Unveiling proton-neutron pairing in homogeneous and isotropic nuclear matter

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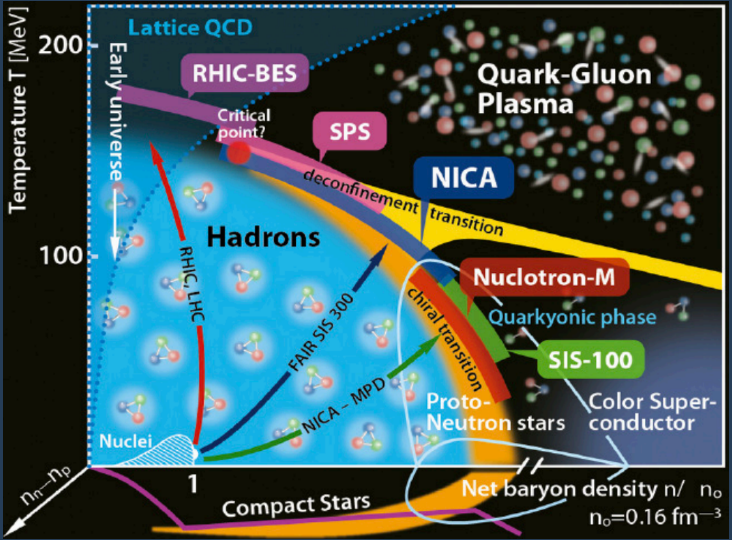
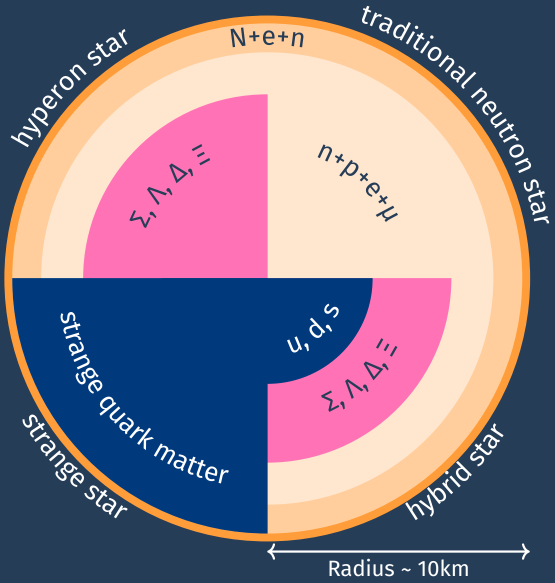
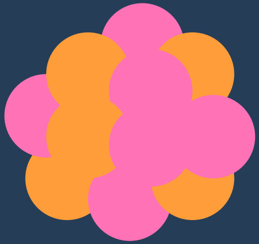
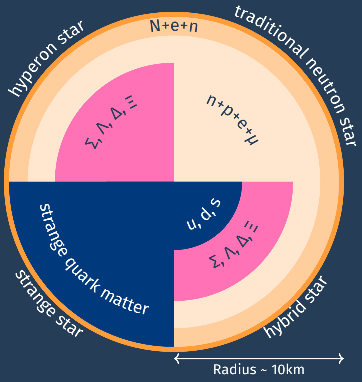

Lattice QCD
Temperature T [MeV]
200
100
Early universe
RHIC-BES
Critical point?
SPS
NICA
Quark-Gluon Plasma
deconfinement transition
Hadrons
Nuclotron-M
Quarkyonic phase
RHIC, LHC
FAIR SIS 300
chiral transition
SIS-100
NICA – MPD
Proto-Neutron stars
Color Super-conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/n_0
$n_0 = 0.16\ fm^{-3}$


hyperon star
N+e+n
traditional neutron star
Σ, Λ, Δ, Ξ
n+p+e+μ
strange quark matter
u, d, s
Σ, Λ, Δ, Ξ
strange star
hybrid star
Radius ~ 10km

