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MULTILAYER MODEL FOR COATINGS WITH ARBITRARY NUMBER OF PLANAR LAYERS FOR SRF APPLICATIONS

International Workshop on Thin Films, Chester, United Kingdom

Monday 21st September, 2026

CONTENTS

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MOTIVATION

Section 1

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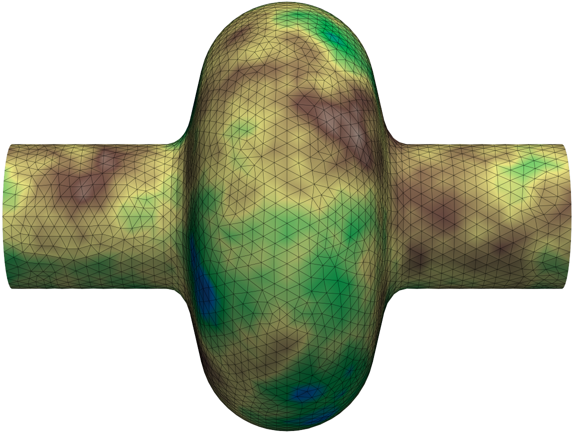
Coating thickness as Gaussian random field on cavity boundary



- Deposition on complex geometry:
  - $\Rightarrow$ Inhomogeneous thin layer thickness distribution
  - $\Rightarrow$ Can we see effects on e.g. quality factor?
- Numerical methods need a surface impedance description $Z(\boldsymbol{r}, \omega, T)$.

How do we obtain such a surface impedance model?

Kubo model + complex Poynting theorem

A. Gobeyn, W. Ackermann, H. De Gersem, arXiv:2605.05505 [Phys. Rev. Accel. Beams (accepted)] (2026).

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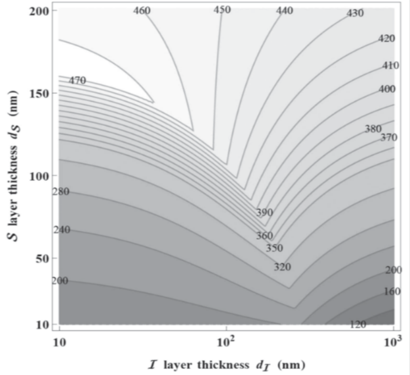

Figure 11. B_{max} of a Nb_3Sn–I–Nb system in units of mT. Assumed parameters are $B_c^{(Nb_3Sn)} = 540$ mT and $\lambda_1 = 120$ nm for the S-layer material, $B_{max}^{(sub)} = 170$ mT and $\lambda_2 = 40$ nm for the substrate. See also [39].

T. Kubo, Supercond. Sci. Technol. **30**, 023001 (2017).

- Can we arbitrarize the number of planar layers?
- Can we arbitrarize the local optical conductivity model $\sigma(\omega, T)$?
- Can we find a configuration with > 3 layers where each individual layer has thickness $< \lambda$?

“[...] The S layer must be as thin as the penetration depth λ, otherwise it may be regarded as just a bulk material and then lead to a thermal quench [...]”

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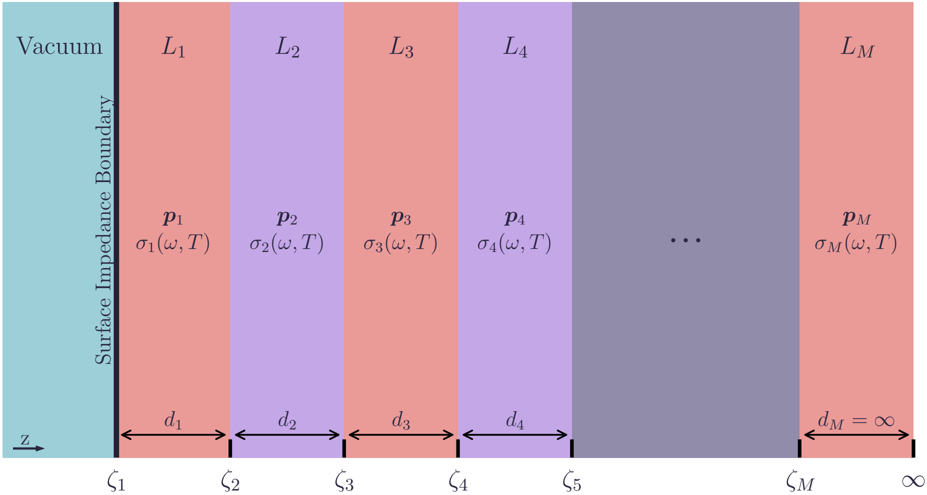
MODEL

