Modelling flux vortex dynamics in type-II superconductors under a thermal gradient-driven phase transition with closed cooling topology

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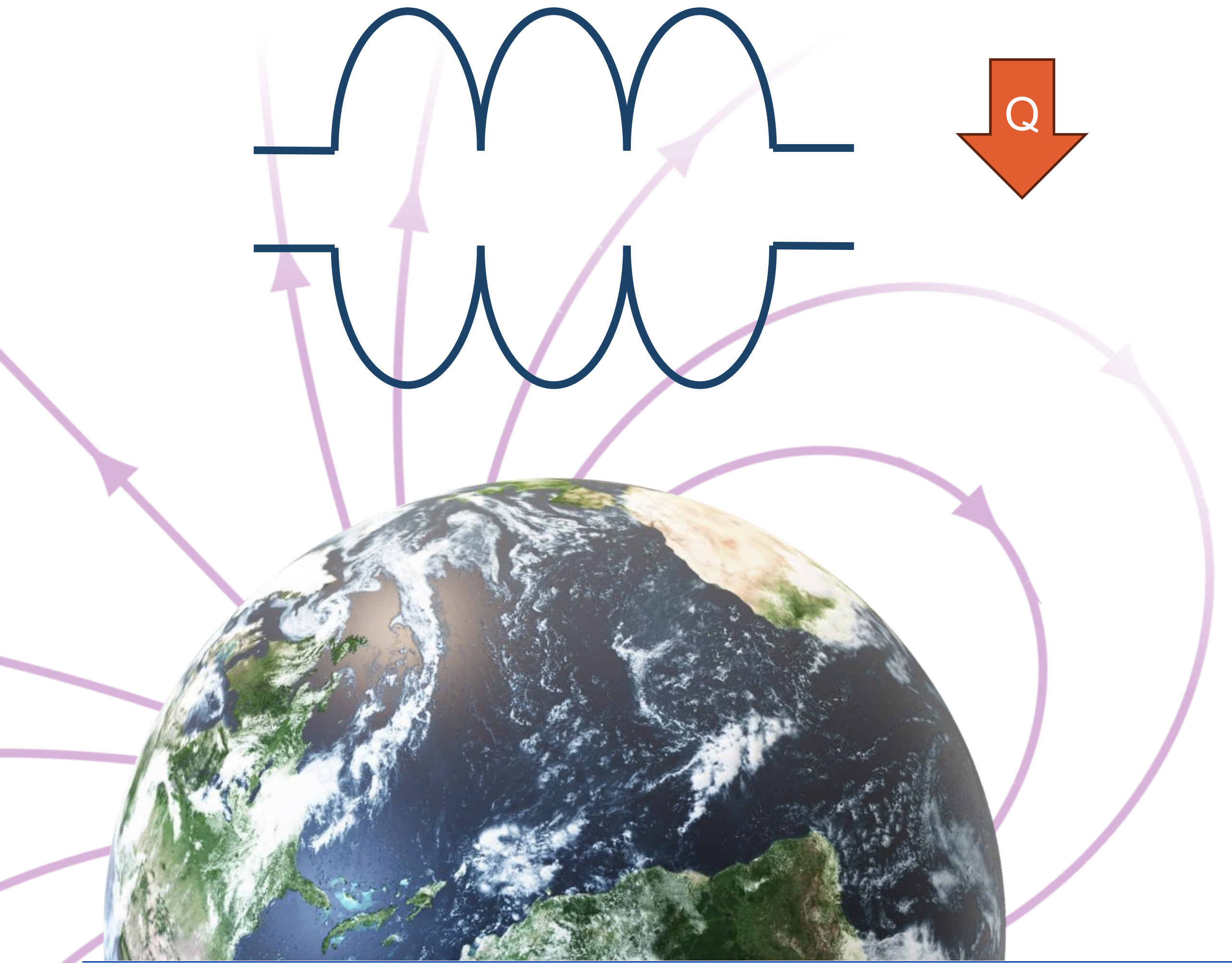
Trapped flux in SRF cavities: decrease in Q



- When cooling down, **trapped field** in SC walls can lead to a **decrease in quality factor**.
- Even DC fields as low as **earth's magnetic field** (tens of μT) can have a significant impact.
- The **cool-down conditions** ($\nabla T, \dot{T}$) have been shown to influence how much flux is trapped. 1

A **closed cooling topology** causes an entirely different response, playing a pivotal role in defining the amount of flux that penetrates the material.

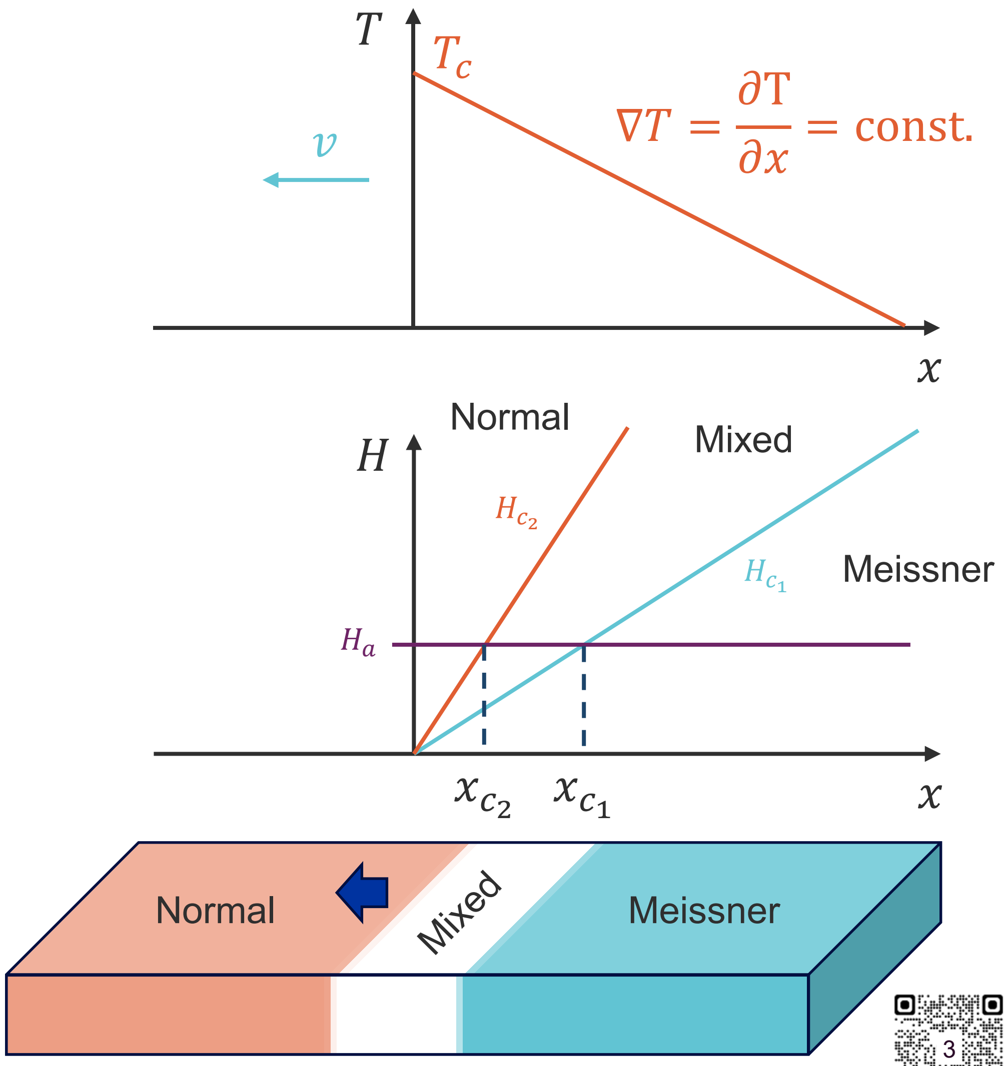
Outline

1. Vortex dynamics during phase transition
2. Numerical model: Time-dependent Ginzburg-Landau equations
3. Results and discussion
4. Summary and outlook

