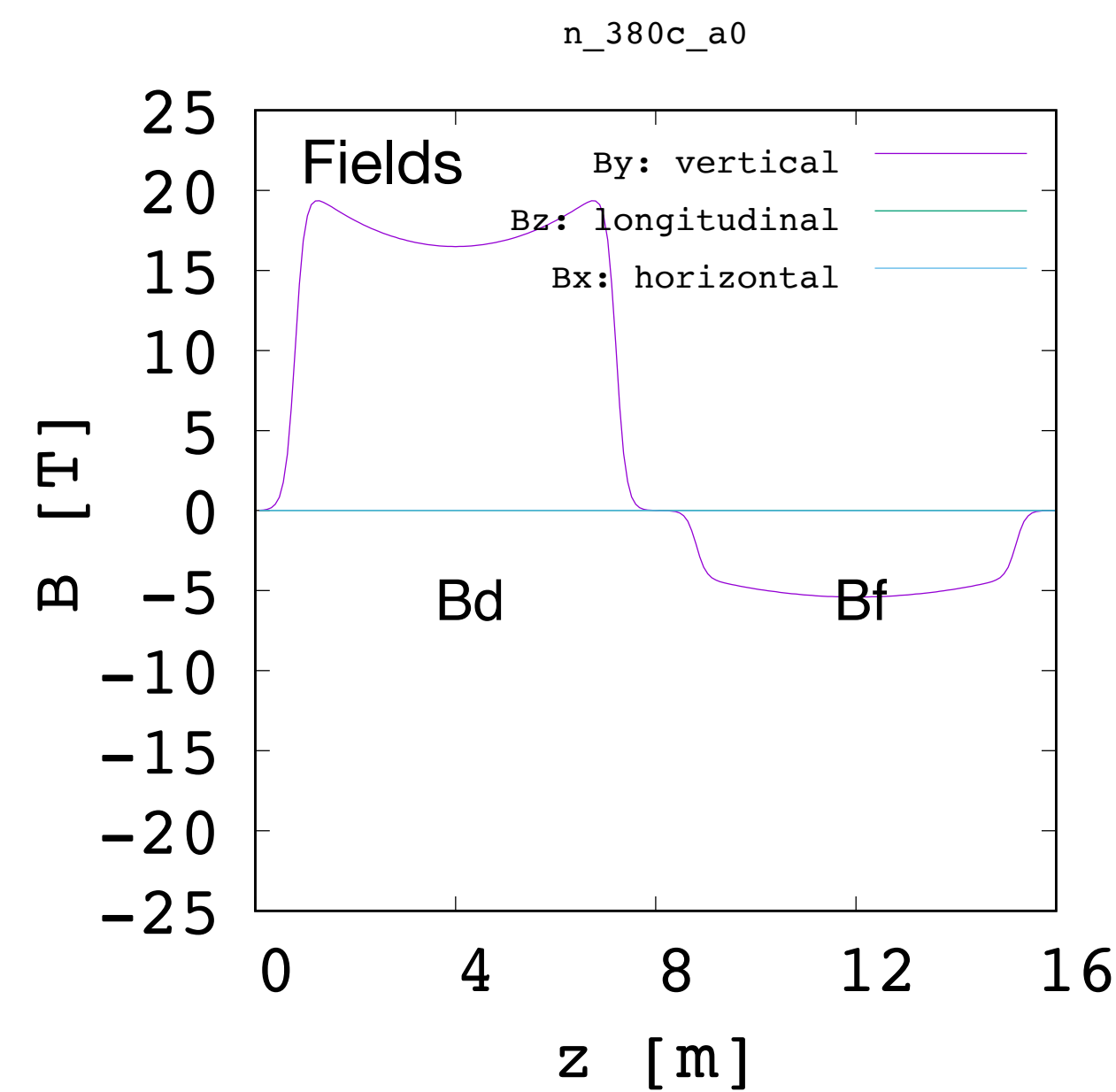
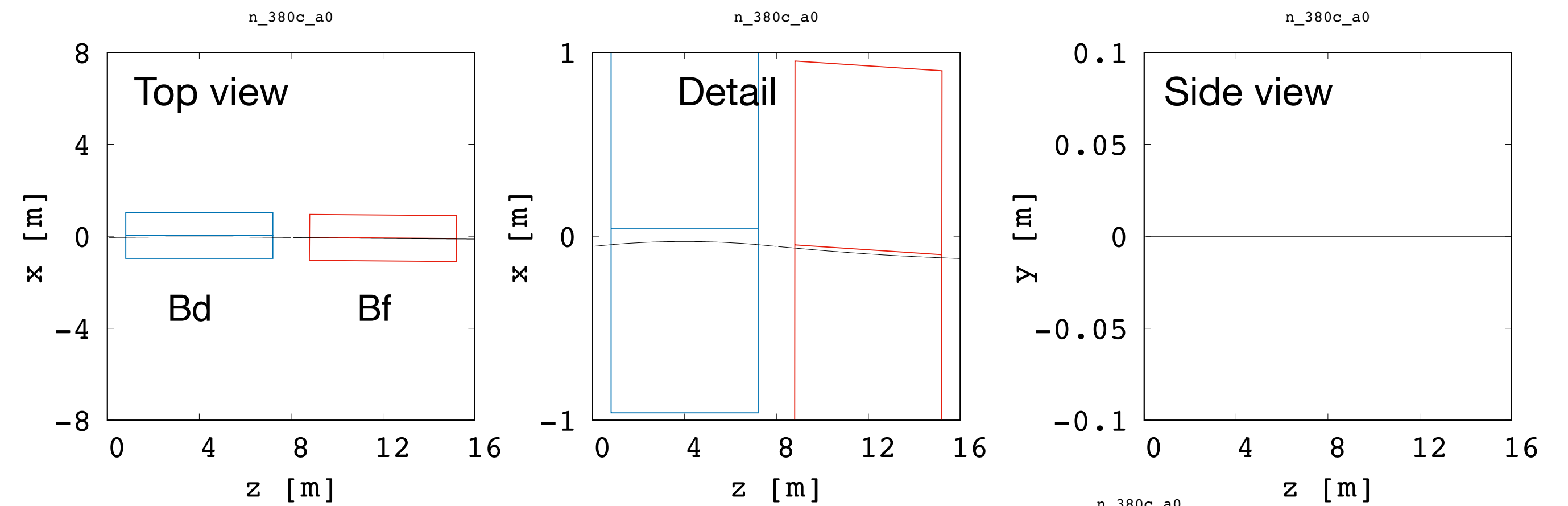