Section 2


Vacuum
L_1
L_2
L_3
L_4
L_M
Surface Impedance Boundary
$\boldsymbol{p}_1$ $\sigma_1(\omega, T)$
$\boldsymbol{p}_2$ $\sigma_2(\omega, T)$
$\boldsymbol{p}_3$ $\sigma_3(\omega, T)$
$\boldsymbol{p}_4$ $\sigma_4(\omega, T)$
$\cdots$
$\boldsymbol{p}_M$ $\sigma_M(\omega, T)$
z
d_1
d_2
d_3
d_4
$d_M = \infty$
ζ_1
ζ_2
ζ_3
ζ_4
ζ_5
ζ_M
∞

In each layer the Helmholtz equations hold

$$\nabla^2 \boldsymbol{\mathcal{E}} + \alpha^2 \boldsymbol{\mathcal{E}} = 0, \quad \nabla^2 \boldsymbol{\mathcal{H}} + \alpha^2 \boldsymbol{\mathcal{H}} = 0,$$

Solution in each layer $k \in \{1, \dots, M\}$ yields

$$\mathcal{E}_x^{(k)}(z) = \boldsymbol{C}_k^T \boldsymbol{u}_k(z), \quad \mathcal{H}_y^{(k)}(z) = \frac{\alpha_k}{\omega \mu_k} \boldsymbol{C}_k^T \boldsymbol{v}_k(z), \quad z \in (\zeta_k, \zeta_{k+1}),$$

where

$$\boldsymbol{C}_k = \begin{pmatrix} A_k \\ B_k \end{pmatrix}, \quad \boldsymbol{u}_k(z) = \begin{pmatrix} e^{\jmath \alpha_k z} \\ e^{-\jmath \alpha_k z} \end{pmatrix}, \quad \boldsymbol{v}_k(z) = \begin{pmatrix} e^{\jmath \alpha_k z} \\ -e^{-\jmath \alpha_k z} \end{pmatrix}.$$

Furthermore, $\alpha_k = \alpha_k(\omega, T)$ depends on the material parameters and local optical conductivity model $\sigma_k(\omega, T)$

$$\alpha_k(\omega, T) = \sqrt{\omega^2 \varepsilon_k \mu_k + \jmath \omega \mu_k \sigma_k(\omega, T)}.$$

Continuity of the fields across interfaces and decay to zero at infinity yields linear relations between unknown coefficients

$$A_1 - B_1 = \frac{\omega\mu_1}{\alpha_1} H_0, \quad B_M = 0, \quad \boldsymbol{C}_k = \tau_{k,k+1}\boldsymbol{C}_{k+1}, \quad k \in \{1, \dots, M-1\}, \quad \tau_{k,k+1} \in \mathbb{C}^{2\times 2}.$$

The first layer coefficients can be related to the final layer coefficients by

$$\boldsymbol{C}_1 = \left[\prod_{k=1}^{M-1} \tau_{k,k+1}\right] \boldsymbol{C}_M := T\boldsymbol{C}_M$$

and using the other conditions solve the unknown coefficient of the last layer

$$A_M = \frac{\omega\mu_1}{\alpha_1 [T_{00} - T_{10}]} H_0$$

Via the recursion relation all other coefficients are fixed, and hence the electromagnetic fields determined.

The maximum applicable field is determined by asking for each superconducting layer L_k:

What must the value of H_0 have been such that the depairing limit is reached when the layer first encounters the field?

