Radiative decay and electromagnetic moments of ^{229}Th from nuclear DFT

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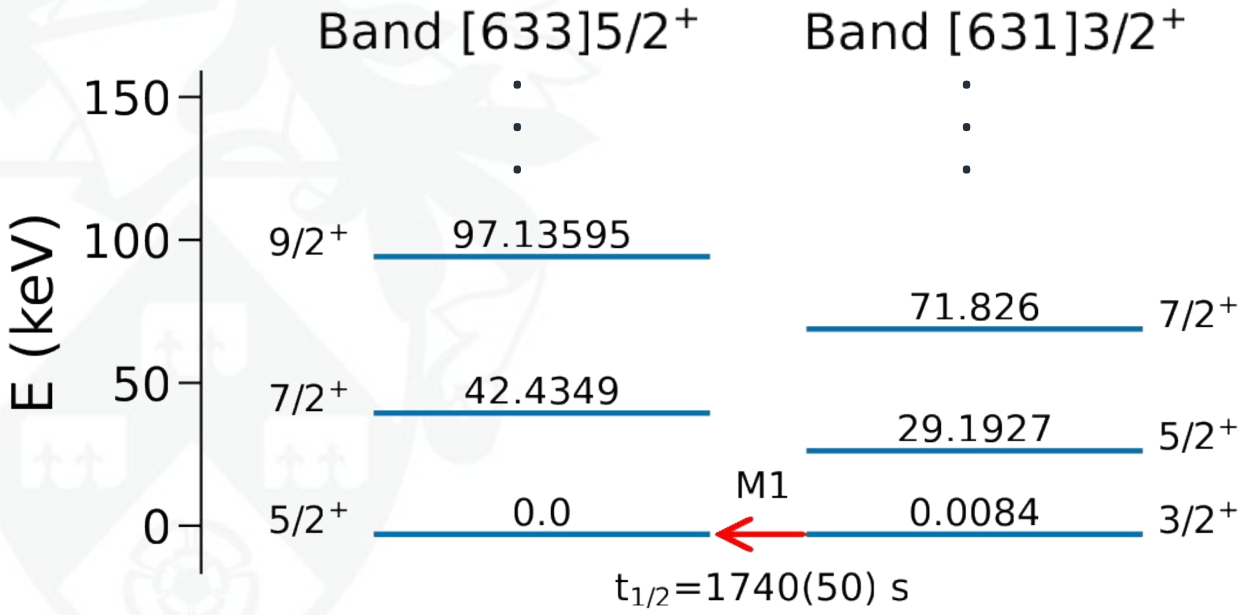
The distinctive property of ^{229}Th

