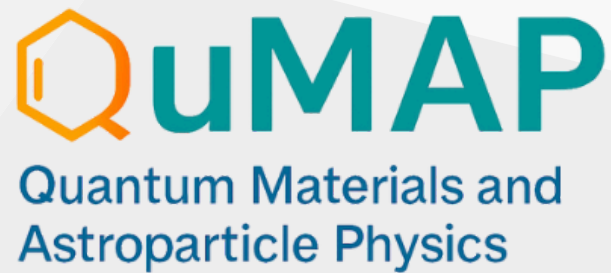


# Heating via nuclear scattering

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# Contents

- Sub-threshold scattering
- Heating channels
- Progress: Where are we now?

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- Sub-threshold scattering ←
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# Heating via sub-threshold scattering

If our DM is too light to break sufficiently many Cooper pairs, can we detect it?

Perhaps! We have access to time-series temperature data.

# Heating via sub-threshold scattering

Suppose we have no *dark* channel for heating:

$$\frac{dT}{dt} \sim P_{\text{bkg}}(t),$$

with  $P_{\text{bkg}}$  some background heating power (e.g. cosmic rays, surroundings).

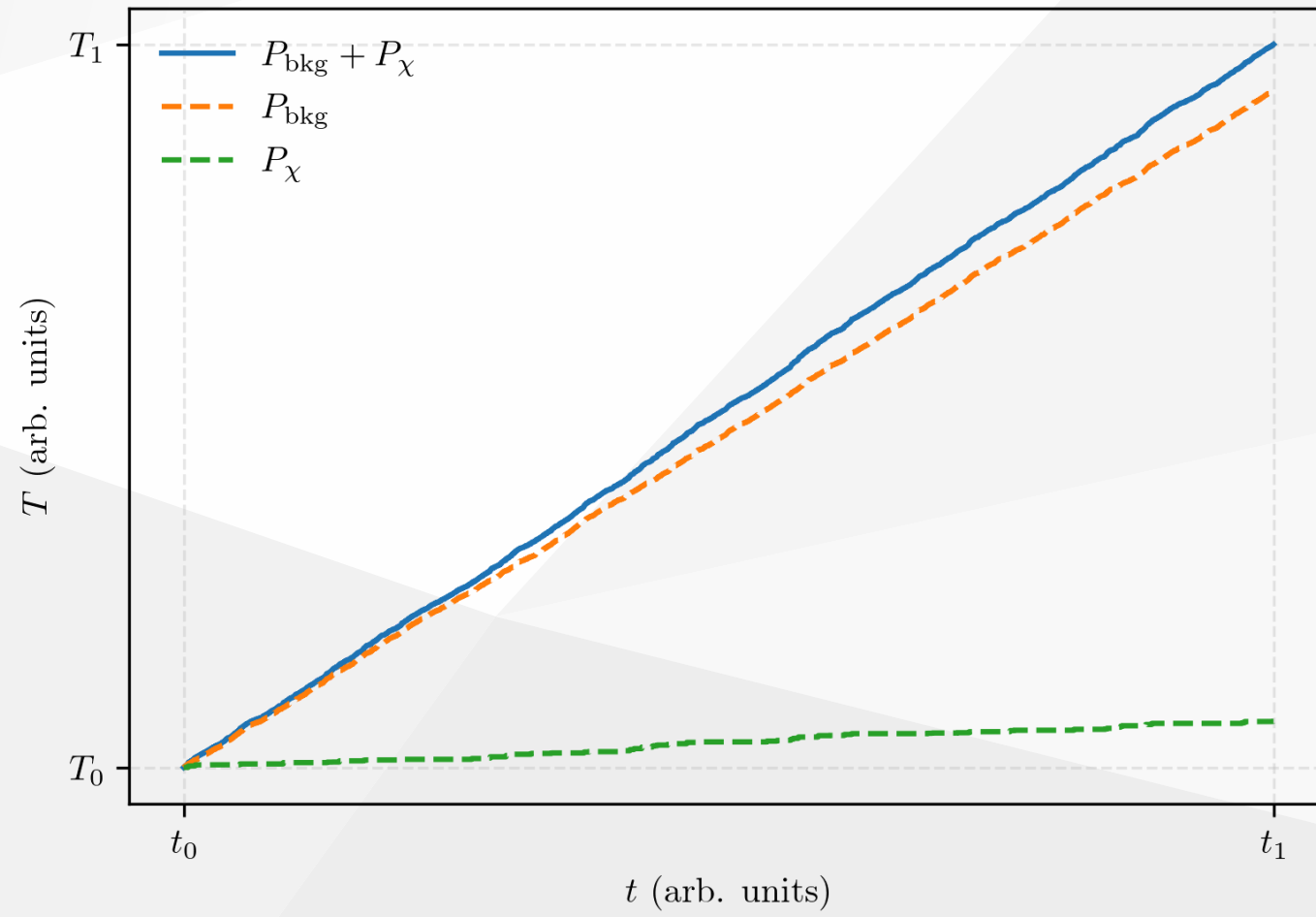
# Heating via sub-threshold scattering

The presence of DM scattering adds an additional heating channel:

