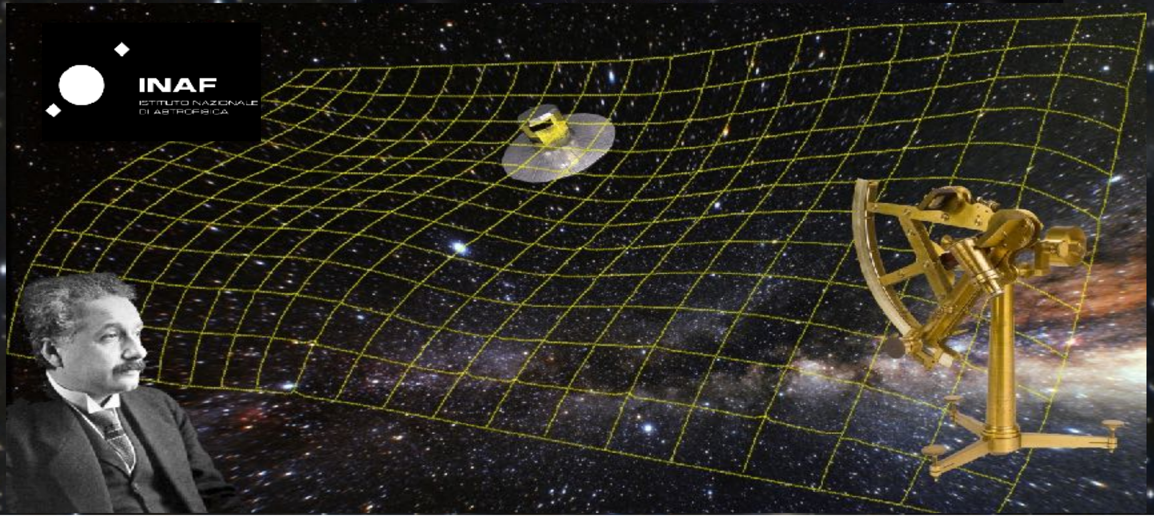
Dark matter cosmology and astrophysics using Gaia

Rutherford Appleton Laboratory
21st January 2026



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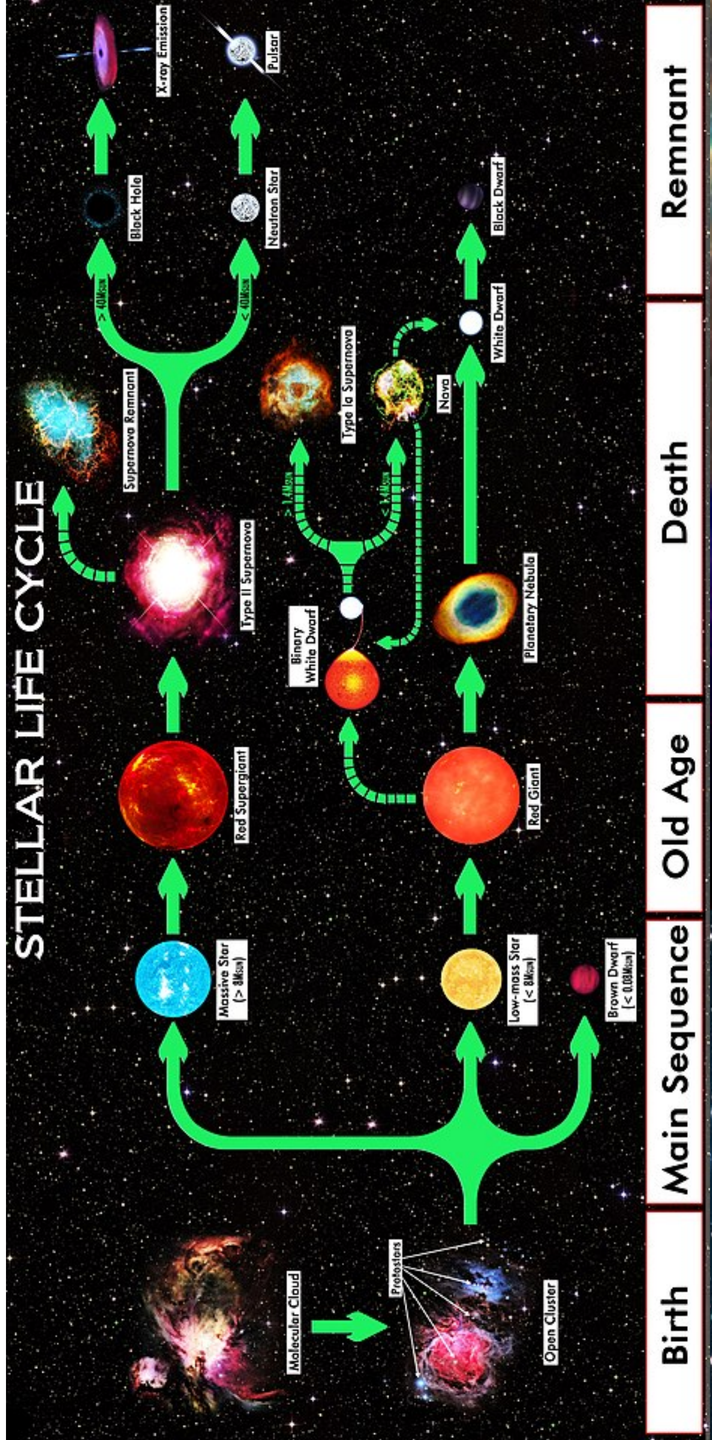
Talk outline

Background

- Motivation: Astrometry from Hipparcus to Einstein, Gaia
- Relativistic/Gravitational Astrometry

Challenging the Galactic Models with Milky Way stars

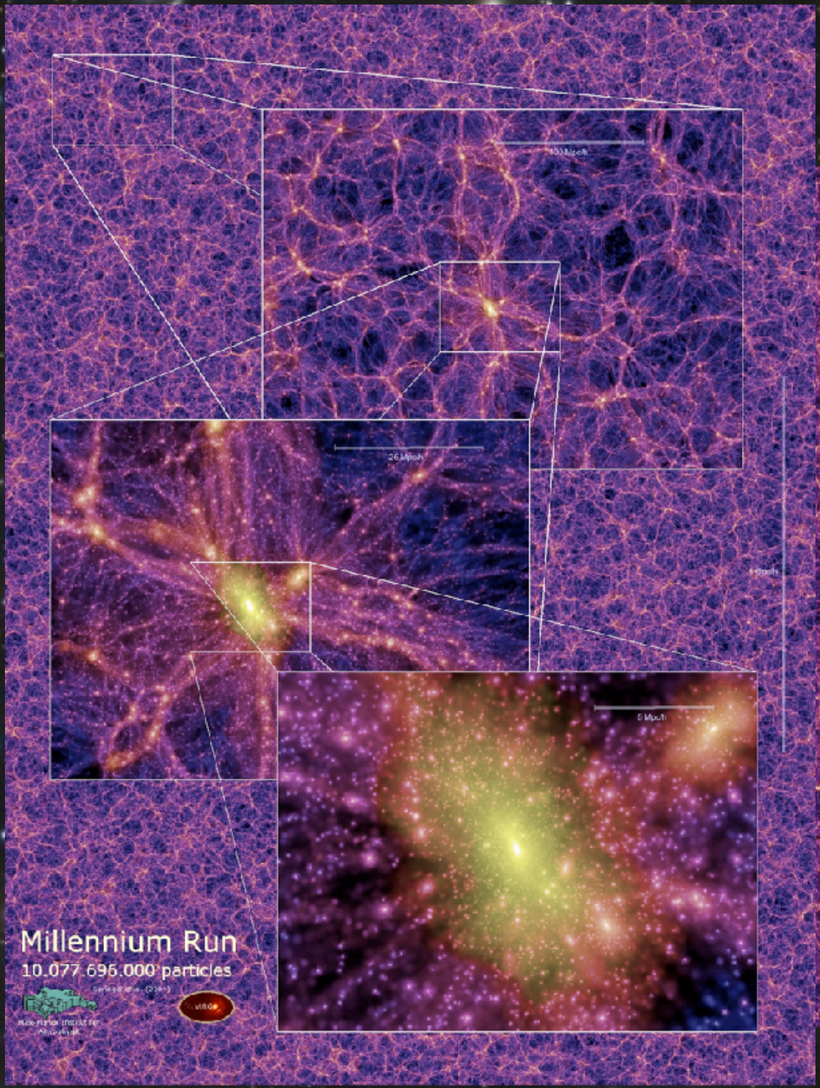
- Local cosmology as ΛCDM laboratory
- Testing General Relativity/Gravity @MilkyWay scale
- Dark Matter interpretation in GR

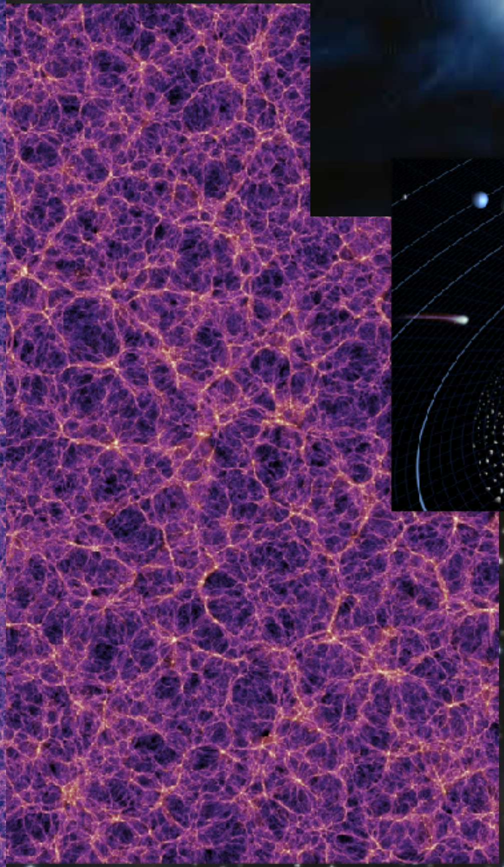


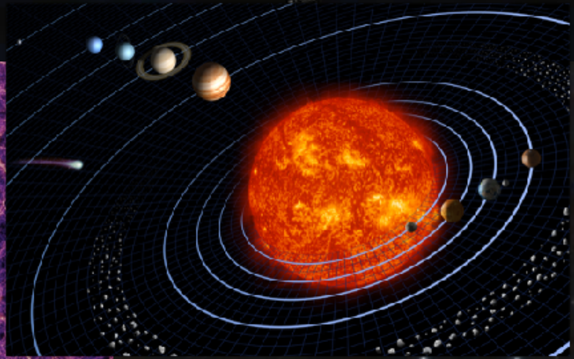
Gravity is not just a force,
it is becoming itself:
the inescapable evolution of all things

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Gravity: A Near and Far Connection





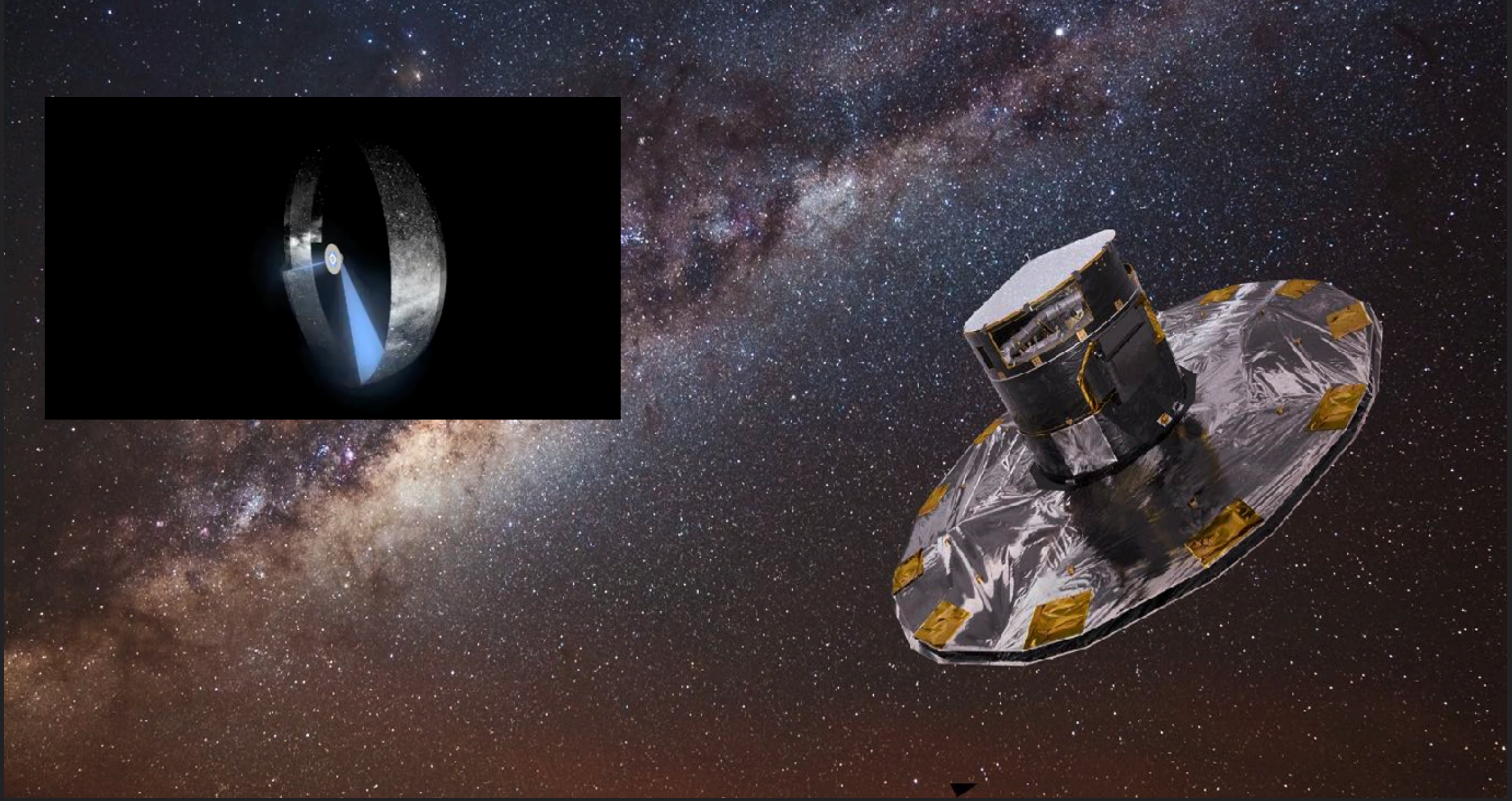


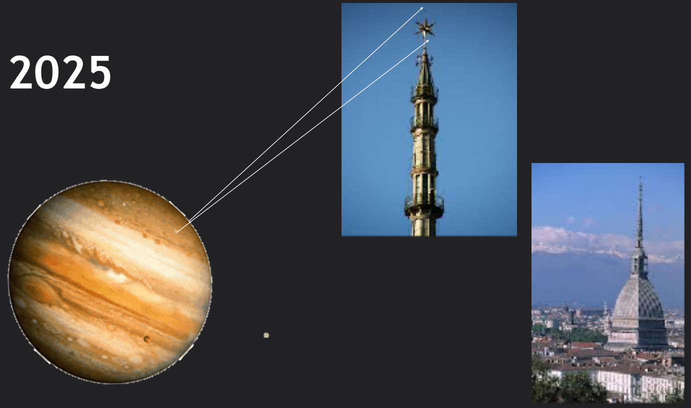
Gravity rules the Universe determining how all its constituents must move:
it creates a distribution of distances and velocities

European mission

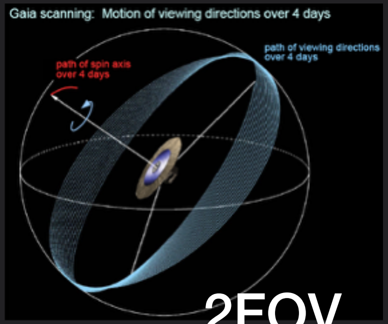
ESA mission launched in 2013, nominal lifetime 5 years, extended up to 2025

Gaia mapped from L2 of the Earth-Sun system position (direction and distance) and velocity of almost 2 billion objects with an accuracy of up to 1 microarcsecond

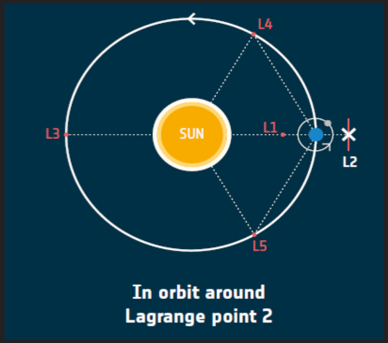


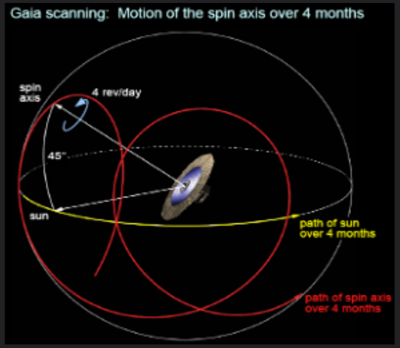


0",000001 = micro(μ) arc sec

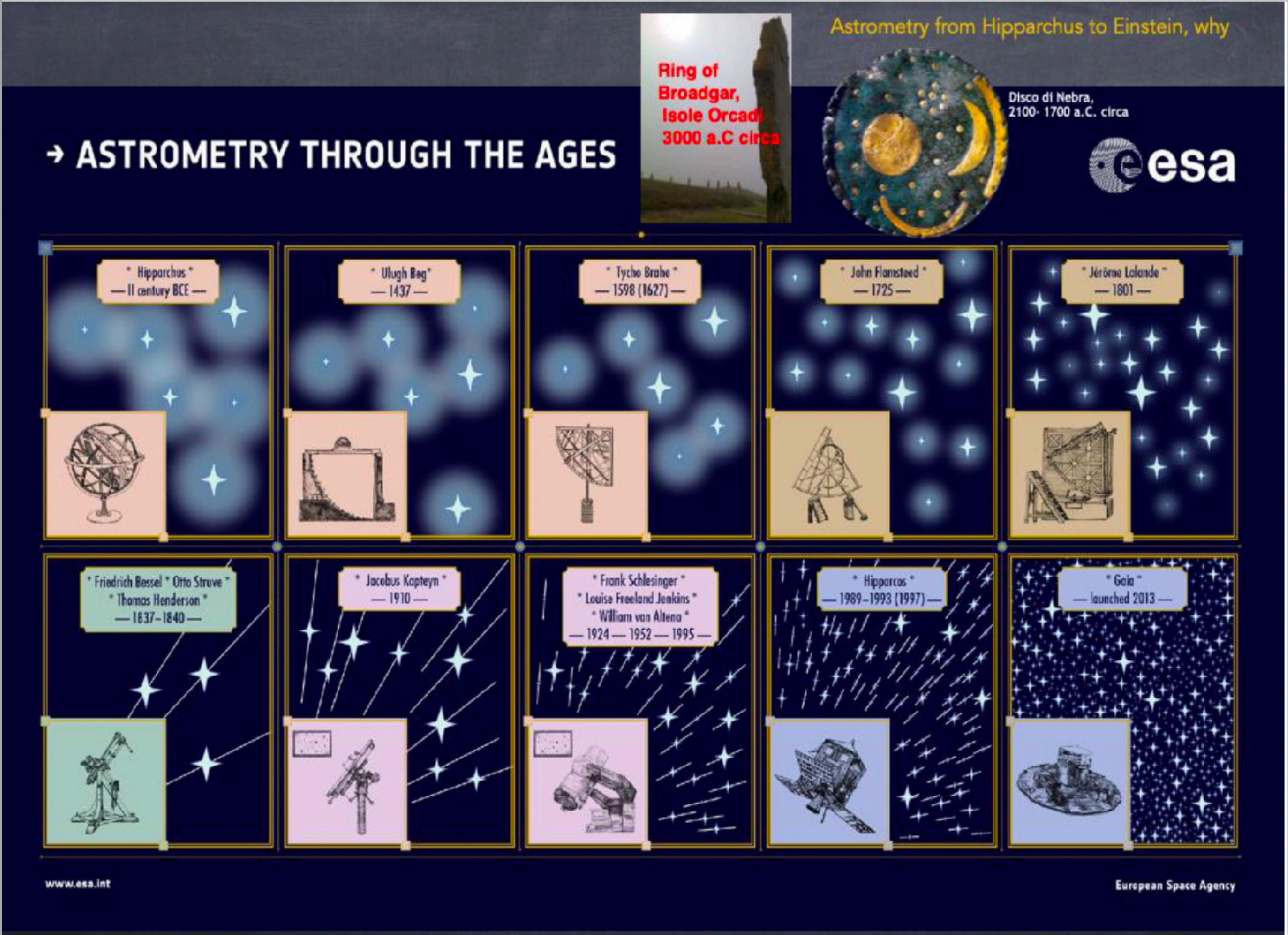


2FOV





sky coverage with precession at fixed angle to the Sun



What is Astrometry

Astrometry is the ancient branch of Astronomy dedicated to the fundamental question: What is our place in the Universe?

-> measurements of angles to provide star positions and their movements in the Universe.

It acquires knowledge through the analysis of photons received over time from all sorts of celestial sources at the observer location.

The location of an object in astrometry is considered reliable if its relative error is less 10%

parallax $\pi(\text{arcsec}) \approx 1(\text{UA})/d^*(\text{pc})$

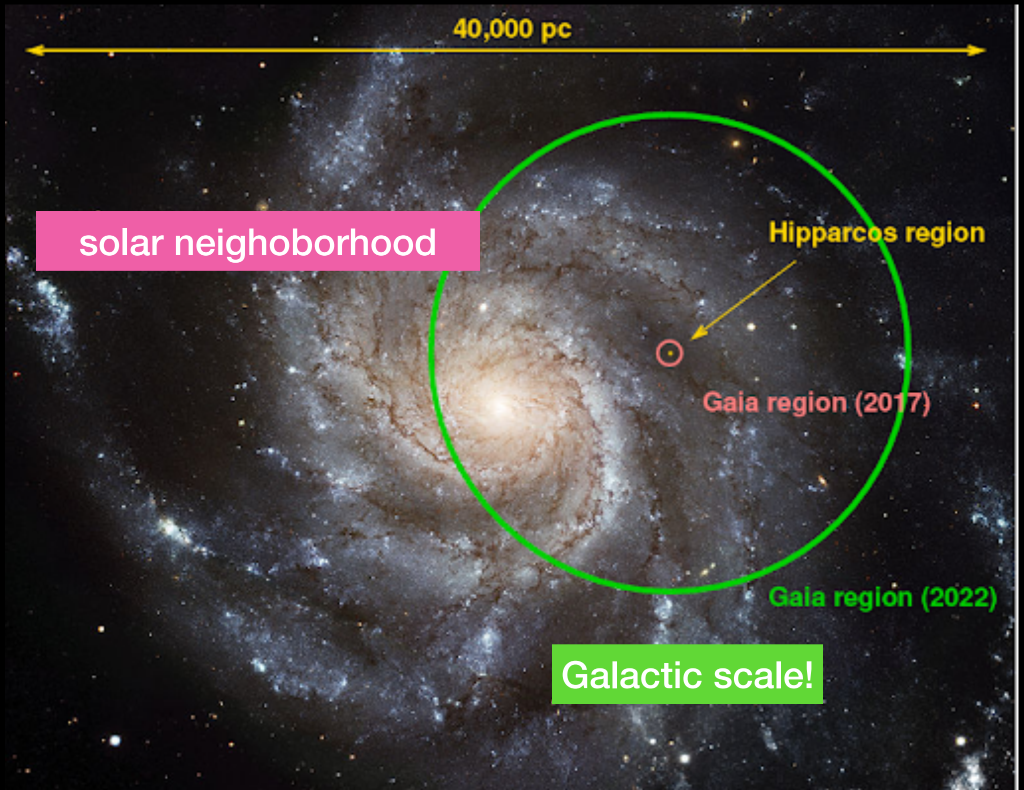
Hipparcos $\sigma_\pi = 1 \text{ mas} = 10^{-3}\text{arcsec}$

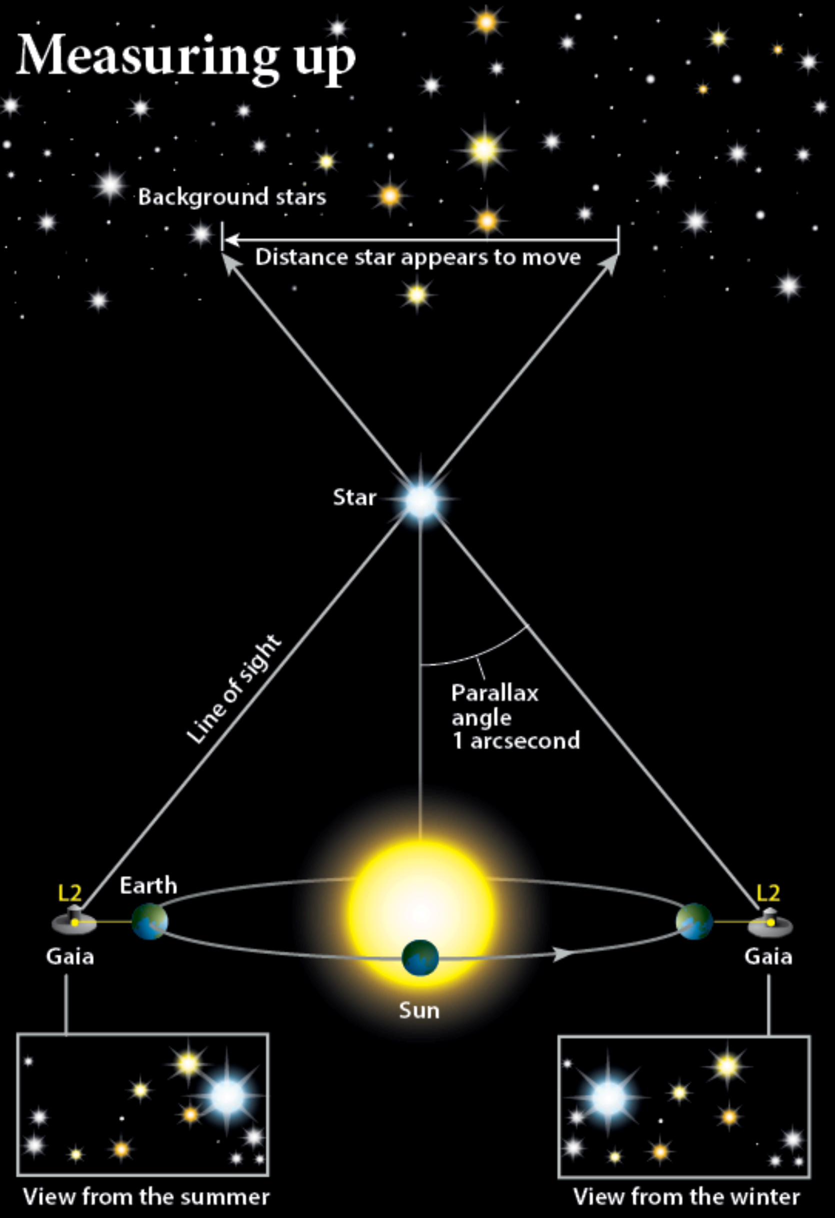
Gaia $\sigma_\pi = 10\ \mu\text{as} = 10^{-5}\text{arcsec}$