1. Vortex Dynamics

During phase transition

Phase transition fronts move with decreasing T



$$T = T_c - \nabla T x \longrightarrow x(T = T_c) = 0$$

$$H_{c_{1,2}} = H^*_{c_{1,2}} \frac{\nabla T}{T_c} x \qquad x_{c_{1,2}} = \frac{H_a T_c}{H^*_{c_{1,2}} \nabla T}$$

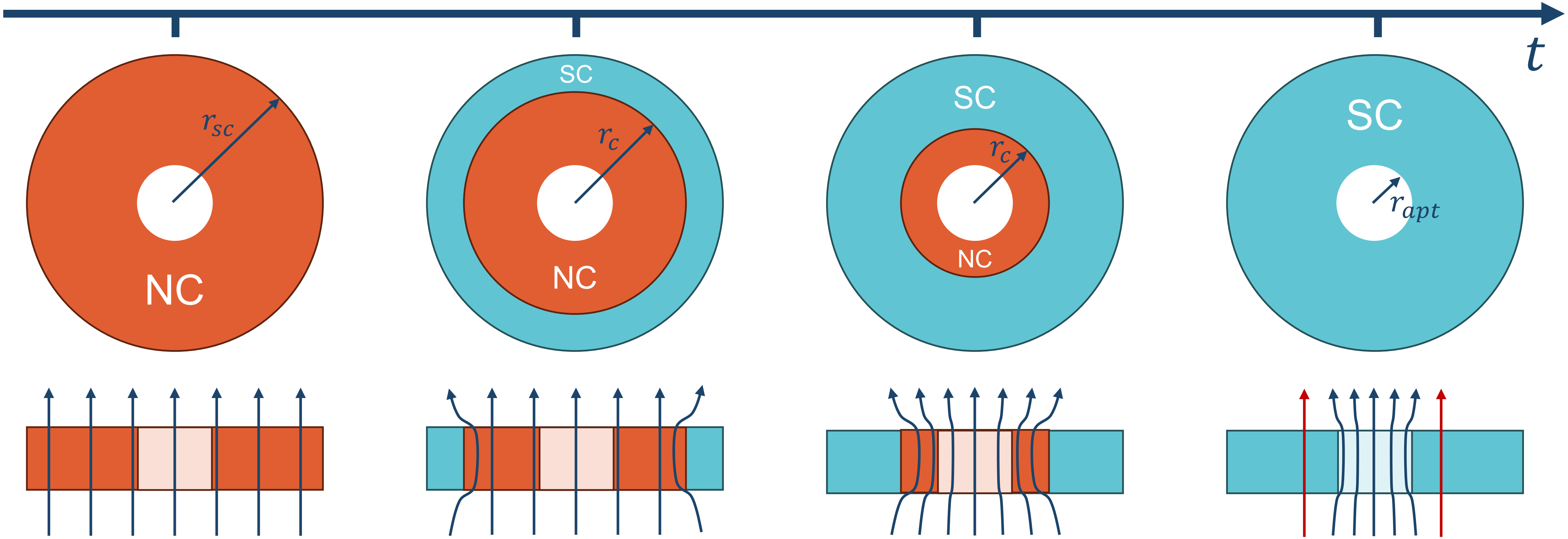
$$\Delta x_c = x_{c_1} - x_{c_2} = \frac{H_a}{\nabla T} T_c \left(\frac{1}{H^*_{c_1}} - \frac{1}{H^*_{c_2}} \right) \longrightarrow \text{Mixed state width}$$

$$\overline{x_c} = \frac{1}{2}(x_{c_1} + x_{c_2}) = \frac{H_a}{\nabla T} \frac{T_c}{2} \left(\frac{1}{H^*_{c_1}} + \frac{1}{H^*_{c_2}} \right) \longrightarrow \text{Mixed state position}$$

- In an open thermal topology, the position and width of the **mixed state** region are **constant** over time.
- Lower **gradients** or higher applied **fields** increase the width and, consequently, **vortex formation**.

Slide based on the work of T. Kubo: https://doi.org/10.1093/ptep/ptw049

Field is enhanced in a closed thermal topology



$$\phi = \pi r_{sc}^2 B_a = \text{const.}$$

$$\pi r_c^2 B_{in} = \pi r_{sc}^2 B_a$$

$$B_{in} = B_a \left(\frac{r_{sc}}{r_c}\right)^2$$

$$ER = \frac{B_{in}}{B_a} = \left(\frac{r_{sc}}{r_{apt}}\right)^2 - \frac{\Delta B}{B_a}$$

Total **flux is compressed** into the aperture; expulsion ratio follows square radii ratio **until vortex formation.**

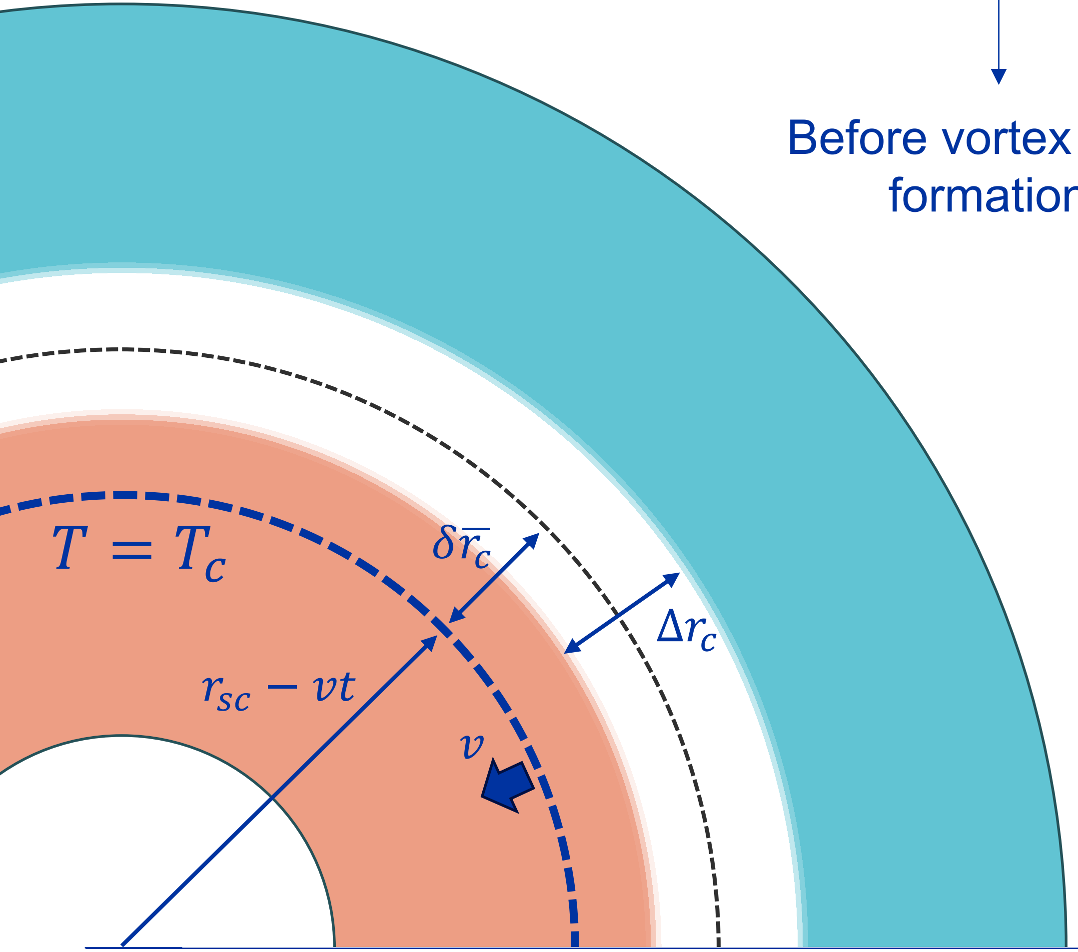
Mixed state changes over time in a closed topology