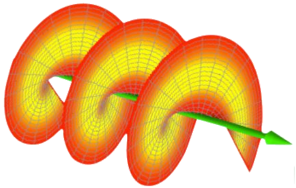


E_γ = 8.35574(3) eV

v = 148.3821(5) nm

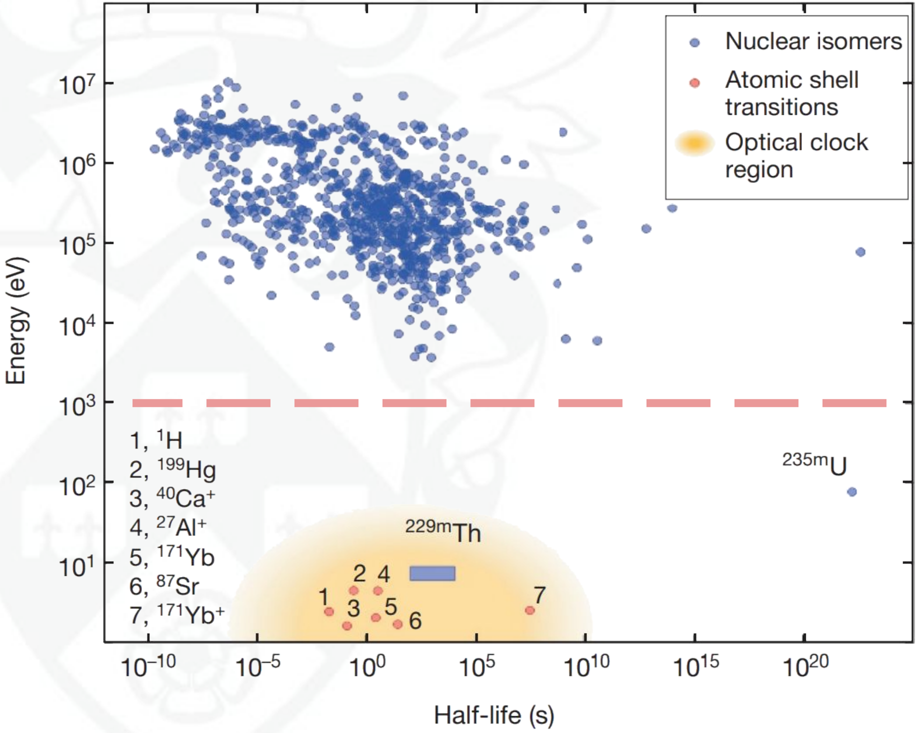
$t_{1/2}$ ≈ 29 min in vacuum

Dominant M1 multipole radiation



Adapted from Wikimedia Commons

^{229}Th as a tool for precise chronometry



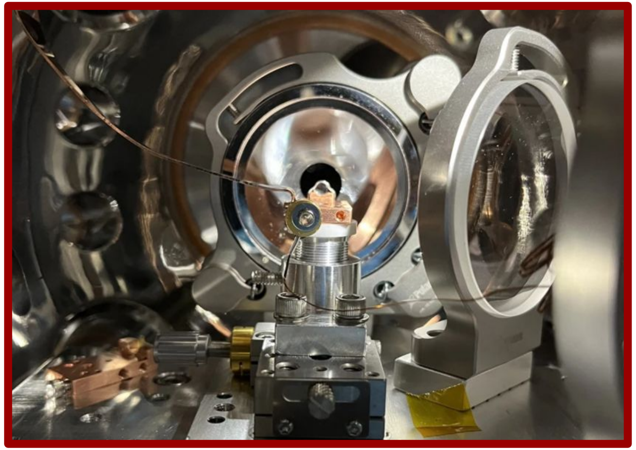
Nature 533 (2016) 47–51

nature

NEWS | 20 March 2026

Elusive 'nuclear clocks' tick closer to reality — after decades in the making

Super-precise timekeepers based on atomic nuclei could be tested as soon as this year.



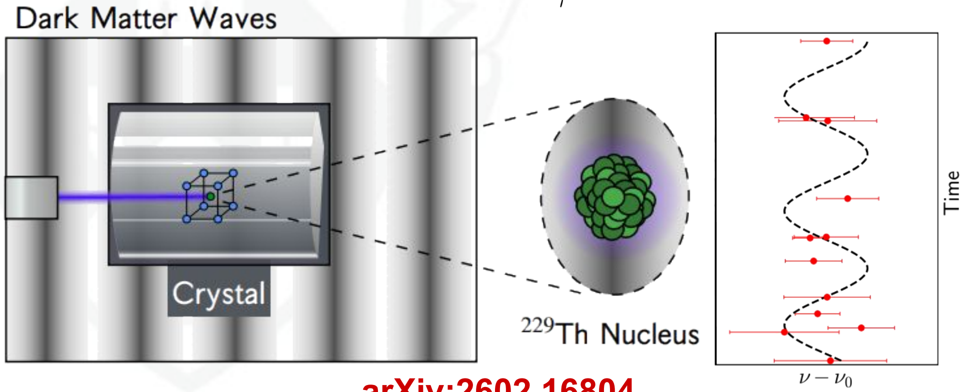
^{229}Th as a probe for new physics

Searches for time variation of the fine structure constant and its mechanism

$$\frac{\dot{\nu}}{\nu} = \frac{\Delta V_C}{E_\gamma}\frac{\dot{\alpha}}{\alpha}$$