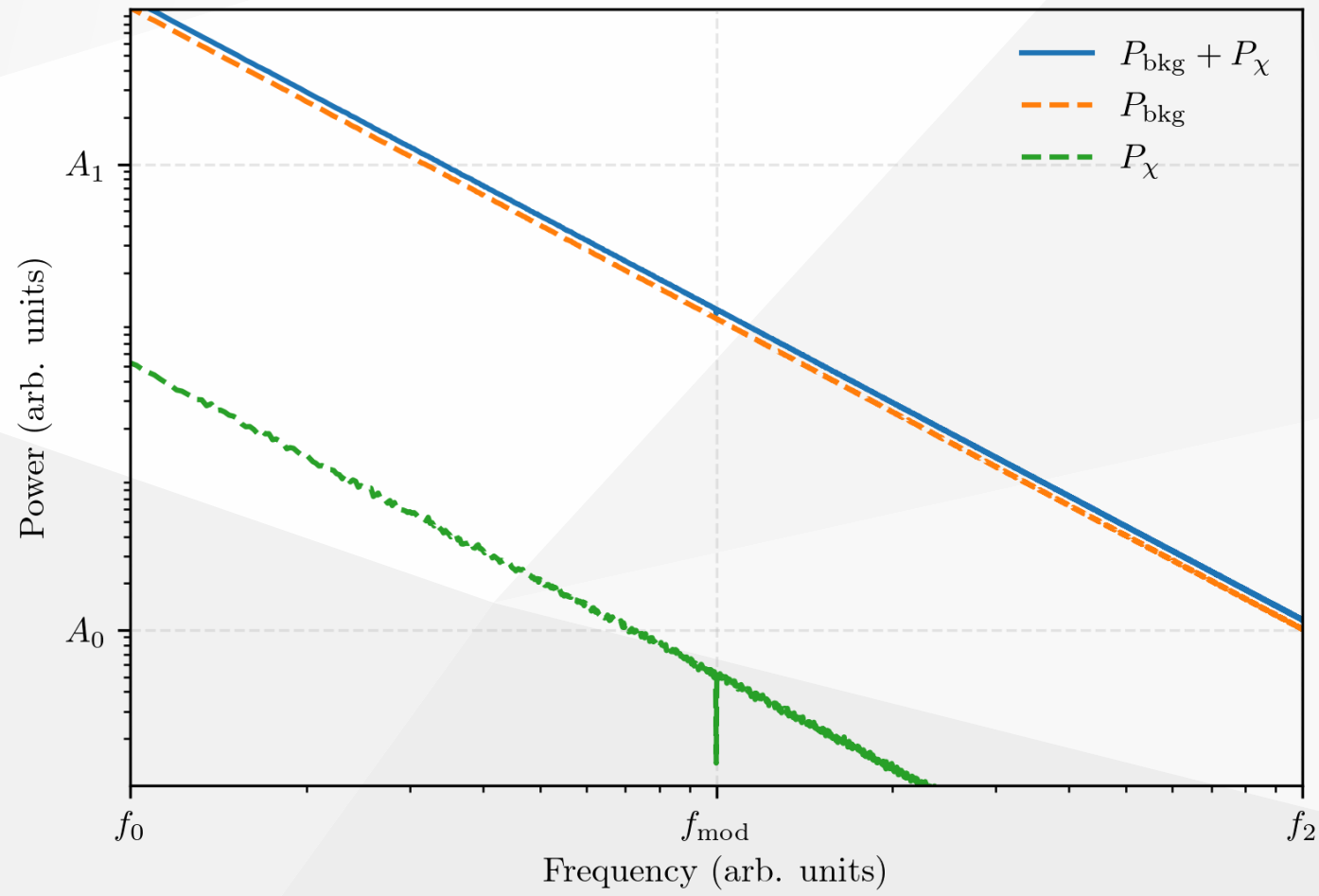
$$\frac{dT}{dt} \sim P_{\text{bkg}}(t) + P_{\chi}(t),$$

where one or both of these terms may be time-dependent.