B.C. Backes




hyperon star
N+e+n
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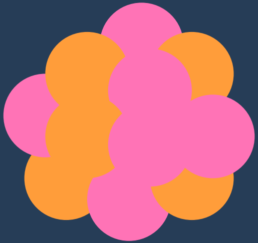
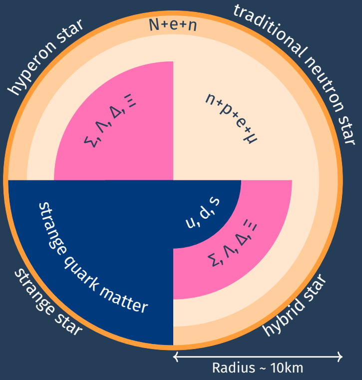




$$E[\rho]$$

a universal Energy Density Functional

Unveiling proton-neutron pairing in homogeneous and isotropic nuclear matter

Unveiling proton-neutron pairing in **homogeneous and isotropic nuclear matter**

$$\underbrace{|Z\rangle = \exp\left\{\hat{Z}^{+}\right\}|0\rangle}_{\text{Thouless wave function}}$$

$$\hat{Z}^{+} = \frac{1}{2}\sum_{\mu\nu} Z^{*}_{\mu\nu} a^{+}_{\mu} a^{+}_{\nu}$$

Thouless pair creation operator

$$\hat{Z}^{+} = \frac{1}{2}\sum_{\mu\nu} Z^{*}_{\mu\nu} a^{+}_{\mu} a^{+}_{\nu}$$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$

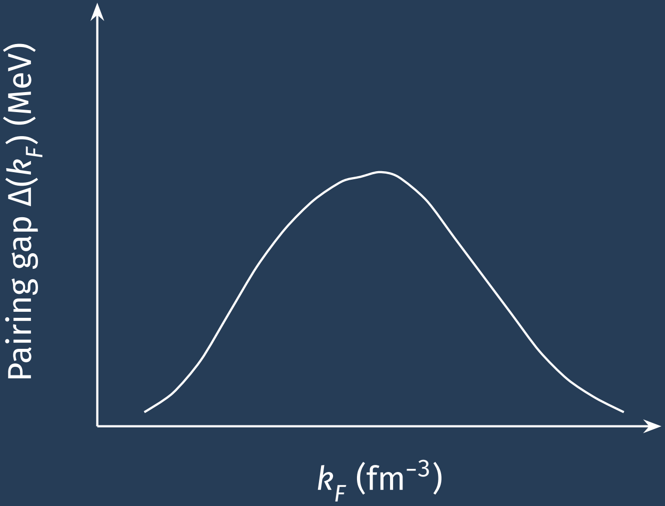
Thouless pair creation operator

$$\hat{Z}^+ = \frac{1}{2}\sum_{\mu\nu} Z^*_{\mu\nu} a^+_\mu a^+_\nu$$

Thouless pair creation operator

$$\kappa_{\mu\nu} = \frac{\langle Z | a_\nu a_\mu | Z \rangle}{\langle Z | Z \rangle}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Pairing tensor}}$$



Pairing gap Δ(k_F) (MeV)
k_F (fm^{-3})

What can we learn about INM building up from the wave functions?

Unveiling proton-neutron pairing in **homogeneous and isotropic nuclear matter**

Unveiling proton-neutron pairing in homogeneous and isotropic nuclear matter

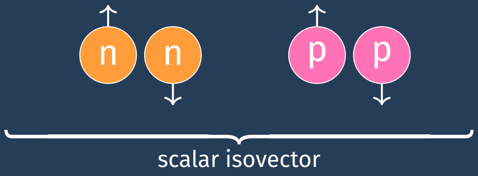
on which ways can we pair two nucleons?