$\pi \approx \sigma_\pi \cdot 10$

d* = 100 pc

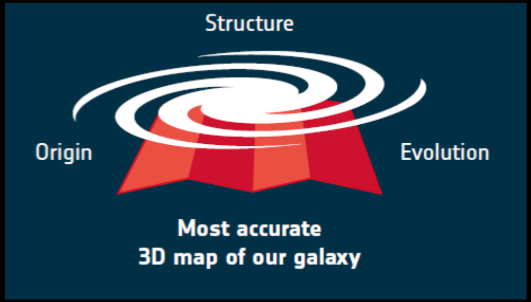
d*= 10 kpc



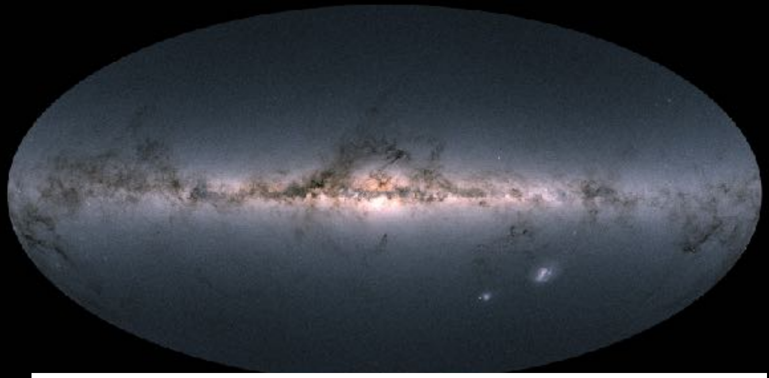


To understand the distances between stars, astronomers rely on a method called parallax. By measuring the distances stars appear to move relative to other stars, astronomers can gauge how far away they are from us and from each other. ASTRONOMY: ROEN KELLY

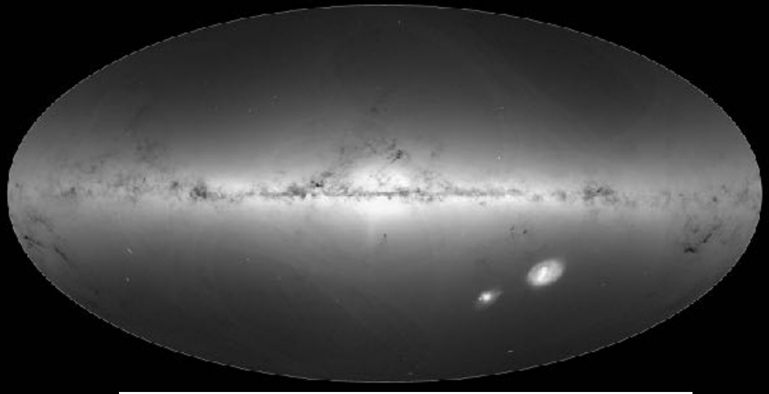
Science with two billion objects in 3D, from structure and evolution of the Milky Way to GR tests



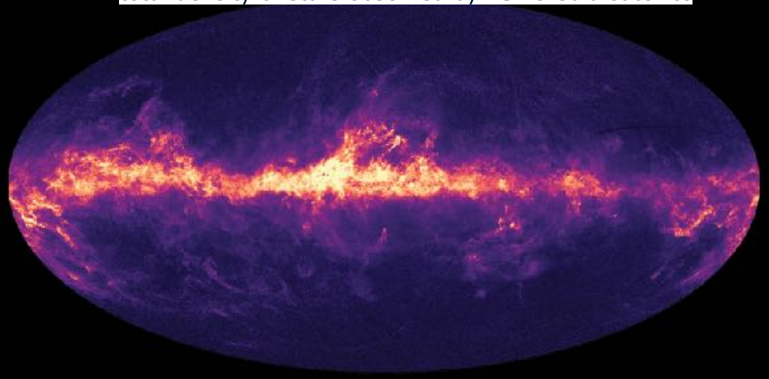
→ GAIA: THE GALACTIC CENSUS TAKES SHAPE

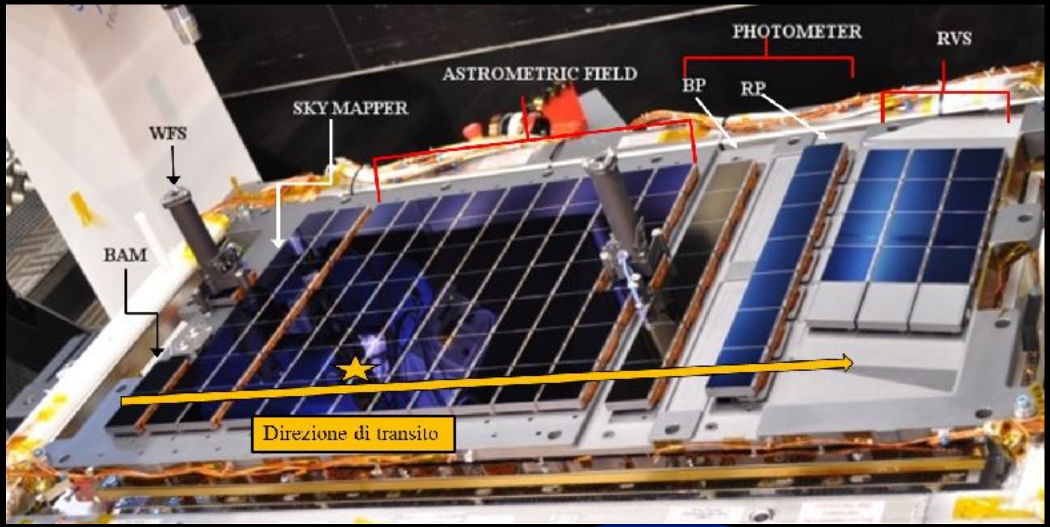


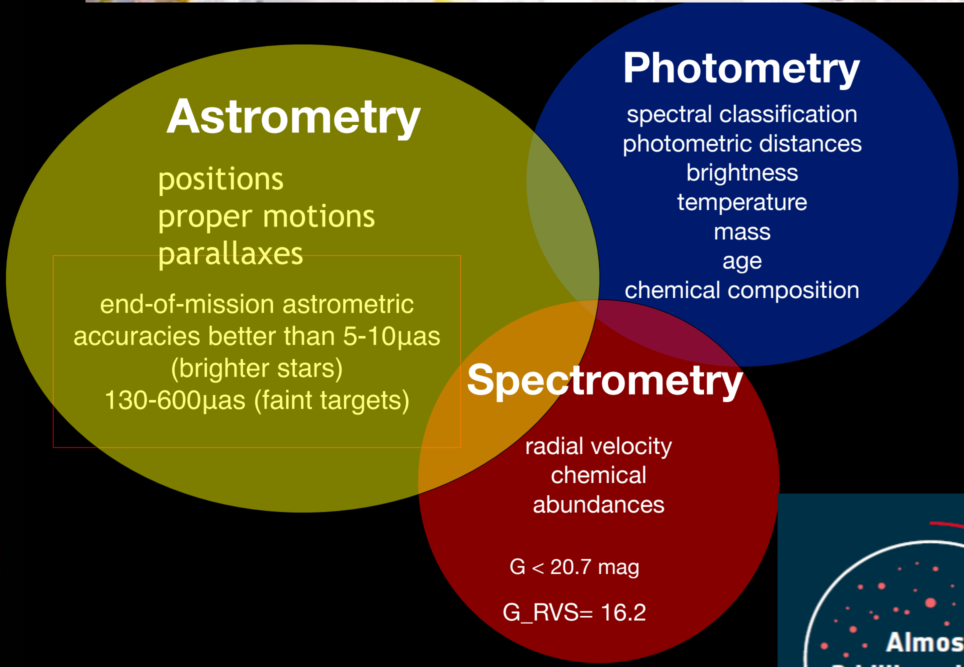
total brightness and colour of stars observed by ESA's Gaia satellite

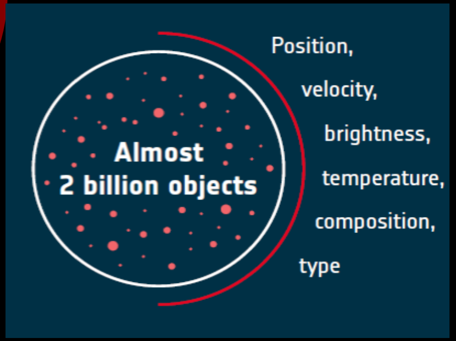


total density of stars observed by ESA's Gaia satellite



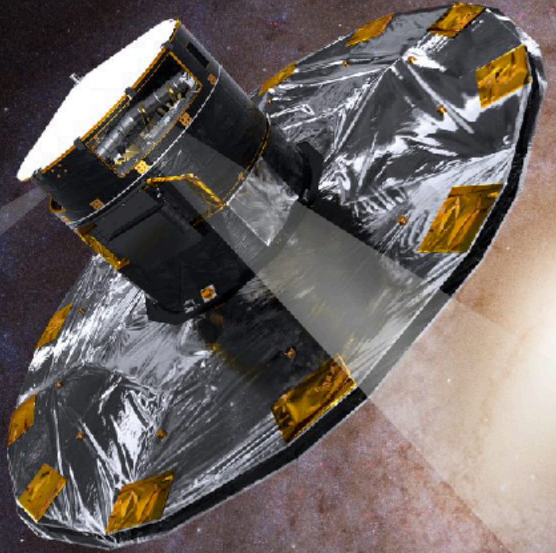






SKY-SCANNING COMPLETE FOR ESA'S MILKY WAY MAPPER GAIA

From 24 July 2014 to 15 January 2025, Gaia made more than three trillion observations of two billion stars and other objects, which revolutionised the view of our home galaxy and cosmic neighbourhood.



3 TRILLION
Observations

2 BILLION
Stars & other objects observed

938 MILLION
Camera pixels on board

15 300
Spacecraft 'pirouettes'

55 KG
Cold nitrogen gas consumed

3827
Days in science operations

50 000 HOURS
Ground station time used

580 MILLION
Accesses of Gaia catalogue so far

13 000
Refereed scientific publications so far

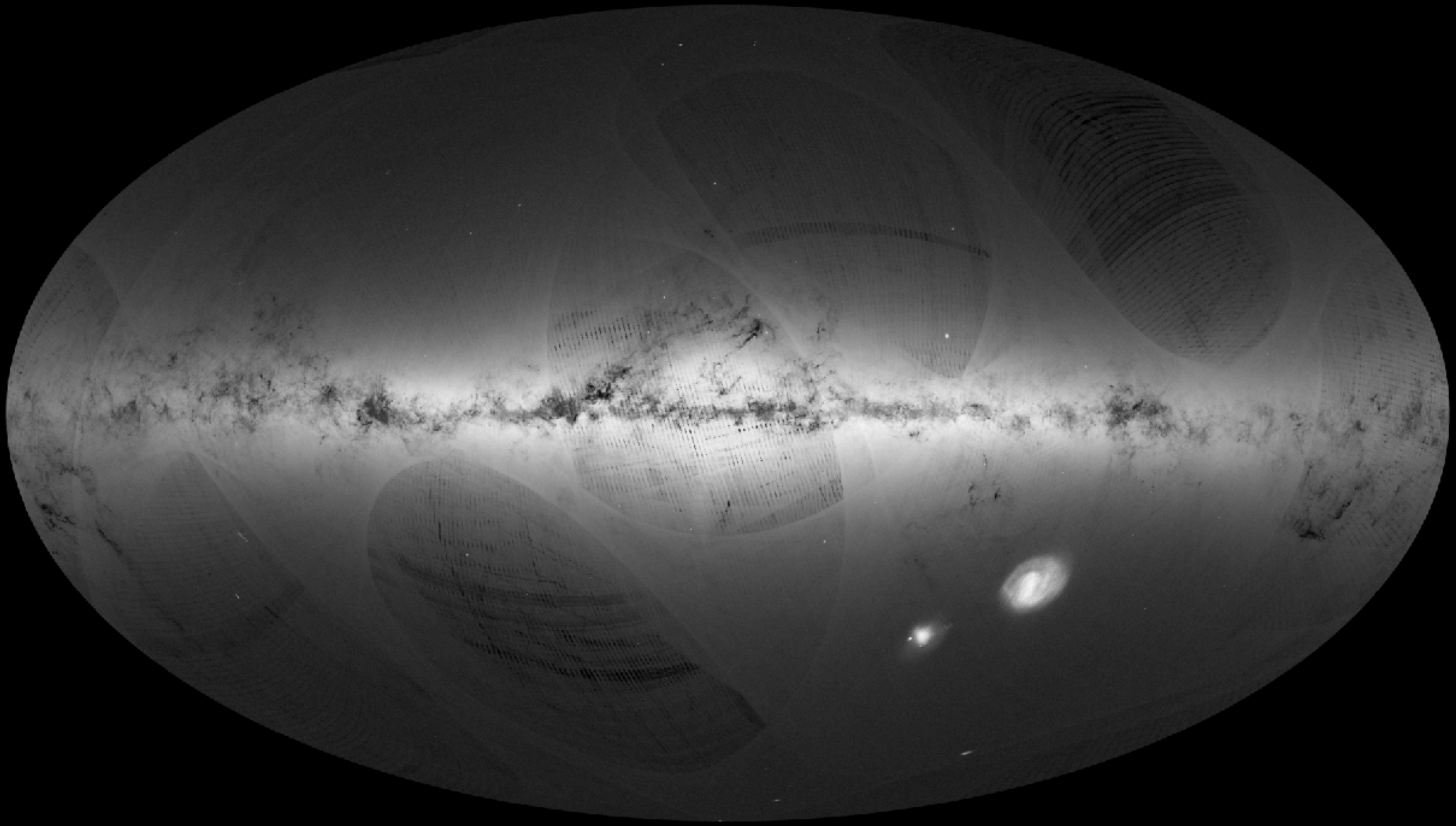
2.8 MILLION
Commands sent to spacecraft

142 TB
Downlinked data (compressed)

500 TB
Volume of data release 4 (5.5 years of observations)

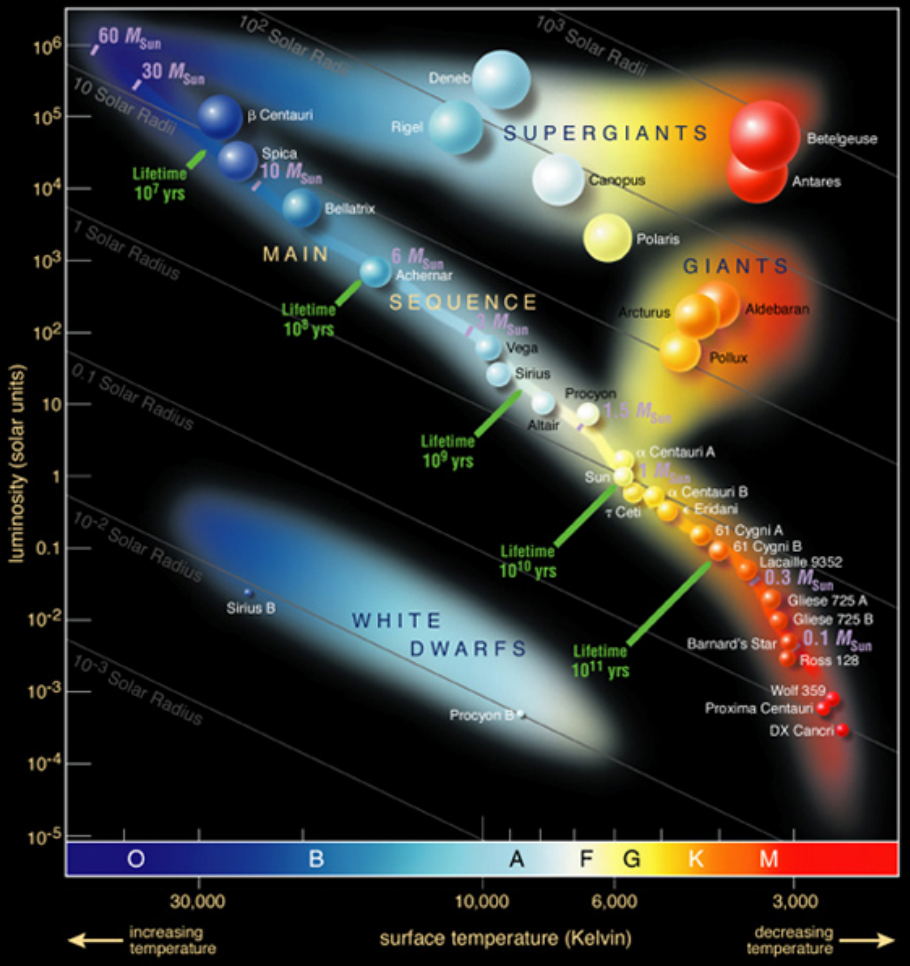
Gaia’s first look into the Milky Way, DR1

observations collected during the first 14 months of Gaia's routine operational phase

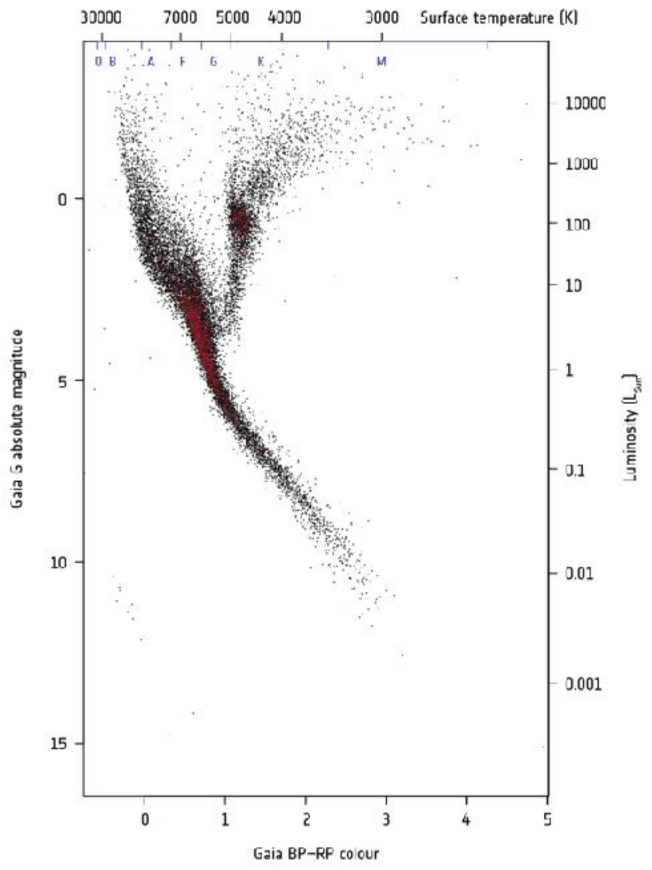


http://www.cosmos.esa.int/web/gaia/science

THE HERTZSPRUNG-RUSSELL DIAGRAM

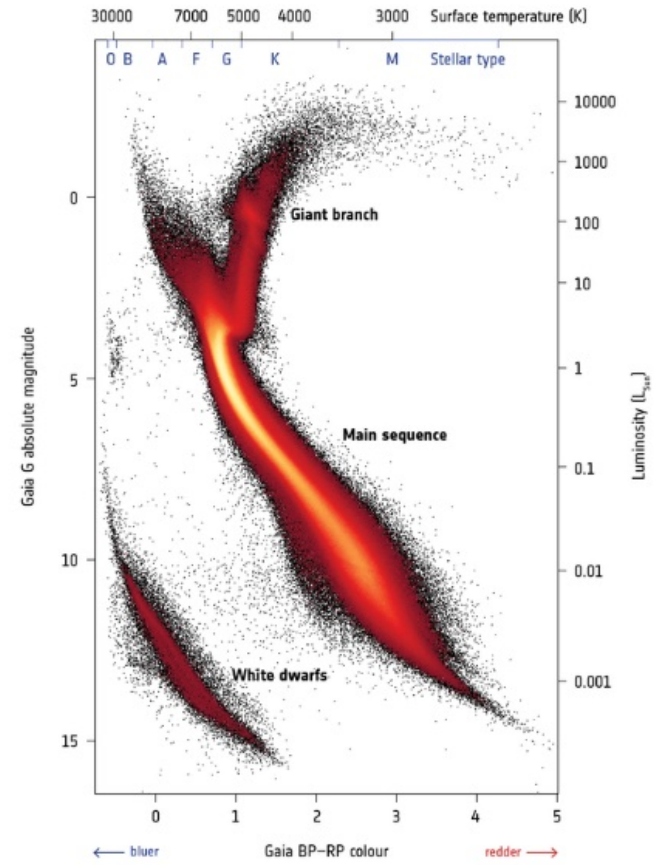


Hipparcos HRD (10^5 stars to 0.1 kpc)



→ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM

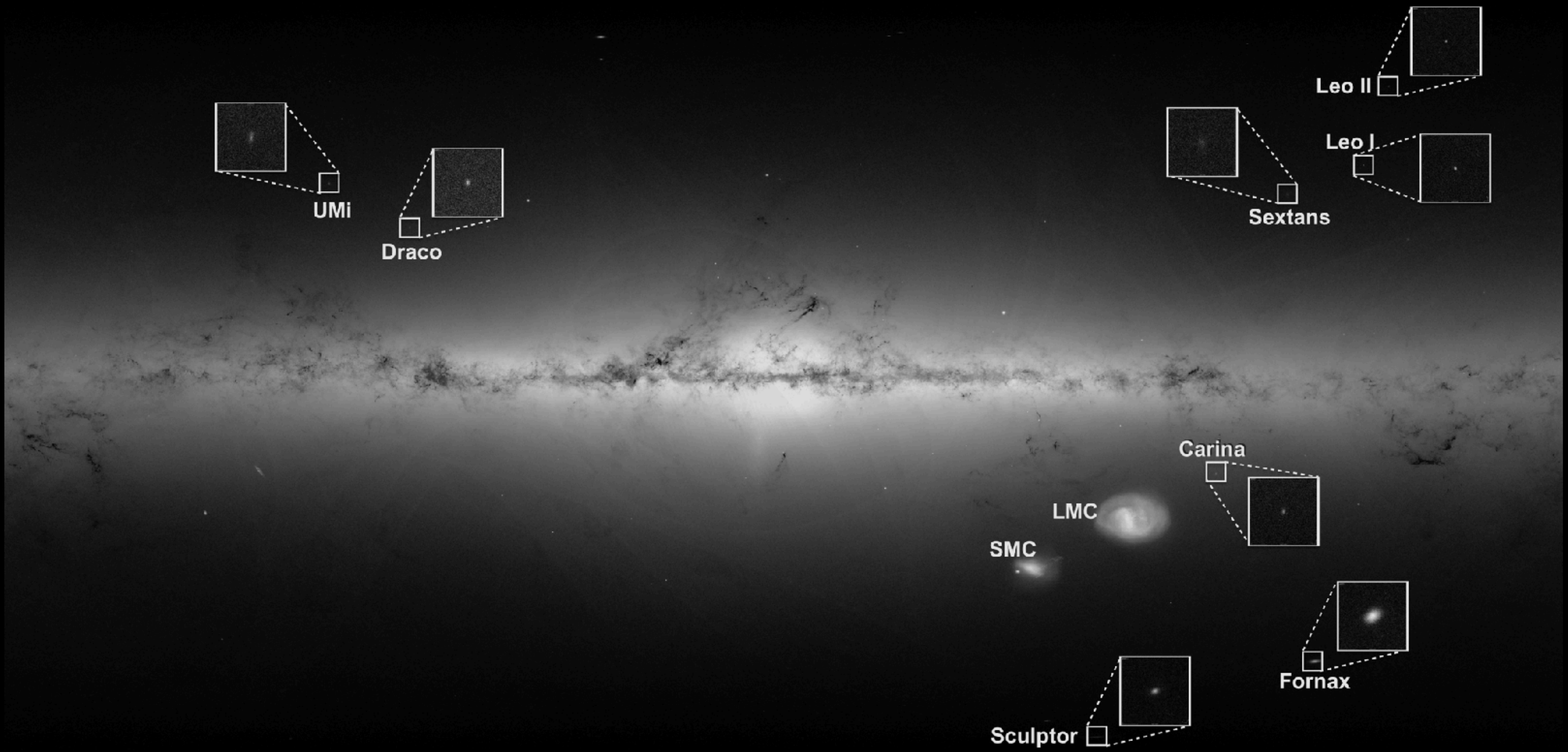
($5\ 10^6$ stars to 1.5 kpc)



(GDR2)

Stellar density from the full Gaia DR3

based on data collected between 25 July 2014 and 28 May 2017, spanning a period of 34 months