# Heating via sub-threshold scattering

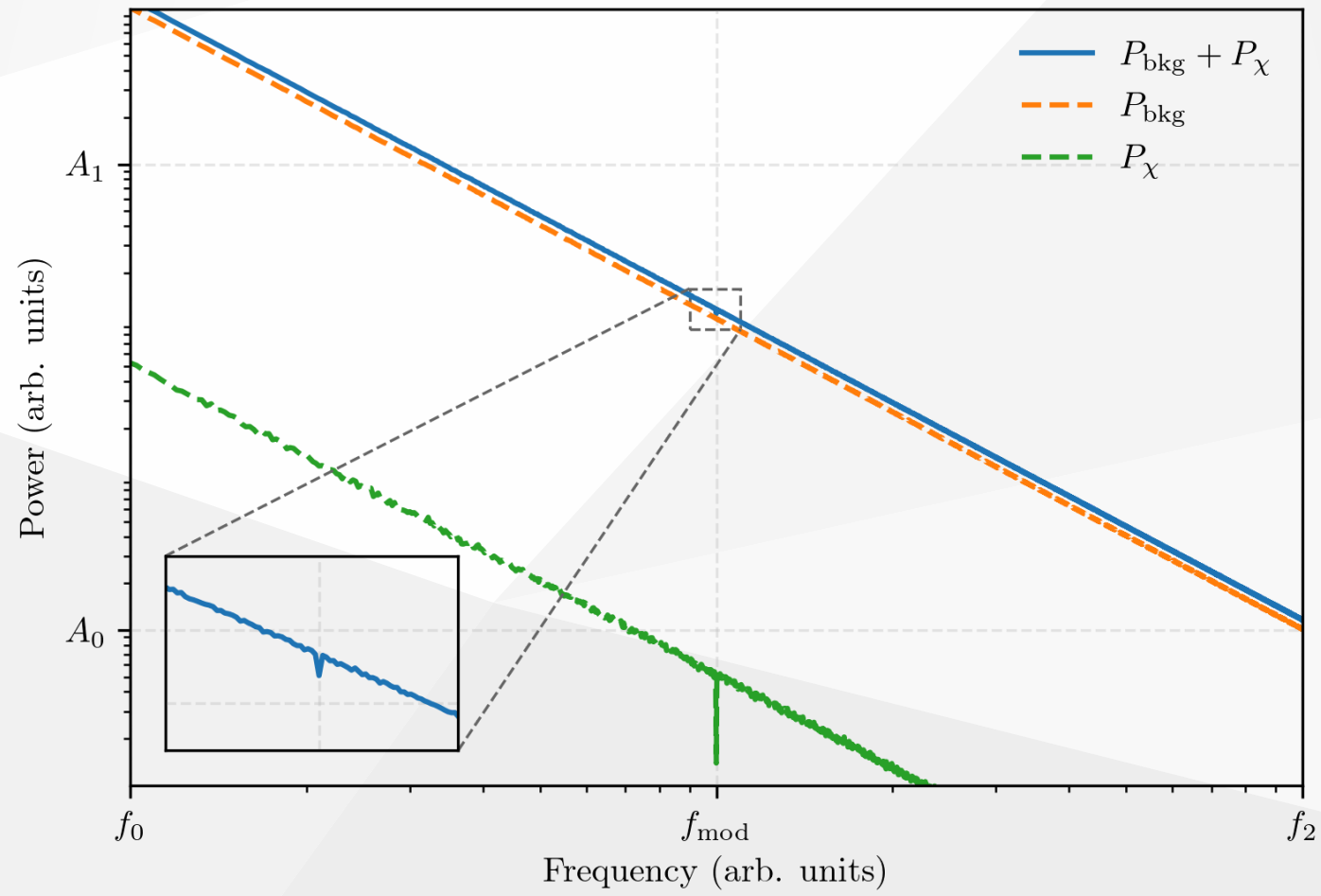


# Heating via sub-threshold scattering





# Heating via sub-threshold scattering



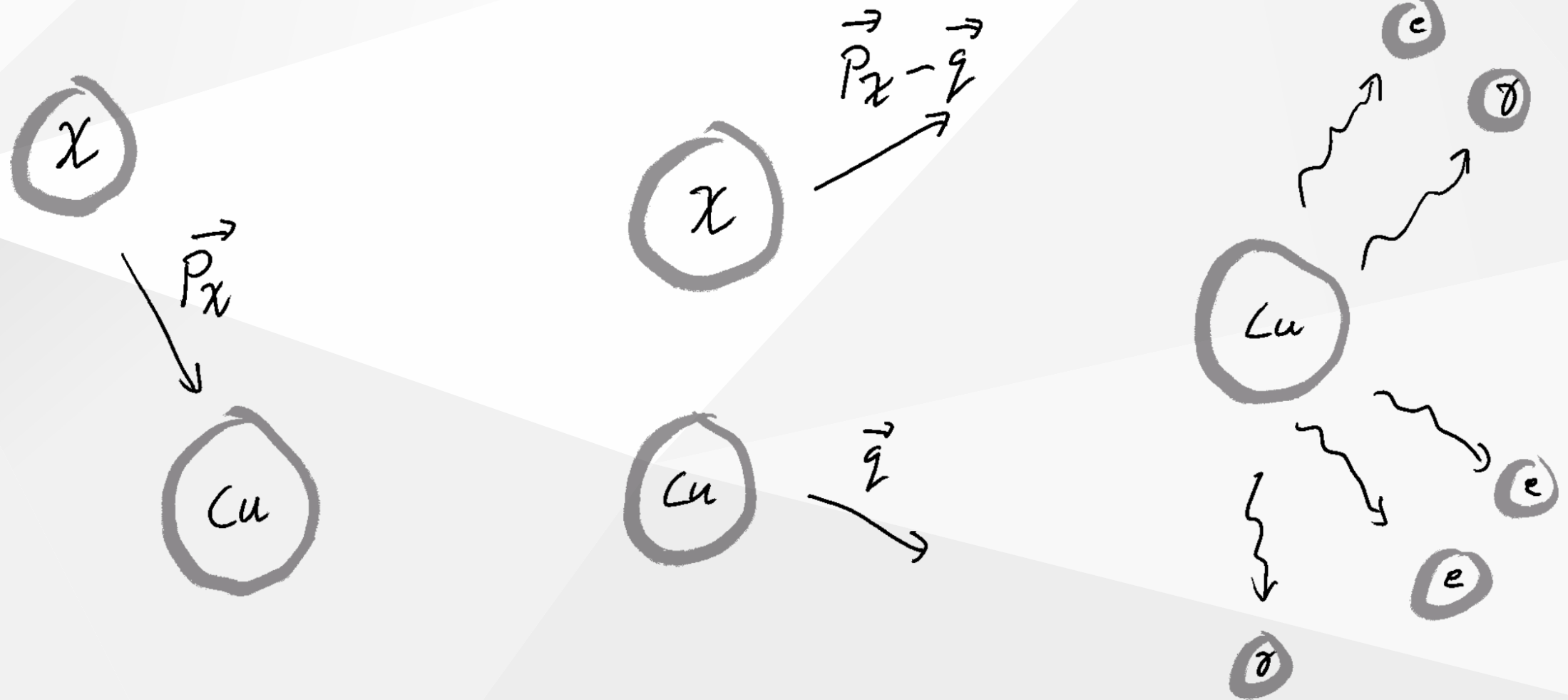
# Contents

- Sub-threshold scattering
- Heating channels ←
- Progress: Where are we now?

# Heating Sources

- Direct scattering on electrons (See Tania's talk)
- Scattering on nuclei:
  - Nuclear recoil
  - The Migdal effect

# Heating Sources: Nuclear Recoil



# Heating Sources: Nuclear Recoil

In the free nucleus picture:

$$Q \simeq E_N \simeq \frac{q^2}{2m_N}$$

For DM of mass  $m_\chi$ :

$$q \simeq m_\chi v_{\text{DM}}$$

# Heating Sources: Nuclear Recoil

In the free nucleus picture:

$$Q \simeq \frac{m_\chi^2 v_{\text{DM}}^2}{2m_N}$$

For a copper nucleus:

$$Q(m_\chi = 10 \text{ MeV}) \simeq 0.8 \text{ meV}$$

# Heating Sources: Nuclear Recoil

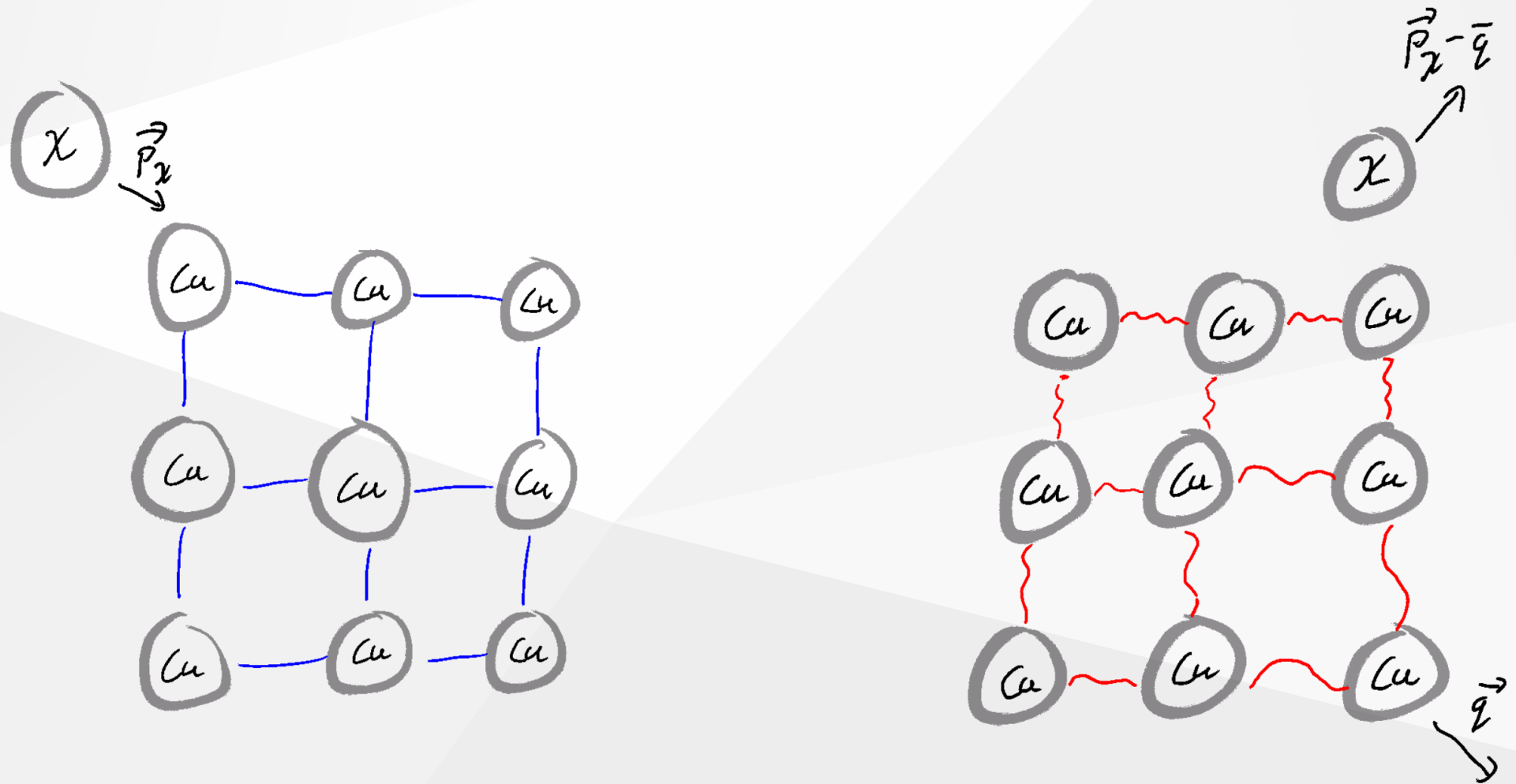
The heating rate is found via:

$$P_{\text{nuc}} = \int dE_N \frac{dR_{\text{nuc}}}{dE_N} E_N$$

In this free nucleus picture:

$$\frac{dR_{\text{nuc}}}{dE_N} \sim \int d^3q d^3p_f \delta(\vec{p}_i - \vec{p}_f - \vec{q}) \dots$$

# Heating Sources: Nuclear Recoil





# Heating Sources: Nuclear Recoil

Taking into account collective effects:

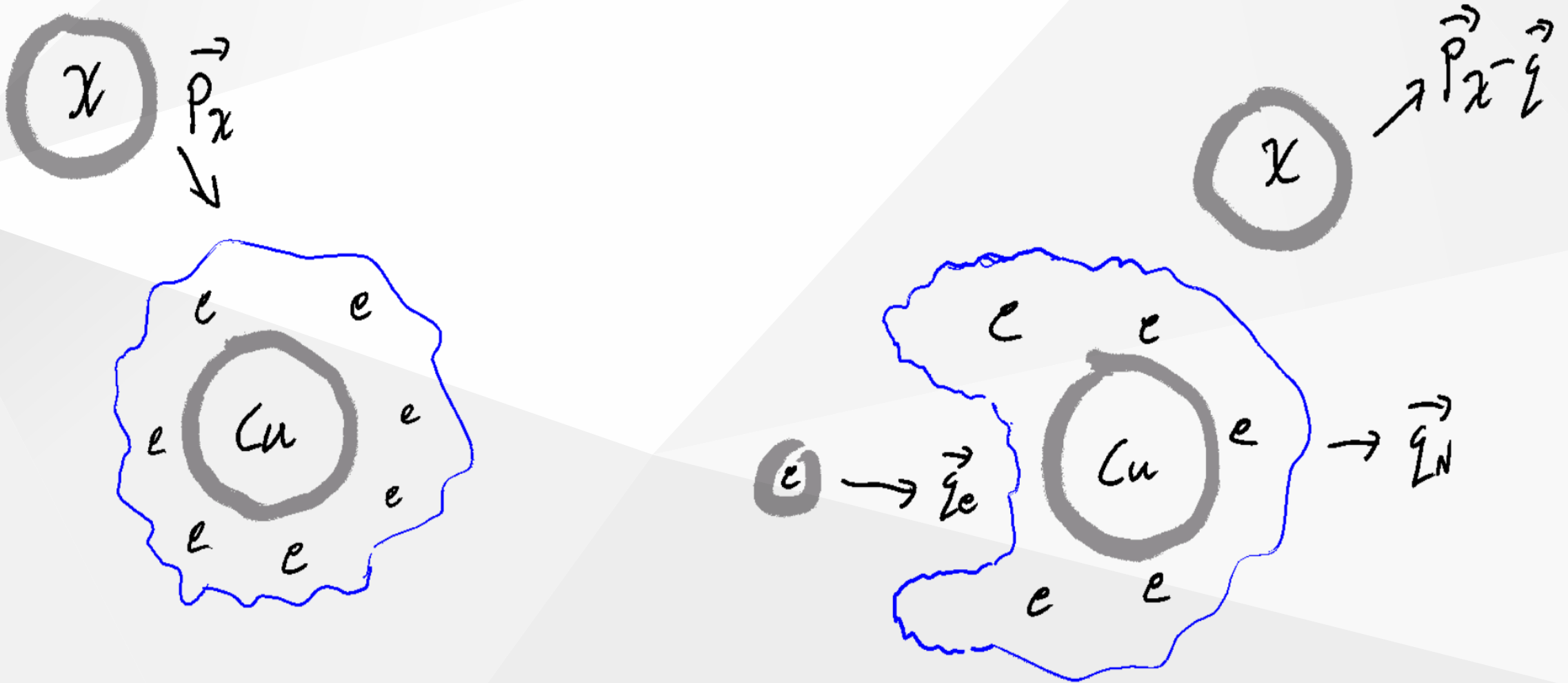
$$\delta(\vec{p}_i - \vec{p}_f - \vec{q}) \rightarrow F(|\vec{p}_i - \vec{p}_f - \vec{q}|),$$

With a simple crystal form factor:

$$F(q) \sim \exp\left(-\frac{q^2}{2m_N\bar{\omega}_{\text{ph}}}\right),$$

with  $\bar{\omega}_{\text{ph}} \sim \mathcal{O}(30 \text{ meV})$ .

# Heating Sources: Migdal Effect



# Heating Sources: Migdal Effect

In Migdal events, energy is efficiently transferred to the electrons:

$$\omega_e \simeq E_{\text{bind}} \sim \mathcal{O}(\text{eV}),$$

assuming this ends up as heat

$$Q \simeq E_{\text{bind}}.$$

Much larger than nuclear recoil!