$$|1\rangle \equiv |{\uparrow}\, n\rangle$$

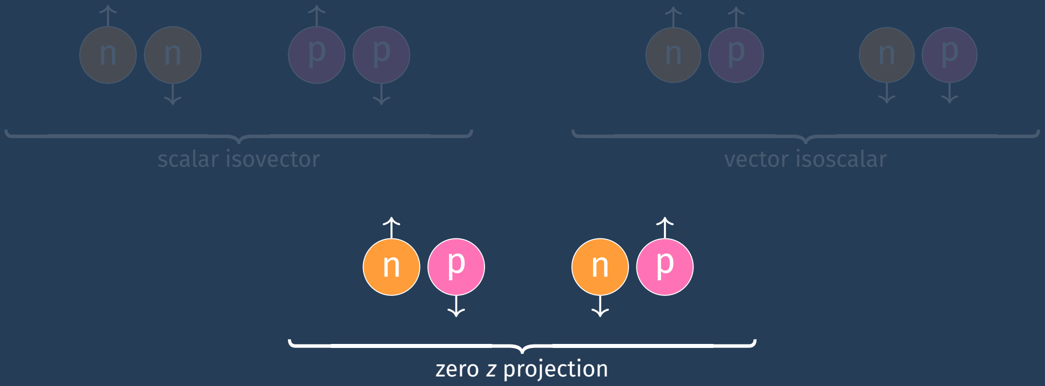
$$|2\rangle \equiv |{\downarrow}\, n\rangle$$

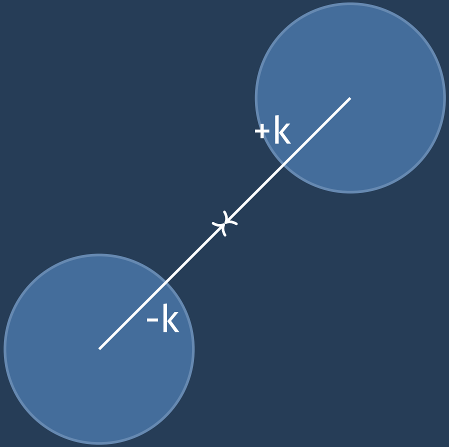
$$|3\rangle \equiv |{\uparrow}\, p\rangle$$

$$|4\rangle \equiv |{\downarrow}\, p\rangle$$


n
n
p
p
scalar isovector

n
n
p
p
scalar isovector
n
p
n
p
vector isoscalar


n n
p p
scalar isovector
n p
n p
vector isoscalar
n p
n p
zero z projection


+k
-k

$$|1\rangle \equiv |{\uparrow} n\rangle$$

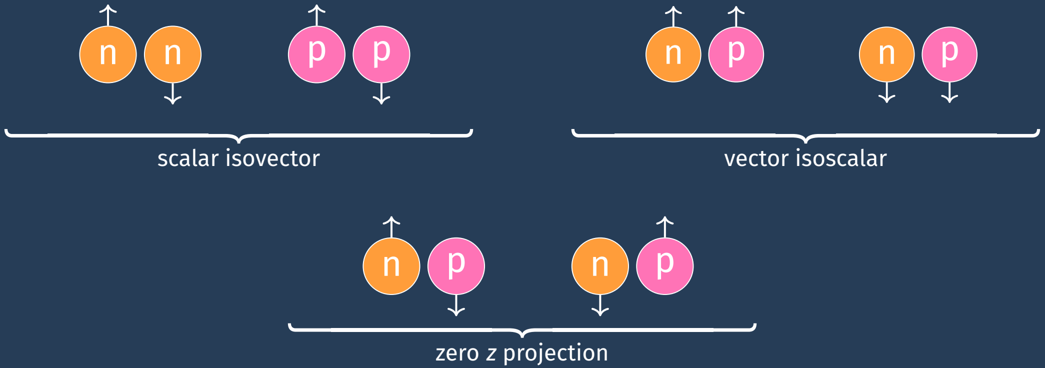
$$|2\rangle \equiv |{\downarrow} n\rangle$$