# 1.5 TeV collider ring

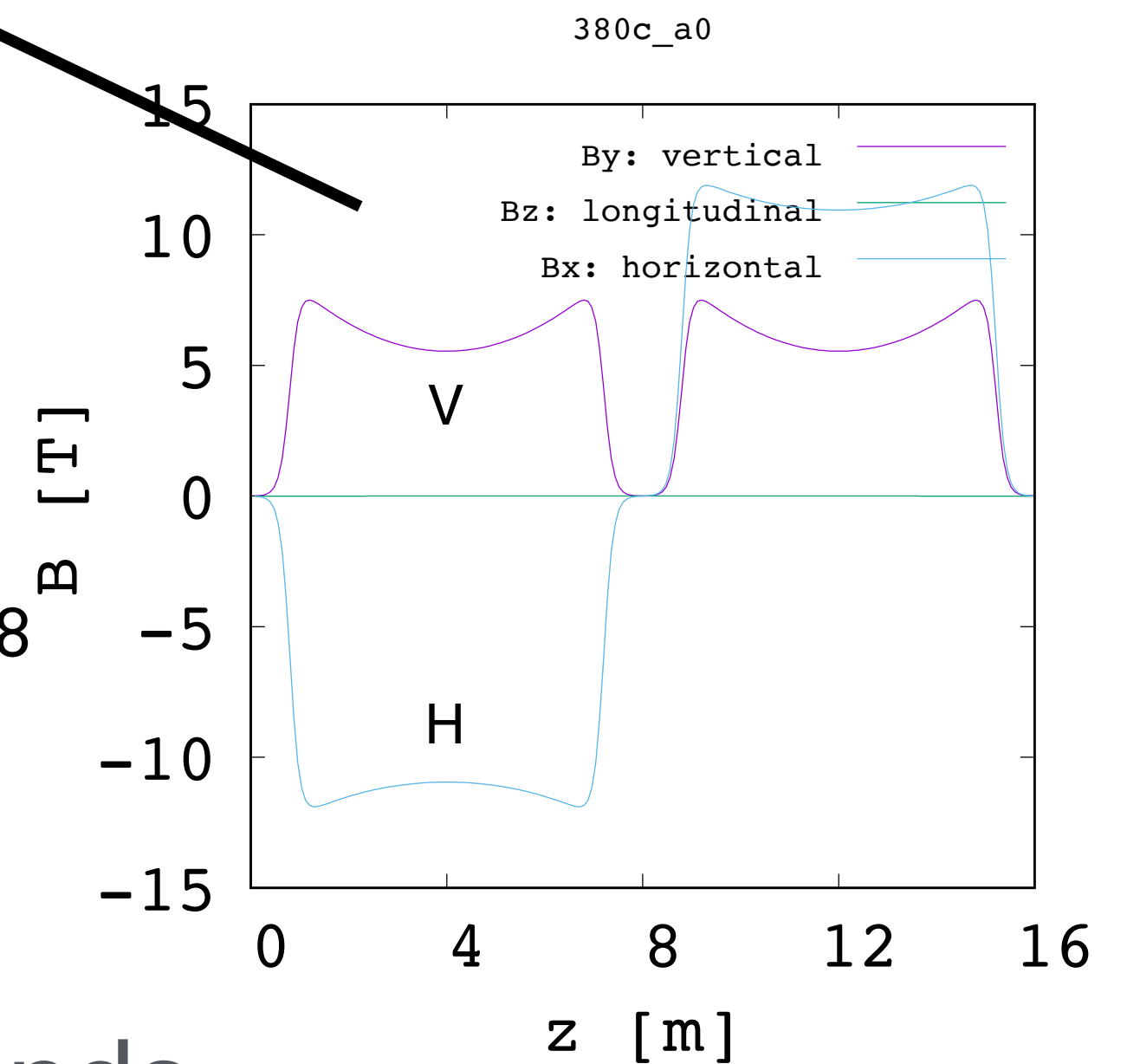
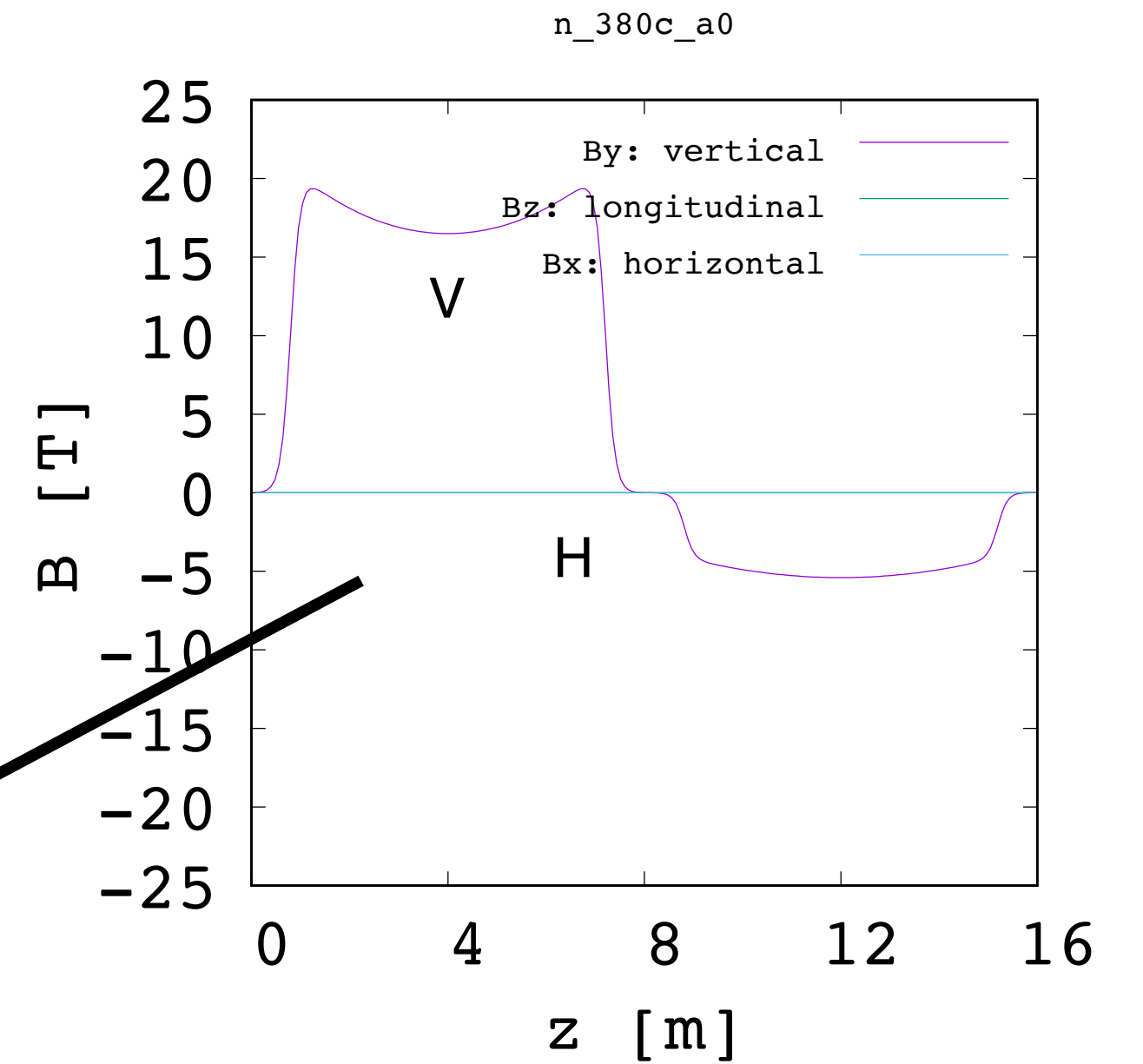