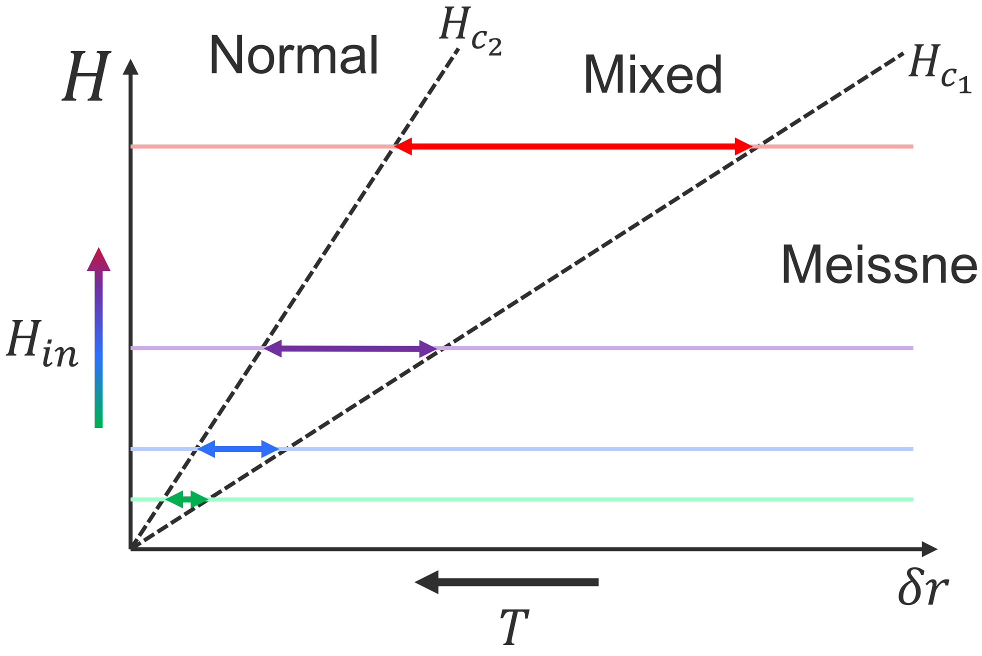
$$H_{in} \approx H_a \left(\frac{r_{sc}}{r_{sc} - vt + \delta\overline{r_c}} \right)^2$$

Before vortex formation

$$\delta\overline{r_c} = \frac{H_{in} T_c}{2\nabla T} \left(\frac{1}{H^*_{c_1}} + \frac{1}{H^*_{c_2}} \right)$$

$$\Delta r_c = \frac{H_{in} T_c}{\nabla T} \left(\frac{1}{H^*_{c_1}} - \frac{1}{H^*_{c_2}} \right)$$





As the transition happens, the inner **field increases** due to the closed topology, pushing the mixed state to a cooler area and **increasing its width** and, consequently, **vortex formation.**

2. Numerical Model

Time-dependent Ginzburg-Landau equations

Time-dependent Ginzburg Landau (TDGL) model

$\kappa = \frac{\lambda}{\xi}$

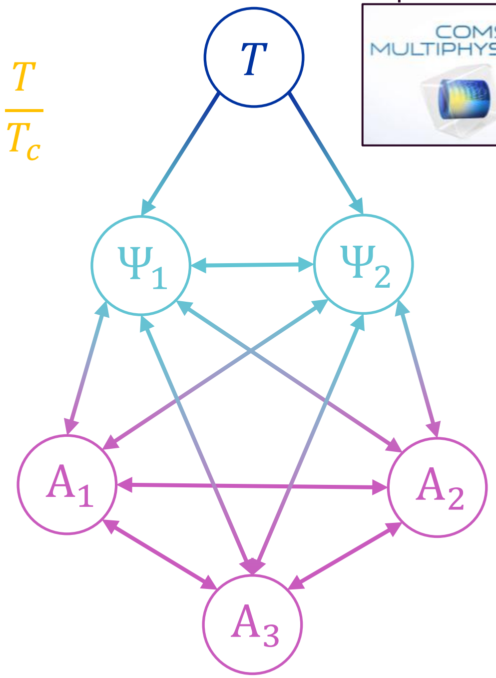
GL Parameter (material-dependent)

Temperature-dependent coefficients

$\alpha = 1 - \frac{T}{T_c}$

$\beta = 1$

Implemented in



Superconductivity and Field

Complex order parameter
$\Psi = \Psi_1 + i\Psi_2$

Magnetic vector potential
$\mathbf{A} = A_1\hat{x} + A_2\hat{y} + A_3\hat{z}$

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{1}{\kappa}\nabla + \mathbf{A}\right)^2 \Psi + (\alpha - \beta|\Psi|^2)\Psi$$

$$\frac{\partial \mathbf{A}}{\partial t} = \overbrace{-\nabla\times\underbrace{\nabla\times\mathbf{A}}_{\mathbf{B}}}^{\mathbf{J}} - |\Psi|^2\left[\mathbf{A} - \frac{1}{\kappa}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)\right]$$

T-dependent material properties:

$$\lambda = \kappa\xi = \frac{1}{\sqrt{\alpha}} = \left(1 - \frac{T}{T_c}\right)^{-\frac{1}{2}}$$

$$\frac{H_{c_1}}{H_{c_1}^*} = \frac{H_{c_2}}{H_{c_2}^*} = \alpha = 1 - \frac{T}{T_c}$$