# Heating Sources: Migdal Effect

But there's an issue:

$$P_{\text{Migdal}} = \int d\omega_e \frac{dR_{\text{Migdal}}}{d\omega_e} \omega_e,$$

where now:

$$\frac{dR_{\text{Migdal}}}{d\omega_e} = \int dE_N \frac{dR_{\text{nuc}}}{dE_N} \mathcal{P}_{\text{Migdal}}(\omega_e | E_N),$$

and  $\mathcal{P}_{\text{Migdal}} \sim 10^{-4} - 10^{-2}$ .

# Heating Sources

As a rough estimate,  $P_{\text{Migdal}} \simeq P_{\text{nuc}}$  around  $m_\chi = 10 \text{ MeV}$ .

Need to consider both effects.

# Contents

- Sub-threshold scattering
- Heating channels
- Progress: Where are we now? ←

# Progress: Overview

- Theory: Done!
  - We have the rates, Migdal probability.
- Computation: On its way.
  - Nuclear recoil channel, not yet programmed.
  - ...but, get it for free from Migdal.
  - Migdal computation is tough.

# Progress: Migdal Effect

The Migdal probability scales like:

$$\mathcal{P}_{\text{Migdal}} \sim \frac{1}{\omega_e^4} \text{Im} \left[ -\frac{1}{\epsilon(q, \omega_e)} \right],$$

which is susceptible to divergences as  $\omega_e \rightarrow 0$ .



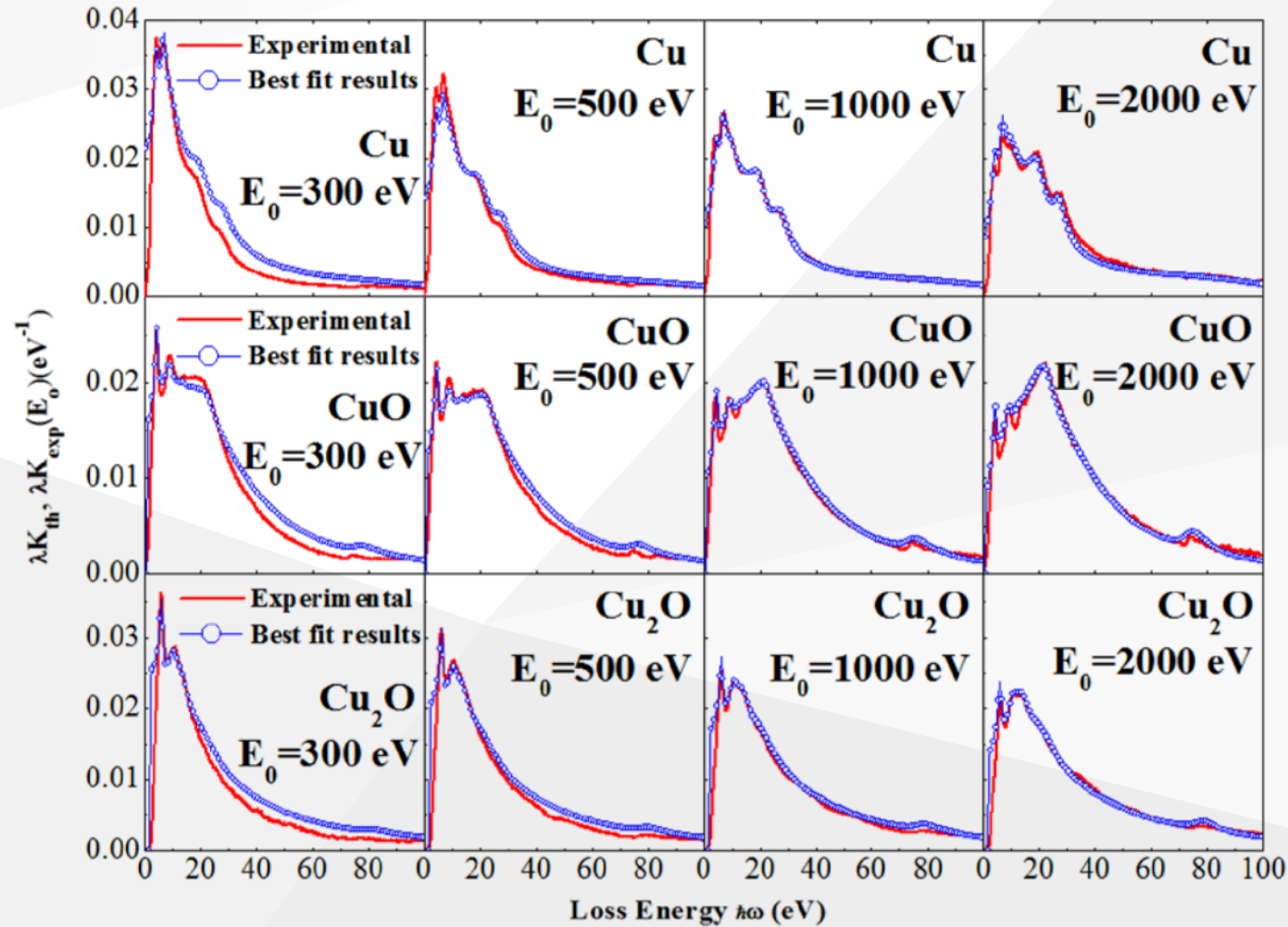
# Progress: Migdal Effect

We need an ELF that properly captures the low  $\omega_e$  behaviour.

The Lindhard method is oversimplified, treating the electron gas as a non-interacting system.