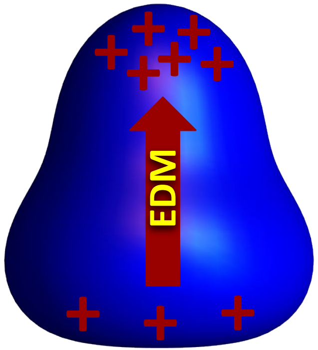
$$\frac{\dot{\alpha}}{\alpha} \lesssim 10^{-17}\ \mathrm{yr}^{-1}$$

$$\frac{\Delta V_C}{E_\gamma} \sim 10^4$$



arXiv:2602.16804

PT violating interactions and the permanent electric dipole moment



Schiff moment

Octupole enhancement

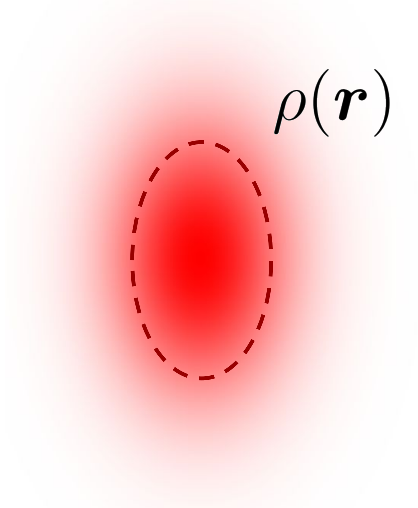
$$d_{\mathrm{SM}}(^{199}\mathrm{Hg}) \sim 10^{-34}\ \mathrm{e\cdot cm}$$
$$d_{\mathrm{Expt.}}(^{199}\mathrm{Hg}) < 10^{-30}\ \mathrm{e\cdot cm}$$

JPS Conf. Proc. 012034 (2018)
Nature 648 (2025) 562–568

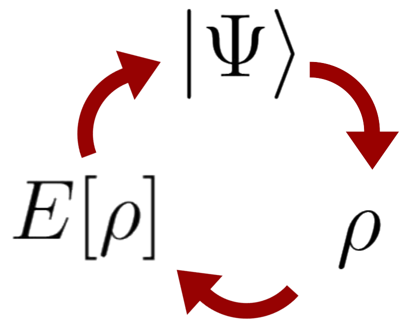
The DFT approach

$$\mathcal{O} \sim \langle \Phi | \hat{O} | \Psi \rangle$$

?



Self-consistency



One-to-one correspondence

$$|\Psi\rangle \iff \rho$$

$$\uparrow$$

$$|\Psi'_{\text{indep.}}\rangle$$

The nuclear energy density functional

$$E[\rho] = E^{\text{Kin.}} + E^{\text{Pot.}} + E^{\text{Coul.}} + E^{\text{Pair.}}$$

The Skyrme EDF

Time-even terms

$$E^{\text{Skyrme}}_{\text{pot.}}[\rho] = \int dV \sum_t \Big\{ C_t^{\rho\rho}\rho_t^2 + C_t^{\rho\Delta\rho}\rho_t\Delta\rho_t + C_t^{\rho\tau}(\rho_t\tau_t - \mathbb{J}_t\cdot\mathbb{J}_t)$$
$$+ C_t^{ss}\boldsymbol{s}_t^2 + C_t^{s\Delta s}\boldsymbol{s}_t\cdot\Delta\boldsymbol{s}_t + C_t^{\rho\nabla J}(\rho_t\nabla\cdot\tilde{\mathbf{J}}_t + \boldsymbol{j}_t\cdot\nabla\times\boldsymbol{s}_t)$$
$$+ C_t^{s\nabla s}(\nabla\cdot\boldsymbol{s}_t)^2 + C_t^{JJ}(\mathbb{J}_t:\mathbb{J}_t - \boldsymbol{s}_t\cdot\boldsymbol{T}_t)$$
$$+ C_t^{J\bar{J}}\Big[(\mathrm{Tr}(\mathbb{J}_t))^2 + \mathbb{J}_t:\mathbb{J}_t^T - 2\mathbf{s}_t\cdot\mathbf{F}_t\Big]\Big\}$$