Heat Transfer in Solids:

$$\rho C_p \dot{T} = \nabla \cdot (k\nabla T) + Q$$

- More realistic approach
- Harder to correlate with ∇T
- Hard coupling

or

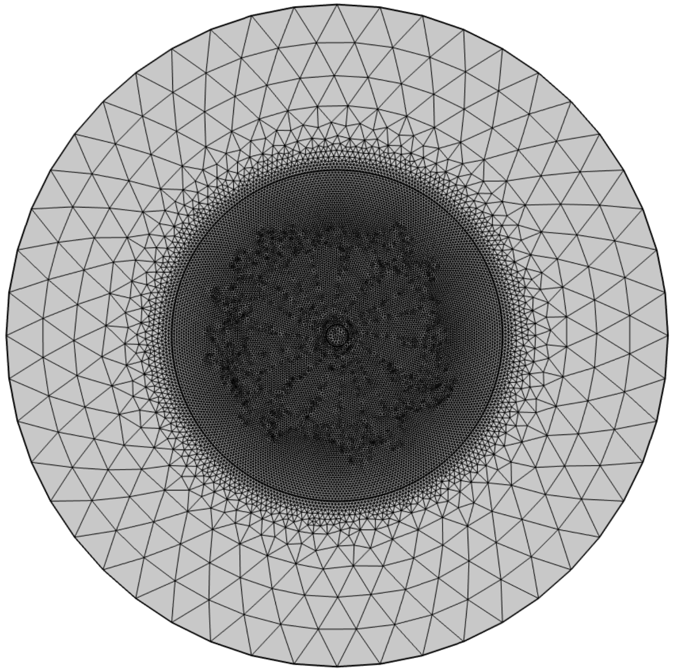
Temperature as an input:

$$T = T_c - \nabla T(r - r_{sc} + vt)$$

- Less realistic approach
- Constant ∇T is an input
- No coupling needed

TDGL Simulations vs. MFL Experiments

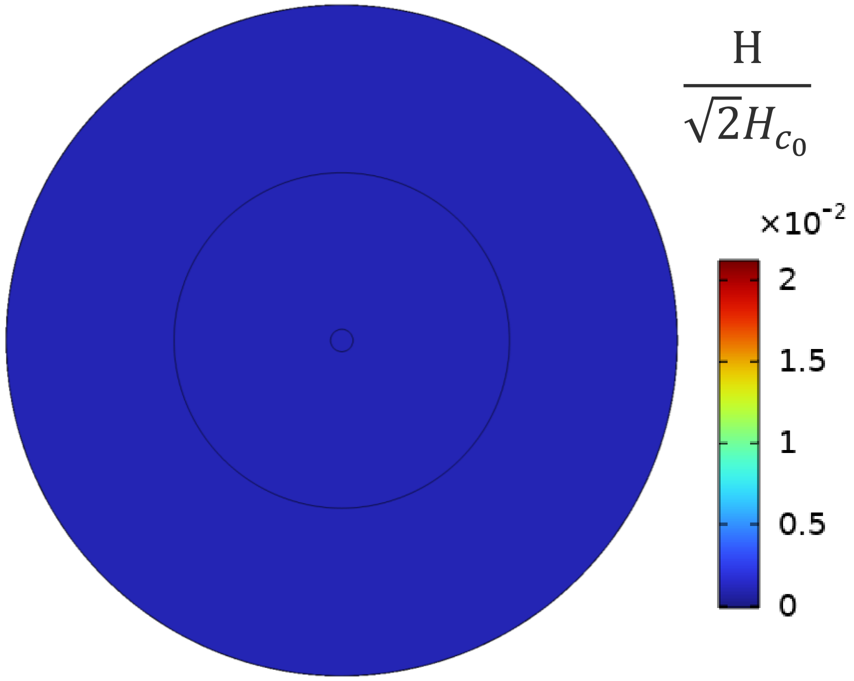
- Extreme spatial scales:
  - Mesh cells $< \lambda, \xi = O(10nm) \ll r_{sc} = O(10mm)$
- Extreme time scales:
  - $(\tau_T \gg \tau_{\Psi,A})$
- Secondary implications:
  - Extreme parameters can lead to **divergence** or **trivial solution**
  - Fine-tuning is necessary but time-consuming

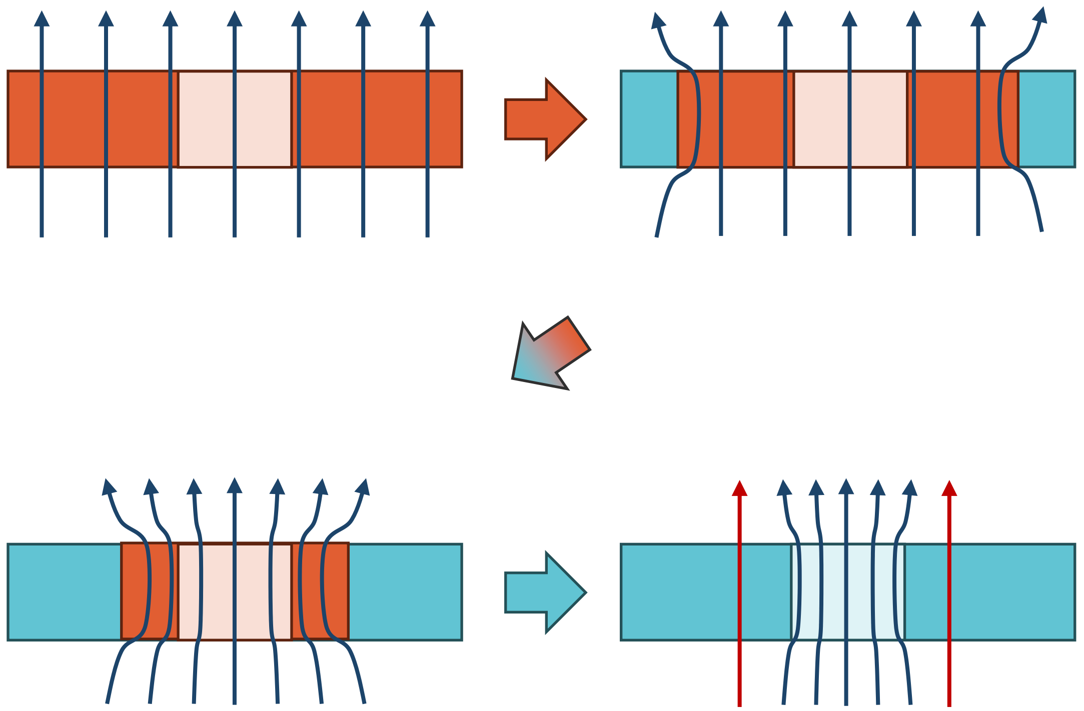


- Too **computationally demanding** to consider life-sized objects
- **Conceptual understanding** applicable to realistic dimensions when **propertly rescaled.**