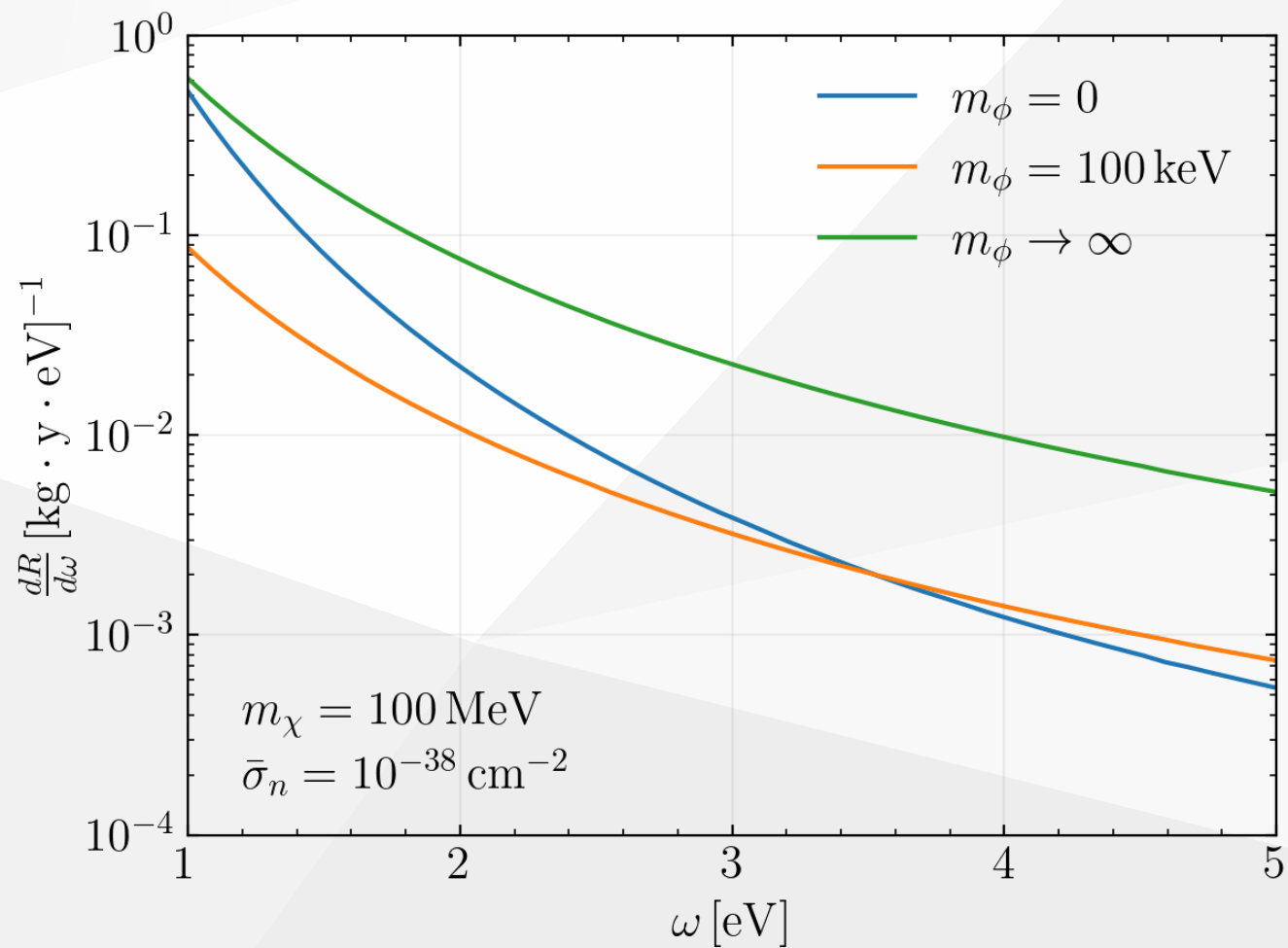
Fits to experimental data fail at properly capturing the low  $\omega_e$  behaviour.