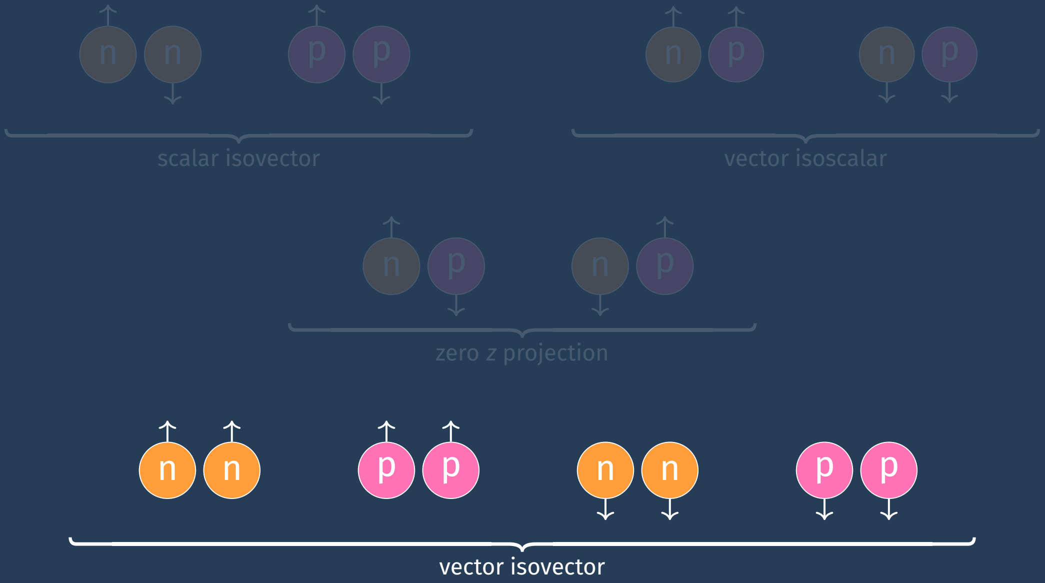
$$|3\rangle \equiv |{\uparrow} p\rangle$$

$$|4\rangle \equiv |{\downarrow} p\rangle$$

$$|1\rangle \equiv |+\mathbf{k} \uparrow n\rangle, \quad |5\rangle \equiv |-\mathbf{k} \uparrow n\rangle,$$
$$|2\rangle \equiv |+\mathbf{k} \downarrow n\rangle, \quad |6\rangle \equiv |-\mathbf{k} \downarrow n\rangle,$$
$$|3\rangle \equiv |+\mathbf{k} \uparrow p\rangle, \quad |7\rangle \equiv |-\mathbf{k} \uparrow p\rangle,$$
$$|4\rangle \equiv |+\mathbf{k} \downarrow p\rangle, \quad |8\rangle \equiv |-\mathbf{k} \downarrow p\rangle$$

$$\tilde{Z}^{+} = \begin{pmatrix} 0 & Z^{+} \\ -Z^{*} & 0 \end{pmatrix}$$


n n
p p
scalar isovector
n p
n p
vector isoscalar
n p
n p
zero z projection



n n p p
scalar isovector
n p n p
vector isoscalar
n p n p
zero z projection
n n p p n n p p
vector isovector

$$\hat{Z}^{+} = z_{n}^{*} a^{+}_{\uparrow n} a^{+}_{\downarrow n} + z_{p}^{*} a^{+}_{\uparrow p} a^{+}_{\downarrow p} + z_{\uparrow}^{*} a^{+}_{\uparrow n} a^{+}_{\uparrow p} + z_{\downarrow}^{*} a^{+}_{\downarrow n} a^{+}_{\downarrow p}$$

$$+ z_{+}^{*} a^{+}_{\uparrow n} a^{+}_{\downarrow p} + z_{-}^{*} a^{+}_{\downarrow n} a^{+}_{\uparrow p} + z_{\uparrow n}^{*} a^{+}_{\uparrow n} a^{+}_{\uparrow n} + z_{\uparrow p}^{*} a^{+}_{\uparrow p} a^{+}_{\uparrow p}$$