3. Results and Discussion

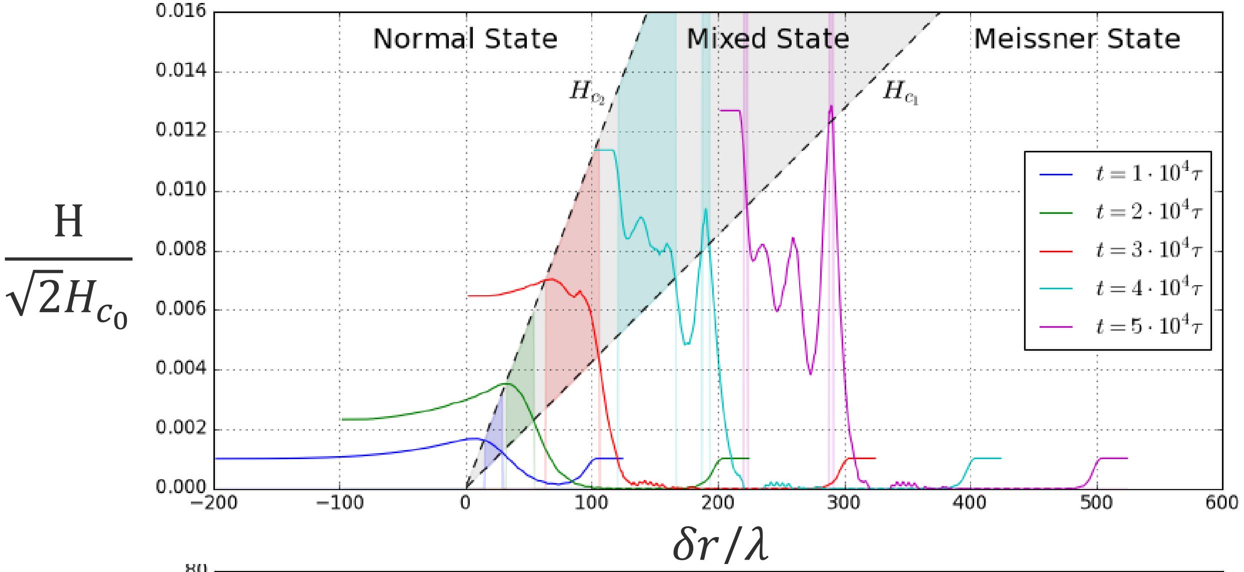
Vortex formation during phase transition

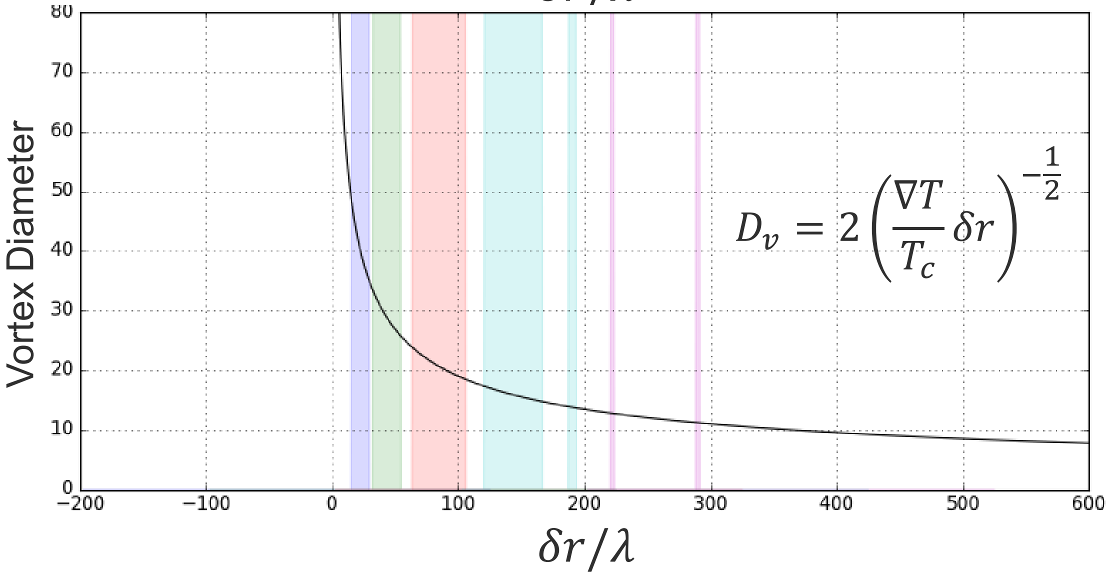


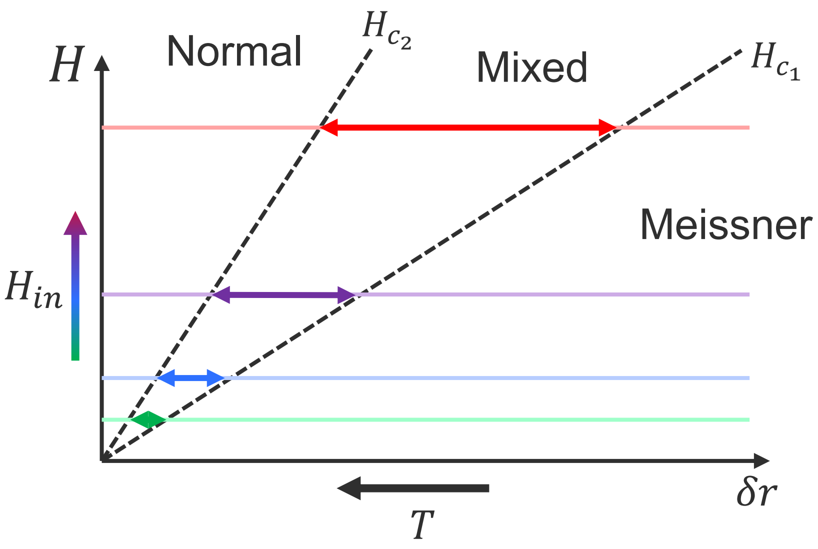


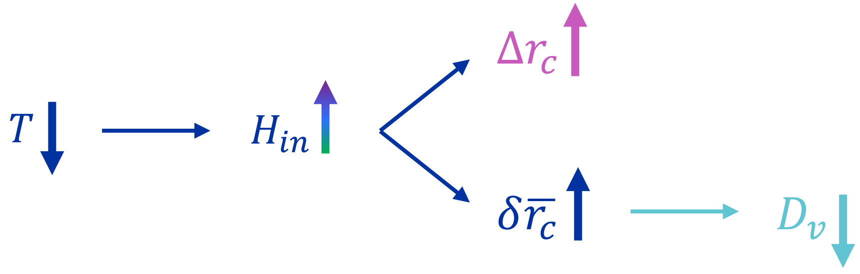
- The externally applied magnetic field is **initially not strong** enough to induce vortex formation
- Field is **compressed towards the center** due to phase transition in a closed topology
- There is a **threshold beyond which vorteices form**. What is this threshold?