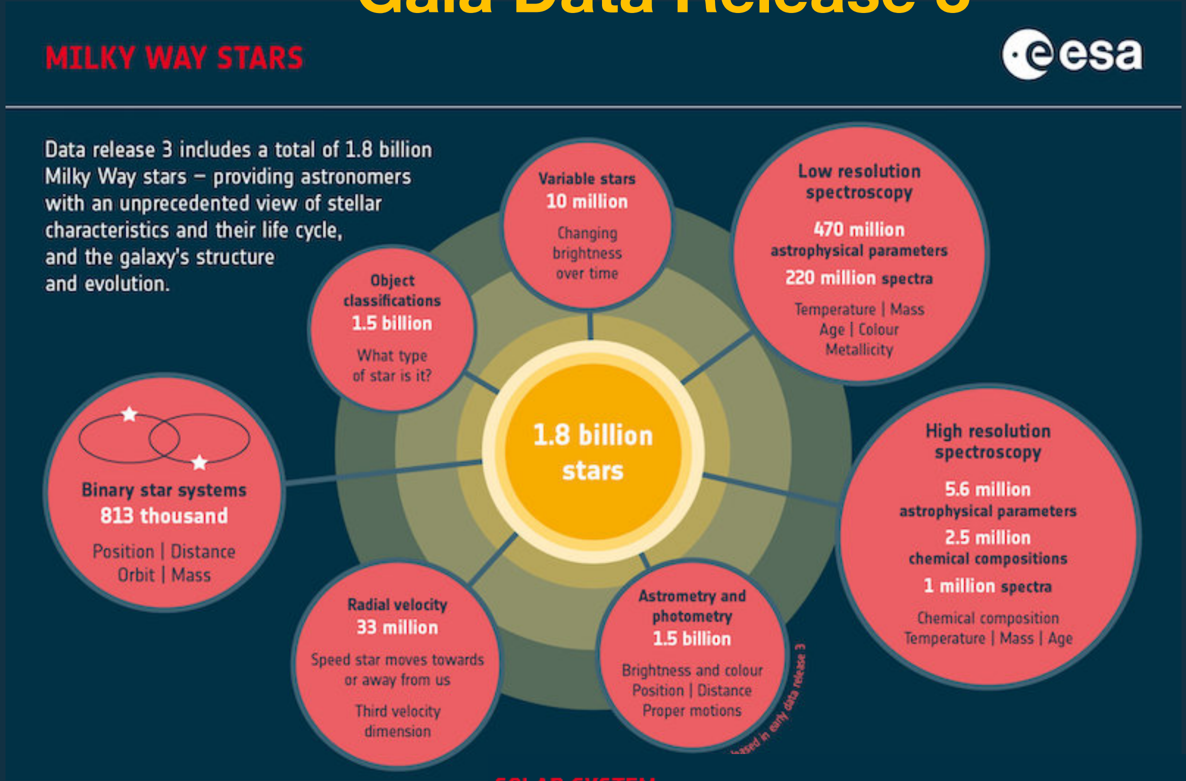


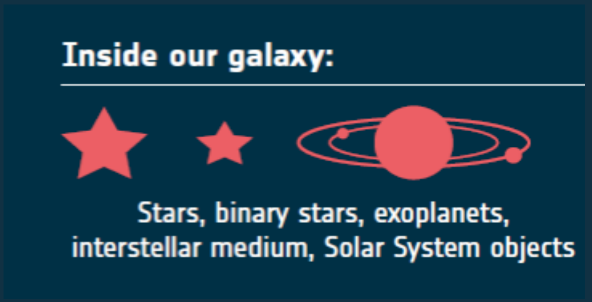
Source count maps based on the Gaia DR3 data.
Image credit: ESA/Gaia/DPAC

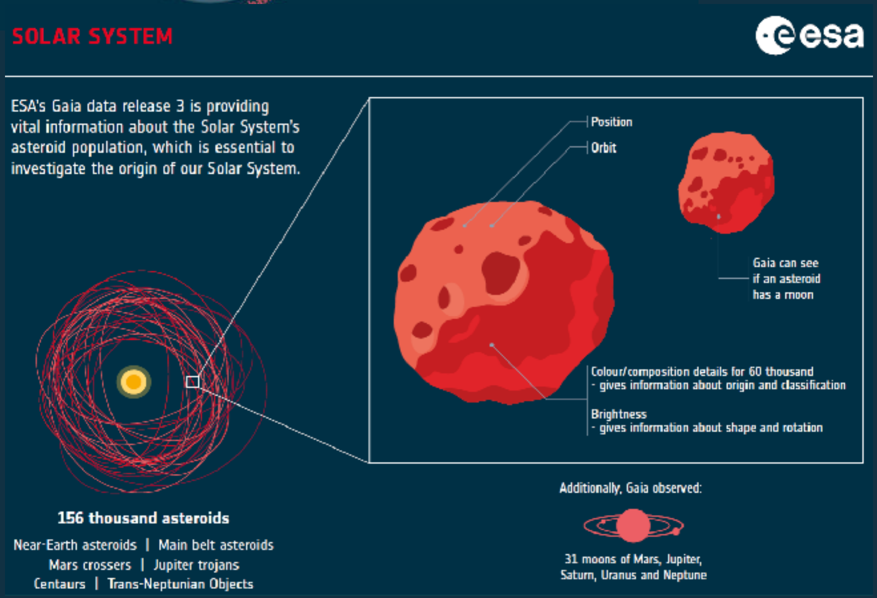
Acknowledgement: Images were created by André Moitinho and Márcia Barros, University of Lisbon, Portugal

https://www.esa.int/Science_Exploration/Space_Science/Gaia

Gaia Data Release 3





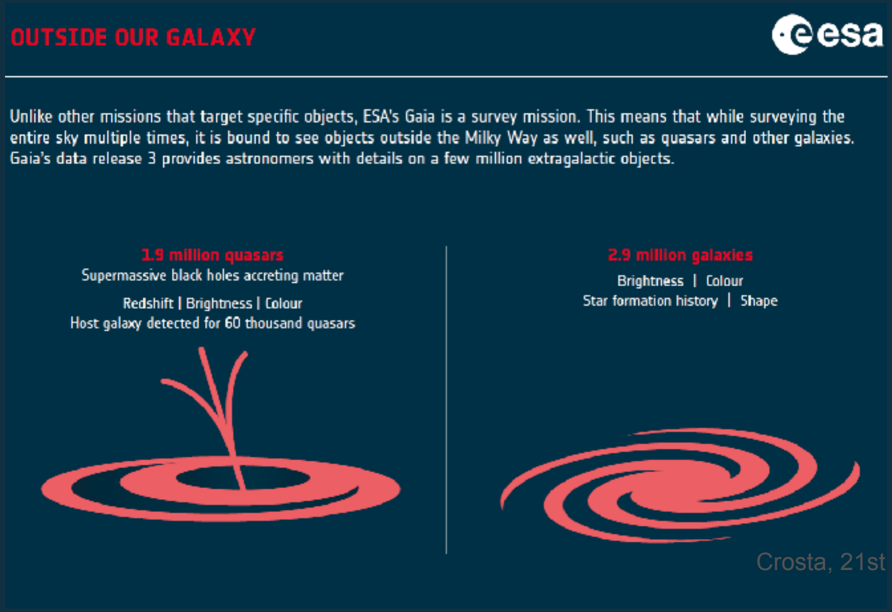


Data Release Scenario
http://www.cosmos.esa.int/web/gaia/release

Next Gaia DR4 (based on 66 months of data) by the end of 2026 will be consisting of:

- Full astrometric, photometric, and radial-velocity catalogues
- All available variable-star and non-single-star solutions
- Source classifications plus multiple astrophysical parameters for stars, unresolved binaries, galaxies, and quasars
- An exoplanet list
- **All epoch and transit data for all sources**!

Gaia DR5 (based on all mission data) not before the end of 2030 will be consisting of **Complete Gaia Legacy Archive of all data**

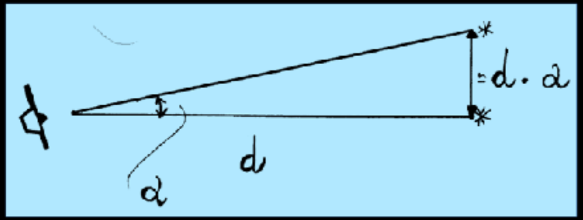


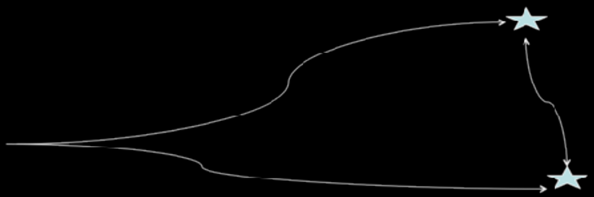
Astrometry nowadays is dominated by Einstein's theory

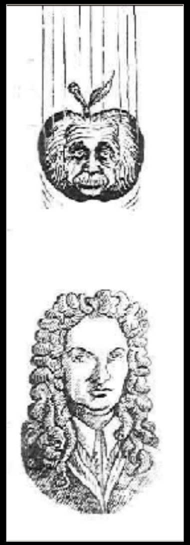
Stars belong to the architecture of spacetime which is dictated by the Einstein equations

Classical Astrometry

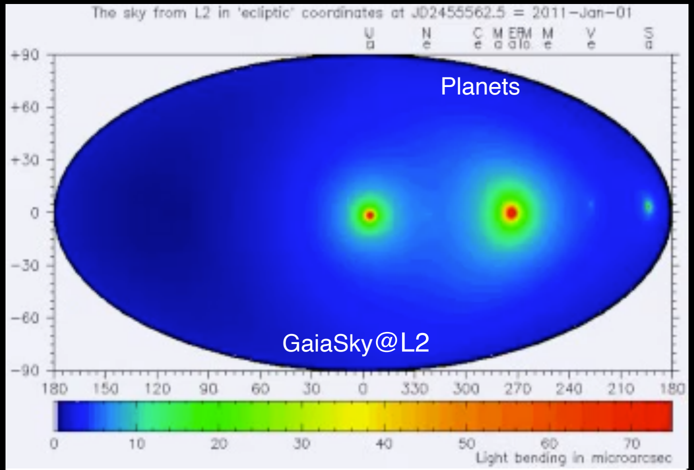
$\alpha, \delta, \mu_{\alpha}, \mu_{\delta}, \pi, \ldots$



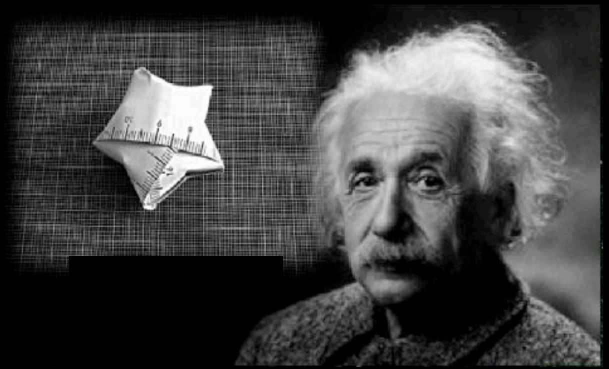




increasingly accurate astronomical data

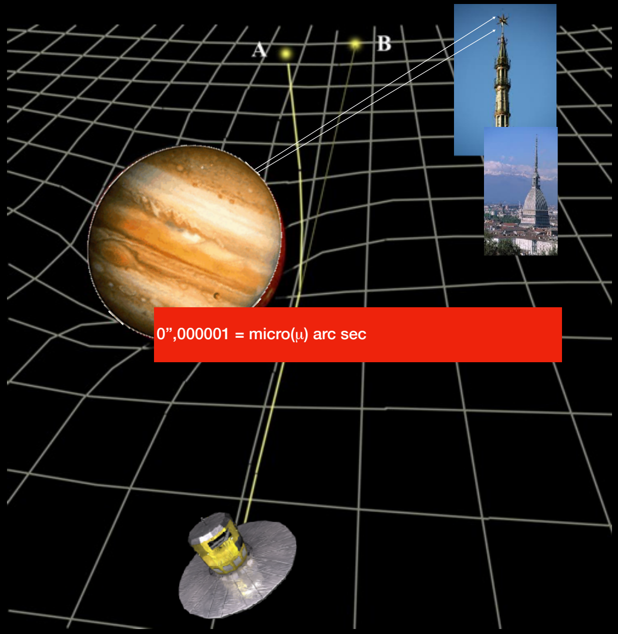


micro-arcsecond accuracy + Solar System gravitational fields => relativistic models for the light-ray propagation, from the observer to the star



Relativistic Astrometry

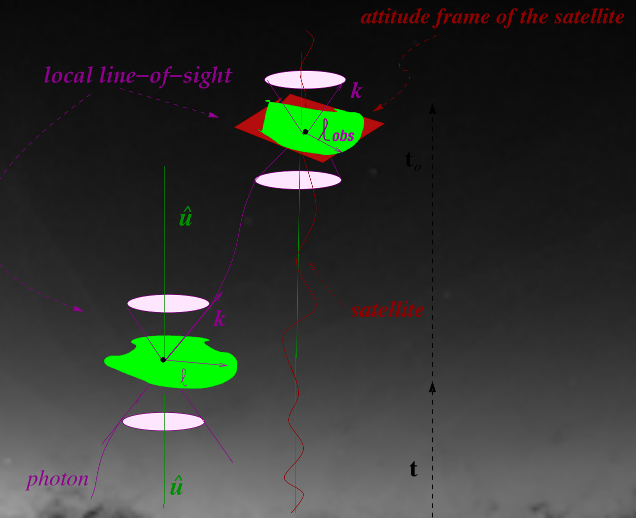
theoretical, analytical and/or numerical models, completely based on General Relativity and relativistic attitude (satellite or ground based observers)



Gaia: the Era of Relativistic Astrometry

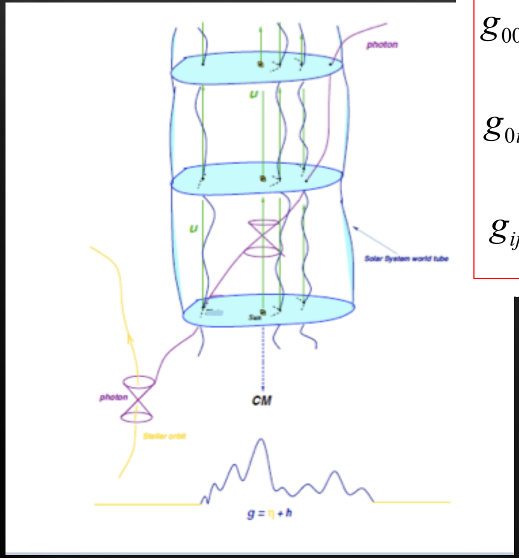
the trajectories of photons emitted by the stars
- null geodesics -

should be as fundamental as
the equation of stellar evolution!



Source count maps based on the Gaia DR3 data.
Image credit: ESA/Gaia/DPAC
Image license: CC BY-SA 3.0 IGO

Acknowledgement: Images were created by André Moitinho and Márcia Barros, University of Lisbon, Portugal



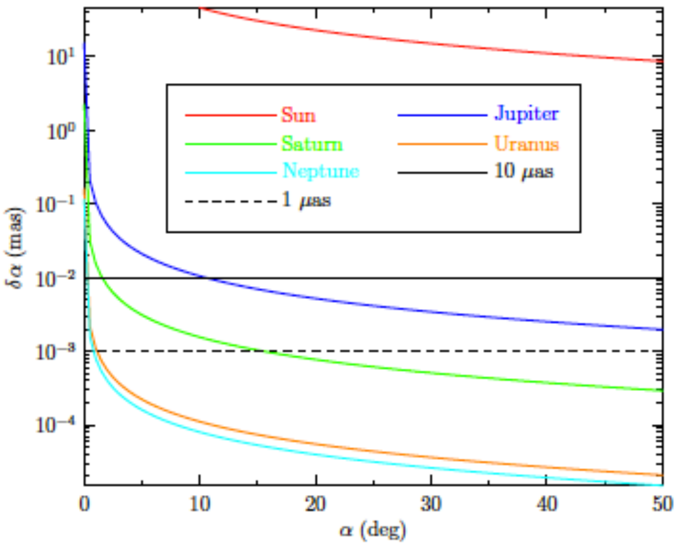
$$g_{00} = -1 + \frac{2}{c^2} w(t, \boldsymbol{x}) - \frac{2}{c^4} w^2(t, \boldsymbol{x}),$$

$$g_{0i} = -\frac{4}{c^3} w^i(t, \boldsymbol{x}),$$

$$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2} w(t, \boldsymbol{x})\right).$$

IAU metric for the definition o

Body	$\delta\alpha_M$ (μas)	$\delta\alpha_Q$ (μas)
Sun	1.75×10^6	~ 1
Mercury	83	
Venus	493	
Earth	574	0.6
Moon	26	
Mars	116	0.2
Jupiter	16270	240
Saturn	5780	95
Uranus	2080	8
Neptune	2533	10



The (Celestial) Sphere Reduction/Reconstruction is Gaia's primary objective

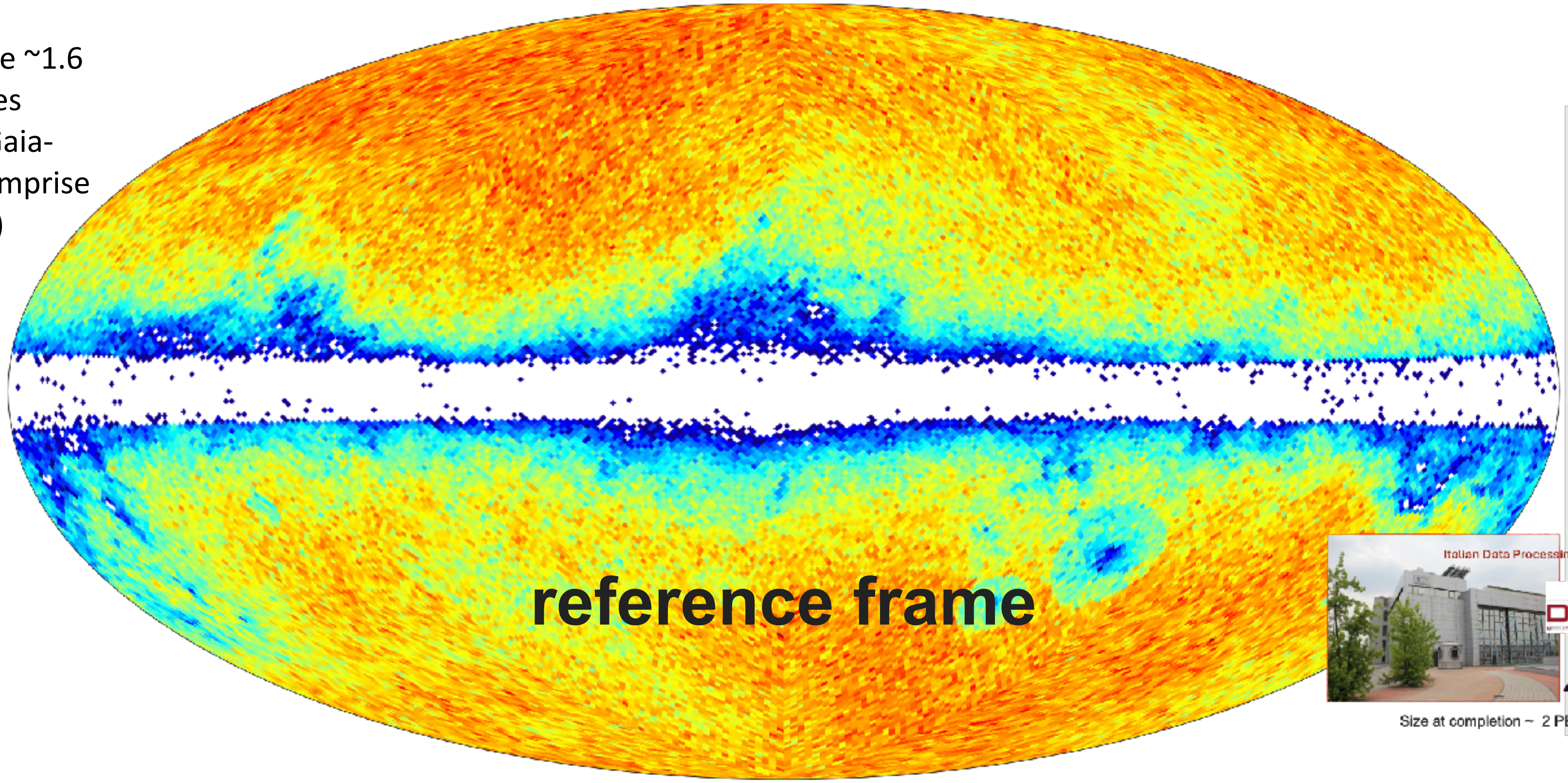
first direct materialization of a dense absolute reference frame at visual bands

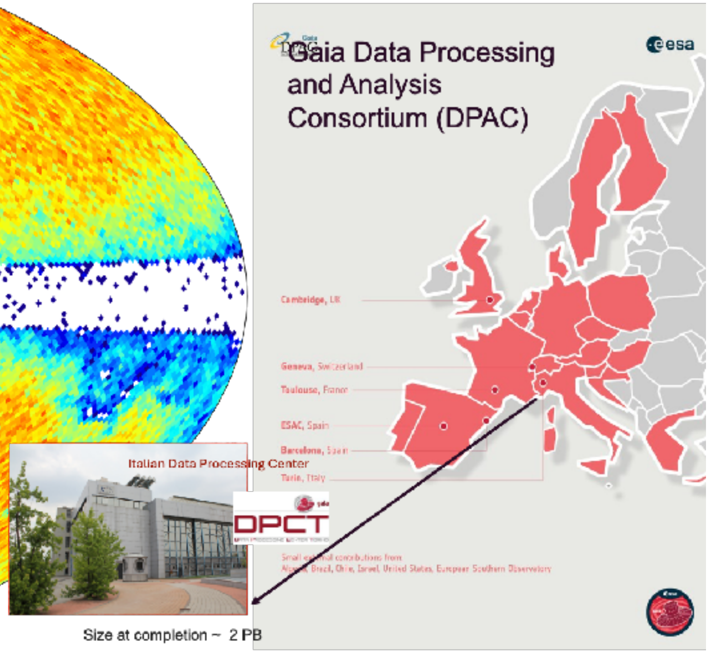
one of the most important fundamental physics task

quasi-inertial kinematically non-rotating global **optical** frame meeting the ICRS prescriptions/IAU recommendations

- Spatial distribution of the ~1.6 million Gaia-CRF3 sources
- A complete version of Gaia-CRF could potentially comprise ~4 million sources (QSOs)

Credits: ESA/Gaia/DPAC





the Consortium constitued for the Gaia data reduction (DPAC) agreed to set up, respectively, two independent global sphere solutions: AGIS and **GSR**

TASK:

link of the optical to the radio reference frame

2 independent GR models:

GREM (Gaia RElativistic Model)

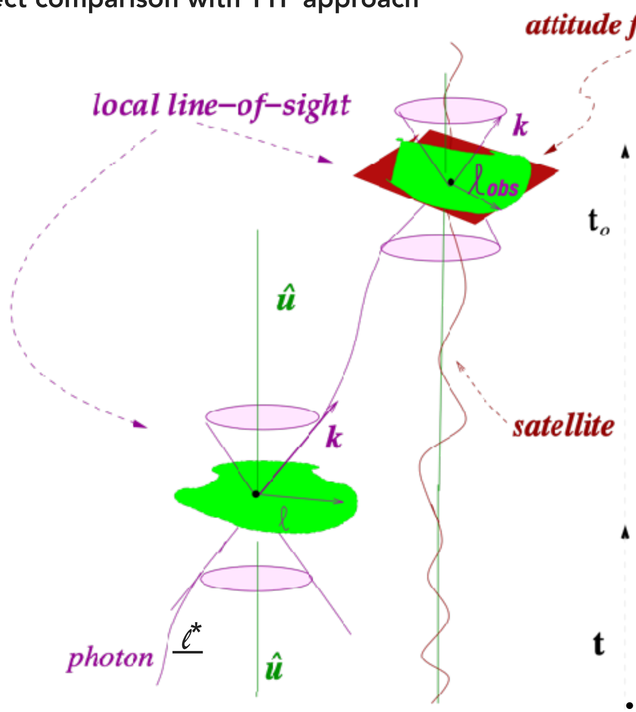
RAMOD (Relativistic Astrometric MODel)

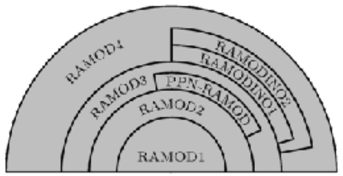
RAMOD is a framework of general relativistic astrometric models with increasing intrinsic accuracy, adapted to many different observer's settings, **interfacing numerical and analytical relativity**

fully based on alghorithms in General Relativity (GR) -> *no a priori approximations*, top-down approach

observations in a curved space-time -> <u>RAMOD applies the measurement protocol in GR</u>

direct comparison with TTF approach





$$\bar{l}^i = n^i\left(1-\frac{h_{00}}{2}\right)+\mathcal{O}\left(\frac{v^4}{c^4}\right)$$

Gravitationally aberrated direction

$h_{\odot} \sim 100\mu as$ @ L2

$x_* = F(x_{obs}, g_{\mu\nu}, \ell_{obs}, E_a, \ldots.)$

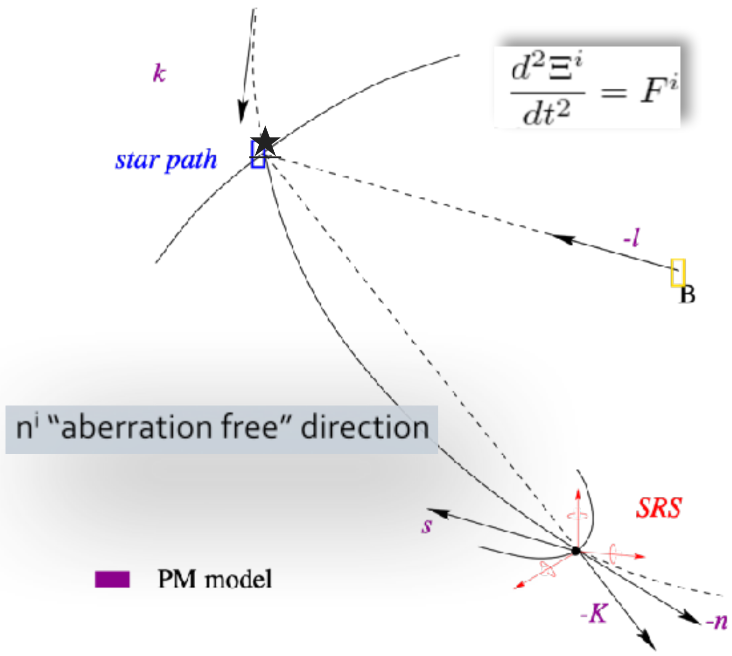
- de Felice F., Crosta M., Vecchiato A. and Lattanzi M. G., Astrophys. J., 607 (2004) 580
- Crosta M., Geralico A., Lattanzi M. G. and Vecchiato A., Phys. Rev. D, 96 (2107) 104030.
- S. Bertone et al. ,2014 Class. Quantum Grav. 31 015021

GREM,

baselined for the Astrometric Global Iterative Solution for Gaia (AGIS), based on post-Newtonian approximations

stellar direction in pN

$$x^i = x^i_o + k^i\Delta t + \Xi^i$$



- Klioner S. A., Astron. Astrophys., 404 (2003) 783.