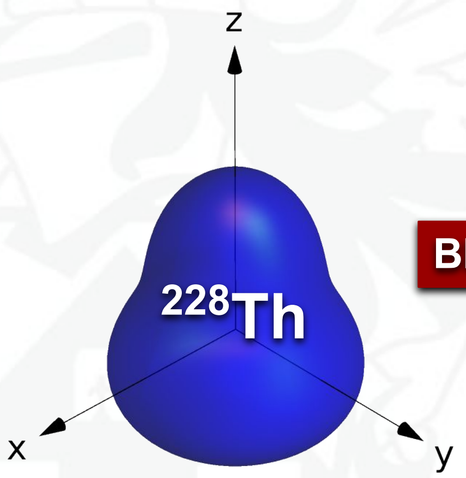
Coupling constants

Time-odd terms

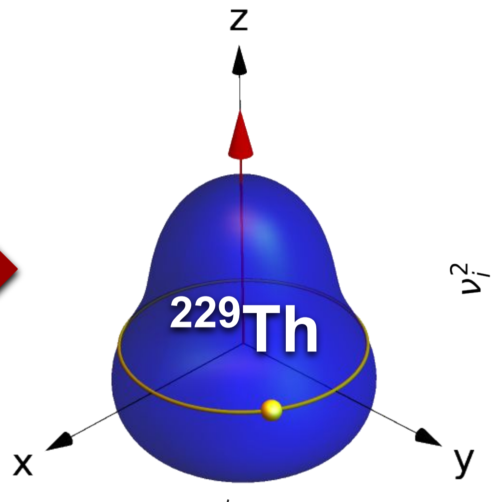
Popular parametrizations

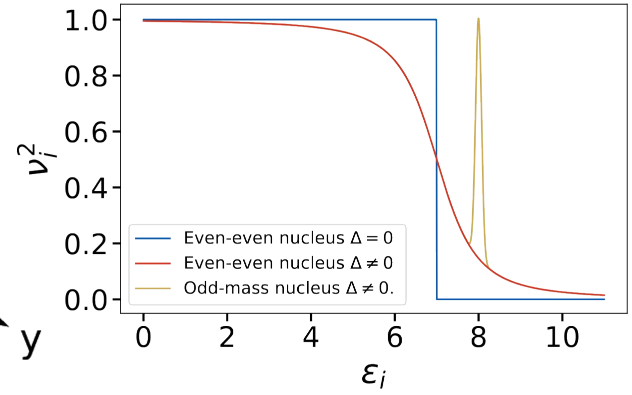
- SIII
- SKM*
- SKO'
- SKXc
- SLYO
- UNEDF0
- UNEDF1

Mean-field calculation



Blocking



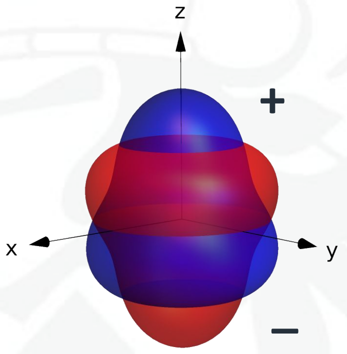


Symmetry breaking even-even core

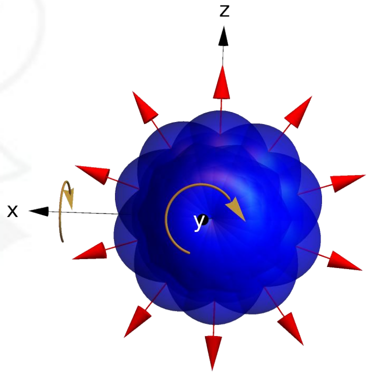
Quasi-particle Nilsson orbitals **$[Nn_z\Lambda]\Omega$**

Symmetry breaking odd mass nucleus

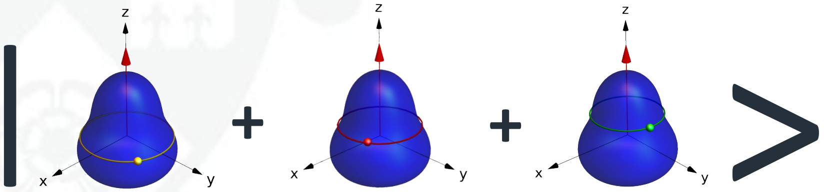
Restoration of symmetries and mixing



Parity projection



Angular momentum projection



Configuration mixing

The generator coordinate method (GCM)

$$|\Phi_\mu\rangle = \int d\boldsymbol{q}\, f_\mu(\boldsymbol{q})|\Phi(\boldsymbol{q})\rangle$$