Transition pushes and widens the mixed state





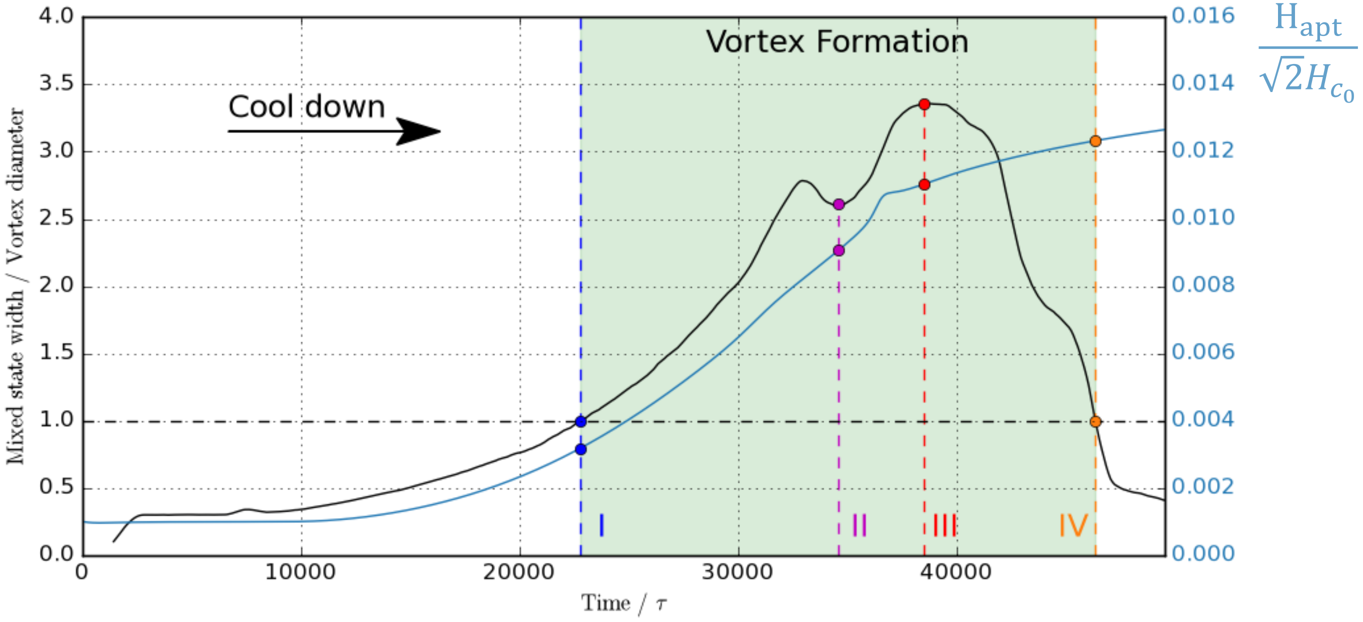


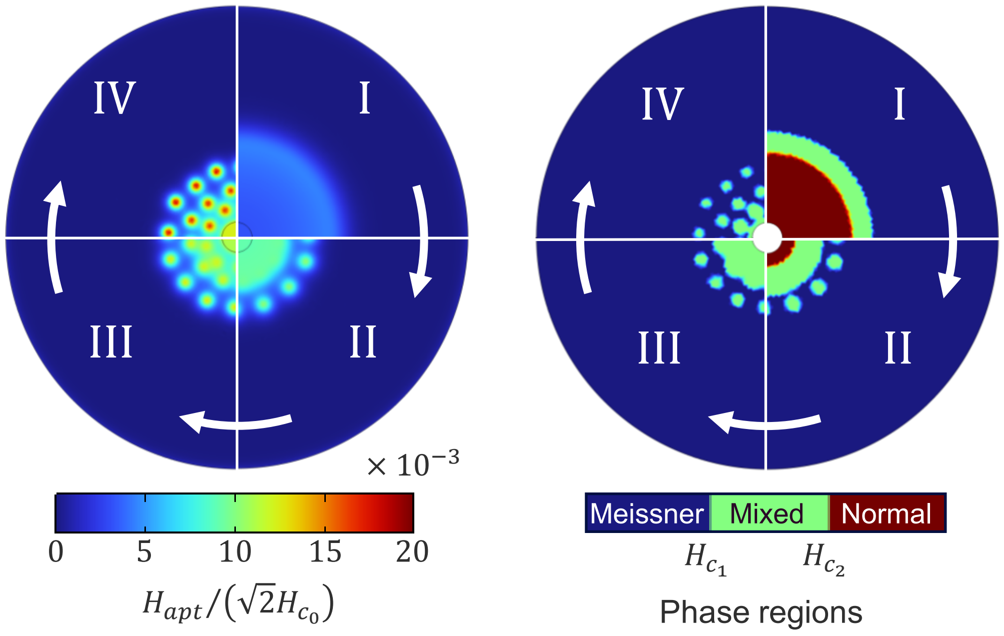


As predicted, the compression of the field

- **Widens the mixed state** region
- Pushes it to a cooler region
  - **Decreasing the vortex** diameter

Vortex form only in wide enough mixed state region





I. Beginning of vortex formation

II. The first row of vortices is fully formed

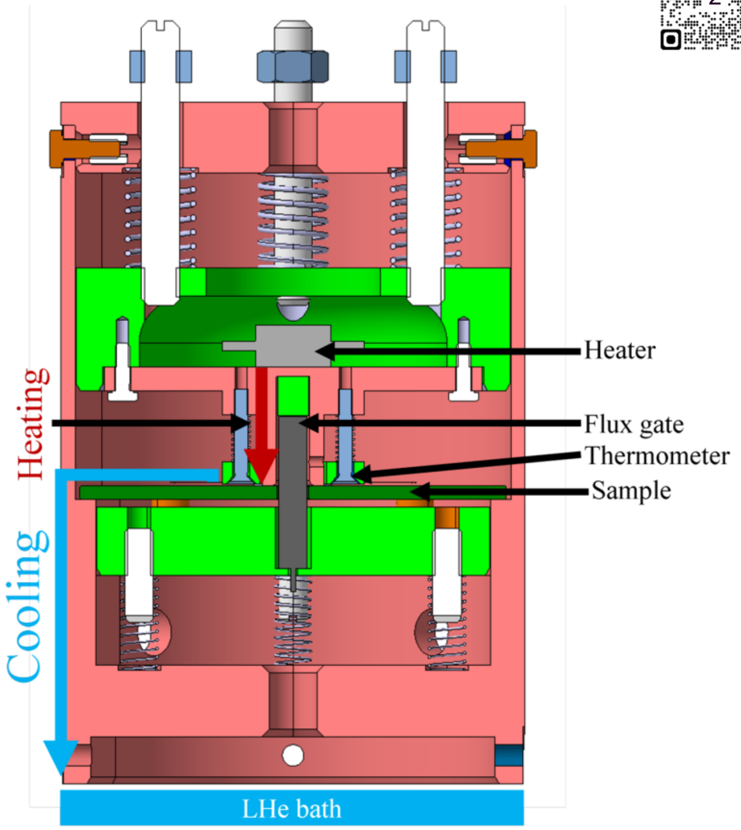
III. The mixed state front reaches the aperture radius

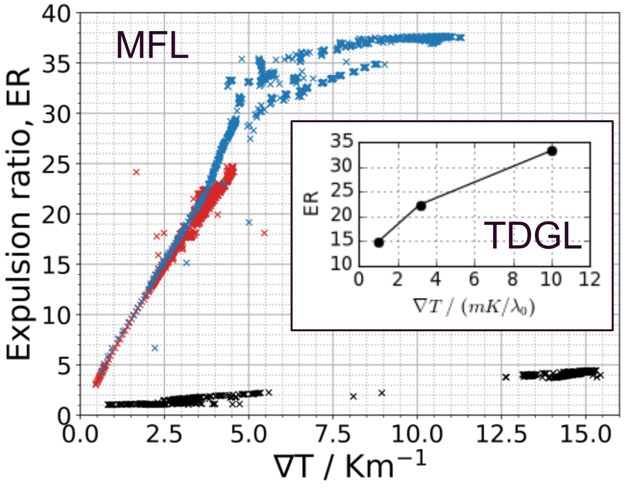
IV. End of vortex formation and phase transition