GREM observed direction converts into a coordinate one via several steps , which separate the effects of the aberration, the gravitational deflection, the parallax, and proper motion-> **bottom-up approach**

RAMOD framework

Coordinates are not "physical observers"
The **observer** is selected according to the chosen measurement, namely by its **specific kinematical status with respect to the background spacetime.**

To any time-like observer **u** we associate to tensorial operators, T and P, so that

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = T_{\alpha\beta}dx^\alpha dx^\beta + P_{\alpha\beta}dx^\alpha dx^\beta$$

dT dL

1+3 decomposition = geometric measurement

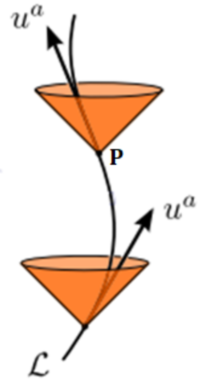
an infinitesimal normal neighborhood of **u** the metric

Space time splitting

$$P(u)_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$$

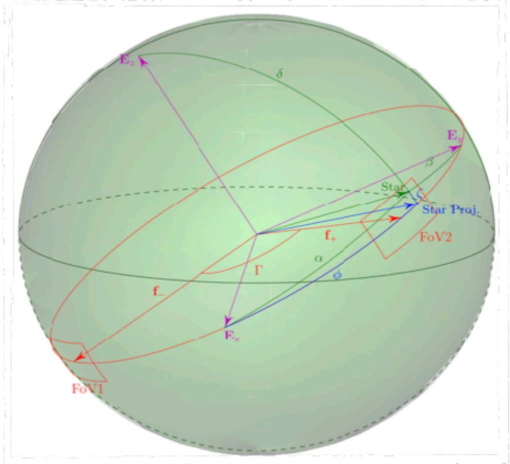
$$T(u)_{\alpha\beta} = - u_\alpha u_\beta$$

$$g_{\alpha\beta} = P_{\alpha\beta} + T_{\alpha\beta}$$



World line

The concept of the Global Sphere reconstruction via general relativistic astrometric observable



Merging repeated observations of the same objects from different satellite orientations and on different times allows to estimate their angular positions, parallaxes, and proper motions, *i.e.* the actual materialization of an absolute Reference Frame.

This process is conventionally called Astrometric Sphere Reconstruction.

Projector operator onto the rest space of the satellite

Theoretical models

$$\cos\psi_{(E_{\hat a},\ell_{obs})} \equiv \mathbf{e}_{\hat a} = \frac{P(u')_{\alpha\beta}k^\alpha \boldsymbol{E}^\beta_{\hat a}}{(P(u')_{\alpha\beta}k^\alpha k^\beta)^{1/2}}$$

E_α^β relativistic attitude tetrad" $E^\alpha_{\hat a} = \mathcal{R}(\alpha_i)_{\hat a}{}^{\hat b}\lambda^\alpha_{\hat b}$

-> essential to define the boundary condition

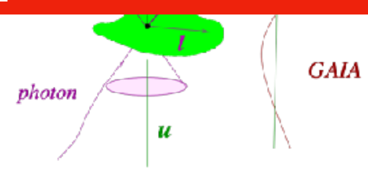
$$\lambda^\alpha_{(bs)\hat a} = P(u_s)^\alpha_\beta[\lambda^\beta_{\hat a} - \frac{\gamma(u_s,u)}{\gamma(u_s,u)+1}\nu(u,u_s)^\alpha(\nu(u,u_s)^\rho\lambda_{\hat a\rho})]$$

Bini , Crosta, and de Felice, Class.Quantum Grav. 20, 4695, 2003

Observation equation

$$\cos\phi \equiv F\left(\underbrace{\alpha_*,\delta_*,\varpi_*,\mu_{\alpha*},\mu_{\delta*}}_{\text{Astrometric parameters}},\underbrace{\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)},\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}}_{\text{Attitude parameters}},\underbrace{c_1,c_2,\ldots}_{\text{Instrument}},\underbrace{\gamma,\ldots}_{\text{Global}}\right)$$

$$\mathbf{x}_* = \frac{1}{\varpi}(\cos\alpha\cos\delta, \sin\alpha\cos\delta, \sin\delta)$$



1 obs. $\Rightarrow$ 1 condition eq. $\alpha(t) = \alpha(t_0) + \mu_\alpha(t - t_0) + O(\Delta t^2)$, $\delta(t) = \delta(t_0) + \mu_\delta(t - t_0) + O(\Delta t^2)$

(linearized) system of solution with dimensions ~$10^{10}\times 10^8$

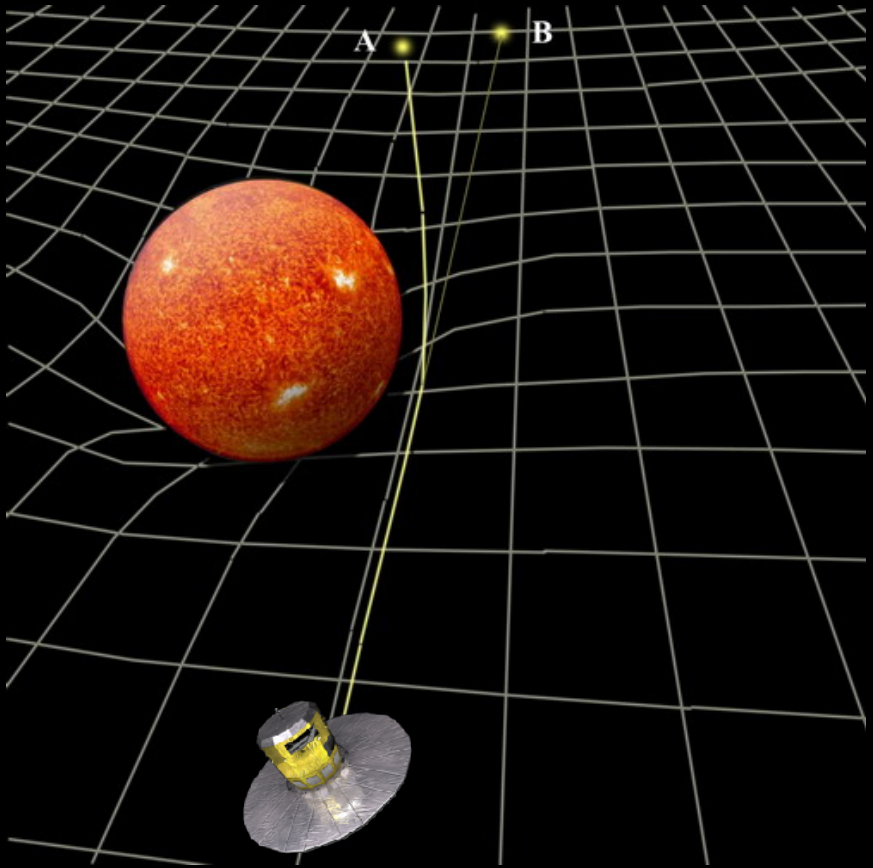
- de Felice F., Crosta M., Vecchiato A. , Lattanzi M. G. And B. Bucciarelli, Astrophys. J., 607 (2004) 580
- Crosta M., Geralico A., Lattanzi M. G. and Vecchiato A., Phys. Rev. D, 96 (2107) 104030.
- Vecchiato A. , B. Bucciarelli, M.G.Lattanzi et al., Astron. Astrophys., 620 (2018) A40

Stars as peering tools @Solar System spacetime

Global astrometry

$$\cos\psi_{(E_{\hat{a}},\ell_{obs})} \equiv \mathbf{e}_{\hat{a}} = \frac{P(u')_{\alpha\beta}k^{\alpha}\boldsymbol{E}^{\beta}_{\hat{a}}}{(P(u')_{\alpha\beta}k^{\alpha}k^{\beta})^{1/2}}$$

$$\cos\phi \equiv F\left(\underbrace{\alpha_*,\delta_*,\varpi_*,\mu_{\alpha*},\mu_{\delta*}}_{\text{Astrometric parameters}},\underbrace{\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)},\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}}_{\text{Attitude parameters}},\underbrace{c_1,c_2,\ldots}_{\text{Instrument}},\underbrace{\gamma,\ldots}_{\text{Global}}\right)$$



•In GR γ=1 -> measures the amount of space curvature generated by a unit mass

•Gravity theories alternative to GR require the existence of a scalar field coupled to gravity and predict it fades with time, so that its residue would manifest itself through very small deviations from Einstein's GR in the weak field regime

deviations from GR depends on the particular scalar-tensor theory adopted

→ *given the number of celestial objects (a real Galilean method applied on the sky!) and directions involved (the whole celestial sphere!), the largest experiment in General Relativity ever made with astrometric methods (since 1919) from space*

A massive repetition of the Eddington et al. astrometric test of GR

⟹ ***estimated error for γ ~10^{-6} with Gaia?***

Butkevich et al. , 2022 (A&A, 663, A71) / *PN gravity and Gaia-like astrometry*
Coupling between PPN gamma and parallax zero point, 2023

Achieving PPN gamma estimation comparable with the result of the Cassini experiment necessitates a parallax zero point at a sub-muas level.

Talk outline

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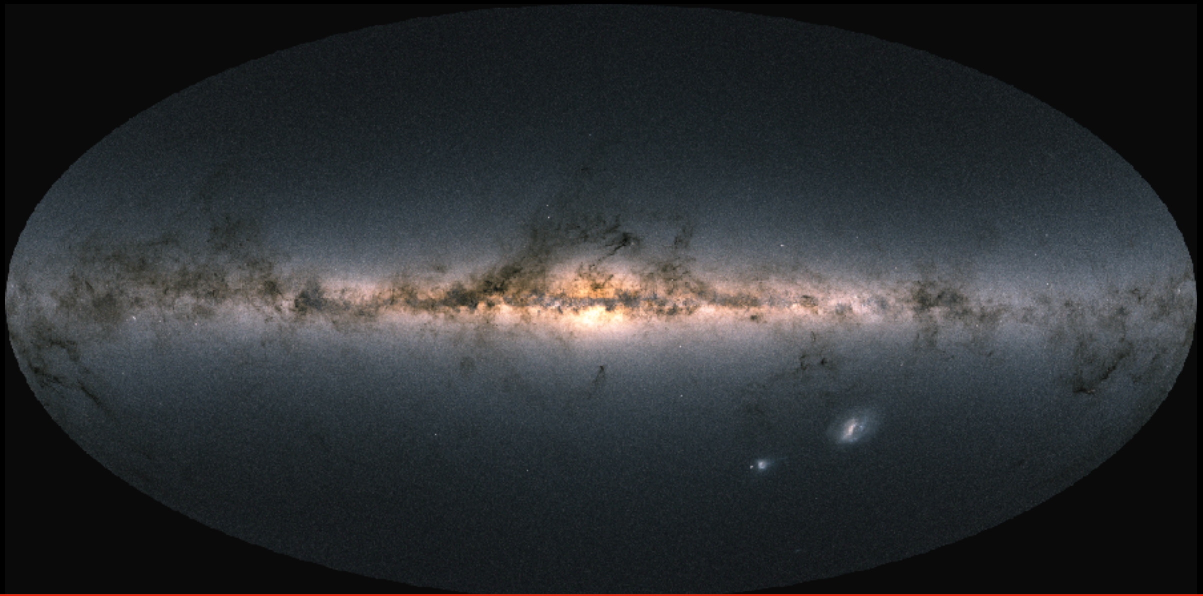
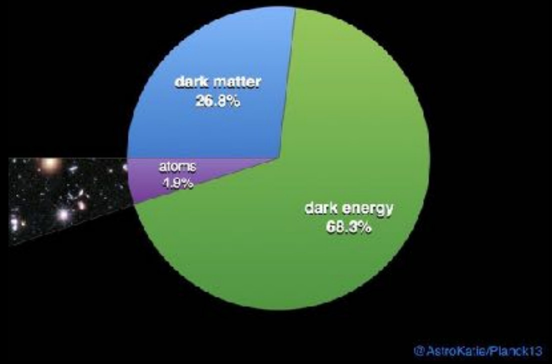
Challenging the Galactic Models with Milky Way stars

- Local cosmology as ΛDCM laboratory
- Testing General Relativity/Gravity @MilkyWay scale
- Dark Matter interpretation



Gravitational astrometry @ Milky Way scale: investigating the effects of gravity on photons at all scales within the Milky Way, and then compare them to the predictions of current Gravity theories and Cosmological formation scenarios including stellar and planetary formation.



MW as the laboratory of «local» Cosmology
(much like what the Sun is to stellar Astrophysics)

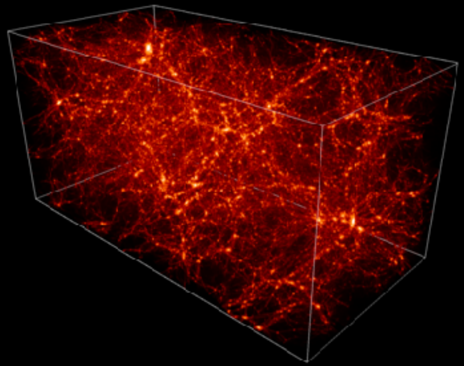
➢ **Local Cosmology:** Lambda-CDM model predictions at the scale of the Milky Way

Gaia can provide values (true observables)
to estimate model parameters

Cosmological Concordance model

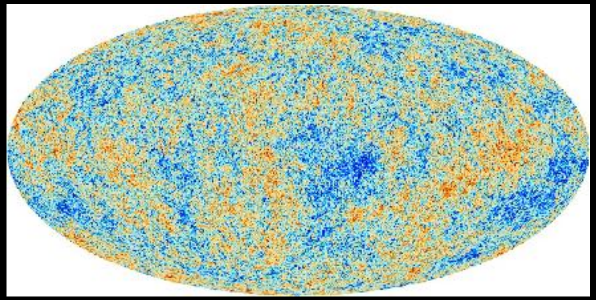
Absence of “evidences” of extra matter

- galaxy cluster
- large structure at Mpc scale
- CMB
- gravitational lensing
- rotation curve at galactic scale



> Candidates

- no baryonic particle (axions, WIMP)/SUSY
- self-interaction
- neutrinos (sterile, massive, etc..)
- scalar fields/modified gravity via MOND etc..

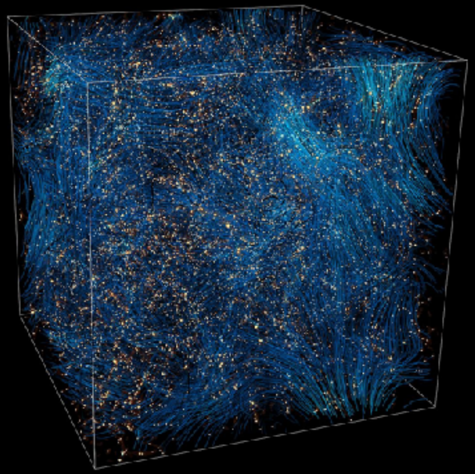


In the most advanced simulations standards Λ-CDM cosmology assumes an average FLRW evolution while growth in structure is treated by Newtonian N-body simulations:

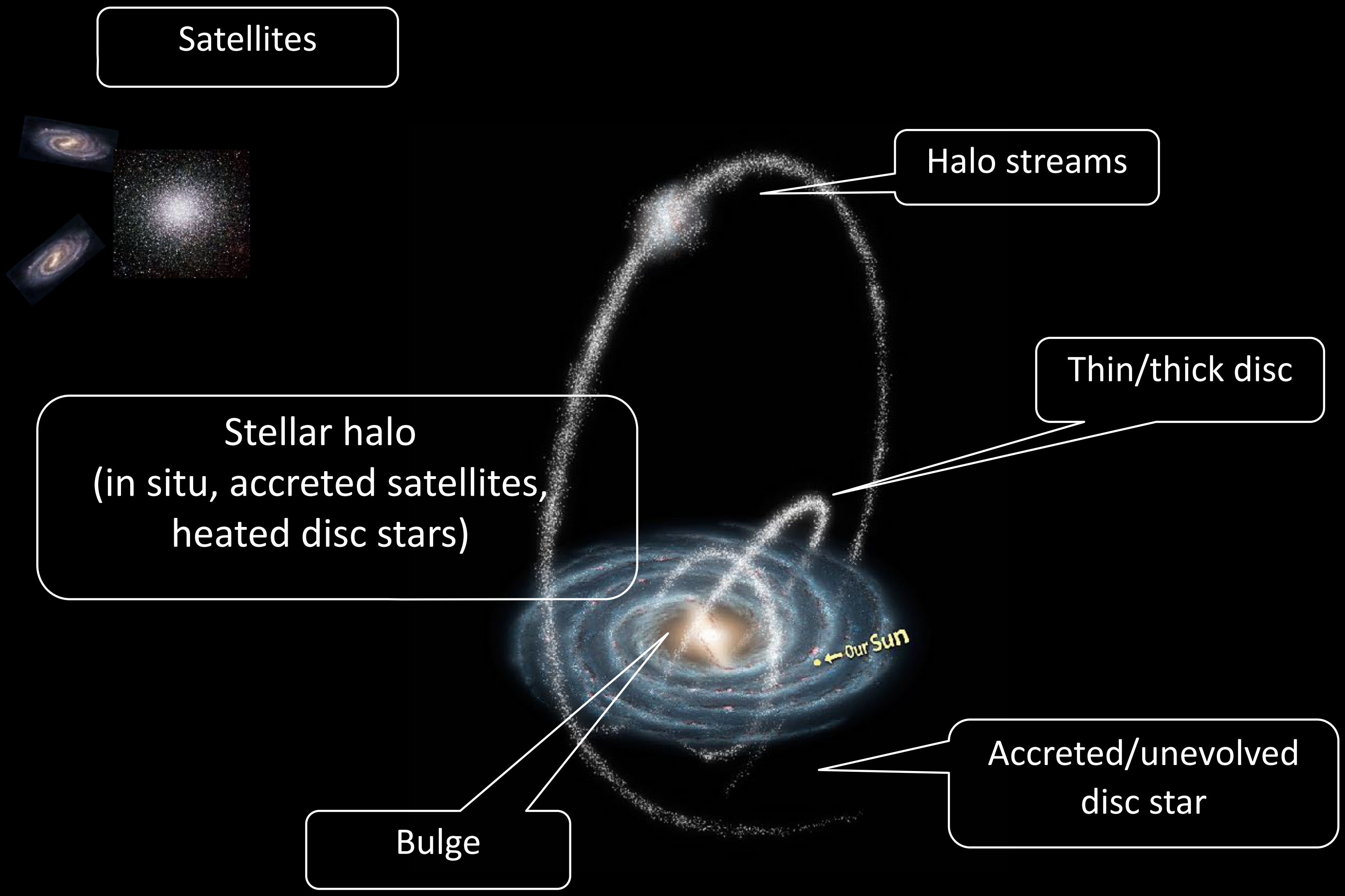
“ Friedman tells space how to curve and Newton tells mass how to move”
arXiv:1612.09309v2

General Relativity (GR) is only partially considered

-> G-evolution: GR code for simulated large structures and expansion in Λ-CDM (Adameck et al. 2016)
-> GRAMSES (Barrera-Hinojosa & Li 2020)

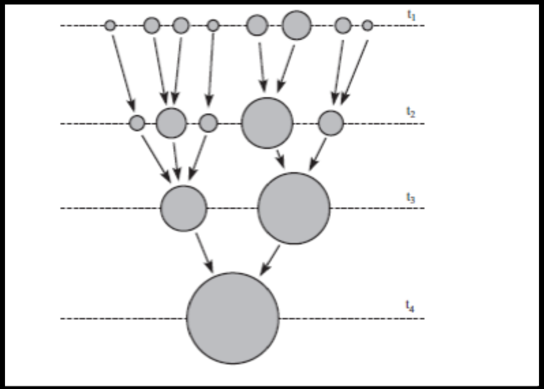


Galactic components



ΛCDM - Hierarchical scenario

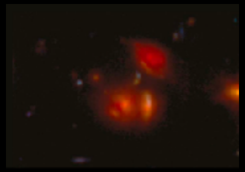
growth of cosmic structures



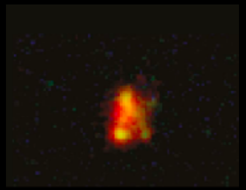
The growth of cosmic structures:

- primordial density fluctuations produced during inflation
- dominant mass component is cold dark-matter (CDM)
- fluctuations grow under the action of gravity
- ΛCDM power spectrum: small objects collapse first
- Gas cooling and star formation
- Galaxy evolution and merging

Examples of galactic building blocks in protogalaxies observed by JWST

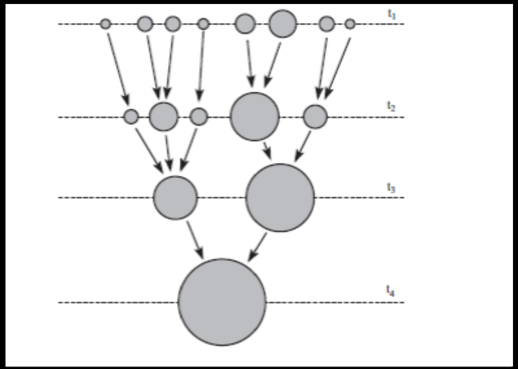


”The cosmic rose” (0.1 Gyr)

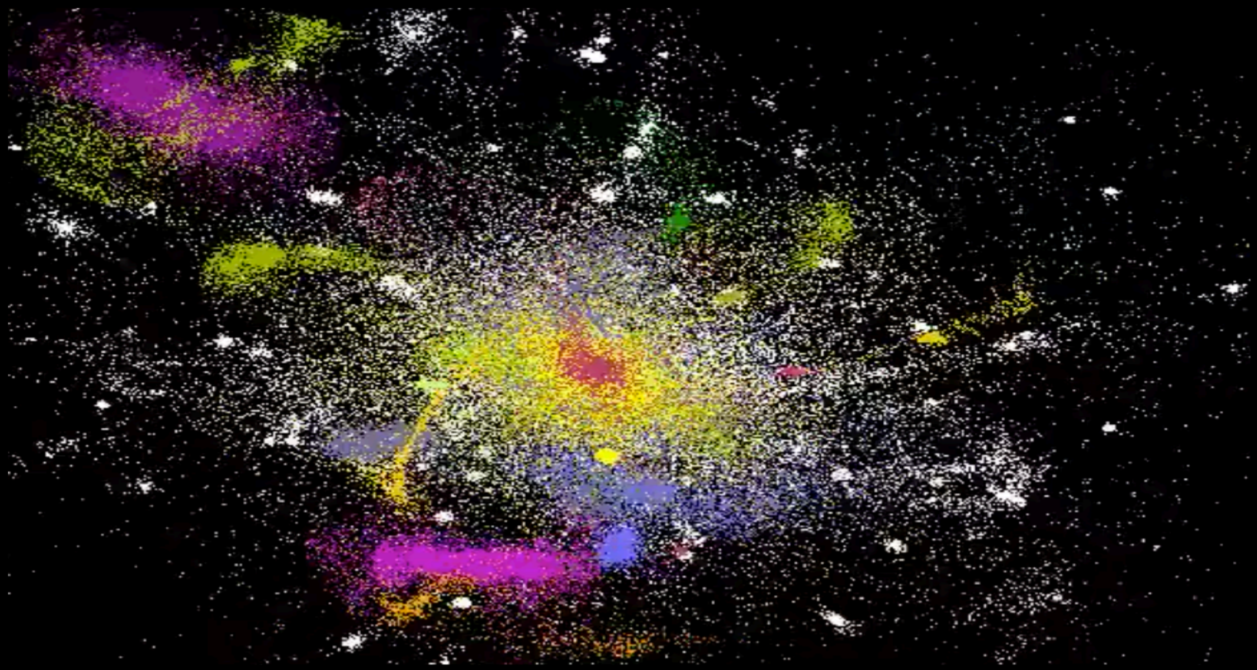


"The big clumpy” (0.3 Gyr)

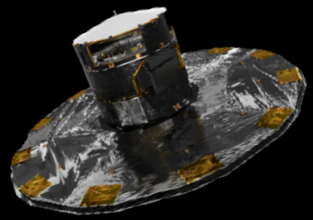
Open questions



- How many mergers in the history of the Milky Way?
- How large were they?
- When did the mergers take place?
- How the mergers have affected the Milky Way?



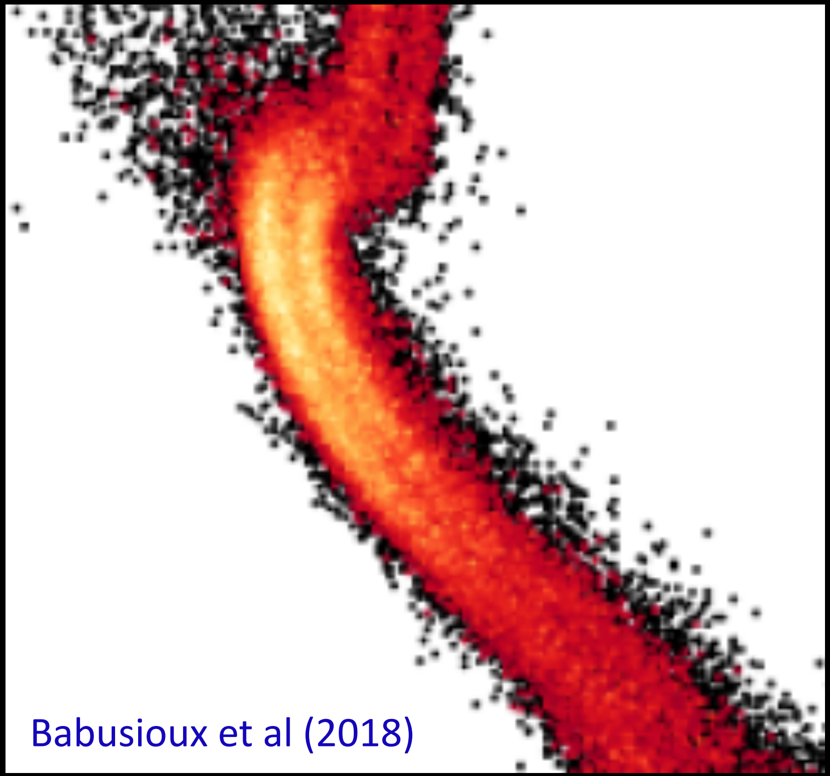
Galactic halo formation - merging contributions



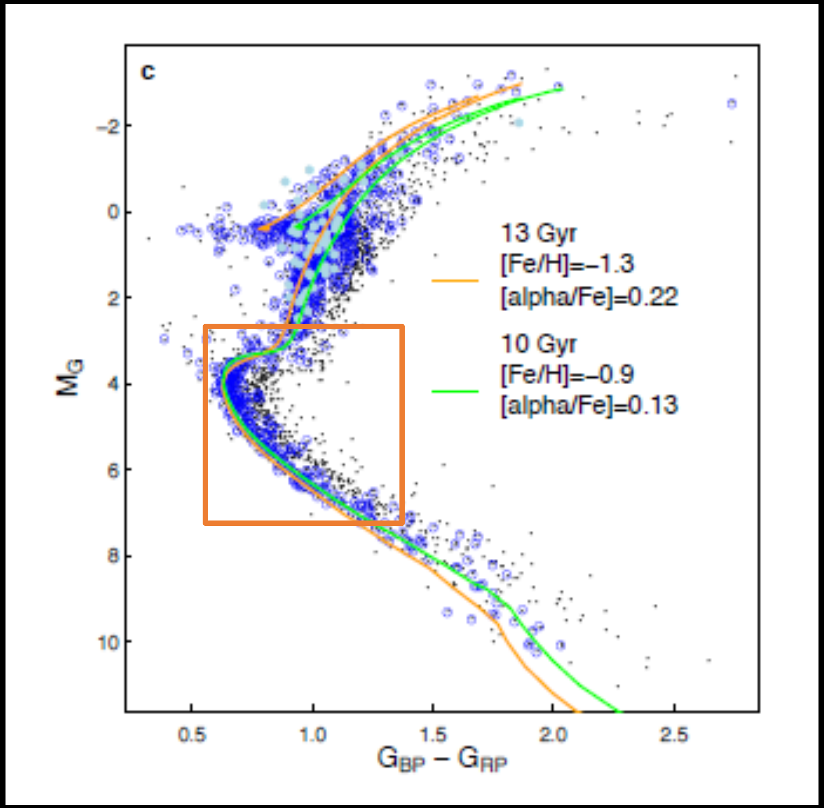
Major merger: *Gaia – Sausage – Enceladus (GSE)*

Amina Helmi et al. 2018, *"The merger that led to the formation of the Milky Way's inner stellar halo and thick disk",* Nature, 563, 85

Abstract. … We demonstrate that the inner halo is dominated by debris from an object which at infall was slightly more massive than the Small Magellanic Cloud.