$$+ z_{\downarrow p}^{*} a^{+}_{\downarrow p} a^{+}_{\downarrow p} + z_{\downarrow n}^{*} a^{+}_{\downarrow n} a^{+}_{\downarrow n}$$

$$\hat{Z}^{+} = z_n^* a^+_{\uparrow n} a^+_{\downarrow n} + z_p^* a^+_{\uparrow p} a^+_{\downarrow p} + z_\uparrow^* a^+_{\uparrow n} a^+_{\uparrow p} + z_\downarrow^* a^+_{\downarrow n} a^+_{\downarrow p}$$

$$+ z_+^* a^+_{\uparrow n} a^+_{\downarrow p} + z_-^* a^+_{\downarrow n} a^+_{\uparrow p} + z_{\uparrow n}^* a^+_{\uparrow n} a^+_{\uparrow n} + z_{\uparrow p}^* a^+_{\uparrow p} a^+_{\uparrow p}$$

$$+ z_{\downarrow p}^* a^+_{\downarrow p} a^+_{\downarrow p} + z_{\downarrow n}^* a^+_{\downarrow n} a^+_{\downarrow n}$$

$$\langle Z|Z\rangle = \underbrace{\det(1 + ZZ^{+})^{1/2}}_{\text{via Onishi theorem}}$$

$$\kappa_{\mu\nu} = \frac{\partial}{\partial Z_{\mu\nu}} \log\langle Z | Z \rangle$$

!

this approach becomes too complicated with the most general proton-neutron pairs

switch to the ST basis,
look at pairs with zero z projection

$|S S_z T T_z\rangle$

$$|0000\rangle = \frac{1}{2}\left(n{\uparrow}\,p{\downarrow} - p{\uparrow}\,n{\downarrow} + p{\downarrow}\,n{\uparrow} - n{\downarrow}\,p{\uparrow} \right)$$

$$|1000\rangle = \frac{1}{2}\left(n{\uparrow}\,p{\downarrow} - p{\uparrow}\,n{\downarrow} - p{\downarrow}\,n{\uparrow} + n{\downarrow}\,p{\uparrow} \right)$$

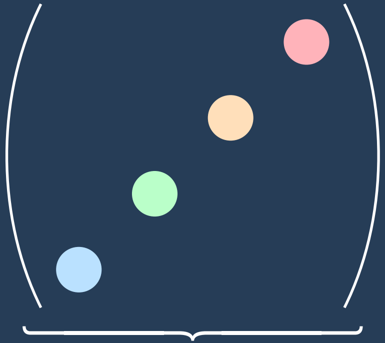
$$|0010\rangle = \frac{1}{2}\left(n{\uparrow}\,p{\downarrow} + p{\uparrow}\,n{\downarrow} - p{\downarrow}\,n{\uparrow} - n{\downarrow}\,p{\uparrow} \right)$$

$$|1010\rangle = \frac{1}{2}\left(n{\uparrow}\,p{\downarrow} + p{\uparrow}\,n{\downarrow} + p{\downarrow}\,n{\uparrow} + n{\downarrow}\,p{\uparrow} \right)$$

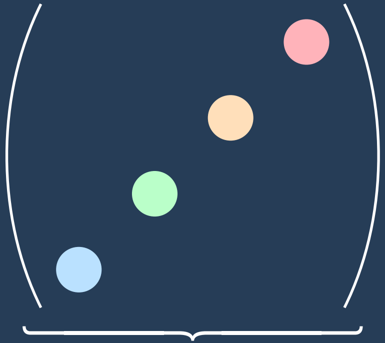
$$Z_z^+ = z_{00}^* |0000\rangle + z_{10}^* |1000\rangle + z_{01}^* |0010\rangle + z_{11}^* |1010\rangle$$

$$Z_z^+ = \begin{pmatrix} 0 & 0 & 0 & z_+^* \\ 0 & 0 & z_-^* & 0 \\ 0 & \bar{z}_-^* & 0 & 0 \\ \bar{z}_+^* & 0 & 0 & 0 \end{pmatrix}$$