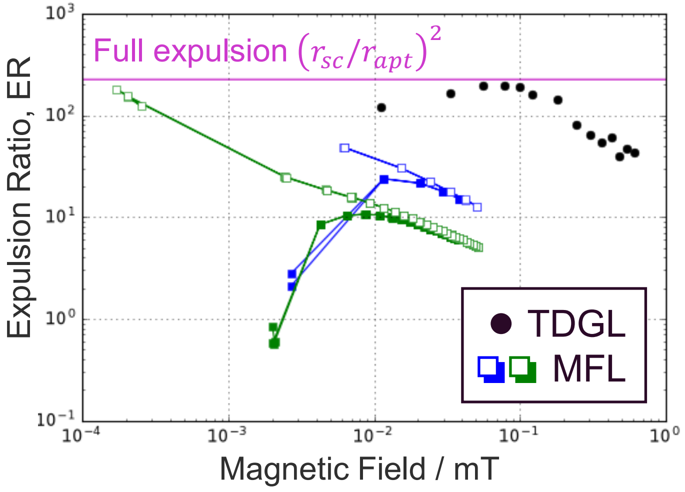
Flux penetrates only when there is **enough room** for vortices in the mixed state

Simulations vs. Experiments: matching behavior

Magnetic Flux Lens (MFL)





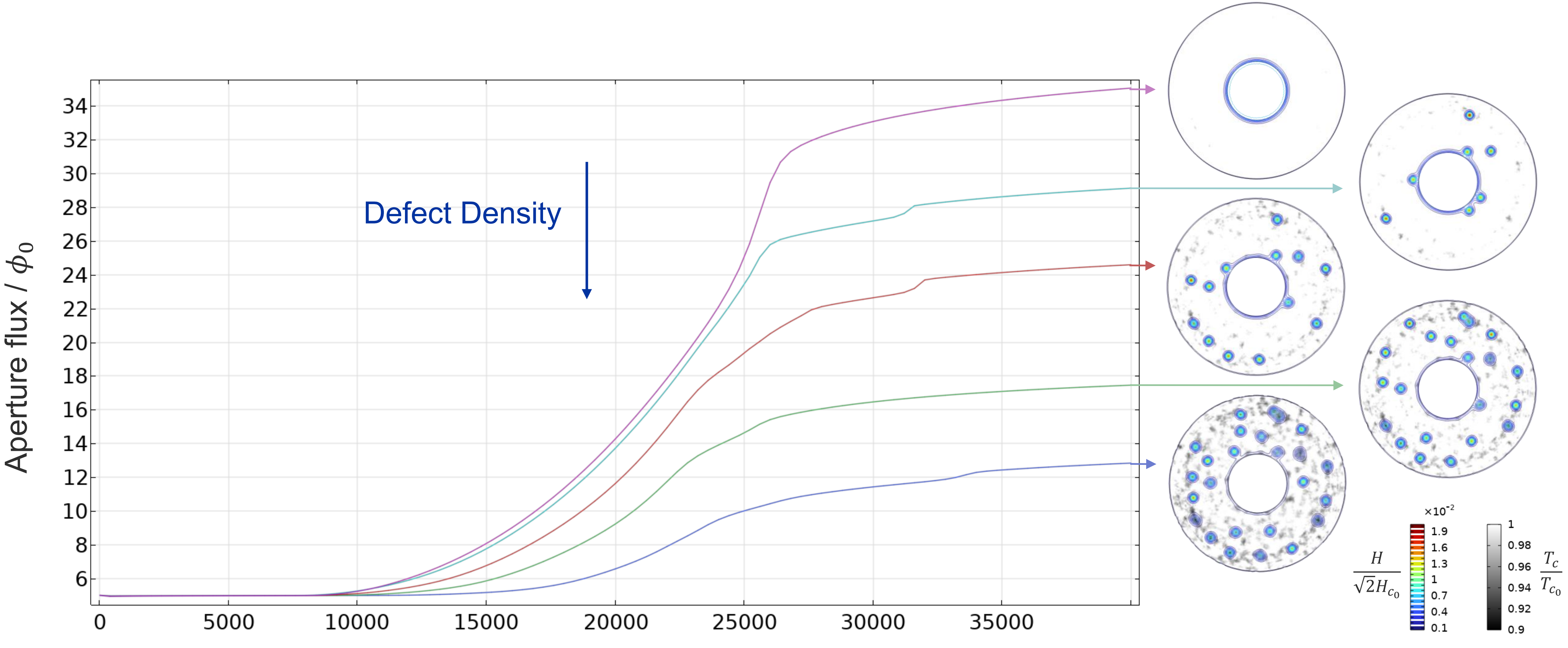


- MFL and TDGL show ER goes **up with ∇T** and **down with H_a**
- Results should be **comparable** if scaled accordingly
  - **Scaling factor** depends on ∇T, H_a and r_{sc}/r_{apt} (studies ongoing)
- Proper scaling will allow mapping experiments to simulations

Beyond the scaling factor, **flux trapping** and **extrinsic thermal current sources** also have to be considered.

MFL schematic and some of the data by D. Turner: https://proceedings.jacow.org/srf2023/papers/mopmb003.pdf

More defects means more trapped field (ongoing)



As expected, dense **pinning landscapes** can have a significant impact on the trapped flux.

4. Summary and Outlook

Summary and Outlook

Thermal Gradient
(analytic)

- Cool-down conditions affect the mixed state
- Vortices form within the mixed state
- Quality factor and performance drop

Closed Topology
(TDGL simulations)

- Inner magnetic flux is compressed
- Mixed state widens and displaces
- Flux penetrates as vortices more easily

Ongoing

- Scaling factor definition
  (mapping simulations to experiments)
- Pinning landscapes

Next Steps

- Thin films, multilayers
- 3D environment
- Extrinsic thermal currents and external field sources

Main take-away message:

- Closed-topology cooling facilitates flux penetration
- Negative impact in the performance of SRF cavities

Thank you!
Merci beaucoup!