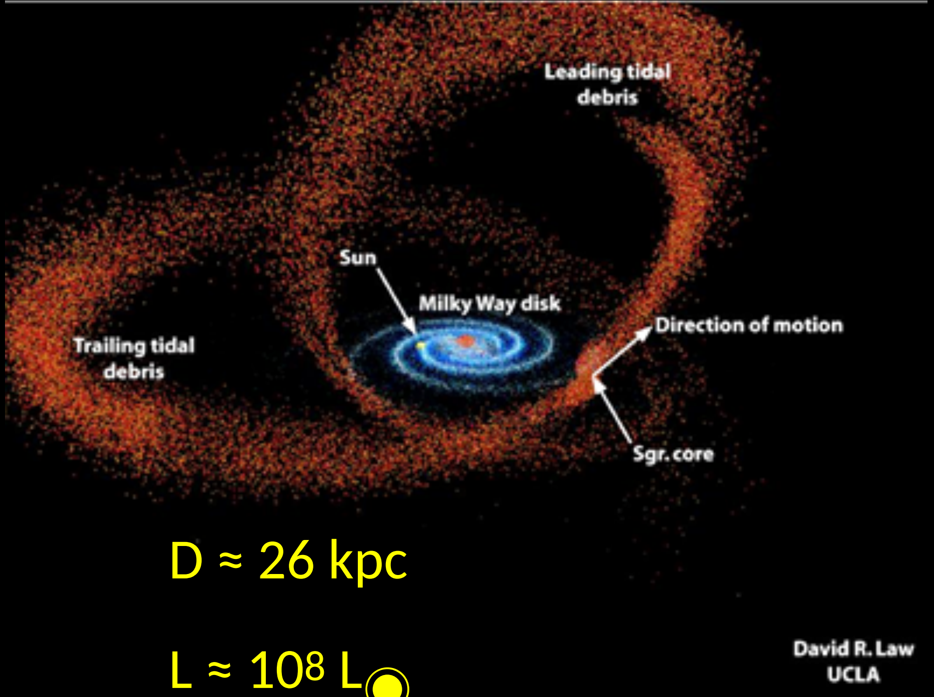


Babusioux et al (2018)



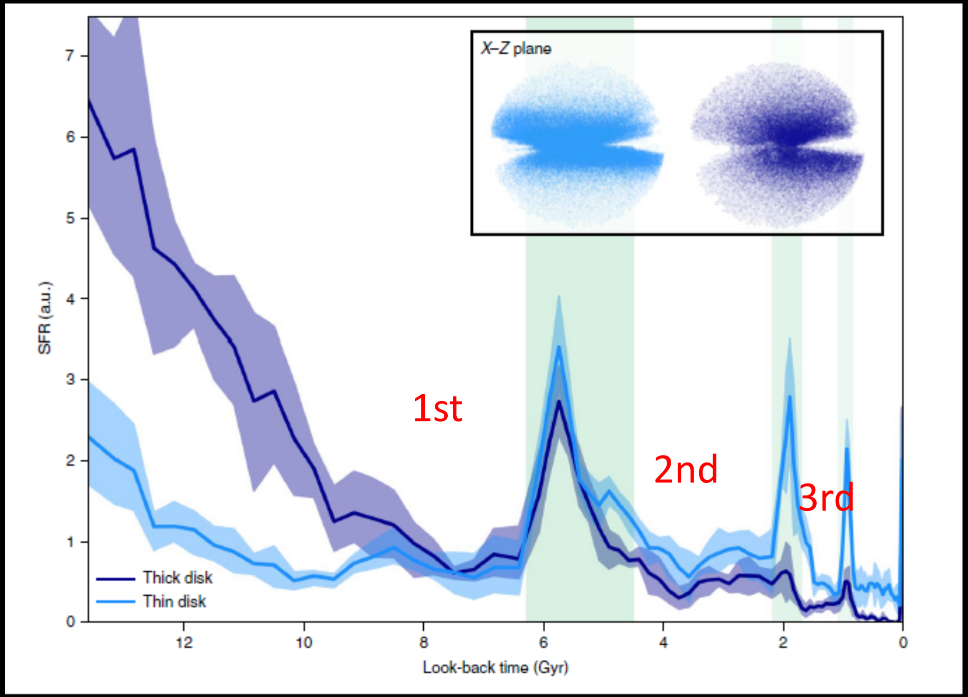
Galactic halo formation - tidal contributions

Sagittarius dwarf galaxy interaction with the MW



D ≈ 26 kpc

L ≈ 10^8 $L_\odot$



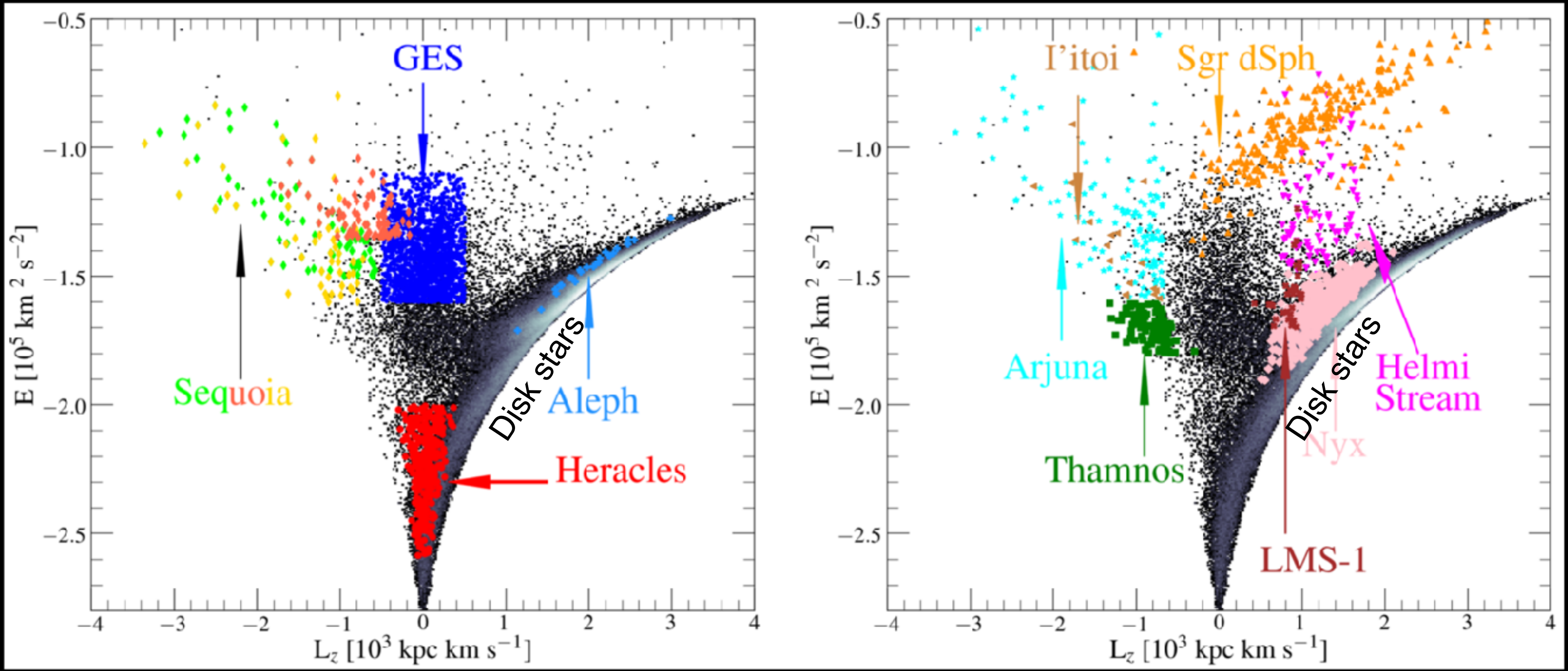
Star Formation History in the ~2-kpc-radius bubble around the Sun distinguishing between the thin and thick disks (selected on the basis of tangential velocity).

Green-shaded areas highlight the location of the detected star-forming bursts.

Three conspicuous and narrow episodes of enhanced star formation that we can precisely date as having occurred 5.7, 1.9 and 1.0 Gyr ago, which coincide with proposed pericentre passages of the Sagittarius dwarf galaxy.

Ruiz Lara et al 2020

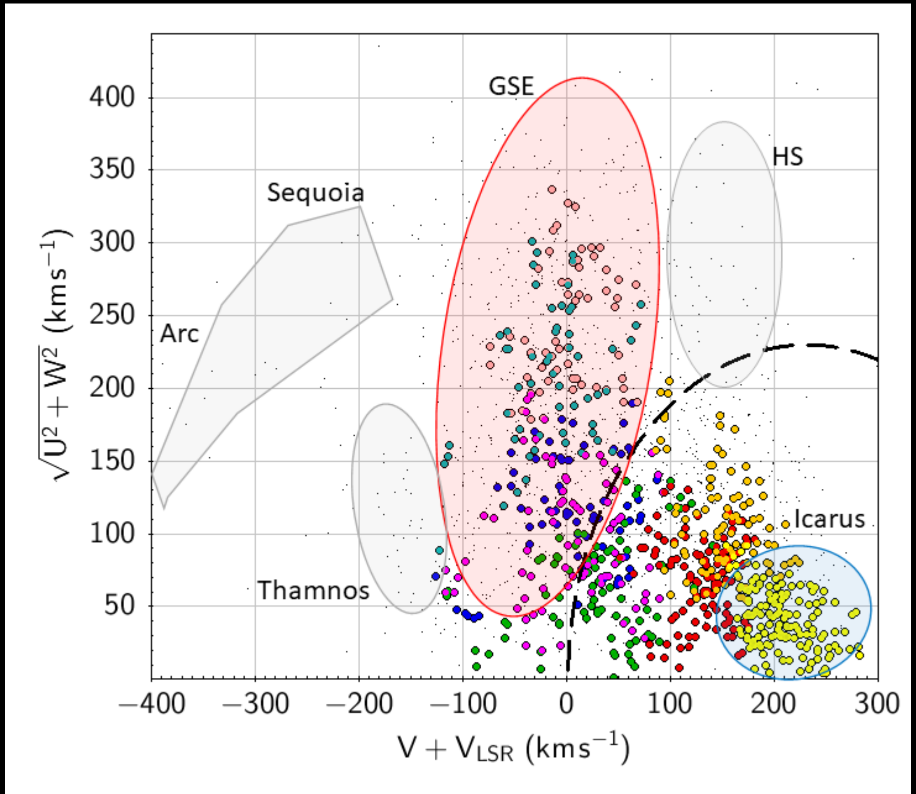
Galactic halo formation - substructures

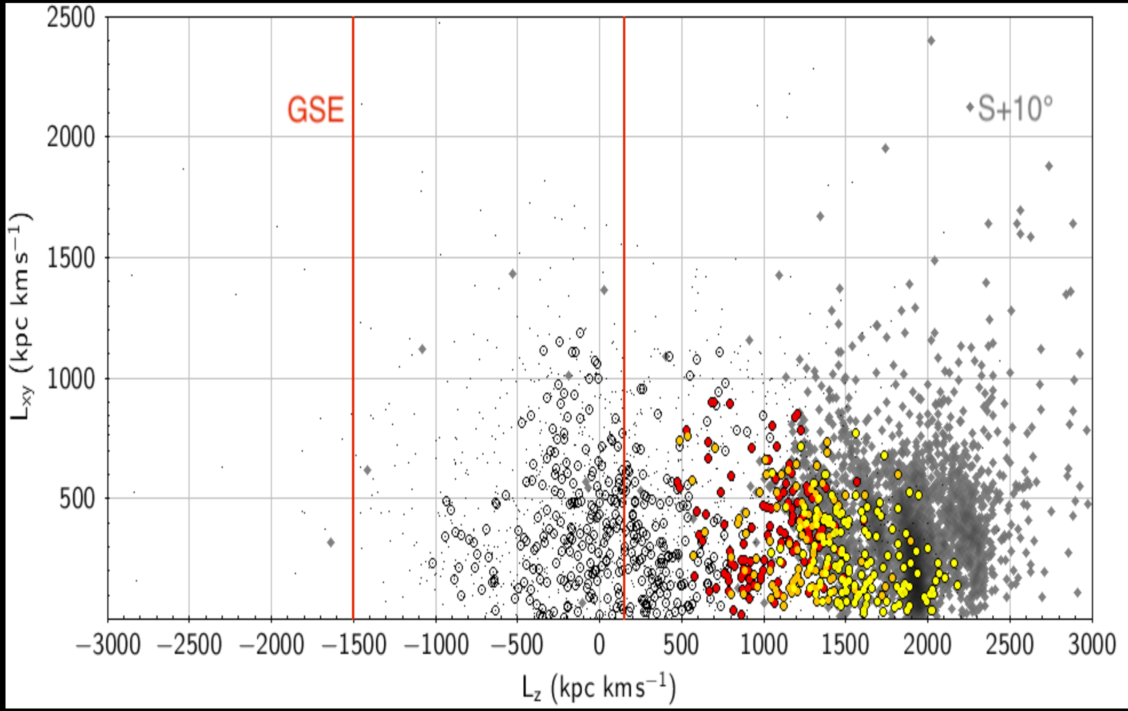


Halo substructures in the orbital energy vs. angular momentum w.r.t. the Galactic disc plane, among the MW sample from Gaia-APOGEE (white/black are high/low density regions). The coloured markers illustrate different halo structures.

(e.g. Ibata+1994; Helmi+1999, 2018; Belokurov+2018; Myeong+2019; Koppelman+2019; Necib+2020; Naidu+2020; Horta+2021,2022)

Galactic disc - Icarus: accreted/unevolved stars





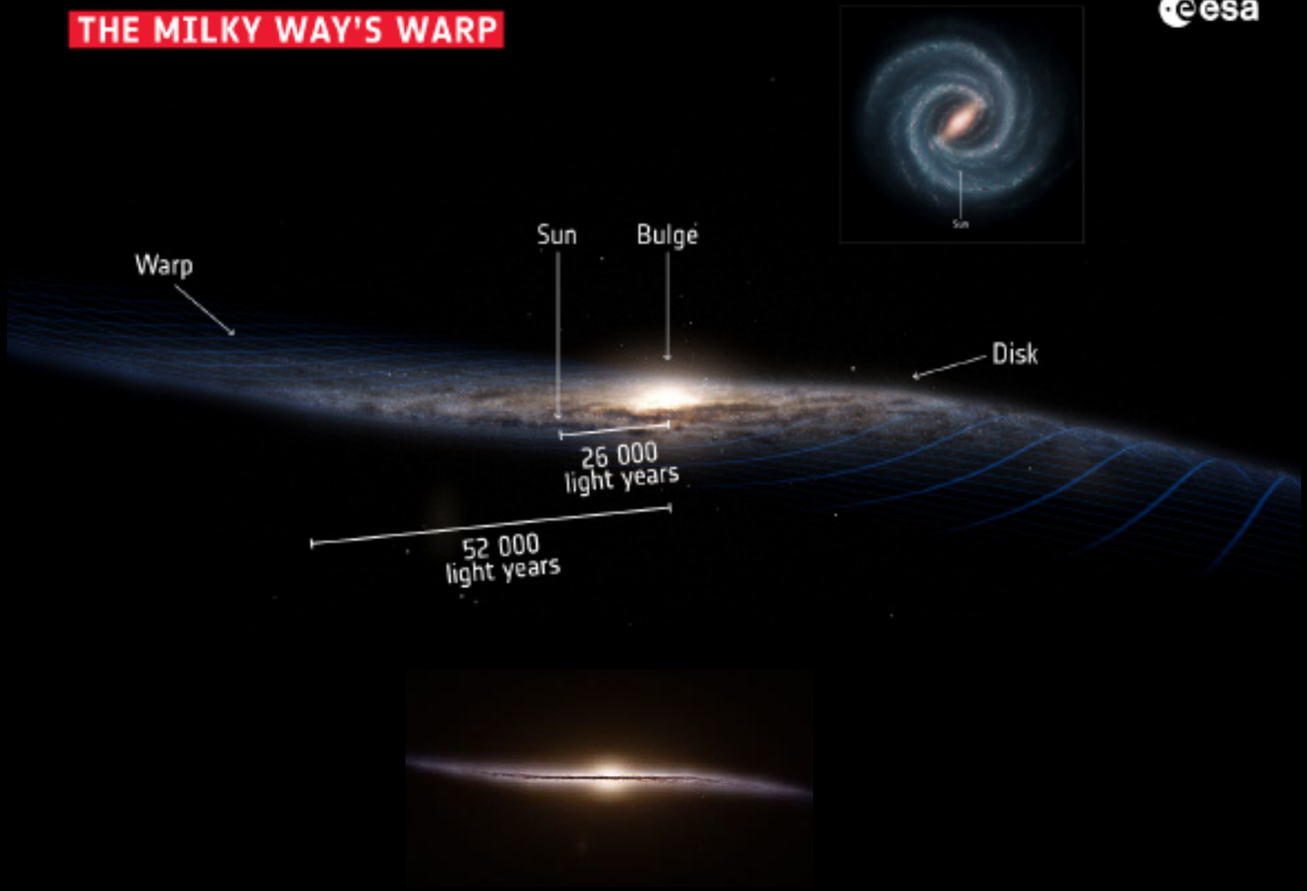
Toomre diagram.

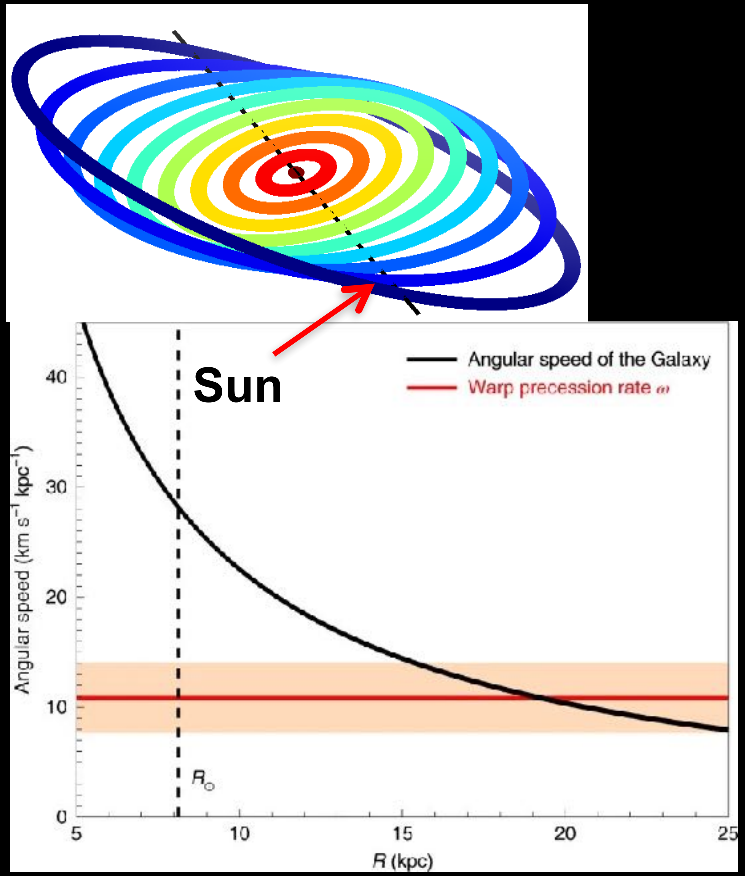
The traditional kinematic selection for halo stars, $|\mathbf{v} - \mathbf{v}_{LSR}| > 230$ km/s, represented by the dashed line.

Re Fiorentin et al (2021, 2024)

L_{XY} vs. L_Z distribution of Icarus stars (yellow and red dots)

The red solid lines indicate the GSE locus (Helmi+2018).
The debris of the simulated 10°-inclination prograde satellite with a stellar mass of **~10⁹M_{Sun}** analysed in Re Fiorentin+2015 are overplotted for comparison (grey diamonds).





Warp is precessing at 10.86 ± 0.03 (statistical) ± 3.20 (systematic) km/s/kpc in the direction of Galactic rotation.

The warp would complete one rotation around the center of the Milky Way in 600 to 700 million years

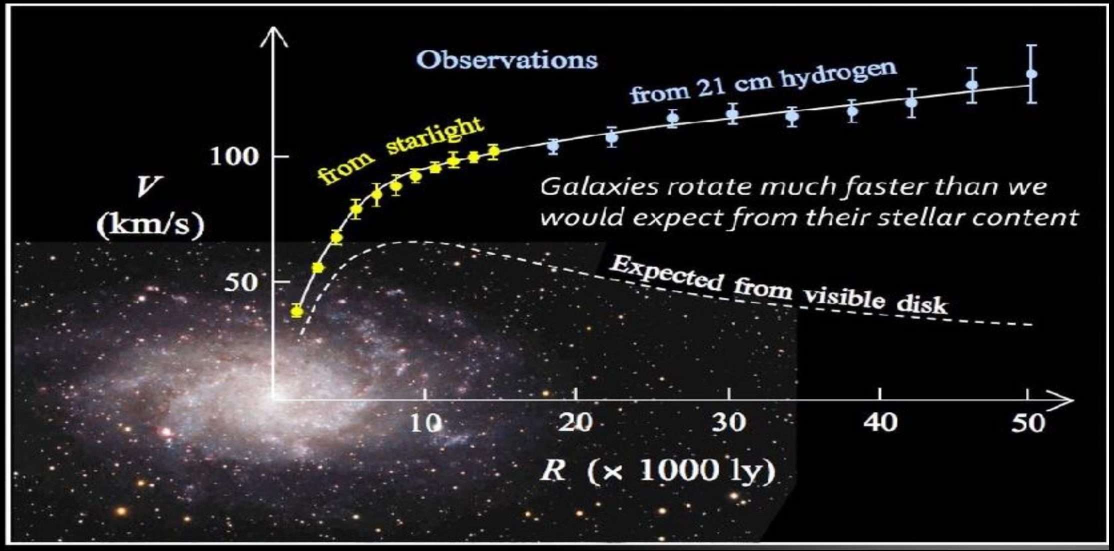
Much faster than expected based on predictions from other models, such as those looking at the effects of the non-spherical halo

The direction and magnitude of the warp's precession rate favor the scenario that the warp is the result of a recent or ongoing encounter with a satellite galaxy, rather than the relic of the ancient assembly history of the Galaxy

Galactic disc: rotation curves

Rotation curves are distinctive features of spiral galaxies like our Milky Way, a sort of a kinematical/dynamical signature, like the HR diagram for the astrophysical content

Flat rotation curves in disk galaxies - a longest outstanding problem in astronomy - provide the main observational support to the **hypothesis of surrounding dark matter.**
Adding a “dark matter” halo **allows a good fit to data**



Stellar kinematics, as tracer of gravitational potential, is the most reliable observable for gauging different matter components

-> the rotation curve of the MW used as a first test for a GR Galaxy

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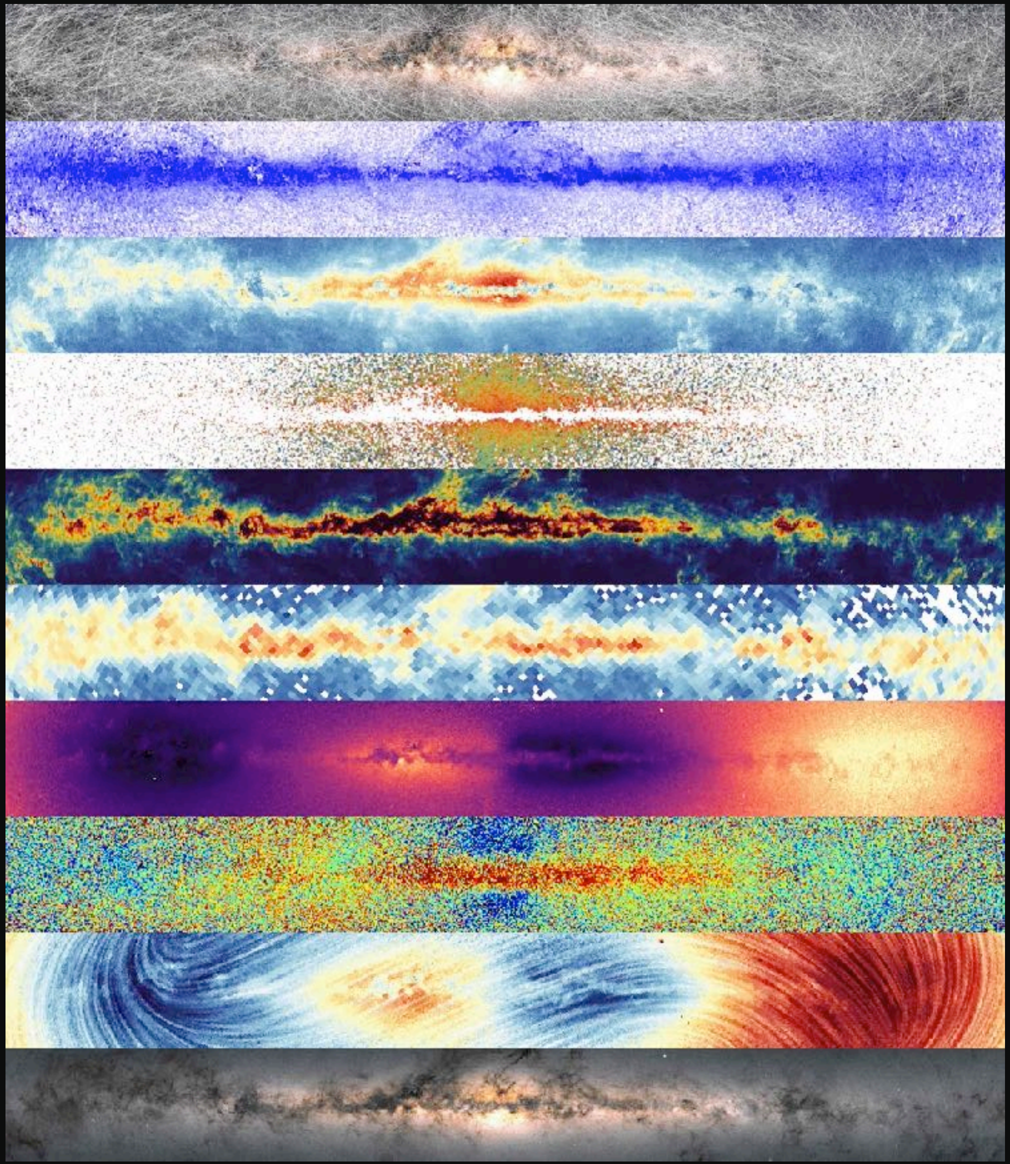


To guarantee Gaia’s scientific outputs, we must rely on General Relativity.