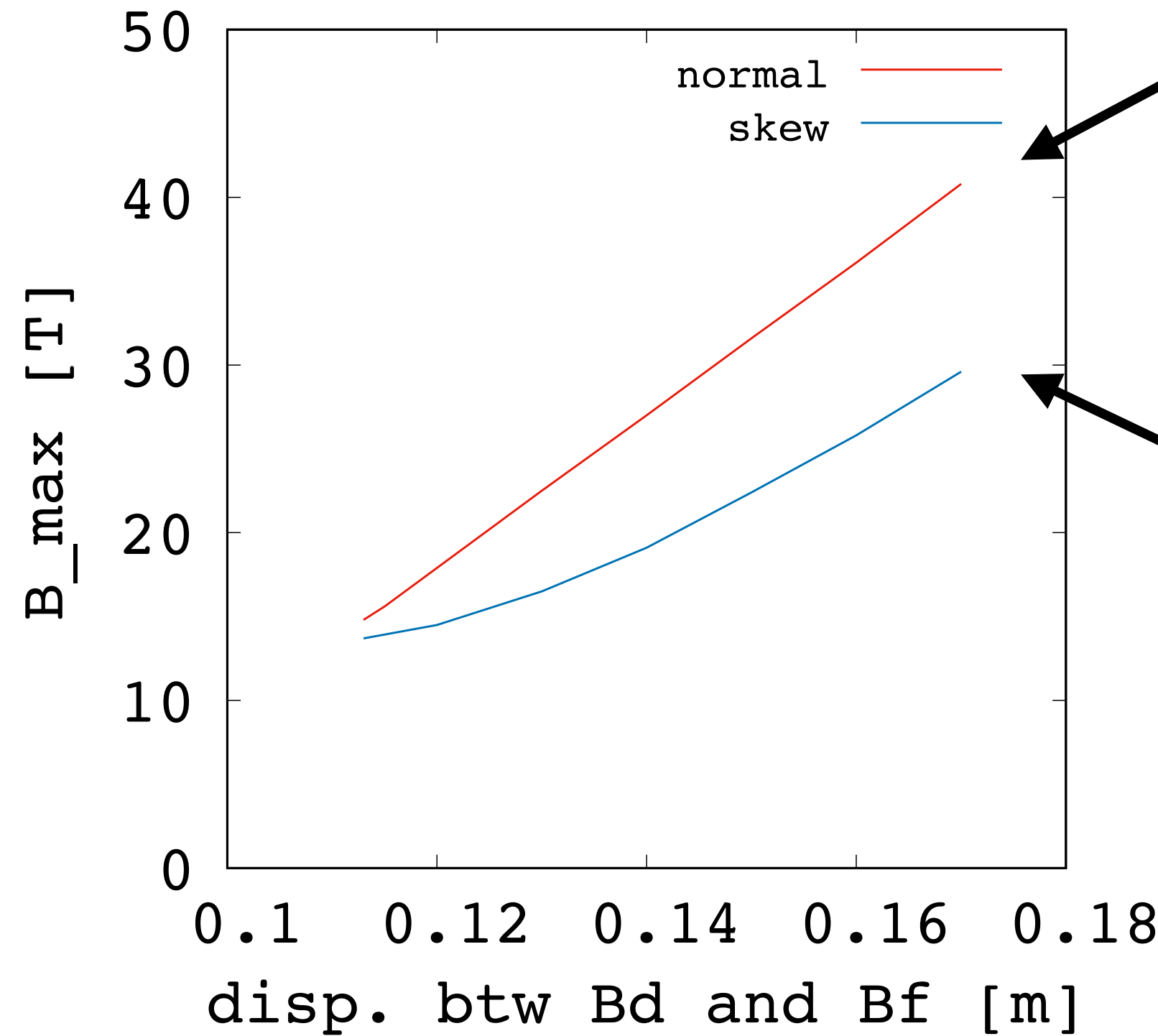
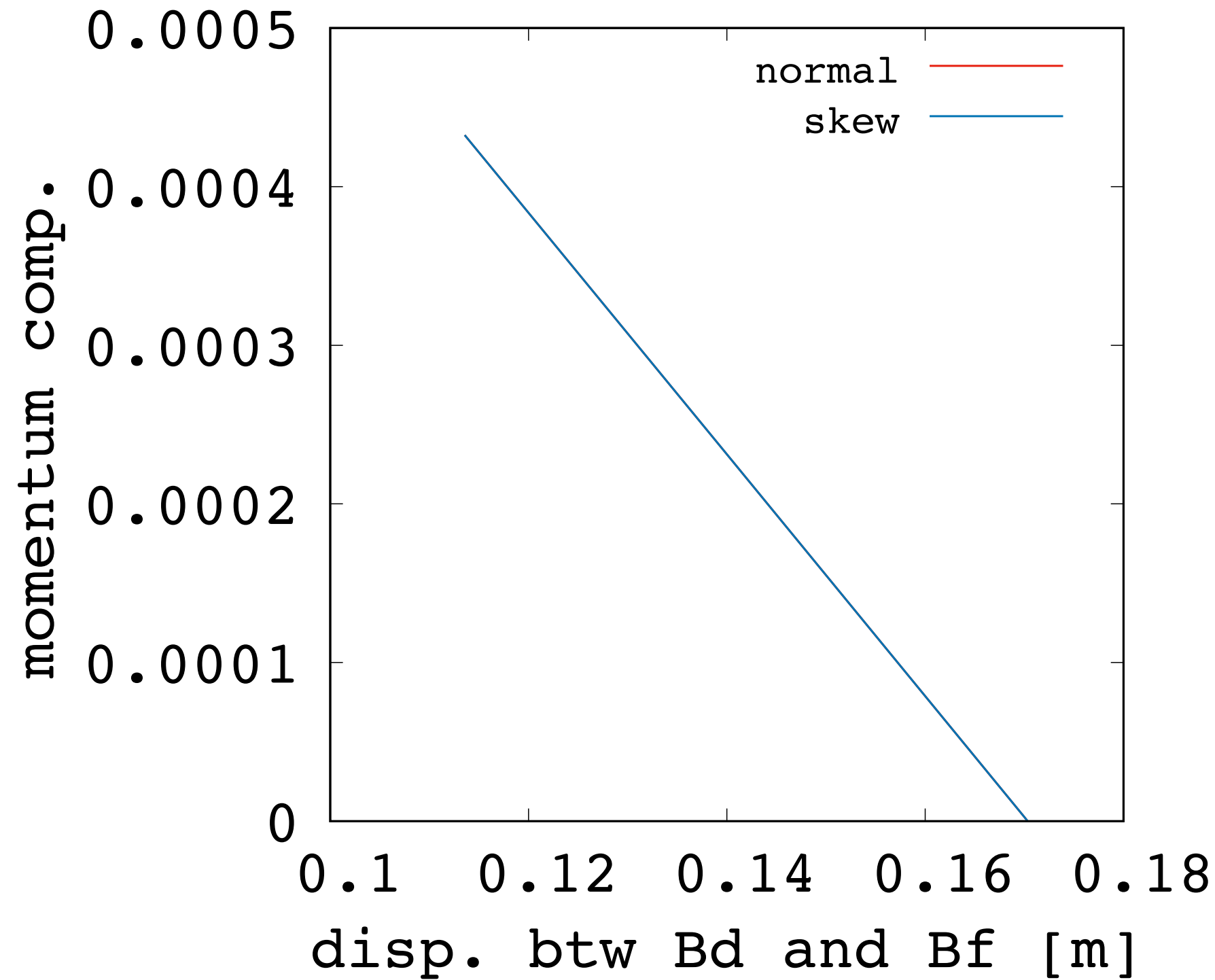
*momentum comp=0, arc only*