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GL Parameter (material-dependent)

Temperature-dependent coefficients

$\alpha = 1 - \frac{T}{T_c}$

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Complex order parameter

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{1}{\kappa}\nabla + \mathbf{A}\right)^2 \Psi + (\alpha - \beta|\Psi|^2)\Psi$$

Magnetic vector potential

$$\frac{\partial \mathbf{A}}{\partial t} = \overbrace{-\nabla \times \underbrace{\nabla \times \mathbf{A}}_{\mathbf{B}}}^{\mathbf{J}} - |\Psi|^2 \left[\mathbf{A} - \frac{1}{\kappa}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*)\right]$$

Temperature-dependent material properties:

$$T = T_c - \nabla T(r - r_{sc} + vt) = T_c - \nabla T \delta r$$

$$\lambda = \kappa\xi = \frac{1}{\sqrt{\alpha}} = \left(1 - \frac{T}{T_c}\right)^{-\frac{1}{2}}$$

$$\lambda = \kappa\xi = \left(\frac{\nabla T}{T_c}\delta r\right)^{-\frac{1}{2}}$$

$$\frac{H_{c_1}}{H_{c_1}^*} = \frac{H_{c_2}}{H_{c_2}^*} = \alpha = 1 - \frac{T}{T_c}$$

$$\frac{H_{c_1}}{H_{c_1}^*} = \frac{H_{c_2}}{H_{c_2}^*} = \frac{\nabla T}{T_c}\delta r$$

Physics coupling in COMSOL Multiphysics®

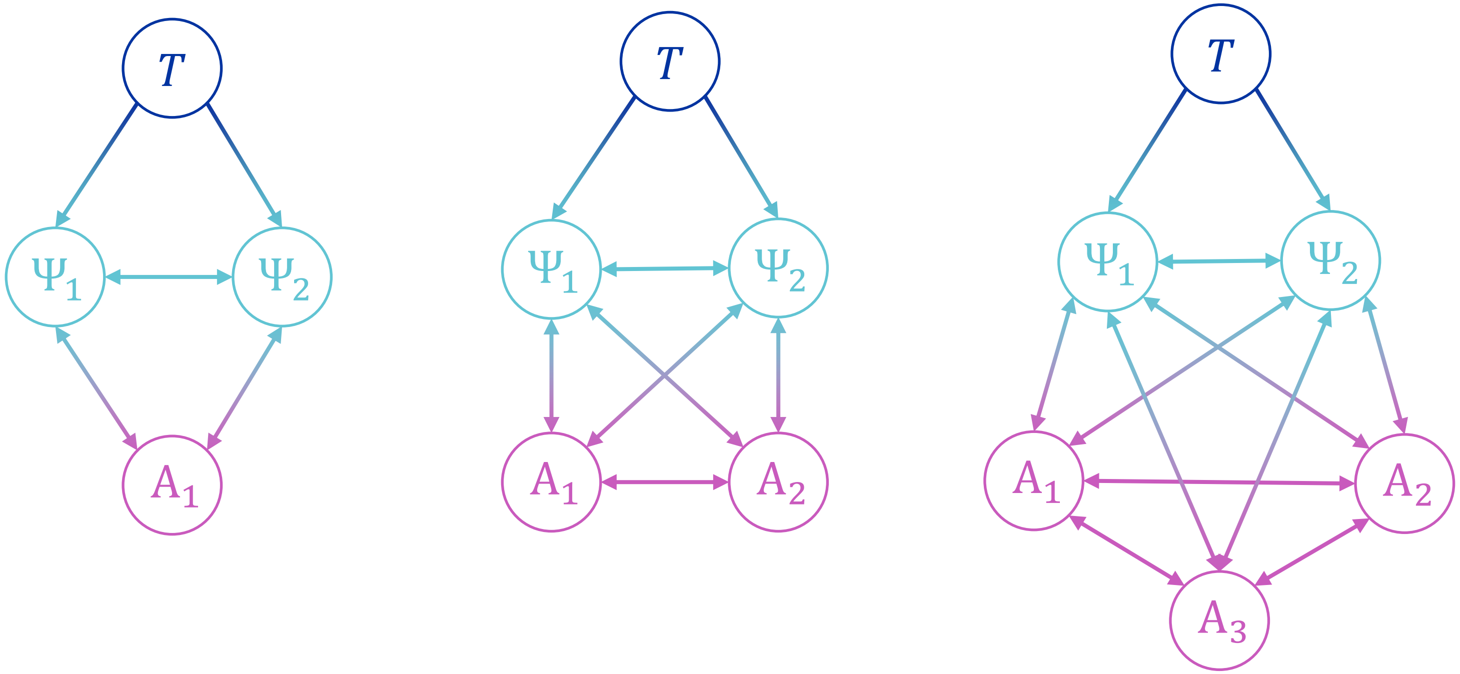
$$\rho C_p \dot{T} = \nabla \cdot (k \nabla T) + Q$$

or

$$T = T_c - \nabla T (r - r_{sc} + vt)$$

$$\Psi = \Psi_1 + i\Psi_2$$

$$\mathbf{A} = A_1\hat{x} + A_2\hat{y} + A_3\hat{z}$$



The model can have from three to six physics interfaces depending on the dimentionality of the problem and whether the temperature is calculated from heat inputs or directly imposed.

- Heat Transfer in Solids:
  - More realistic approach
  - Harder to correlate with ∇T
  - Hard coupling
- Temperature as an input:
  - Less realistic approach
  - Constant ∇T is an input
  - No coupling needed

Both options were implemented in a **2D** environment

The model in COMSOL Multiphysics®

$$\dot{\Psi}_1 = \frac{1}{\kappa^2}\left(\Psi_{1.xx} + \Psi_{1.yy}\right) + \frac{2}{\kappa}\left(A_1\Psi_{2.x} + A_2\Psi_{2.y}\right) + \frac{\Psi_2}{\kappa}\left(A_{1.x} + A_{2.y}\right) - \Psi_1(A_1^2 + A_2^2) + \Psi_1\alpha - \Psi_1(\Psi_1^2 + \Psi_2^2)$$

$$\dot{\Psi}_2 = \frac{1}{\kappa^2}\left(\Psi_{2.xx} + \Psi_{2.yy}\right) + \frac{2}{\kappa}\left(A_1\Psi_{1.x} + A_2\Psi_{1.y}\right) + \frac{\Psi_1}{\kappa}\left(A_{1.x} + A_{2.y}\right) - \Psi_2(A_1^2 + A_2^2) + \Psi_2\alpha - \Psi_2(\Psi_1^2 + \Psi_2^2)$$

$$\dot{A}_1 = -\frac{1}{\kappa}(\Psi_2\Psi_{1.x} - \Psi_1\Psi_{2.x}) - (\Psi_1^2 + \Psi_2^2)A_1 + A_{1.yy} - A_{2.xy}$$

$$\dot{A}_2 = -\frac{1}{\kappa}\left(\Psi_2\Psi_{1.y} - \Psi_1\Psi_{2.y}\right) - (\Psi_1^2 + \Psi_2^2)A_2 + A_{2.yy} - A_{1.xy}$$

$$\rho C_p\dot{T} = \nabla\cdot(k\nabla T) + Q$$

Summary and Outlook (in text)

- Cavity **performance is affected by trapped field**, which can change **depending on the cool-down** conditions.

- **Vortex formation** during transition is determined by the **phase interfaces delimiting the mixed state**, which in turn is impacted by the **applied magnetic field and the thermal gradient**.

- To evaluate transition in a closed thermal topology, a detailed **multiphysics model** based on **time-dependent Ginzburg-Landau** equations was implemented accouting for the **temperature dependence** of material properties.

- Analytical development and simulation results agree in that, in a closed topology, the **inner field** delimited by the expanding SC region **increases over time**, changing the **width and position of the mixed state**. This leads to an increasing ratio between mixed state width and vortex diameter, up to a point when the mixed state is **wide enough to hold vortices**, enabling their formation.

- Investigation on the effects of **pinning and a precise definition of the scaling factor are ongoing**, to achieve a better comparison between simulation and experiments.

- The simulation model will be expanded into a **three-dimensional environment**, to account for assymetries along the z-axis, **extrinsic sources of thermal currents** and **thin films and multilayer superconductors**.