Given that the data analysis and processing follow a GR approach, any subsequent exploitation of the results must remain consistent with the theoretical framework underlying the astrometric model.

A fully relativistic model for the Milky Way (MW) should be pursued!

weak field regime @Milky Way scale

In general one assumes that:
gravitational potential or "relativistic effects" at the MW scale are usually "small", then

✓negligible..

✓locally Newton approximation is retained valid at each point..

but $(v_{Gal}/c)^2 \sim 0{,}69 \times 10^{-6}$ (rad) ~100 mas

$(v_{Gal}/c)^3 \sim 0{,}57 \times 10^{-9}$ (rad) ~ $120\mu as$

the individual astrometric error is $\leq 100\mu as$
throughout most of its magnitude range

"weakly" relativistic effect could be relevant?

Lesson from Gaia: For the Gaia-like observer the weak gravitational regime turns out to be "strong" when one has to perform high accurate measurements

ε ~ v^2/c^2 ~ GM/rc^2 ~ mas accuracy

which requires determination of

g_{oo} even terms in ε, lowest order ε^2~mas

g_{oj} odd terms in ε, lowest order ε^3~μ-as

g_{ij} even terms in ε, lowest order ε^2~mas

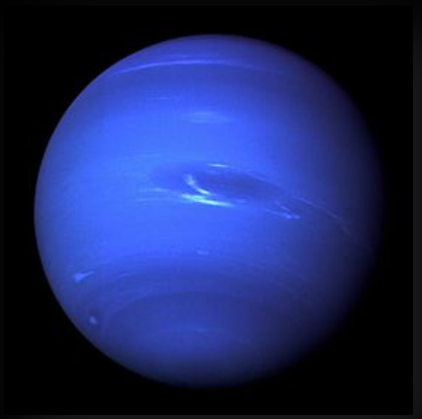
The small curvature limit in General Relativity may not coincide with the Newtonian regime

need to compare the GR model and the classical one

Lesson from the past

- Neptune (as "dark" planet in the orbit of Uranus….a new "Newtonian" planet!

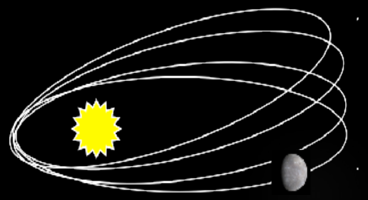
1846 observed by Johann Galle within a degree from the position predicted by Le Verrier



- advancement of Mercury's perihelion: instead of correcting the dynamics by adding a "dark planet" (Vulcano) following the case of Neptune, GR cured the anomalous precession by accounting for the weak non-linear gravitational fields overlapping nearby the Sun.

It amounts to only 43"/century, because of the small curvature, however the effect was "strong" enough to justify a modification of the Newtonian theory

excess of the perihelion shift of Mercury 43"/100yr



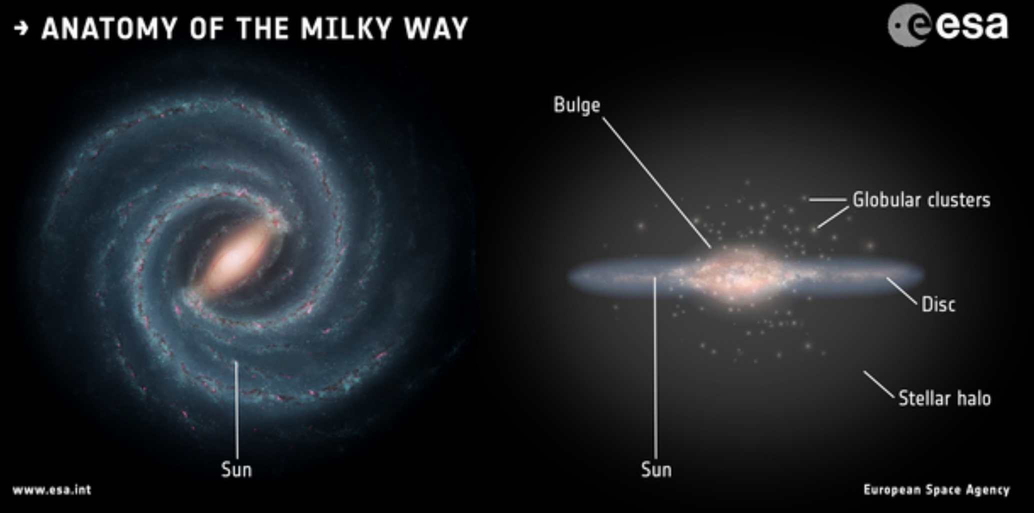
Lense-Thirring effect, the distortion of space-time due to rotating masses: new (weak) relativistic effect!

Lense-Thirring effect

"Classic" Milky Way (MWC) model with NFW dark matter halo

- Newtonian limit applied for Galactic dynamics -> Poisson's equation

$$\nabla^2\Phi_{tot} = 4\pi G(\rho_b + \rho_{td} + \rho_{Td} + \rho_h)$$



1. Plummer bulge

$$\rho_b = \frac{3b_b^2 M_b}{4\pi(r^2 + b_b^2)^{5/2}}$$

Pouliasis, E., Di Matteo, P.

Haywood, M. 2017, A&A, 598, A66

2. Miyamoto-Nagai thin and thick discs

$$\rho_d(R,z) = \frac{M_d b_d^2}{4\pi} \frac{\left[a_d R^2 + (a_d + 3\sqrt{z^2 + b_d^2})(a_d + \sqrt{z^2 + b_d^2})^2\right]}{\left[R^2 + (a_d + \sqrt{z^2 + b_d^2})^2\right]^{5/2} (z^2 + b_d^2)^{3/2}}$$

Bovy, J. 2015, ApJs, 216, 29
Korol, Rossi & Barausse (2019)

3. Navarro-Frank-White DM halo

$$\rho_h(r) = \rho_0^{halo} \frac{1}{(r/A_h)(1 + r/A_h)^2}$$

McMillan, P. J. 2017, MNRAS, 465, 76-94

Navarro, J. F., Frenk, C. S. and White, S. D. M. 1996, ApJ, 462, 563

M_b, M_{td}, M_{Td}, a_{td}, a_{Td}, b_b, bd, ρ_0^{halo} and **A_h** correspond to the bulge mass, the masses and the scale lengths/heights of the thin and thick discs, the halo scale density, and the halo radial scale

$$\nabla^2\Phi_{tot} = 4\pi G(\rho_b + \rho_{td} + \rho_{Td} + \rho_h) \quad \Rightarrow \quad V_c^2 = R\left(d\Phi_{tot}/dR\right)$$

MWC velocity profile

Same baryonic distribution of MWC

MOND

$$g_{MOND} = \eta\left(\frac{g_N}{g_0}\right) g_0$$

gravitational acceleration

g_N conventional Newtonian acceleration, baryonic matter alone

$$\eta\left(\frac{g_N}{g_0}\right) = (1 - e^{-\sqrt{g_N/g_0}})^{-1}$$

interpolation function

setting the transition between the Newtonian and the deep MOND regimes through the acceleration scale g_0

$$g_0 = (1.20 \pm 0.02) 10^{-10} \mathrm{ms}^{-2}$$

acceleration scale

constrained by the observed Radial Acceleration Relation of external galaxies (Lelli et al. 2017)

gravitational acceleration g_{MOND} = centripetal acceleration

$$V_{MOND}(R, V_{bar}) = \frac{V_{bar}}{\sqrt{1 - e^{-V_{bar}/\sqrt{R g_0}}}}$$

EINASTO DENSITY PROFILE

$$\rho_{\mathrm{Einasto}}(r) = \rho_s \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right\}$$

Cold dark matter distribution

parameters of the Einasto profile: ρ_s characteristic density, r_s scale radius, and α shape parameter

$C_{200} \equiv r_{200}/r_s$. **halo concentration** Li et al. (2019)

virial radius r_{200} : the enclosed average density is 200 times the critical density of the Universe (Planck Collaboration et al. 2014; Dutton & Maccio` 2014)

$$V_{200} = 10 C_{200} r_s H_0$$

rotation velocity

$$M_{200} = \frac{V_{200}^3}{10 G^2 H_0^2}$$

enclosed halo mass at the virial radius

GR model for the Milky Way

<u>Einstein equation</u> are very difficult to solve analytically and Galaxy is a multi-structured object making it even the more difficult to detail a metric for the whole Galaxy

in a stationary and axisymmetric space-time there exist two commuting Killing vector fields, **k** (time-like) and **m** (always zero on the axis of symmetry), and a coordinate system adapted to the symmetries whose line element takes the form (Stephani et al.2009, de Felice & Clarke 1990)

Lewis-Weyl-Papapetrou class

$$m^\alpha = \partial^\alpha_\phi,\; k^\alpha = \partial^\alpha_t,\; \partial_t g_{ij} = \partial_\phi g_{ij} = 0,\; g_{\phi a} = g_{ta} = 0,$$

$$-e^{2U} = (k|k),\; -Ae^{2U} = (k|m),\; e^{-2U}W^2 - A^2 e^{2U} = (m|m).$$

$$ds^2 = -\,e^{2U}(dt + Nd\phi)^2 + e^{-2U}\left[e^{\nu}\left(dr^2 + dz^2\right) + r^2 d\phi^2\right]$$

$$T^{\mu\nu} = \rho u^\mu u^\nu \qquad \nabla_\mu(\rho u^\mu) = 0 \qquad u^\mu \nabla_\mu u^\nu = 0$$

Regularity condition, if violated singularities on the axis

$$\lim_{r\to 0}\left[r^{-1}e^{U-\gamma}(e^{-2U}W^2 - e^{2U}A^2)^{1/2}\right] = 1$$

Galactic metric-disc

<u>Stationarity and axisymmetry spacetime</u>

Reflection symmetry (around the galactic plane)

$|z| < r_{in}$

<u>Disc is an equilibrium configuration of a pressure-less rotating perfect fluid</u> (a GR dust)

<u>Masses</u> inside a large portion of the Galaxy <u>interact only gravitationally</u> and <u>reside far from</u> the central bulge region

Stellar encounters become effective below the parsec scale, Galaxy can be considered globally isolated around 25 kpc.

Observer in circular motion

$$u^\alpha = \Gamma\left(k^\alpha + \beta m^\alpha\right)$$ β angular velocity, Γ normalization factor

or

$$u^\alpha = \gamma\left(e^\alpha_{\hat{0}} + \zeta^{\hat{\phi}} e^\alpha_{\hat{\phi}}\right)$$ orthonormal frame adapted to the ZAMO

ZAMO frames = locally non-rotating observers, move on worldlines orthogonal to the hypersurfaces t=constant

(de Felice and Bini, "Classical measurements in curved space-time")

Crosta M., Gammaria M., Lattanzi M. G., Poggio E., (2020)

$$\gamma = M\Gamma$$

γ Lorentz factor

spatial velocity w.r.t the local non-rotating observer $Z^\alpha = (1/M)(\partial_t - M^\phi \partial_\phi)$

$$\zeta^{\hat{\phi}} = \frac{\sqrt{g_{\phi\phi}}}{M}(\beta + M^\phi) = \zeta^{\hat{\phi}}_k + \zeta^{\hat{\phi}}_d$$

$$ds^2 = -M^2 dt^2 + (r^2 - N^2)\left(d\phi + M^\phi dt\right)^2 + e^\nu (dr^2 + dz^2)$$

Geometric terms

β coordinate angular velocity

a suitable foliation of the space time manifold that reflects the assumed symmetries

$$\zeta^{\hat{\phi}}_k = \frac{\sqrt{g_{\phi\phi}}}{M}\beta \qquad \zeta^{\hat{\phi}}_d = \frac{\sqrt{g_{\phi\phi}}}{M} M^\phi$$

- **non local correlation of local time, namely synchronisation of times in different points of space**

Relativistic kinematics, valid regardless the geometry

GR model for the Milky Way disc

the function $N(r,z)$ was solved by Balasin & Grumiller (BG)

with $e^{2U}=1$ -> rigidly rotating dust

$$M = r/\sqrt{(r^2 - N^2)}, \quad M^{\phi} = N/(r^2 - N^2)$$

$$N(r,z) = V_0(R_{out} - r_{in}) + \frac{V_0}{2}\sum_{\pm}\left(\sqrt{(z \pm r_{in})^2 + r^2} - \sqrt{(z \pm R_{out})^2 + r^2}\right)$$

(Balasin and Grummiler, Int.J. Mod. Phys., 2008)

✓Einstein equation allows to treat separately velocities and density

$$r\partial_z \nu + \partial_r N \partial_z N = 0$$

$$2r\partial_r \nu + (\partial_r N)^2 - (\partial_z N)^2 = 0$$

$$2r^2(\partial_r\partial_r \nu + \partial_z\partial_z \nu) + (\partial_r N)^2 + (\partial_z N)^2 = 0$$

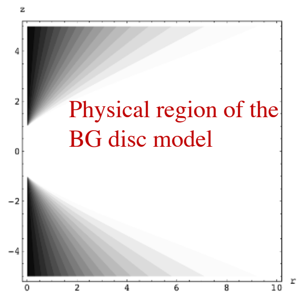
$$r(\partial_r\partial_r N + \partial_z\partial_z N) - \partial_r N = 0$$

$$(\partial_r N)^2 + (\partial_z N)^2 = k r^2 \rho e^{\nu}$$

Einstein field Eq.

physical boundaries: for $r >> N$, far from $r = 0$, and $|z| < r_{in}$

- r_{in} = **bulge size**
- **R_{out} = extension of the MW disk**
- **V_0 = velocity in the flat regime**
- + $e^{\nu(r,z)}$ **conformal factor (new parameter)**



$$\rho(R,z) = e^{-\nu(R,z)}\frac{1}{8\pi R^2}\left[(\partial_R N(R,z))^2 + (\partial_z N(R,z))^2\right]$$

Stationarity and axisymmetry spacetime may include Kerr solution for the bulge as well as different disc solutions

Regions around the bulge and the bar need relativistic hydrodynamics, where equilibrium conditions are not possible

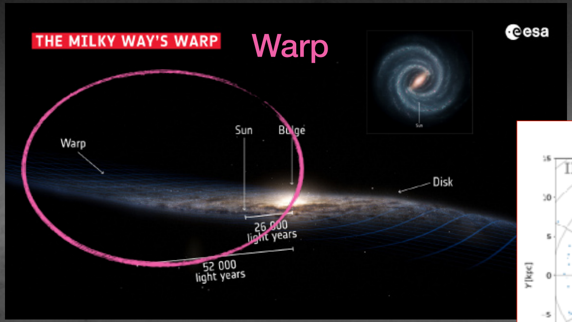
$$\zeta^{\hat{\phi}} = \frac{\sqrt{g_{\phi\phi}}}{M}(\beta + M^{\phi})$$

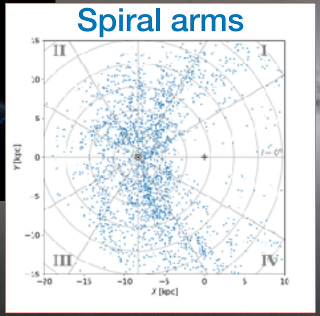
If β is negligible or zero ▶

$$\zeta_d^{\hat{\phi}}(r,z) = N(r,z)/r \propto g_{0\phi}$$

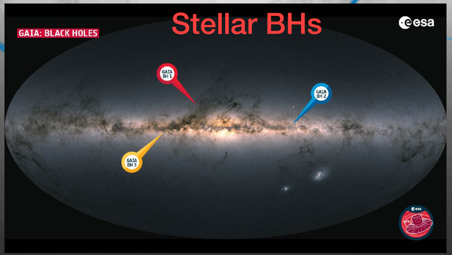
Different from the IAU metric!

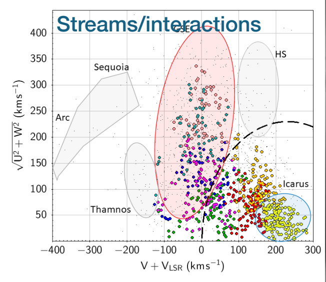
Gravitational dragging working at disc scale?





MW core
Relativistic hydrodynamics for the bulge/bar?





Flaring

Disc

On testing CDM and geometry-driven Milky Way rotation curve models with *Gaia* DR2- Crosta M., Giammaria M., Lattanzi M. G., Poggio E.,MNRAS, Volume 496, Issue 2, August 2020, Pages 2107–2122

The Galaxy is a multistructured object, global solutions are unrealistic

Peering into hidden parts is utmost fundamental to establish boundary matching conditions between internal/external Einstein's solutions

Our ansatz: the flatness of MW rotation curve is geometry driven?

- 3 congruences of observers within our framework:

i) the local barycentric observer tied to the BCRS metric (based on the post-Newtonian approximation to GR) and relativistic modelling for Gaia (based on the measurement protocol in GR involving splitting formalism)

ii) the co-rotating static observer associated with the BG metric in the stationary axisymmetric spacetime

iii) the ZAMO observers, which locally do not rotate with respect to the local geometry

- It is expected that the static observer and the locally barycentric observer at infinity coincide. However, the BCRS is connected to a quasi-inertial rather than inertial system. Therefore, our ansatz could turn into verifying whether asymptotically these observers can indeed coincide.

- In this context, the ZAMOs are employed as gauges of a potential dragging. The local barycentric observer aligns at infinity with the congruence of curves that are orthonormal, vorticity-free and expansion-free -> the threading and slicing point of views coincide

- With static dust, this relative spatial velocity inherently reflects the angular velocity attributed solely to the gravitational dragging effects within the BG spacetime -> our assumption is to compare this rotational velocity with the observed rotation curve measured by Gaia, i.e. with respect to an observer at rest with the distant quasars

Data sample: full reconstruction of disc kinematics based on Gaia data only

i. **Complete Gaia astrometric dataset** ($\alpha, \delta, \mu_\alpha, \mu_\delta$, parallax) and corresponding covariance matrix

ii. **Three Gaia photometric bands (G, BP, RP)** all available and RUWE < 1.4 [to discard sources with problematic astrometric solutions, astrometric binaries, and other anomalous cases]

iii. **Parallaxes good to 20%** (i.e. parallax_over_error ≥ 5) [parallaxes to better than 20% allow to deal with similar (quasi–gaussian) statistics when transforming to distances]

iv. **Gaia-measured** velocity along the line of sight, i.e. **radial velocity, with better than 20% uncertainties**

i.+ii.+iii.+iv—> **proper 6D reconstruction of the phase-space location occupied by each individual star as derived by the same observer**

1. Full transformation (including complete error propagation) from the ICRS equatorial to heliocentric galactic coordinates
2. translation to the galactic center

-> independency from the local standard of rest

angular-momentum sustained stellar population of the Milky Way that better traces its observed RC

DR2: **very homogenous sample of 5277 early type stars and 325 classical type I Cepheids.**

DR3: **a much larger sample of high-quality astrometric and spectro-photometric data of unprecedented homogeneity of**
719143 young disc stars within |z| < 1 kpc and up to R = 19 kpc
241’918 OBA stars, 475’520 RGB giants, and 1’705 Cepheides
radial cut at 4.5 kpc to avoid the bar influence

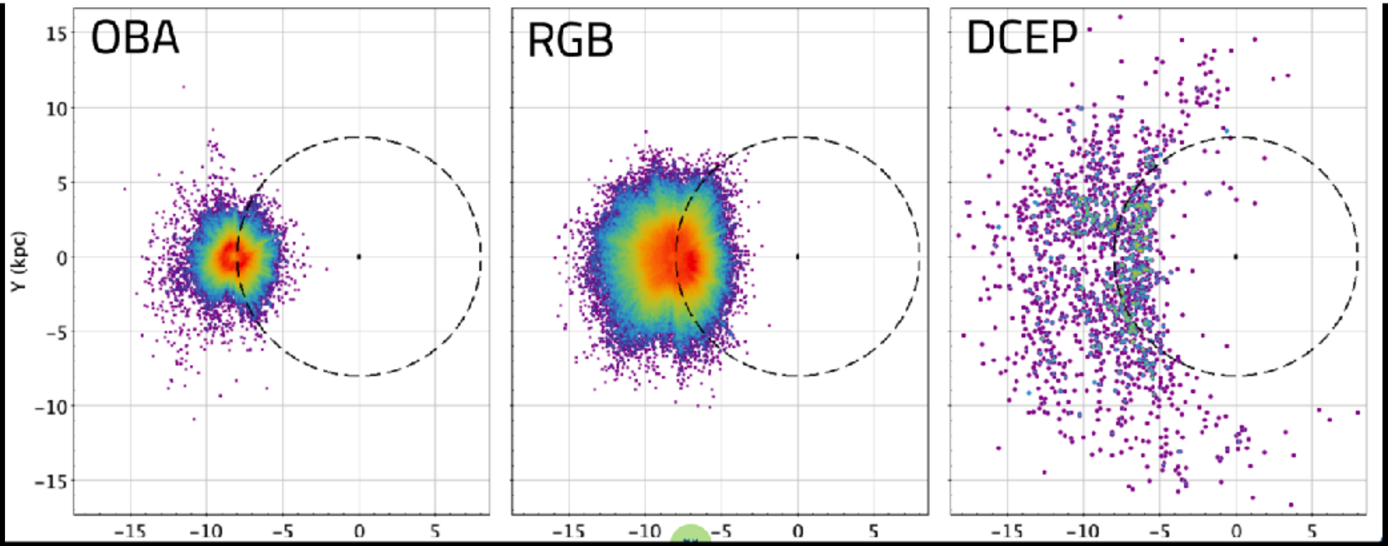
v. Cross-matched entry in the 2MASS catalogue for the actual characterization of the sample in case of DR2 and EDR3

Ref: On testing CDM and geometry-driven Milky Way rotation curve models with *Gaia* DR2- Crosta M., Giammaria M., Lattanzi M. G., Poggio E.,MNRAS, Volume 496, Issue 2, August 2020, Pages 2107–2122

Gaia DR3 disc tracers

From ~33 million stars with high-precision astrometry and spectroscopic LOS velocities, we focus on three disc populations, namely:

O-,B-,A-type stars	Red Giants	Classical Cepheids
6D phase space: 5 astrometric parameters + spectroscopic radial velocities		
Trigonometric distances (parallax error < 20%)		Photometric distances
Kinematic selection: thin disk stars with low velocity dispersion, to minimize possible halo contaminants $\sqrt{V_R^2 + (V_\phi - V_\phi^{LSR})^2 + V_z^2} < 180\ \mathrm{km\ s^{-1}}$	Disc-like kinematics and nearly-circular orbits (eccentricity < 0.1) $\lvert V_R \rvert < 50\ \mathrm{km\ s^{-1}}$ $\lvert V_z \rvert < 30\ \mathrm{km\ s^{-1}}$ and $100 < V_\phi < 350\ \mathrm{km\ s^{-1}}$	
Close to the galactic plane and far from the bar: $\lvert z \rvert < 1\ kpc$ and $R > 4.5\ kpc$		



Spatial distribution for the three samples of tracers. OBA stars, RGB giants with (quasi) circular orbits, DCEP in the Galactic plane. The position of the Galactic centre is shown by the black dot on the right; the dashed line represents a Galactocentric circle passing through the Sun's position at (x,y)=(-8.249 kpc, 0 kpc).

To avoid the influence of the MW bar a radial cut at 4.5 kpc is set, while halo stars are further discarded requiring IzI<1 kpc. The final sample comprises 719'143 stars including 241'918 OBA, 475'520 RGB and 1'705 DCEP.