# Progress: Experimental Fits

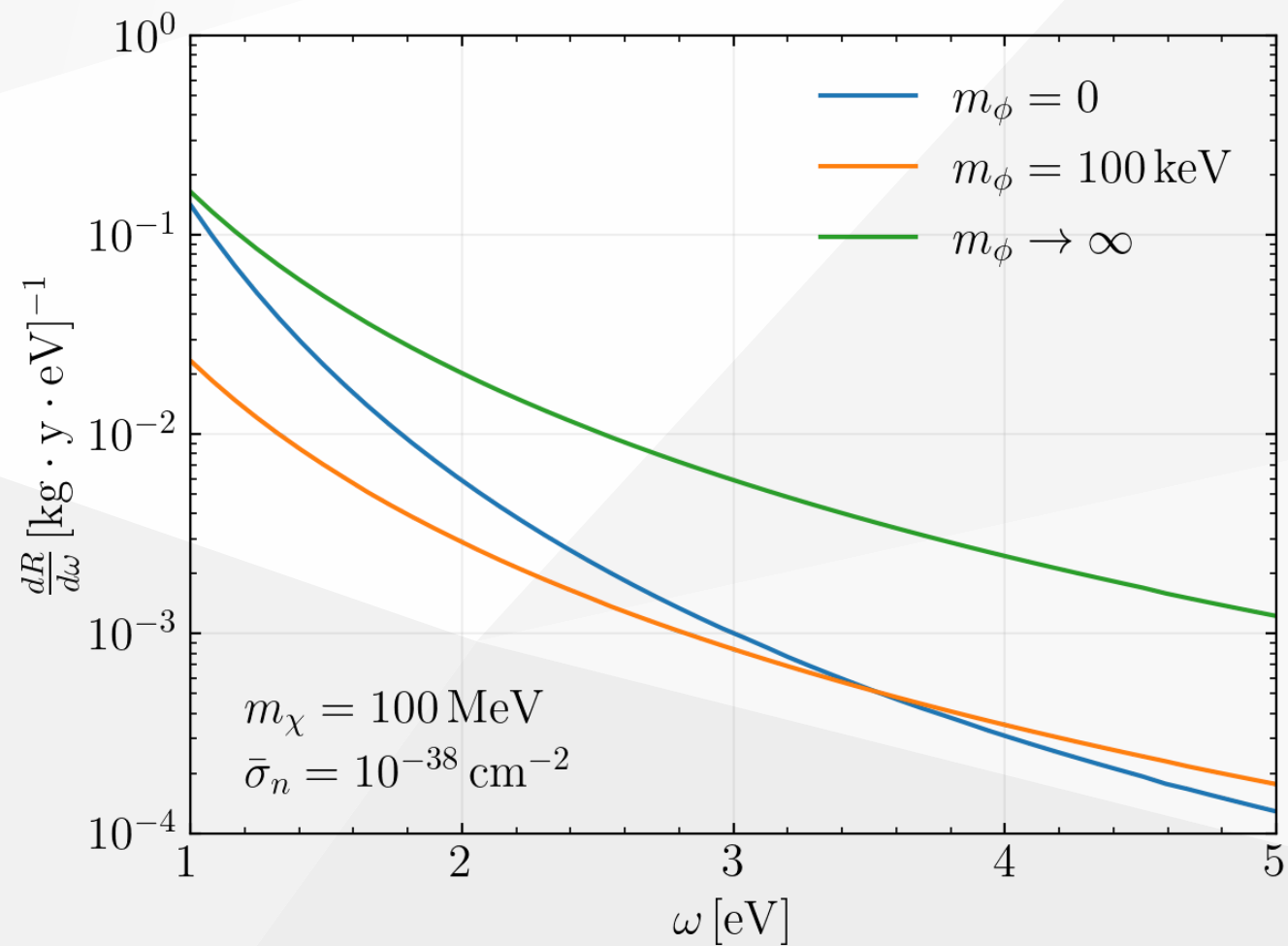


J. Phys.: Condens. Matter 24  
(2012) 175002 (8pp).

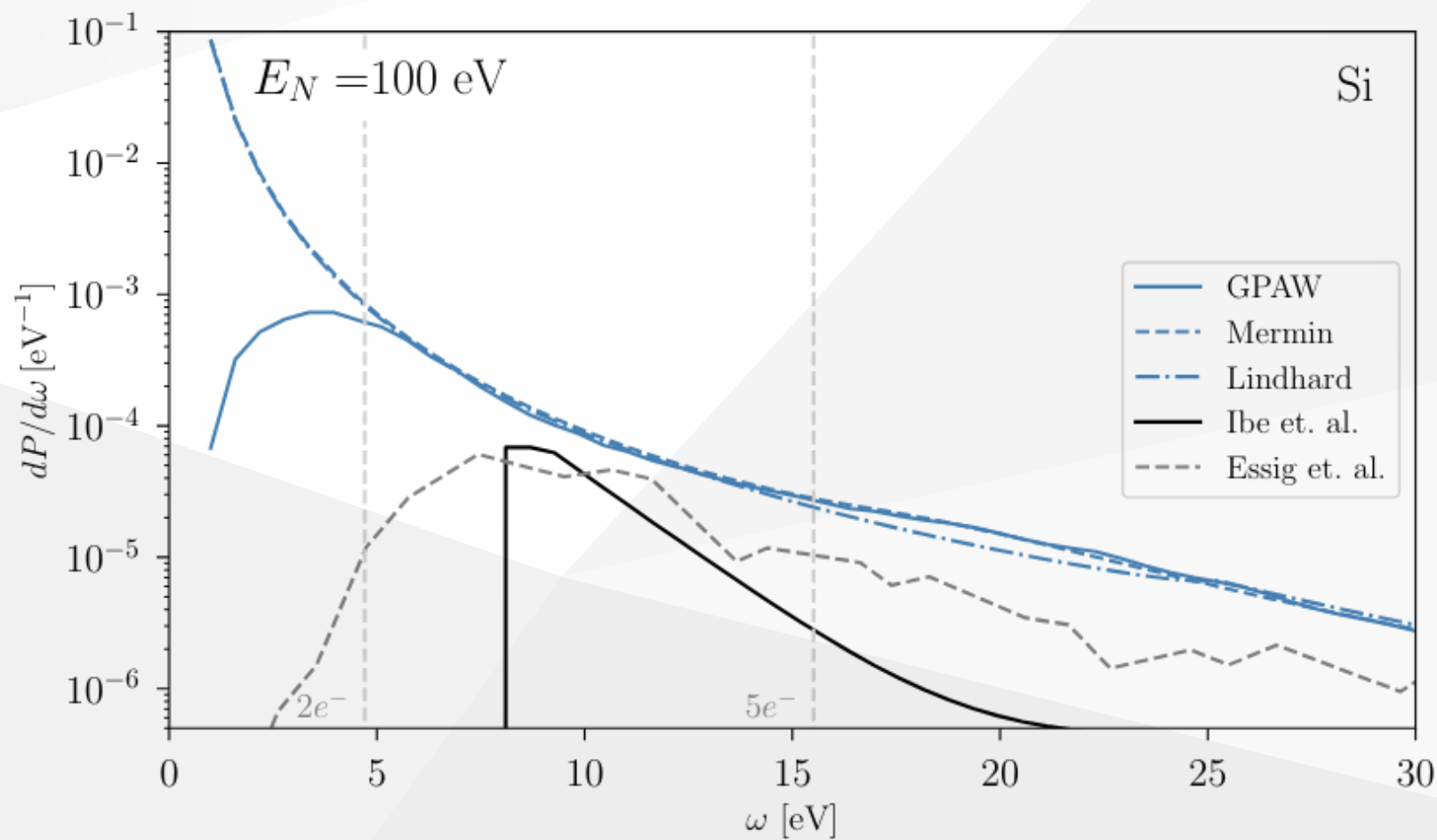
# Progress: Experimental ELF



# Progress: Lindhard ELF

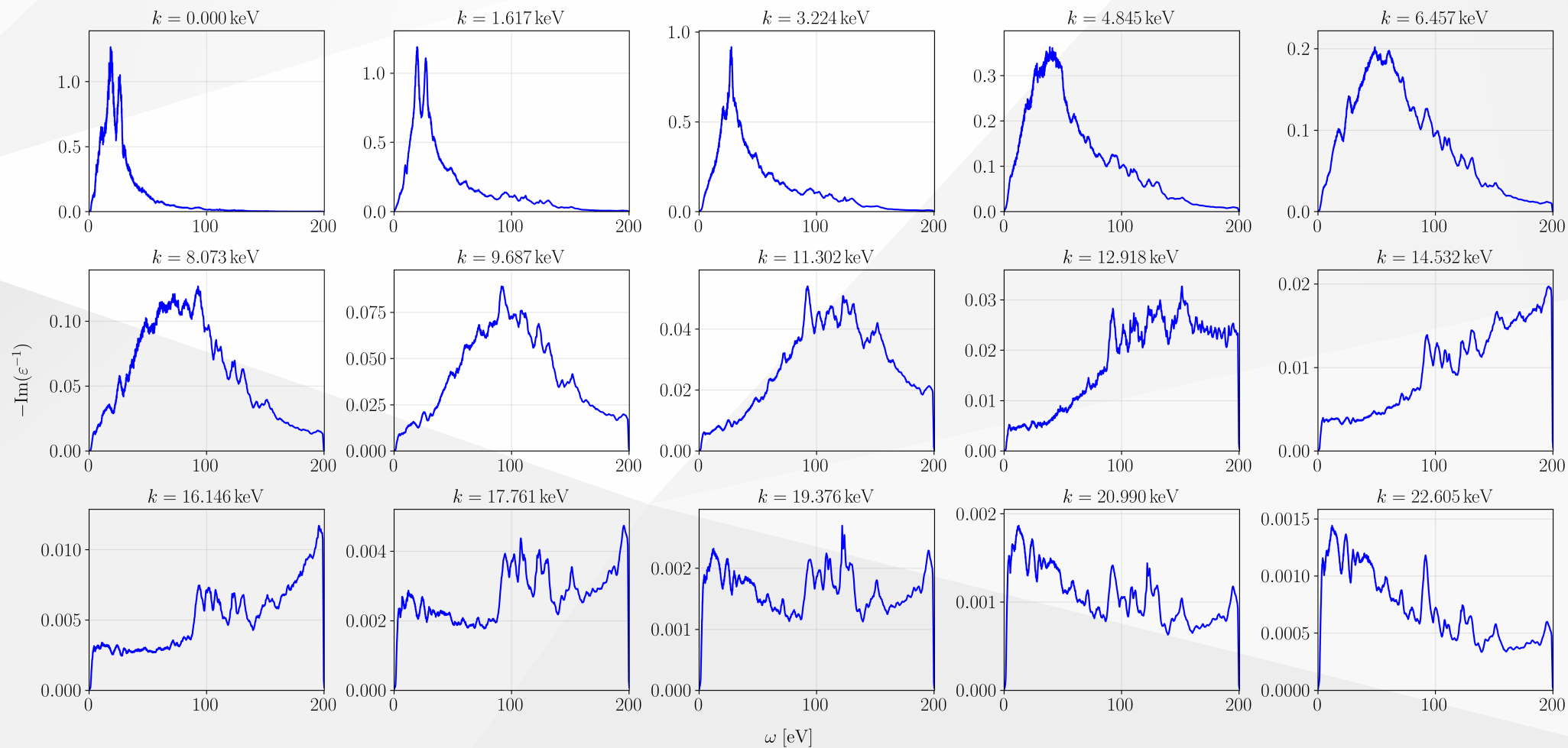


# Progress: Solution?

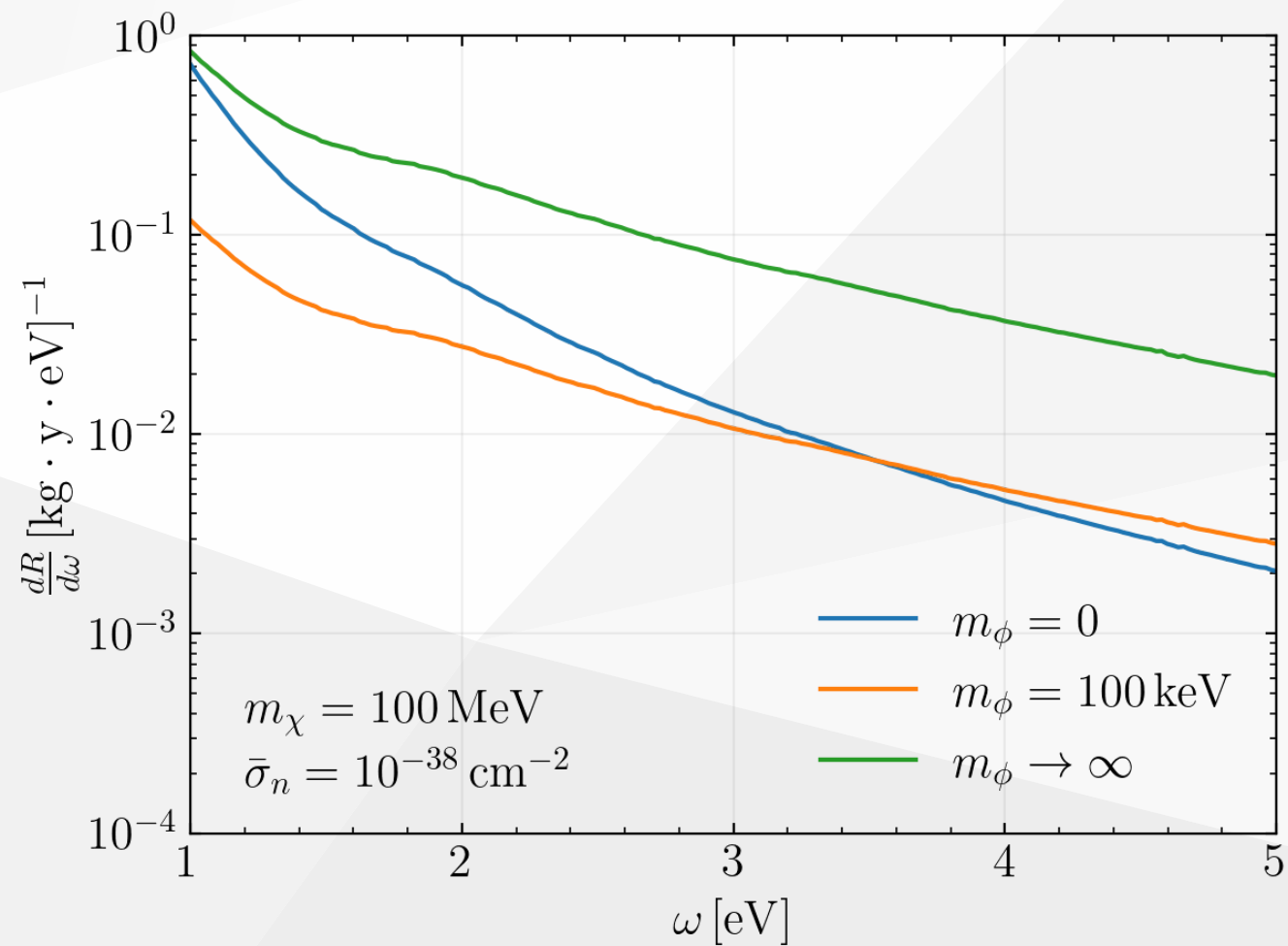


Phys. Rev. D 105,  
015014 (2022).

# Progress: GPAW ELF



# Progress: DFT ELF



# Progress: What's next?

This took around half a day to run.

What's next? We need to:

- Push to higher  $q$ , ideally up to  $q \sim 30$  keV. This is expensive,  $t \sim q_{\max}^{4.76}$ . Push to denser  $\vec{k}$  grids too,  $t \sim N_k^{4.08}$ .
- Try different xc functionals. More precision, more expensive. However, the GS computation is cheap compared to the ELF currently ( $\sim 10$  m vs 12 h).



**Thank you!**  
**Questions?**