	Normal FODO
Energy	1.5 TeV
Momentum compaction	0
Circumference	6080 m
Cell length	16 m
Magnet length	2 x 6.4 m
# of cell	380
Maximum field	20 T
Field gradient	240 T/m
Cell tune	0.3131 / 0.3131



# 1.5 TeV collider ring

## *momentum compactor and maximum field strength*



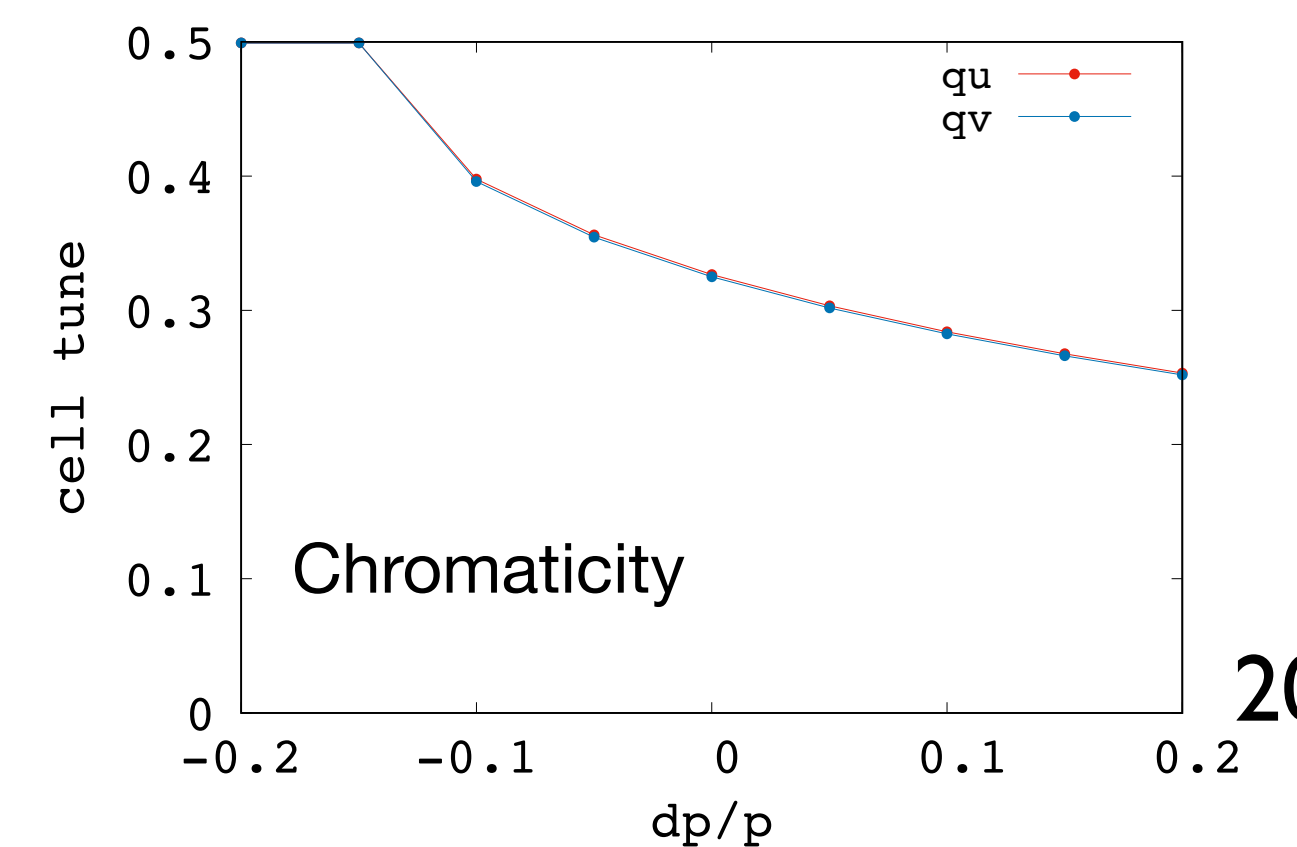
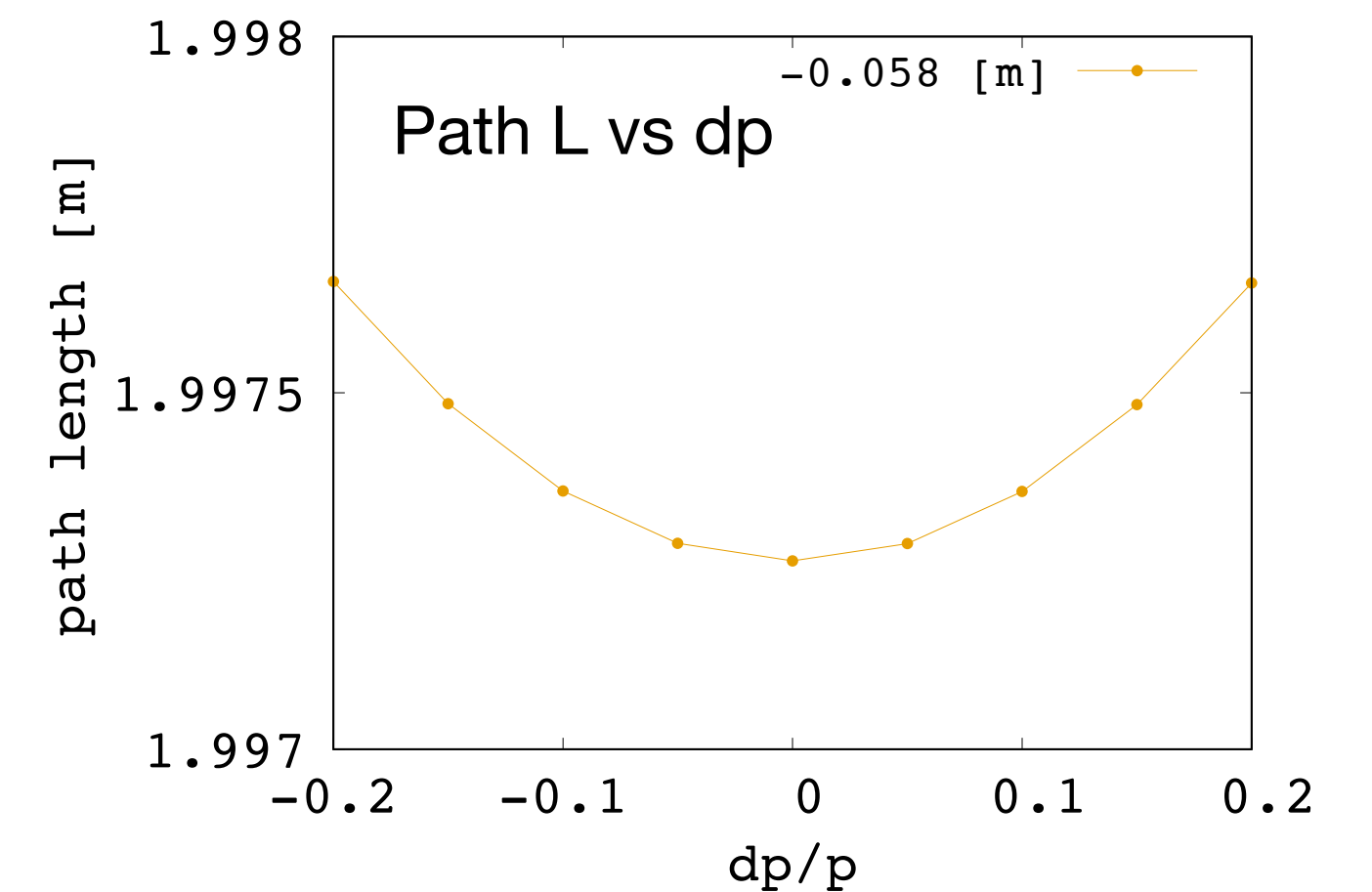