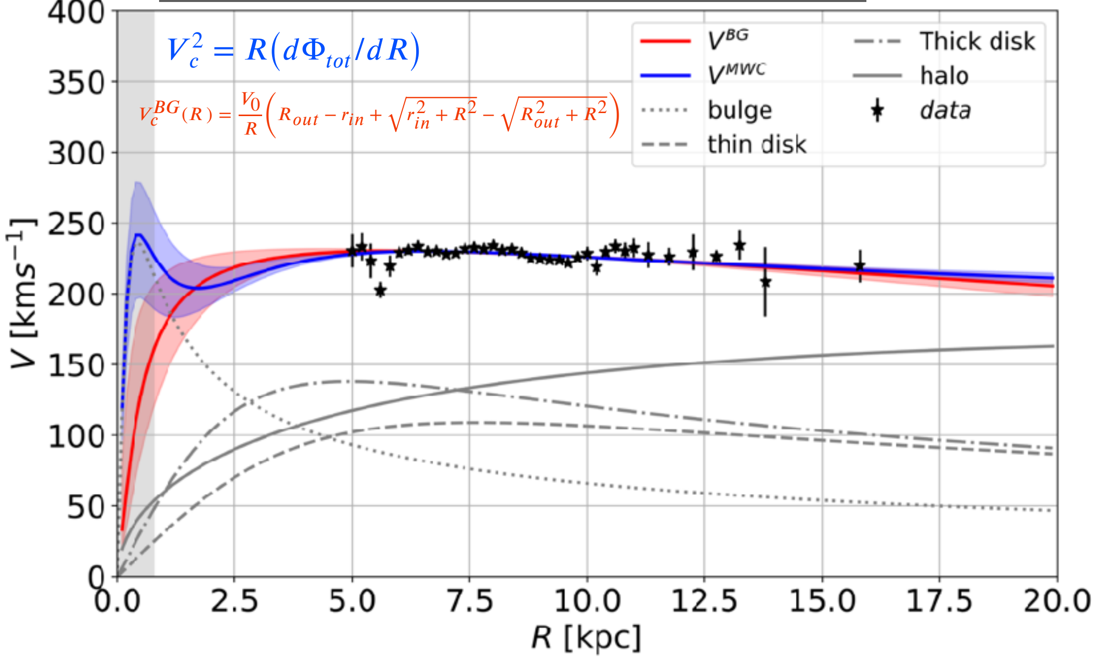
MCMC fit with DR2 data

Best fit estimates as the median of the posteriors and their 1σ level credible interval

BG model	θ	σ_θ^-	σ_θ^+
r_{in} [kpc]	0.39	-0.25	+0.36
R_{out} [kpc]	47.87	-14.80	+23.96
V_0 [km/s]	263.10	-16.44	+25.93
$e^{-\nu}$ [$\cdot 10^{-7}$]	3.59	-0.47	+0.65

MWC model	θ	σ_θ^-	σ_θ^+
$M_b [10^{10} M_\odot]$	1.0	-0.4	+0.4
$M_{td} [10^{10} M_\odot]$	3.9	-0.4	+0.4
$M_{Td} [10^{10} M_\odot]$	4.0	-0.5	+0.5
a_{td} [kpc]	5.2	-0.5	+0.5
a_{Td} [kpc]	2.7	-0.4	+0.4
$\rho_0^{halo} [M_\odot pc^{-3}]$	0.009	-0.003	+0.004
A_h [kpc]	17	-3	+4

Velocity profiles - Classical (MWC) and GR (BG) RC



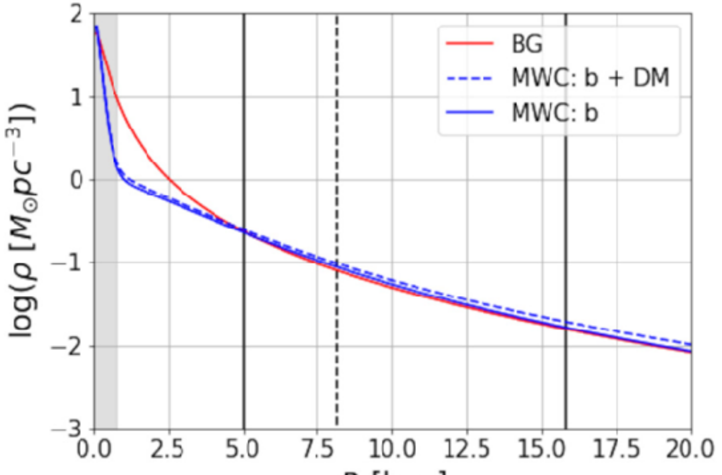
Stars = dust grains in axysimmetric and stationary spacetime (circular motion)

5277 **early-type stars** e **325 classical type I Cepheides**

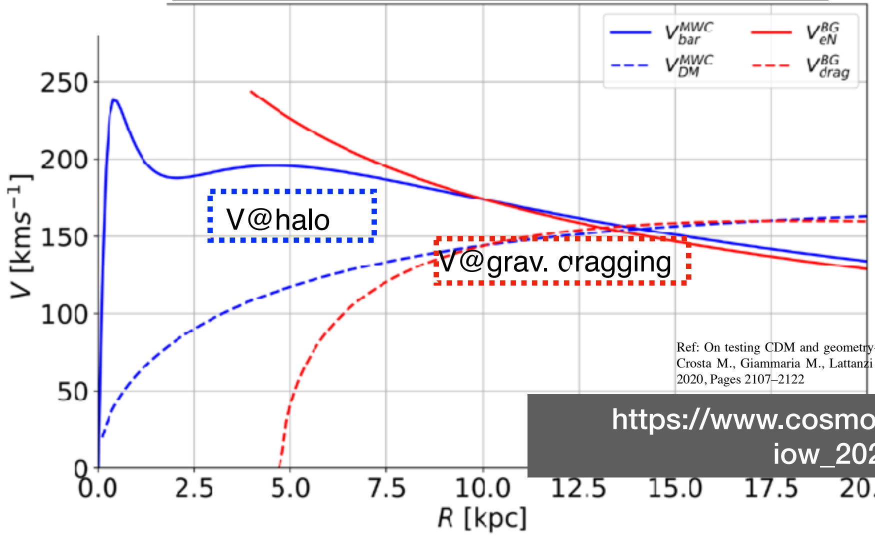
parallax/sigma_parallax > 5

RV/sigma_RV > 5 dalla Gaia DR2

barionic density profiles: relativistic (red)/classic (blu)



Dragging effect vs. halo effect

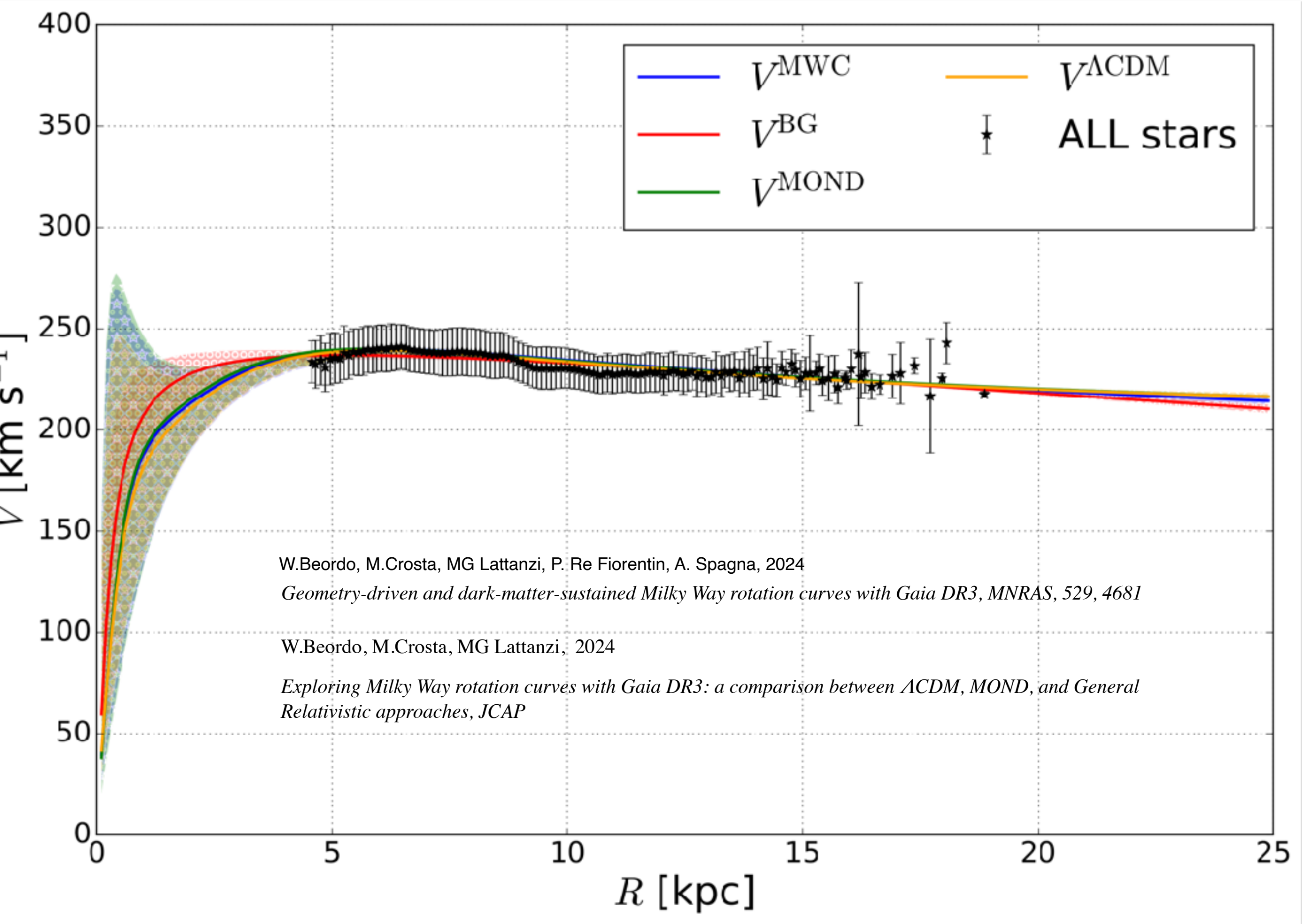


Ref: On testing CDM and geometry-driven Milky Way rotation curve models with *Gaia* DR2- Crosta M., Giammaria M., Lattanzi M. G., Poggio E.,MNRAS, Volume 496, Issue 2, August 2020, Pages 2107–2122

https://www.cosmos.esa.int/web/gaia/iow_20200716)

MCMC fit to the Gaia DR3 data - Classical (MWC) MOND, EINASTO GR
I.Results: azimuthal velocity profile of the MW

The red, blue, green, and yellow curves show the best-fitting to the BG, MWC, MOND, and ΛCDM models, respectively



W.Beordo, M.Crosta, MG Lattanzi, P. Re Fiorentin, A. Spagna, 2024
Geometry-driven and dark-matter-sustained Milky Way rotation curves with Gaia DR3, MNRAS, 529, 4681

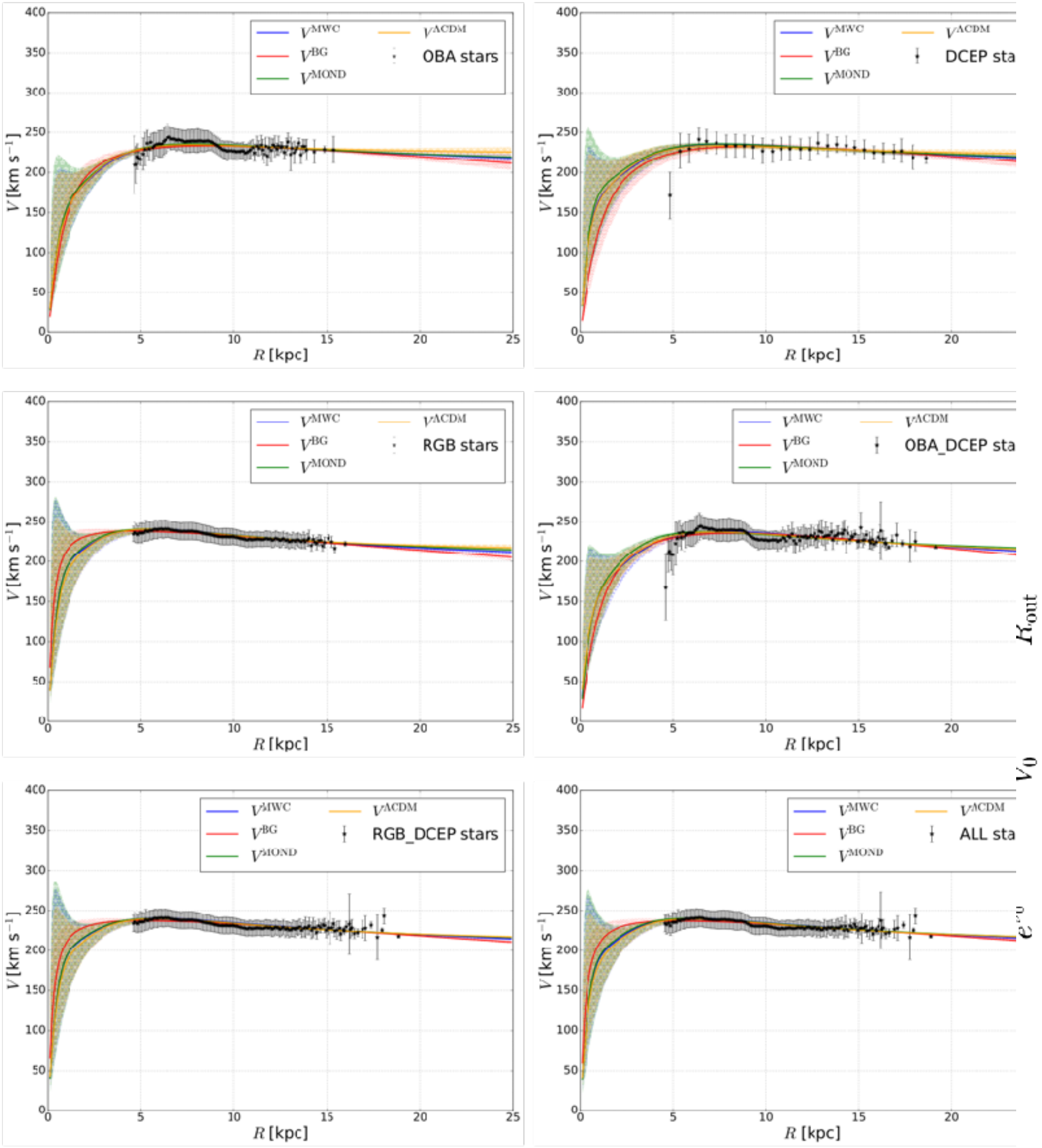
W.Beordo, M.Crosta, MG Lattanzi, 2024
Exploring Milky Way rotation curves with Gaia DR3: a comparison between ΛCDM, MOND, and General Relativistic approaches, JCAP

The four models are found to be statistically equivalent

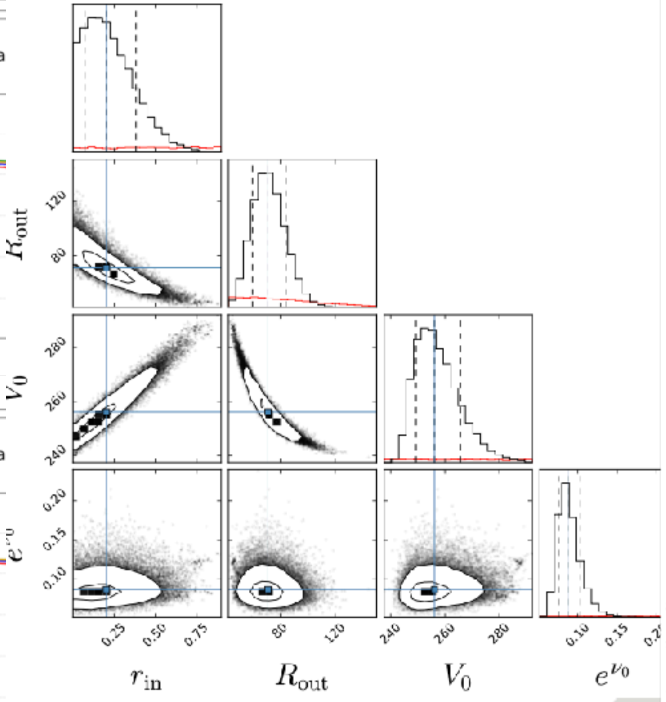
The four velocity profiles are all good representations of the observed (binned) data.

- Black starred symbols represent the median azimuthal velocity at the median distance from the galactic centre of the stellar population within each of the radial bins
- Robust Scatter Estimate (RSE) adopted as a robust measure of the azimuthal velocity dispersion of the population in each radial bin
- The filled areas represent the 68 per cent reliability intervals of each rotation curve
- For R ≲ 4.5 kpc both the classical and the relativistic curves are very uncertain because of the lack of data in that region

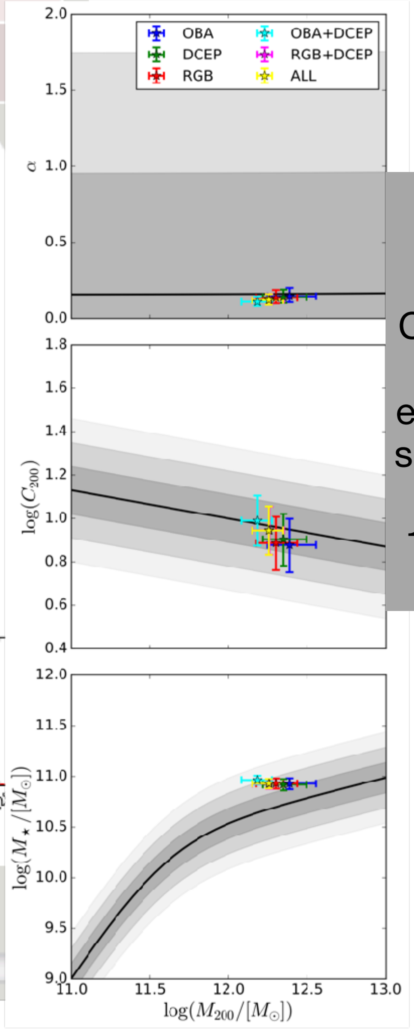
MCMC fit to the Gaia DR3 data - Classical (MWC) and GR (BG) RC- velocity profile for each sample



BG model	
r_0 [kpc]	$0.20^{+0.18}_{-0.13}$
R [kpc]	71^{+13}_{-11}
V_0 [km/s]	256^{+10}_{-7}



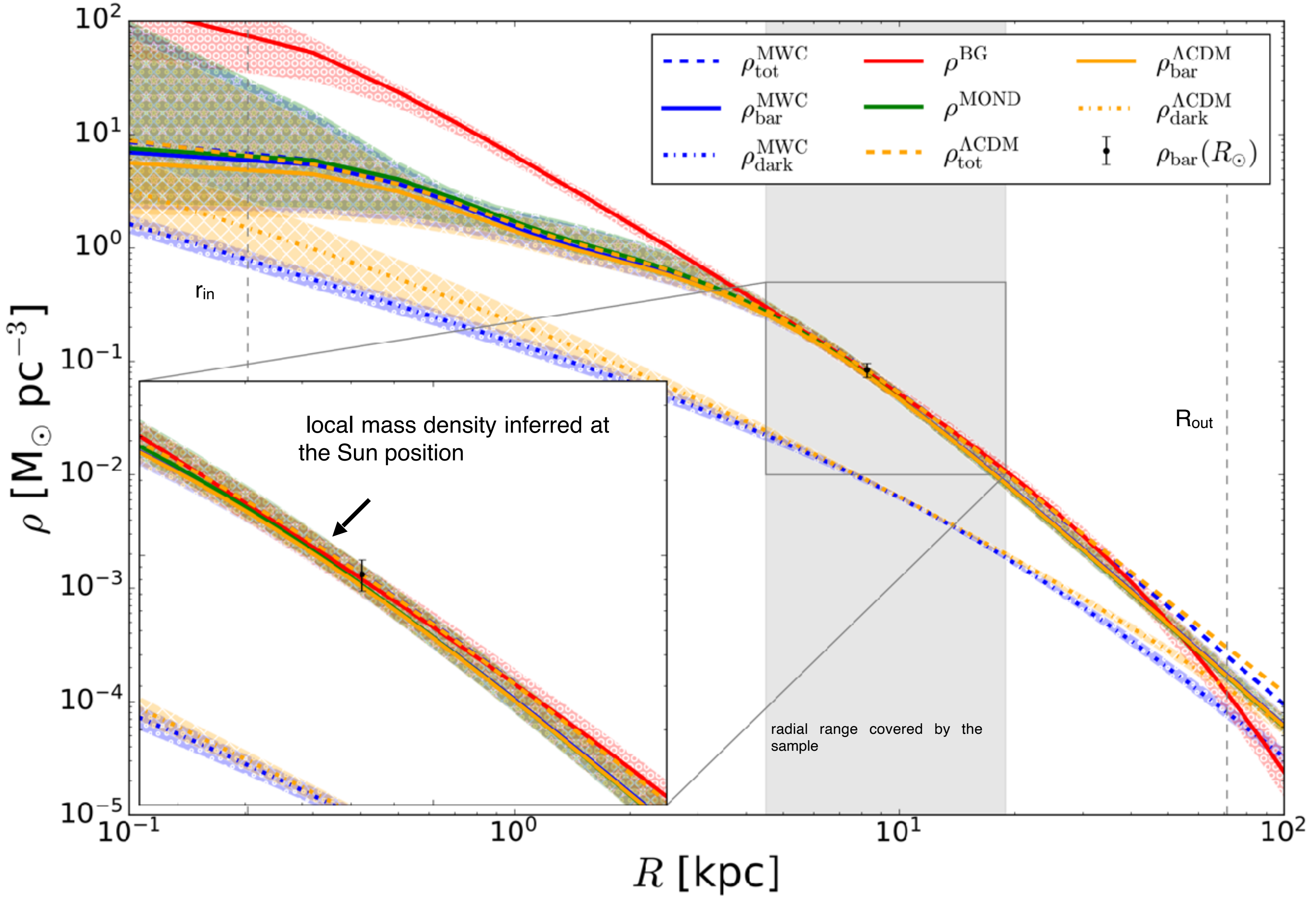
best-fit estimates
the medians of the posteriors
and their 1σ credible intervals



Black lines ΛCDM cosmological priors.

Coloured data points are the best parameter estimates for each stellar sample. The shaded grey regions represent the 1-,2-,3-σ ranges around the mean relations.

MCMC fit to the Gaia DR3 data - II. Results: radial density profile of the MW at z=0



solid lines baryonic matter contributions

BG and MOND density profiles are consistent with both the baryonic and total density profiles of MWC

MWC and ΛCDM total matter density profiles (dashed lines) are almost coincident while departing from each other only at very large radii

Einasto profile of the ΛCDM model results larger than the NFW one both in the inner and outer parts of the Galaxy (dash-dotted lines)

baryonic matter density observed at the Sun

$\varrho_{bar}(R_\odot) = 0.084 \pm 0.012\ M_\odot pc^{-3}$

estimates of the local baryonic density $\varrho^{\Lambda CDM}$ and ϱ^{MOND} around $0.080 M_\odot pc^{-3}$

Crosta et. al, 2020, Beordo et al. 2024, Garbari et al. 2012; Bienaymé et al. 2014; McKee et al. 2015

$$\log \mathcal{L} = -\frac{1}{2}\sum_i \left(\frac{\left[V_\phi(R_i) - V_\phi^{\exp}(R_i|\theta)\right]^2}{\sigma^2_{V_\phi}} + \log\left(\sigma^2_{V_\phi}\right) \right) - \frac{1}{2}\left(\frac{\left[\rho(R_\odot) - \rho^{\exp}(R_\odot|\theta)\right]^2}{\sigma^2_{\rho_\odot}} + \log\left(\sigma^2_{\rho_\odot}\right) \right),$$

$$\rho(R,z) = e^{-\nu(R,z)} \frac{1}{8\pi R^2}\left[\left(\partial_R N(R,z)\right)^2 + \left(\partial_z N(R,z)\right)^2\right]$$

BG model	
e^{ν_0}	0.09 ± 0.01

Only one reliable estimate for the baryonic matter density

$$\frac{1}{r^2}\left(N_r^2 + N_z^2\right) = k\rho e^{\nu}$$

We can constrain $e^{\nu(r,z)}$ only at the Sun (e^{ν_0})

Komar integral: $$M = -2\int (T_0^0 - \frac{1}{2}T)\sqrt{-g}\; d^3x,$$

$M_{\rm bar}^{\rm MWC}$ [10^{10} $M_\odot$] ~ 1.48

$M^{\rm BG}$ [10^{10} $M_\odot$] ~ 1.54

MCMC fit to the Gaia DR3 data - III. Results: Total mass estimates

Density profile in agreement between all four models within the region of validity of BG

GR mass in agreement with the baryonic mass within the region of validity of the model

The **total** baryonic mass predicted by the ΛCDM scenario is slightly smaller than the values expected for the MWC and MOND models

The dynamical mass is supplied by more dark matter in the ΛCDM scenario compared to a NWF halo (with no cosmological constraints)

-> the values are an order of magnitude larger than what proposed recently in the literature (Keplerian fall-off of the RC)

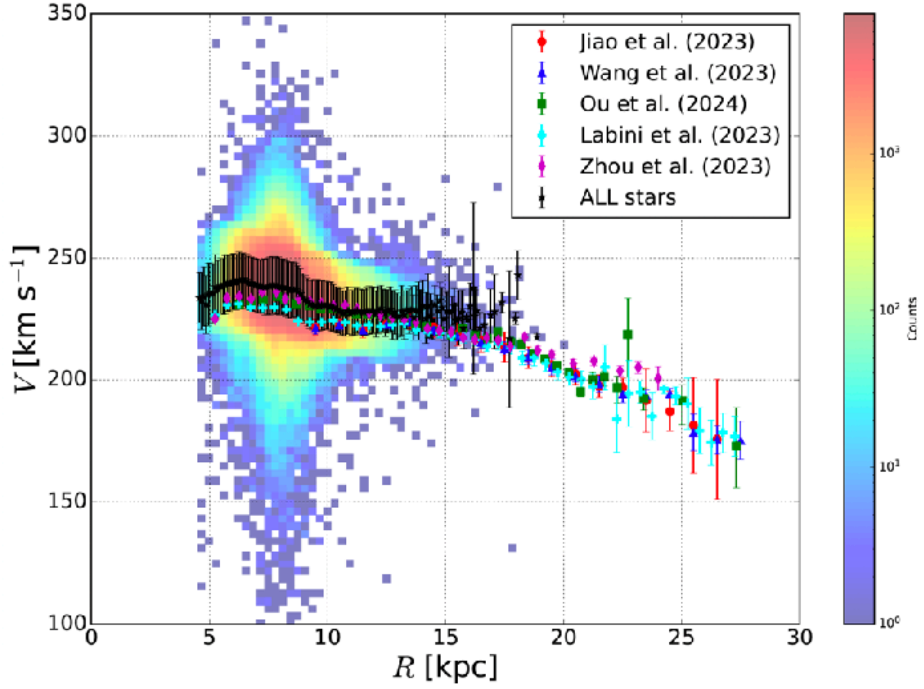
Quantity	OBA	DCEP	RGB	OBA + DCEP	RGB + DCEP	ALL
$\rho^{\rm MWC}_{\rm bar,\odot}$ [$\rm M_\odot\,pc^{-3}$]	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$
$\rho^{\rm MWC}_{\rm h,\odot}$ [$\rm M_\odot\,pc^{-3}$]	$0.0092^{+0.0009}_{-0.0009}$	$0.0092^{+0.0009}_{-0.0009}$	$0.0084^{+0.0007}_{-0.0007}$	$0.0083^{+0.0007}_{-0.0007}$	$0.0088^{+0.0006}_{-0.0007}$	$0.0088^{+0.0006}_{-0.0007}$
$\rho^{\rm BG}_{\odot}$ [$\rm M_\odot\,pc^{-3}$]	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.013}_{-0.012}$	$0.080^{+0.013}_{-0.012}$	$0.081^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$
$M^{\rm MWC}_{\rm bar}$ [$10^{10}\,\rm M_\odot$]	~1.62	~1.83	~1.25	~1.96	~1.36	~1.48
$M^{\rm BG}$ [$10^{10}\,\rm M_\odot$]	~1.81	~2.39	~1.11	~2.37	~1.39	~1.54
$M^{\rm MWC}_{\star}$ [$10^{10}\,\rm M_\odot$]	$9.24^{+1.07}_{-1.01}$	$9.30^{+1.12}_{-1.10}$	$9.35^{+0.95}_{-0.93}$	$10.15^{+0.99}_{-0.95}$	$9.22^{+0.94}_{-0.91}$	$9.27^{+0.90}_{-0.95}$
$M^{\rm MWC}_{\rm vir}$ [$10^{10}\,\rm M_\odot$]	~114	~109	~103	~85	~105	~103
$R^{\rm MWC}_{\rm vir}$ [kpc]	~222	~218	~214	~201	~216	~215