We call the answer to that question $H_{0,\lim}^{(k)}$. The maximum applicable field is then determined by the smallest of these values, i.e.

$$B_{\max} = \min_k \left\{ \mu_k H_{0,\lim}^{(k)} \right\}$$

The surface impedance can be determined by applying the Leontovich boundary condition

$$\boldsymbol{n} \times \boldsymbol{\mathcal{E}} = Z(\omega, T)[\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{\mathcal{H}})],$$

which yields

$$Z(\omega, T) = \frac{\omega \mu_1}{\alpha_1} \frac{T_{00} + T_{10}}{T_{00} - T_{10}}$$

S. V. Yuferev and N. Ida, *Surface impedance boundary conditions: A comprehensive approach* (CRC Press, 2010).

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RESULTS

Section 3

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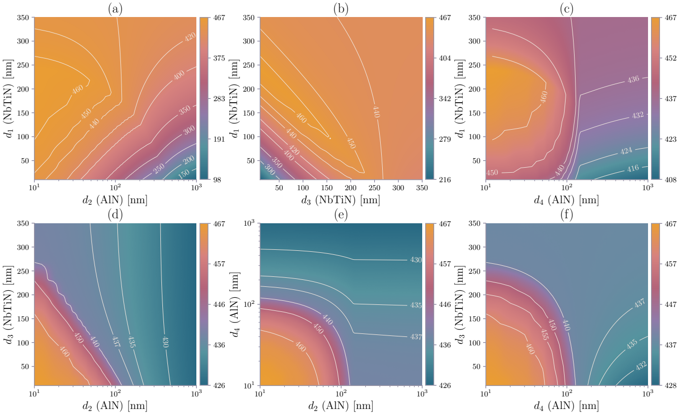
Maximum applicable field of $(NbTiN\text{-}AlN)^2\text{-}Nb$ in mT



- Optimally, reduce to three layers with $d_{\mathrm{NbTiN}} = 260\,\mathrm{nm}$ and $d_{\mathrm{AlN}} = 10\,\mathrm{nm}$, but ...
- using two NbTiN layers of $150\,\mathrm{nm}$ thickness, only $15\,\mathrm{mT}$ less than optimum.
- Penetration depth of NbTiN is $180.57\,\mathrm{nm}$!

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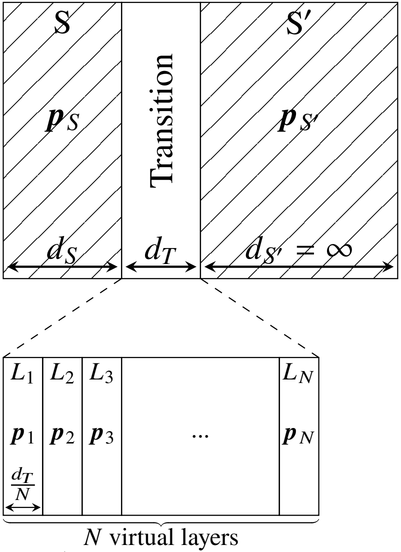
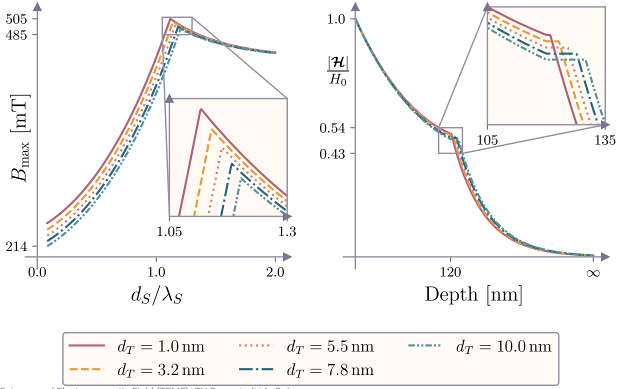
Model transition between two superconductors with a number of virtual layers with individually assignable material parameters

(Nb_3Sn-Nb)





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CONCLUSION

Section 4

- Developed a model analogous to Kubo model but with arbitrary number of planar layers with arbitrary local optical conductivity model.
- Determined the electromagnetic fields, maximum applicable field and surface impedance for this model.
- According to the model, adding an additional pair of SI layers can decrease the individual superconducting thin layer thicknesses to below their penetration depth.
- Proposed idea to use the model for modeling transition layers by inserting virtual layers with different material parameters.

We have written a paper on the topic, but have not submitted it yet. We would like the input of experts on the topic (you!) before doing so. Thanks in advance.

A. Gobeyn, W. Ackermann and H. De Gersem, arXiv:2605.07958 (2026).