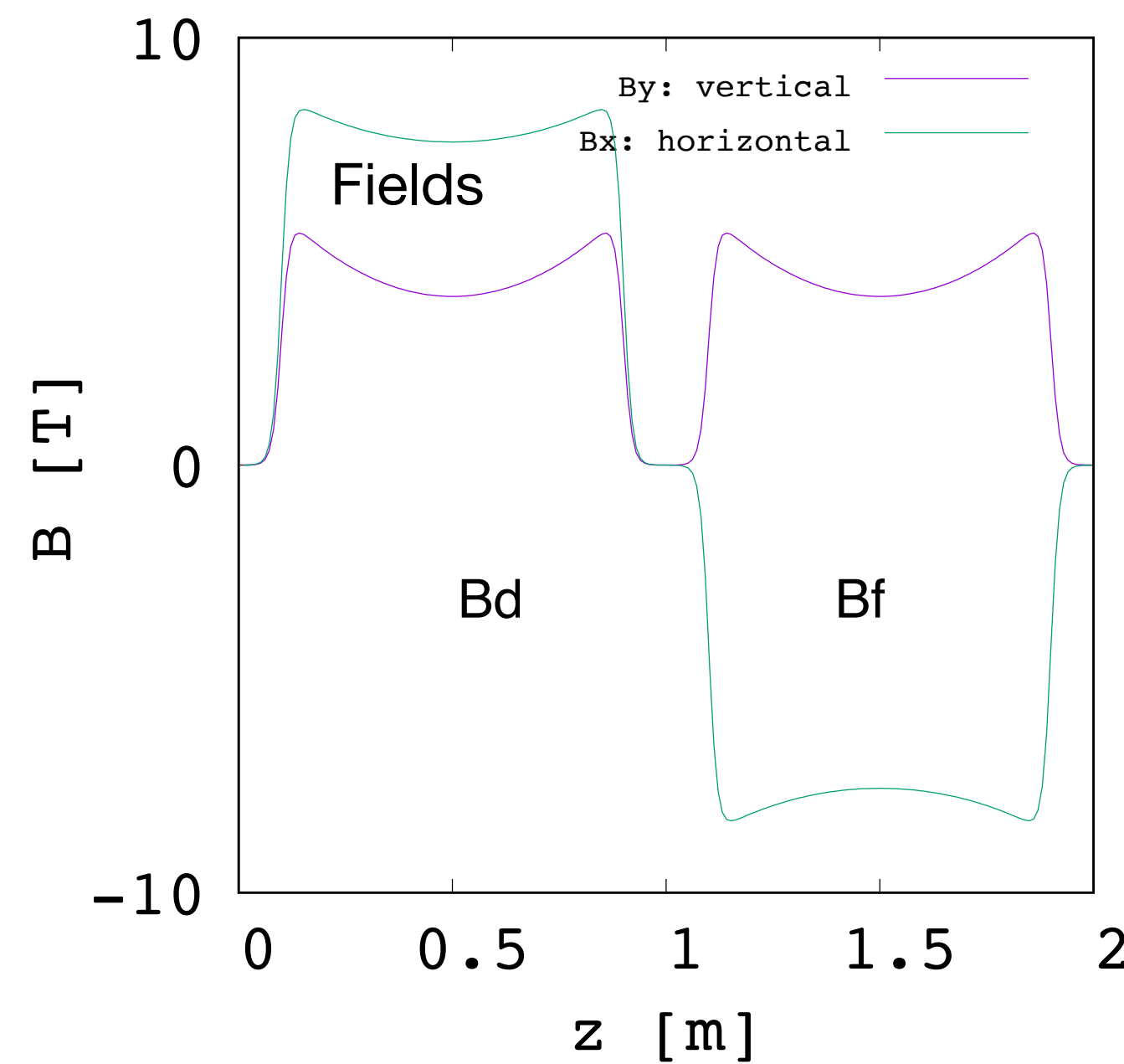
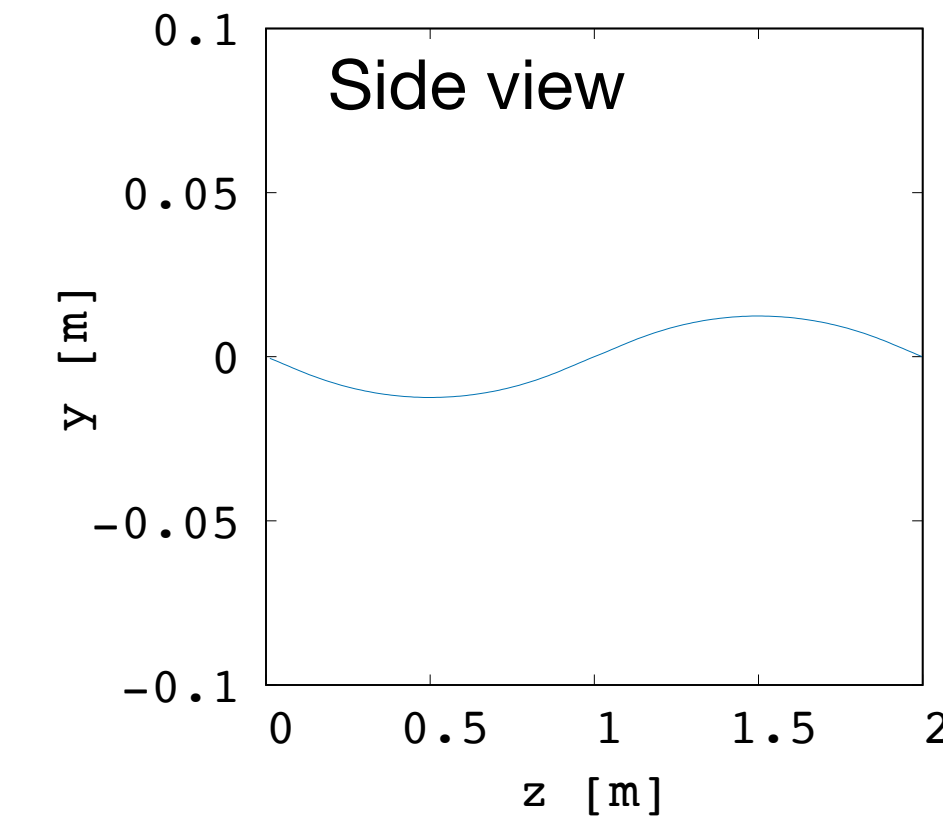
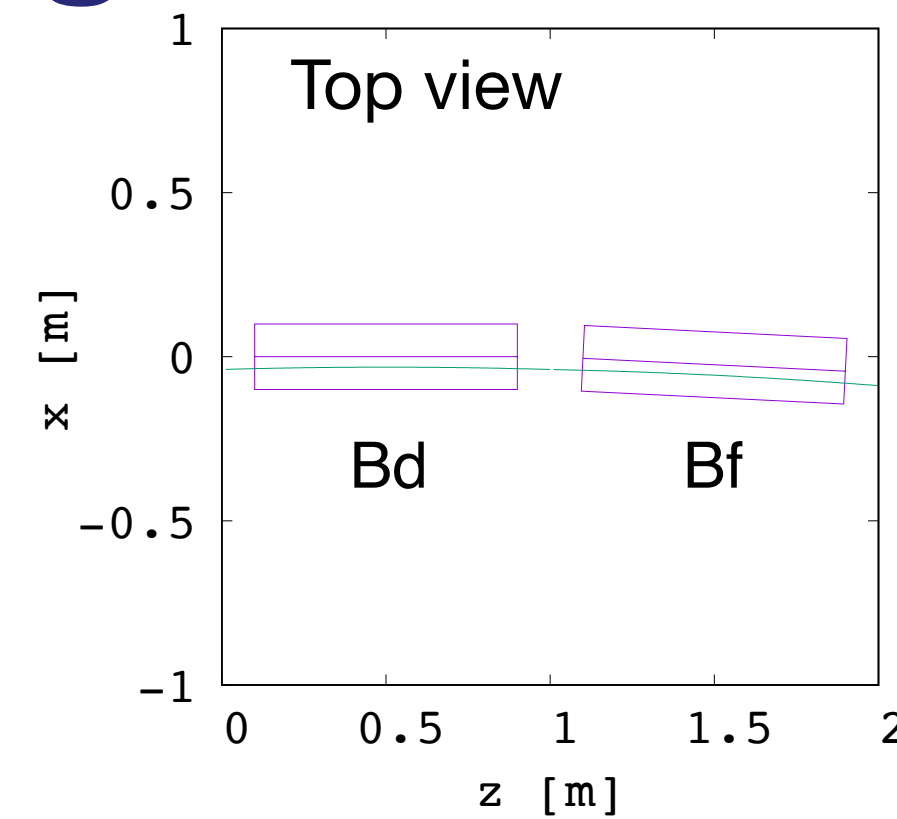
Skew CF does not need negative bends.