(0.34 ± 0.02) GeV cm^{-3}

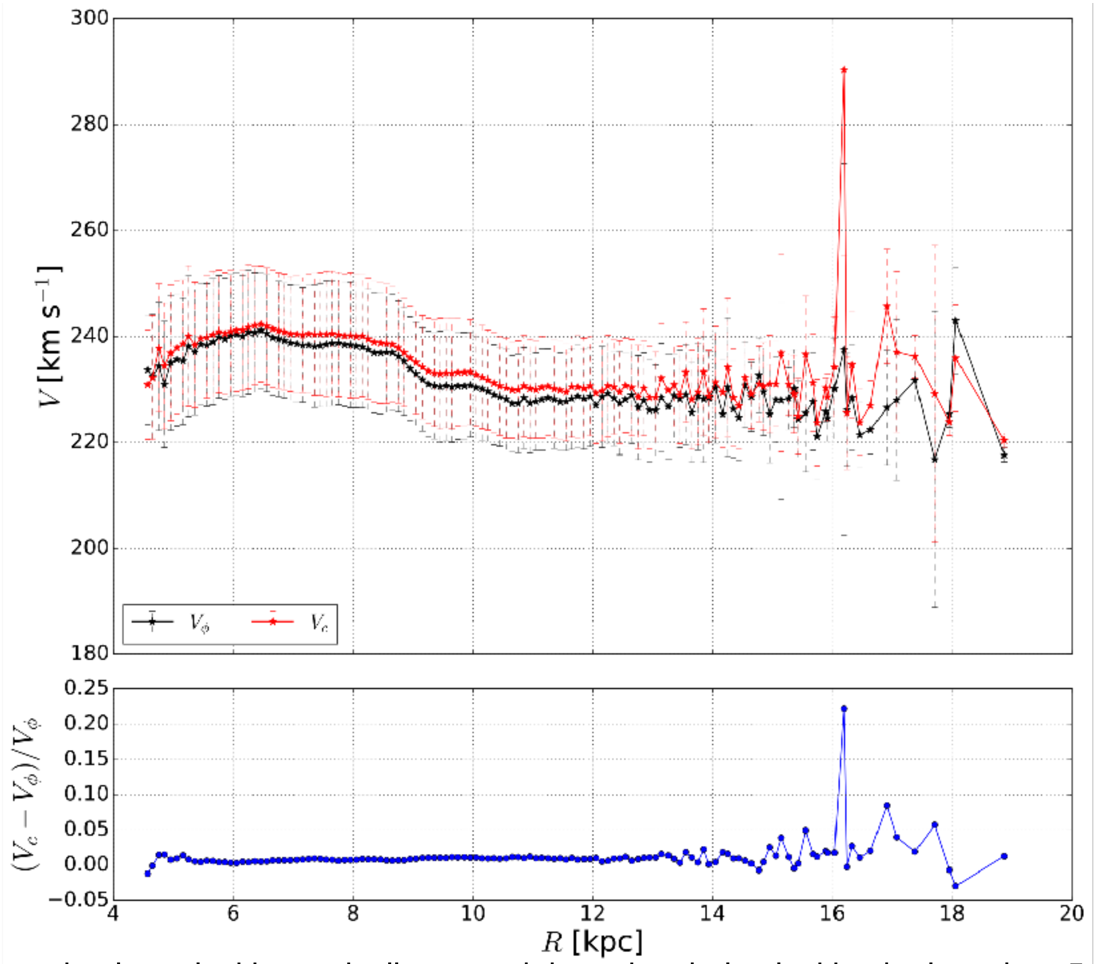
Quantity	OBA	DCEP	RGB	OBA + DCEP	RGB + DCEP	ALL
$\rho^{\Lambda\rm CDM}_{\rm bar,\odot}$ [$\rm M_\odot pc^{-3}$]	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$
$\rho^{\Lambda\rm CDM}_{\rm h,\odot}$ [$\rm M_\odot pc^{-3}$]	$0.0099^{+0.0008}_{-0.0008}$	$0.0098^{+0.0008}_{-0.0008}$	$0.0088^{+0.0078}_{-0.0007}$	$0.0085^{+0.0006}_{-0.0006}$	$0.0091^{+0.0006}_{-0.0006}$	$0.0090^{+0.0006}_{-0.0006}$
$\rho^{\rm MOND}_{\odot}$ [$\rm M_\odot pc^{-3}$]	$0.080^{+0.012}_{-0.012}$	$0.080^{+0.012}_{-0.012}$	$0.081^{+0.012}_{-0.012}$	$0.081^{+0.012}_{-0.012}$	$0.081^{+0.012}_{-0.012}$	$0.081^{+0.012}_{-0.012}$
$M^{\Lambda\rm CDM}_{\rm bar}$ [$10^{10}\,\rm M_\odot$]	$8.49^{+1.10}_{-1.03}$	$8.31^{+1.11}_{-1.04}$	$8.51^{+1.06}_{-0.95}$	$9.23^{+0.95}_{-0.93}$	$8.43^{+0.96}_{-0.91}$	$8.49^{+0.96}_{-0.89}$
$M^{\rm MOND}$ [$10^{10}\,\rm M_\odot$]	$10.12^{+0.33}_{-0.30}$	$10.05^{+0.48}_{-0.45}$	$9.32^{+0.27}_{-0.24}$	$9.59^{+0.19}_{-0.18}$	$9.59^{+0.21}_{-0.19}$	$9.56^{+0.21}_{-0.19}$
$M^{\Lambda\rm CDM}_{200}$ [$10^{12}\,\rm M_\odot$]	$2.45^{+1.16}_{-0.68}$	$2.24^{+0.88}_{-0.59}$	$1.99^{+0.74}_{-0.49}$	$1.53^{+0.35}_{-0.33}$	$1.82^{+0.44}_{-0.40}$	$1.80^{+0.43}_{-0.39}$

Within the overlapping range of 10–18 kpc, our rotation curves exhibit slightly declining profiles, aligning with recent findings that indicate a pronounced decline only beyond 18–19 kpc

The Jeans analysis on our selected sample shows a further slight increase within error bars suggesting that the lack of the Jeans analysis in our procedure is unlikely to be the cause of the discrepancy observed at around 15 kpc.

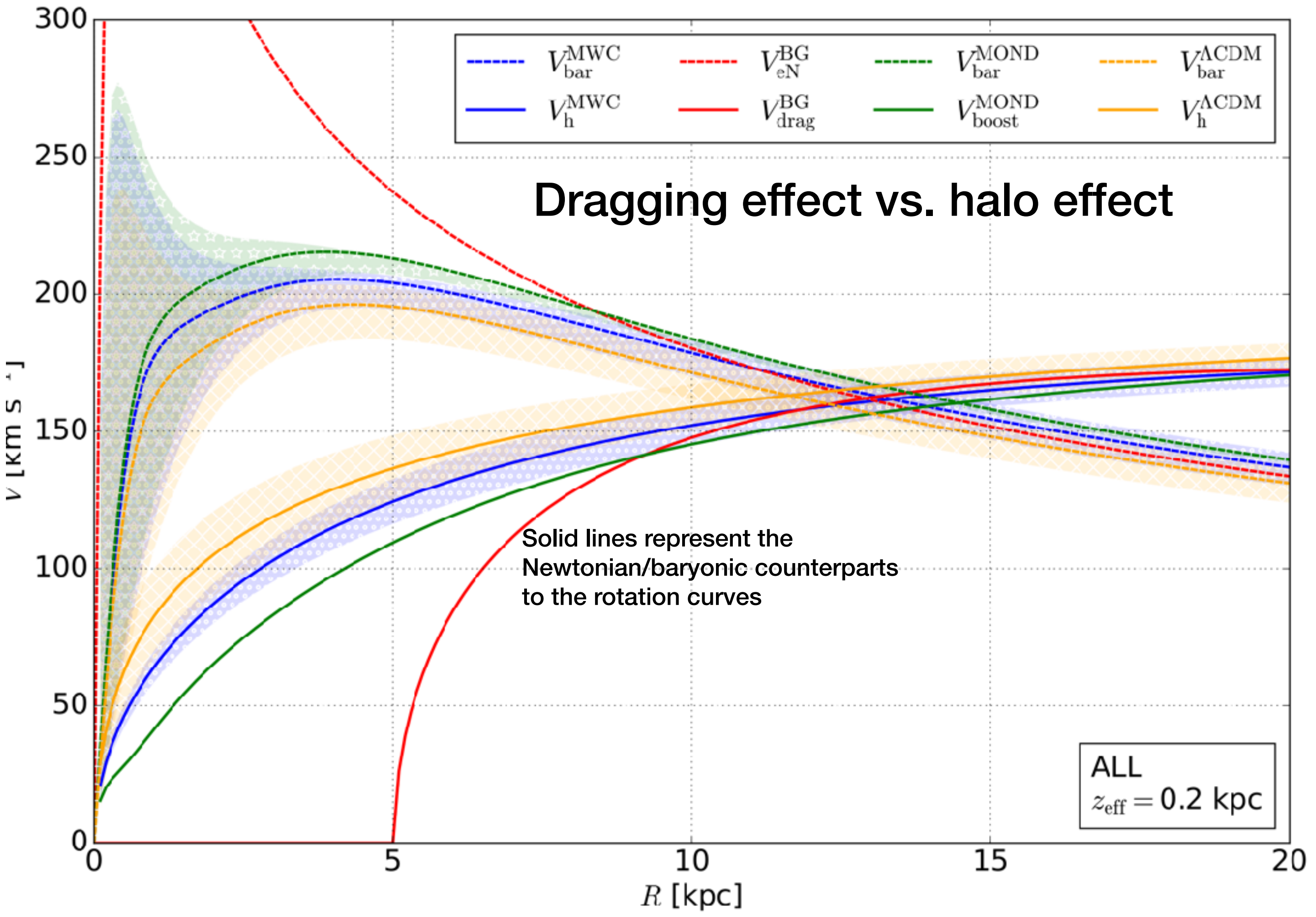


Instead of employing various techniques to extend the measured rotation curve to 30 kpc, we imposed a stringent requirement of errors on parallaxes smaller than 20%.



The resulting circular velocities typically exceed the azimuthal velocities by less than 5 per cent and fall well within the error bars- The eccentricity selection for the orbits of RGB stars removes the effects of the asymmetric drift to match the OBA and DCEP rotation curves

MCMC fit to the Gaia DR3 data - IV.Results: Non-Newtonian contributions vs dark matter halo



Non-Newtonian contributions to the rotation curve are consistent with that of the dark matter halo: they become predominant over the classical baryonic counterpart from 10-15 kpc outwards and, at the Sun distance, they are responsible for the 30-37% of the velocity profile.

The relativistic dragging effect has no newtonian counterpart, thus we compared:

(i) the MWC baryonic-only contribution with the effective Newtonian profile (Binney & Tremaine 1988) calculated by using the BG density: $\nabla^2 \Phi_{eN} = 4\pi G \rho_{BG}$

(ii) the MWC dark matter-only contribution (halo) with the "dragging curve" traced by subtracting *effective Newtonian profile* to *V*$_{BG}$.

$$\sum_{i=1}^{N} \frac{[V_{bar}^{MWC}(r_i) - V_{eN}^{BG}(r_i; z_k)]^2}{N} \quad |z_k| < r_0$$

$$\longrightarrow \; |z_{eff}| = 0.20\ kpc = r_{in}$$

$$(V_{drag}^{BG}(R_i; |z|_{eff})) = \sqrt{(V^{BG}(R))^2 - (V_{eN}^{BG}(R; |z|_{eff}))^2}$$

$$V_{\rm drag}^{\rm BG} = \sqrt{V_{\rm BG}^2 - V_{\rm eN}^2}$$

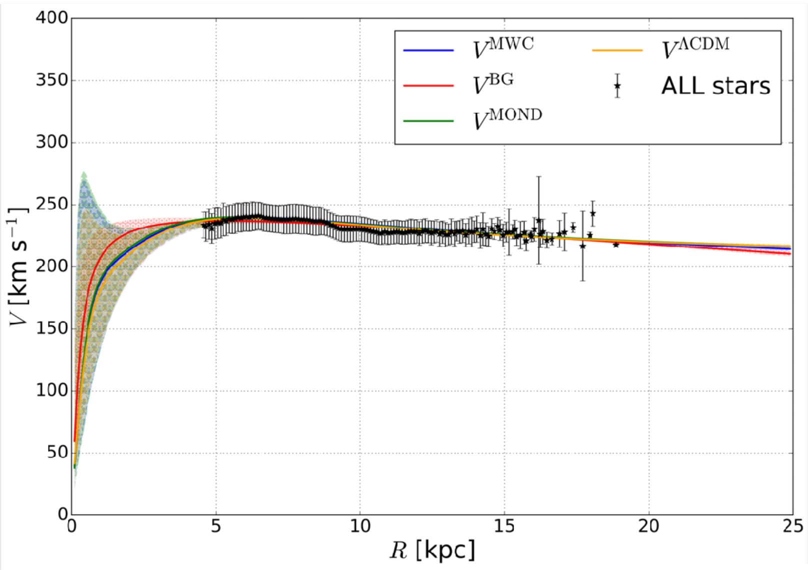
amount of rotational velocity across the MW plane due to gravitational dragging

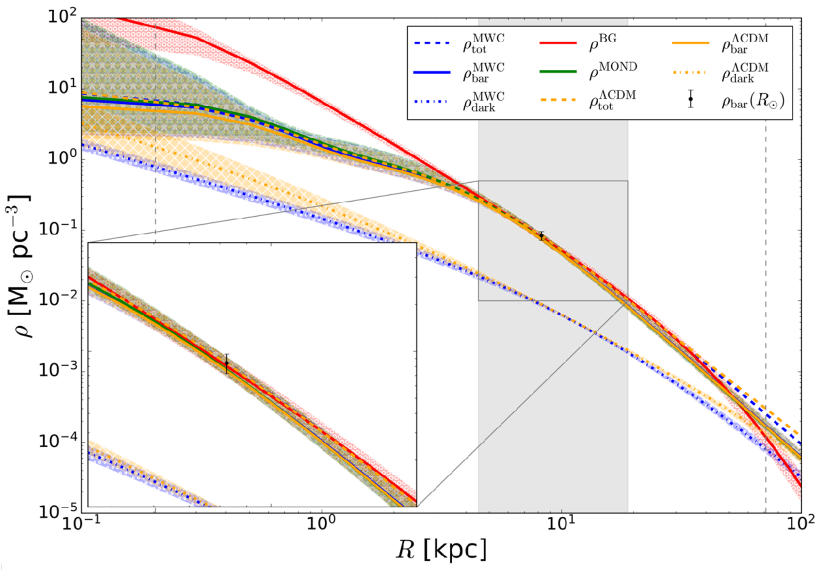
$$V_{\rm boost}^{\rm MOND} = \sqrt{V_{\rm MOND}^2 - V_{\rm bar}^2} = V_{\rm bar}\sqrt{\eta(R, V_{\rm bar}) - 1}$$

pure Mondian boost

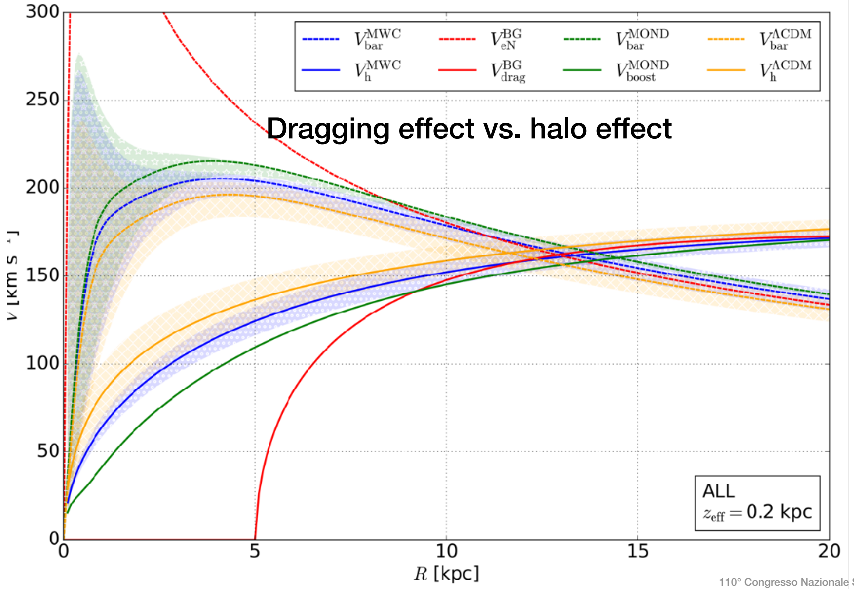
MCMC fit to the Gaia DR3 data - Classical (MWC) MOND EINASTO GR

Best fit estimates as the median of the posteriors and their 1σ level credible interval





The four models are found to be statistically equivalent



110° Congresso Nazionale SIF, 9- 13Sept. 2024 , Crosta

All this again favourably points to the fact that a gravitational dragging-like effect could sustain a flat rotation curve

Crosta M., Giammaria M., Lattanzi M. G., Poggio E.,
On testing CDM and geometry-driven Milky Way rotation curve models with Gaia DR2, MNRAS, Volume 496, Issue 2, 2020

W.Beordo, M.Crosta, MG Lattanzi, P. Re Fiorentin, A. Spagna, 2024
Geometry-driven and dark-matter-sustained Milky Way rotation curves with Gaia DR3, MNRAS, 529, 4681

Exploring Milky Way rotation curves with Gaia DR3: a comparison between ΛCDM, MOND, and General Relativistic approaches, JCAP 2024

Talk outline

Background

- Motivation: Astrometry from Hipparcus to Einstein, Gaia
- Relativistic/Gravitational Astrometry

Challenging the Galactic Models with Milky Way stars

- Local cosmology as ΛDCM laboratory
- Testing General Relativity/Gravity @MilkyWay scale
- The Dark Matter interpretation in GR

Wrap-up

Data are independent from the theoretical models that we use for the predictions and that is exactly why they constitute the testing ground

For our likelihood analysis the four models appear almost identically consistent with the data

- GR model has only 4 parameters, the classical model needs at least 10 parameters +1 for MOND , +3 for Lambda CDM

- **DM does not absorb or emit light but it exerts and responds only to the gravity force; it enters the calculation as extra mass (halo) required to justify the flat galactic rotational curves**

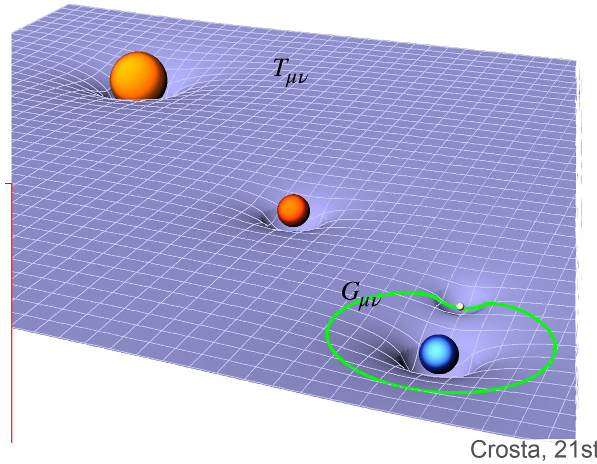
- **MOND requires an adjustment in the low acceleration regime**

- **Einasto ΛCDM** model results larger than the NFW one, dynamical mass supplied by more dark matter in the ΛCDM scenario compared to the case of an NFW halo with no cosmological constraints

- **GR** **could imply a gravitational dragging "DM-like" effect driving the Galaxy velocity rotation curve, i.e. the geometry - unseen but perceived as manifestation of gravity according to Einstein's equation - is responsible of the flatness at large Galactic radii**

Our interpretation with Gaia DR2/DR3 **depends only on the background geometry**

"space tells mass how to move"



GR is the standard theory of gravity

Hypotehsis non fingo&Occam's razor

By setting a coherent GR framework, we are pursuing to:

- ✓ Treat separately velocities and density with Einstein's equations [contrary to what is done in classical models]
- ✓ Analyze potential modifications or extensions of General Relativity (GR) using GR itself as the fundamental background theory.
- ✓ Establish to what extent the MW structure is dictated by the standard theory of gravity [avoiding replica of the common assumption that invalidate GR, i.e the GR effects are small in the linear approximation] or, conversely, determine the conditions under which GR might fail, necessitating Newtonian or alternative dynamics.
- ✓ Identify further exact solutions to Einstein's equations. At Galactic scale MW dynamics can be dominated, e.g., by Weyl, Lewis-Papapetrou spacetimes at large scale, whereas the Newtonian approximation is valid locally (e.g in the Solar System, binary systems, …)

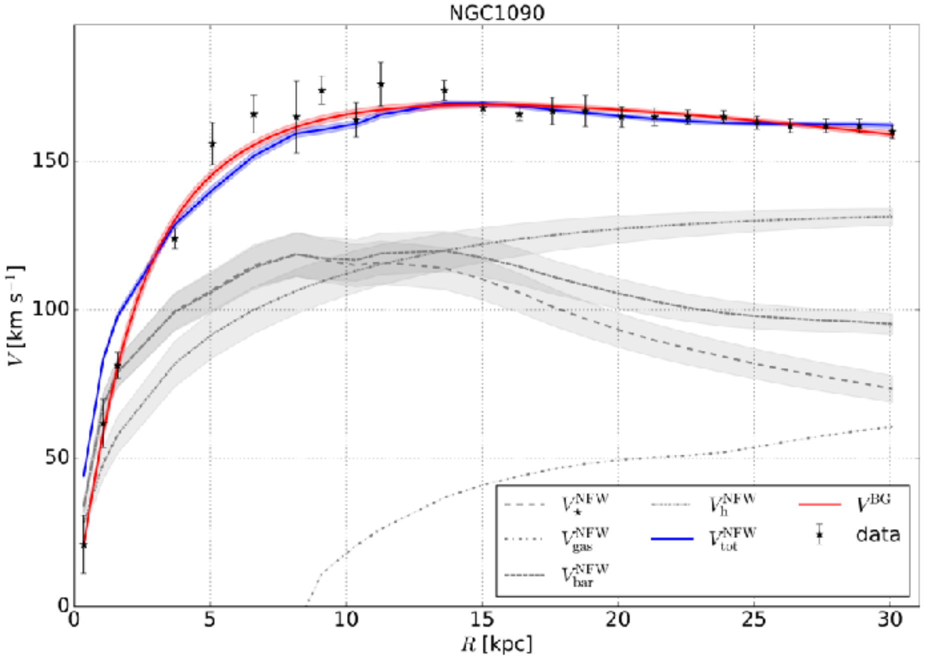
✓ Extend the MW "geometries" to other galaxies:, the "geometries" of the Galaxy can play a reference role for other galaxies, just like the Sun for stellar models

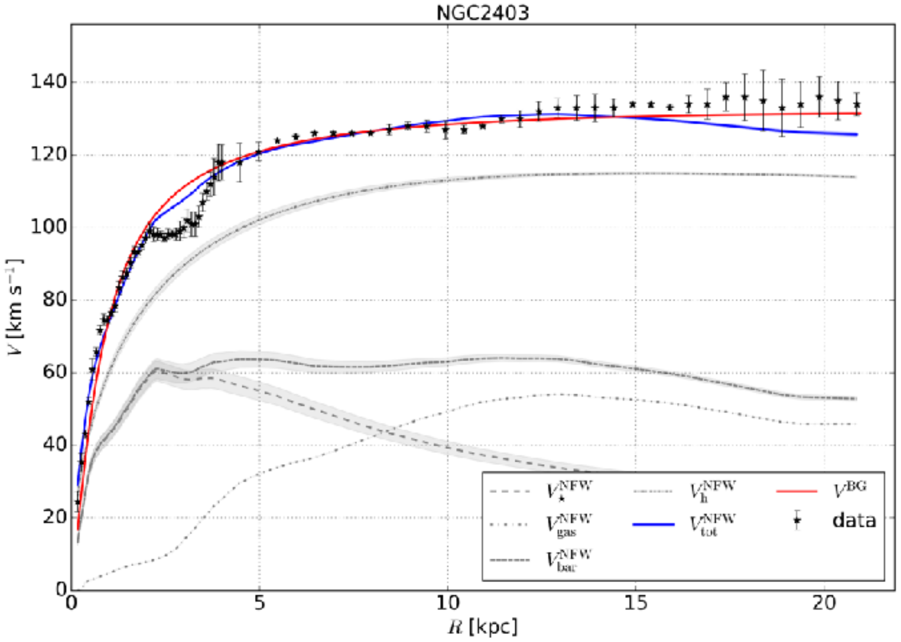
MCMC fit to external Galaxies

Velocity profiles (SPARC data)

Classical (MWC) GR (BG)

Best fit estimates as the median of the posteriors and their 1σ level credible interval





Next developments

Inside-out approach

- Improve the GR models by including more realistic solutions, e.g. metric solutions to describe the structure and evolution of a multi-structured Galaxy and its local Universe
- Fix boundary matching conditions between internal/external Einstein's solutions [e.g. Mars and Senovilla "On the construction of global models describing rotating bodies; uniqueness of the exterior gravitational field" (gr-qc/9806094v1)]
- Using hydrodynamical simulation for the inner part, exploring semi-analytical solutions or Einstein-Vlasov system
- Adopt suitable gluing procedures among different exact solutions [see J. Corvino 2025, S. Czimek and I. Rodnianski, 2022, P. T Chruściel and Wan Cong 2023, Aretakis S, Czimek S and Rodnianski I 2021]
- Explore more GR observables enabling to prove the Milky Way formation and evolution
- Export the fine-tuned template of our Galaxy to other galaxies and set the limits, if any

Outside-in approach

- Set comparisons at the scale of the Milky Way with the ΛCDM model simulations [e.g., cosmological hydrodynamical simulation of MW-like disc galaxies (Giammaria et al. (2021) MNRAS, 502, 2), or ΛCDM DM-only Newtonian N-body simulations at galactic scale (Beordo et al. (2026), MNRAS]

Conclusions
the Gaia legacy and its legacy science

How did the Universe form and evolve? What is our place in it?
What do we really know about the dark matter in the Universe?
What is the deep nature of the fabric of spacetime?

- The mandatory use of GR for high-accuracy astrometry in space has opened new possibilities and strategies to apply Einstein's Theory in classical astronomy domain, providing new coherent methods and "laboratories" to exploit at best the standard theory of gravity and the LDCM scenario, i.e. any modification of GR is done with GR as background theory

- Any GR tests performed by using Gaia Local & Galaxy scale can play a reference role for other tests, much like the Sun for the stars, the Earth/Jupiter for exoplanets, our Galaxy for other similar galaxies, and so on..

- For the first time, quantitative evidence of the differences between the Newtonian and GR approaches to MW dynamics pushing towards more mathematical/numerical solutions of Einstein's equations

From Relativistic astrometry

the "ether" was cured by a new kinematics (i.e. special relativity) instead of "new" dynamic as inspired by the FitzGerald-Lorentz contraction phenomena ("extra molecular force", FitzGerald, Science, 1889)

From Gravitational astrometry

Einstein, 1917, letter to de Sitter: "*One day, our actual knowledge of the composition of the fixed stars sky, the apparent motion of the fixed stars, and the position of the spectral lines as a function of the distance will probably have come far enough for us to be able to decide empirically the question whether or not Λ vanishes*"

Thanks for your attention!