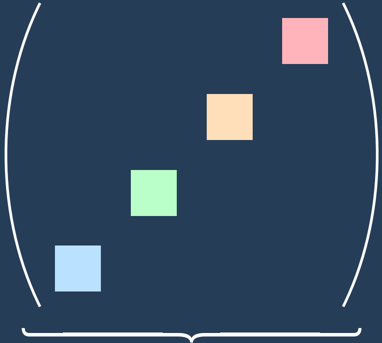
$$\langle Z|Z\rangle = (1 + |z_+|^2)(1 + |z_-|^2)(1 + |\bar{z}_+|^2)(1 + |\bar{z}_+|^2)$$



structure of Z_z^+



structure of Z_z^+



structure of κ

$$\kappa_{\mu\nu} = \frac{\partial}{\partial Z_{\mu\nu}} \log \det(1 + ZZ^{+})^{1/2}$$

$$\kappa_{\mu\nu} = \frac{\partial}{\partial Z_{\mu\nu}} \log \underbrace{\det(1 + ZZ^{+})^{1/2}}_{\text{invariant under rotations}}$$

$$\kappa_{\mu\nu}^{\mathrm{rot}} = \frac{1}{\langle Z_z | Z_z \rangle} \frac{\partial}{\partial Z_{\mu\nu}^{\mathrm{rot}}} \langle Z_z | Z_z \rangle$$

$$\kappa_{\mu\nu}^{\text{rot}} = \frac{1}{\langle Z_z | Z_z \rangle} \frac{\partial}{\partial Z_{\mu\nu}^{\text{rot}}} \langle Z_z | Z_z \rangle$$

!

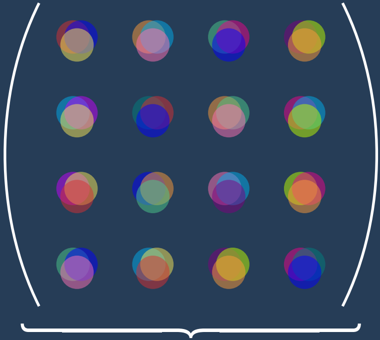
much richer structure!

all pairing channels are mixed

an example: $z^*_{11} = 0$ (no $|1010\rangle$ pairing)

structure of Z^{+}_{rot}

structure of Z^{+}_{rot}



structure of κ_{rot}

interaction-independent analysis of pairing in nuclear matter

interaction-independent analysis of pairing in nuclear matter

derived properties of the most general paired wave function

interaction-independent analysis of pairing in nuclear matter

derived properties of the most general paired wave function

!

pairing channels are incredibly mixed in nuclear matter

interaction-independent analysis of pairing in nuclear matter

derived properties of the most general paired wave function

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pairing channels are incredibly mixed in nuclear matter

!

cannot infer a structure for κ from the wave function

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backup slides

$$(Z^{+}_{\mathrm{rot}})_{11} = \frac{z^{*}_{11}}{2} \sin \beta_T{}^{-i\alpha_T} \sin \beta_S{}^{-i\alpha_S}$$

$$z^{*}_{11} = z^{*}_{+} + z^{*}_{-} + \bar{z}^{*}_{+} + \bar{z}^{*}_{-}$$

$$\kappa_{11}^{\mathrm{rot}} = C(\alpha_{S,T}, \beta_{S,T}) \left[\frac{z_+^*}{1+|z_+|^2} + \frac{\bar{z}_+^*}{1+|\bar{z}_+|^2} + \frac{z_-^*}{1+|z_-|^2} + \frac{\bar{z}_-^*}{1+|\bar{z}_-|^2} \right]$$