# Muon accumulator ring (LEMMA)

# 22.5 GeV accumulator ring for LEMMA

*arc only*

	Skew FODO
Energy	22.5 GeV
Momentum compaction	0
Arc	128 m
Cell length	2 m
Magnet length	2 x 0.8 m
# of cell	64
Maximum field	10 T
Field gradient	238 T/m
Cell tune	0.3269 / 0.3261



# Low beta insertion

## *skew quadrupole channel*

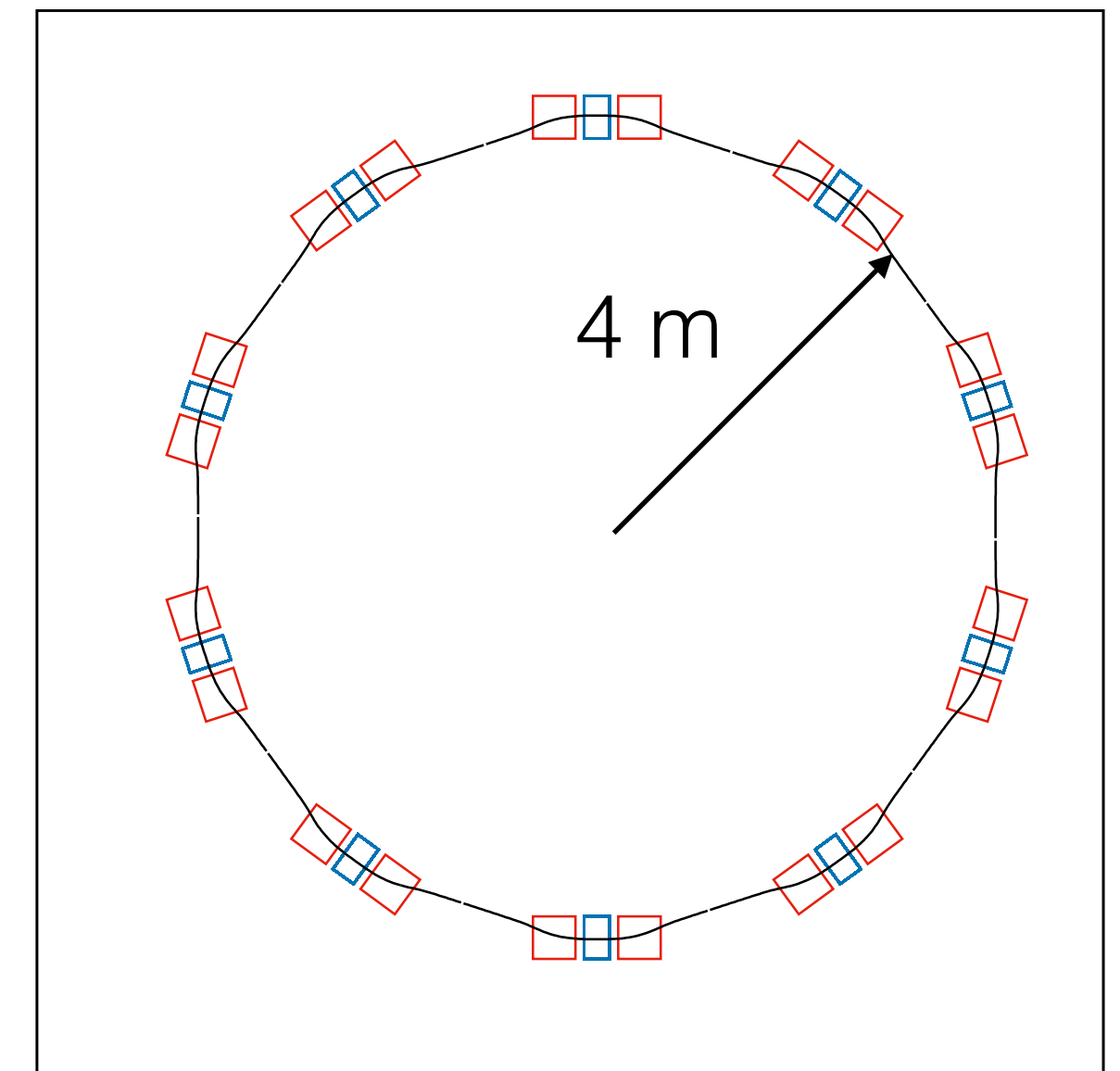
- No design yet.
- Conventional low beta straight of normal quadrupole with 45 degree rotation.
- or Skew combined function with wiggling orbit but zero net bending.
- Under the frame with 45 degree rotation, every element for the whole ring is normal quadrupole except weak focusing at bending which give 'skew' in that frame.
- Normal quadrupole magnets between arc and straight will work as correction of coupling.

