Scattering Rate from the Structure Function of Superfluid ^{3}He

Adam Ting

In collaboration with Neda Darvishi and Stephen West

Royal Holloway University of London

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Signal Event Rate

We want a signal rate that takes into account the many-body physics of superfluid ^{3}He.
A DM interaction potential $\mathcal{V}_{\text{int.}}(x) = (\mathcal{V}_{\text{Atom}} * n)(x)$, gives

$$\frac{dR}{d\omega} = \frac{\rho_\chi}{\rho_T m_\chi} \frac{\pi\bar{\sigma}}{\mu^2} \left\langle \int \frac{d^3 q}{(2\pi)^3} F(q) \underbrace{\frac{2\pi}{V} \sum_f |\langle f | \hat{n}_T(-q) | i \rangle|^2 \, \delta(\Delta E_{fi} - \omega_q)}_{S(q,\omega_q)} \delta(\omega - \omega_q) \right\rangle_v$$

$$= \frac{\rho_\chi}{\rho_T m_\chi} \frac{\bar{\sigma}}{4\pi\mu^2} \int q \, dq \Big[S(q,\omega) \, F(q) \, \eta(v_{\min}(q,\omega)) \Big]$$

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The Classical Gas has

$$S(q,\omega) = 2\pi \frac{N_A}{V} \delta\left(\frac{q^2}{2m_N} - \omega\right), \text{ s.t. } \frac{dR}{d\omega} = \frac{\rho_\chi \bar{\sigma}}{2m_\chi \mu^2} \eta(v_{\text{min}}(q,\omega)) \, F(q) \Big|_{q=\sqrt{2m_N\omega}}$$

Coherence Factor

The DM sees the atoms; inital and final states are described in terms of quasiparticles.

$$\Longrightarrow \quad \hat{n}_{-q} \supset \sum_{c,d} U^*_{p-q,\uparrow c} V_{p,\uparrow d}\, \hat{\alpha}^\dagger_{p-q,c} \hat{\alpha}^\dagger_{-p,d} + U^*_{p-q,\downarrow c} V_{p,\downarrow d}\, \hat{\alpha}^\dagger_{p-q,c} \hat{\alpha}^\dagger_{-p,d}$$

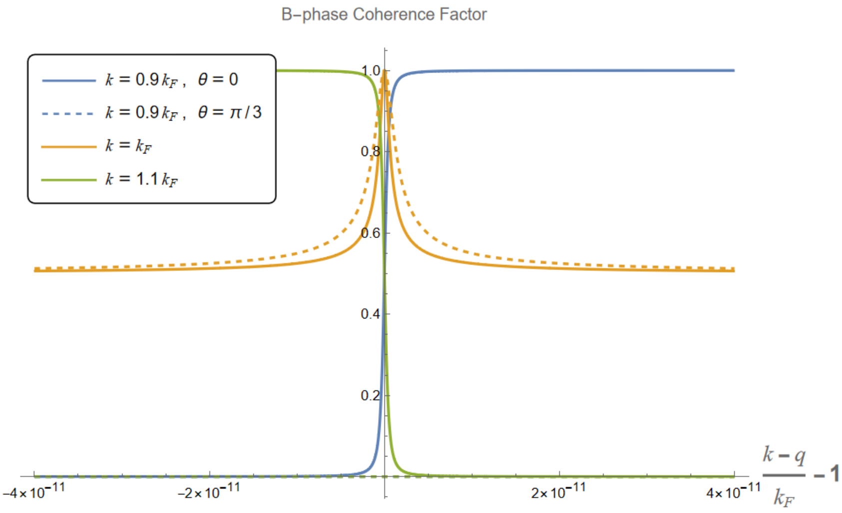
For the s-wave pairing, $\Delta_{\boldsymbol{k}} = \Delta_0(i\sigma_2)$, we reproduce the result of Hochberg et al. 2023

$$|\langle f; -\boldsymbol{p} - \boldsymbol{q}, \boldsymbol{p}|\hat{n}_{-q}|i\rangle|^2 = 1 - \frac{\xi_p \xi_{-p-q}}{E_p E_{-p-q}} + \frac{\Delta^2}{E_p E_{-p-q}}$$

For the B-phase, p-wave pairing, $\Delta_{\boldsymbol{k}} = \Delta_0\, \hat{\boldsymbol{k}} \cdot \boldsymbol{\sigma}\, (i\sigma_2)$,

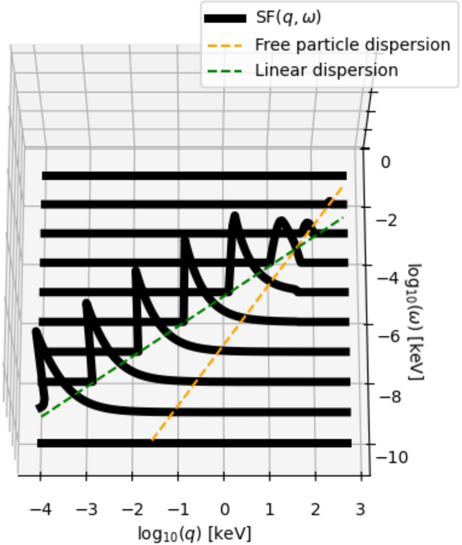
$$|\langle f; -\boldsymbol{p} - \boldsymbol{q}, \boldsymbol{p}|\hat{n}_{-q}|i\rangle|^2 = 1 - \frac{\xi_p \xi_{-p-q}}{E_p E_{-p-q}} + \frac{\Delta^2}{E_p E_{-p-q}} \frac{\boldsymbol{p} \cdot (-\boldsymbol{p} - \boldsymbol{q})}{|\boldsymbol{p}||\boldsymbol{p} + \boldsymbol{q}|}.$$

Coherence Factor



Structure Function

- From the Coherence factor we can evaluate the structure function and find the differential scattering rate and power injection.
- This reproduces the classical structure function - we have the usual scattering at high-q (near the threshold).
- The superfluid gap Δ provides an absolute cutoff corresponding to $m_\chi \simeq 10^{-4}$eV.



Scope

- We have a more complete description of nuclear scattering signal that verifies the classical limit.
- This can be used for various DM scattering interactions. Would need to be extended for DM absorption rate.
- This can be built upon to be used alongside the copper heating calculations to investigate sensitivity 'sub-threshold' masses through power injection.