# Proton driver

# Proton driver

*Not for muon facility, but as a prototype for ISIS upgrade plan*

Energy	3 to 12 (17) MeV
Number of cell	10
Cell length	2.5 m
Length of Bd	0.24 m
Length of Bf	0.40 m
Space between Bd and Bf	0.08 m
Relative displacement btw Bd and Bf	+/- 0.0 m
Length of straight section	1.30 m
Fringe length L (Tanh x/L)	0.200 m
-Bd/Bf	0.2
Field index m	1.28 m <sup>-1</sup>
Cell tune	(0.217844, 0.134228)
Order of expansion	10



# Summary

- vFFA and novel skew optics have great potential for muon collider accelerator complex.
- vFFA for muon accelerator
  - All advantage listed.
  - Negative bend increases the circumference.
  - Current design needs 8.7 T magnet to fit the ring in LHC tunnel to accelerate to 1.5 TeV
- Novel skew optics for muon collider and muon accumulator
  - Clear advantage on the required magnet strength compared with normal combined function magnet.
  - Need low beta insertion design.



# Thank you for your attention

# Backup slides

# vFFA

## 3D magnetic fields

All 3D magnetic fields have to satisfy the same dependence on the vertical coordinate.

$$B_z(z, x, y) = B_0 \exp(mz) \sum_{i=0}^{\infty} b_{zi}(x) y^i$$

$$B_x(z, x, y) = B_0 \exp(mz) \sum_{i=0}^{\infty} b_{xi}(x) y^i$$

$$B_y(z, x, y) = B_0 \exp(mz) \sum_{i=0}^{\infty} b_{yi}(x) y^i$$

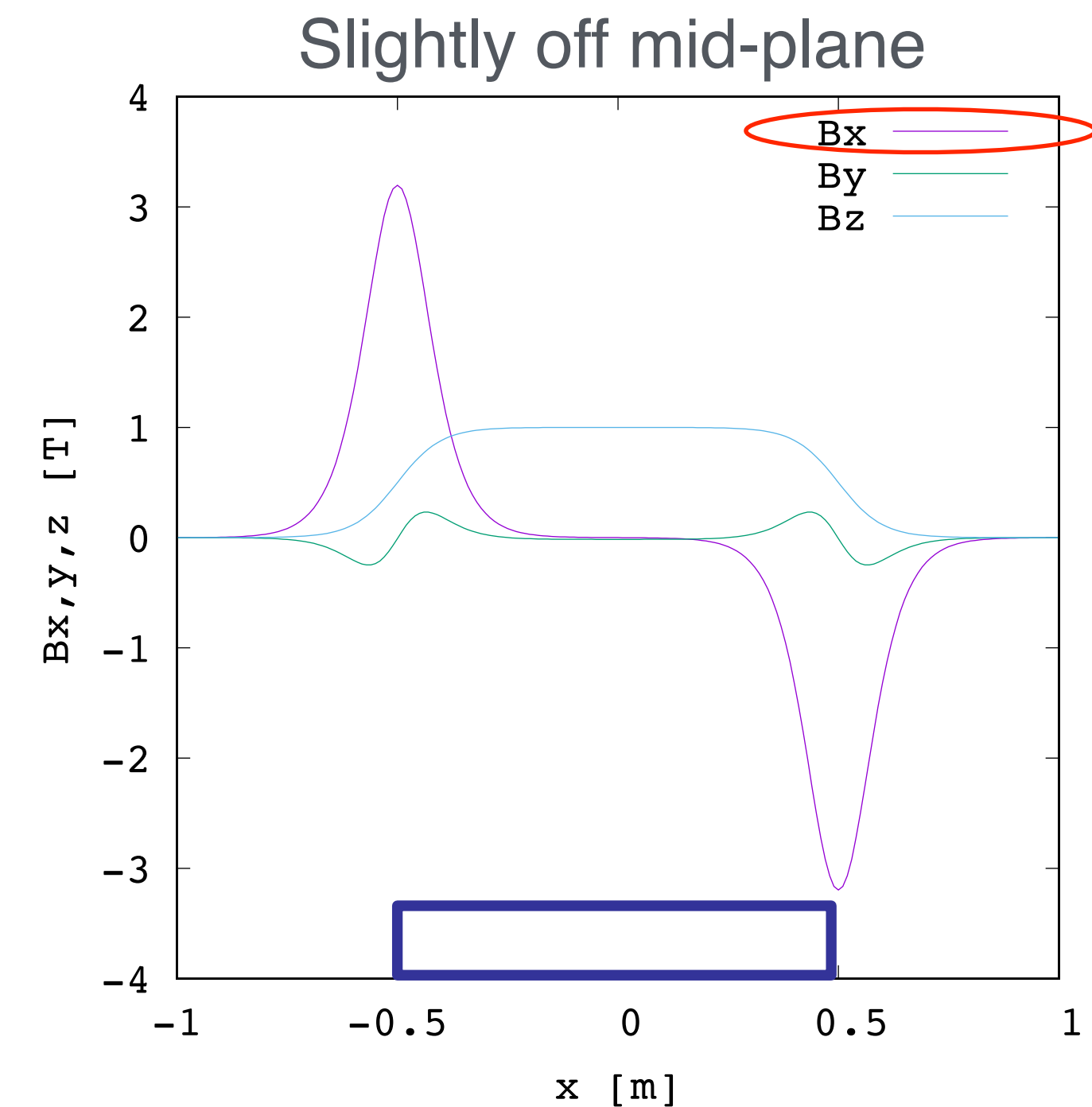
Off mid-plane field is expanded as polynomial with  $y$ .

where

$$b_{z0}(x) = g(x)$$

$$b_{x0}(x) = \frac{1}{m} \frac{\partial g}{\partial x}$$

$$b_{y0}(x) = 0$$



**Non zero longitudinal field on mid-plane is something we did not have in a conventional accelerator.**

Skew quadrupole (body) + Solenoidal field (ends)