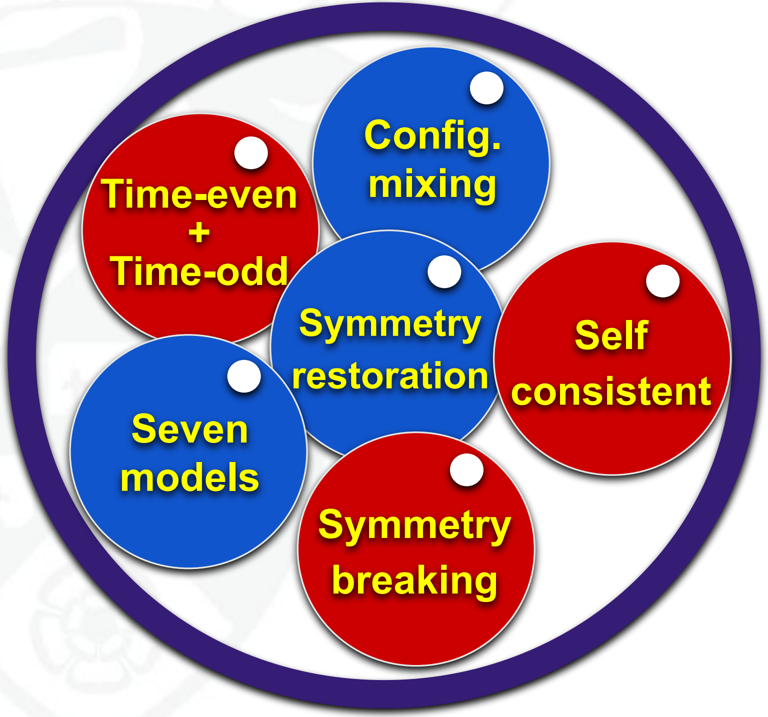
Parity and angular momentum projection

$$|\Phi_{nJ^+J}\rangle = \hat{P}^+\hat{P}^J_{JK}|\Phi_n\rangle$$

Configuration mixing

$$|\Phi_{J^+J}\rangle = \sum_n a_{nJ^+J}|\Phi_{nJ^+J}\rangle$$

Ingredients of our calculations

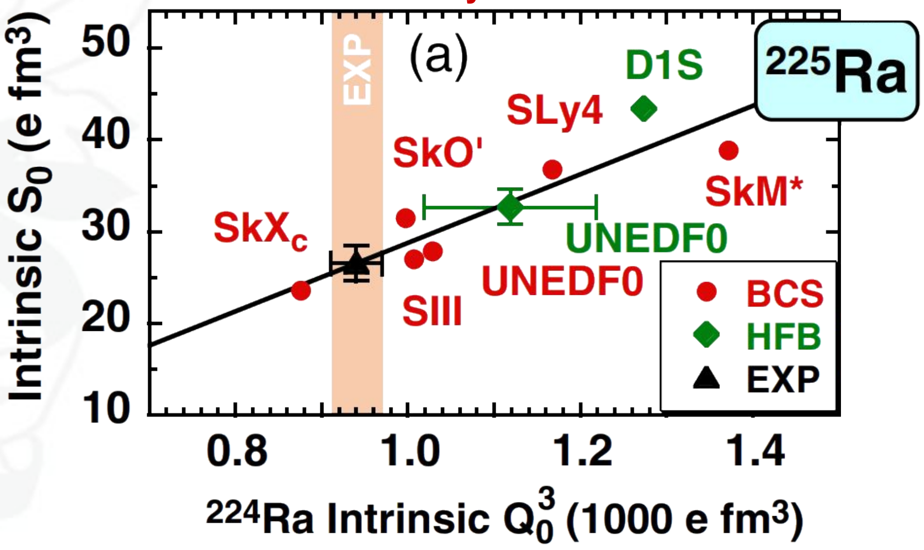


J. Phys. G 48 102001 (2021)

<u>Pairing volume strengths</u> fitted from mass staggering and <u>Landau parameters</u> from dipole moments

The “committee of models”

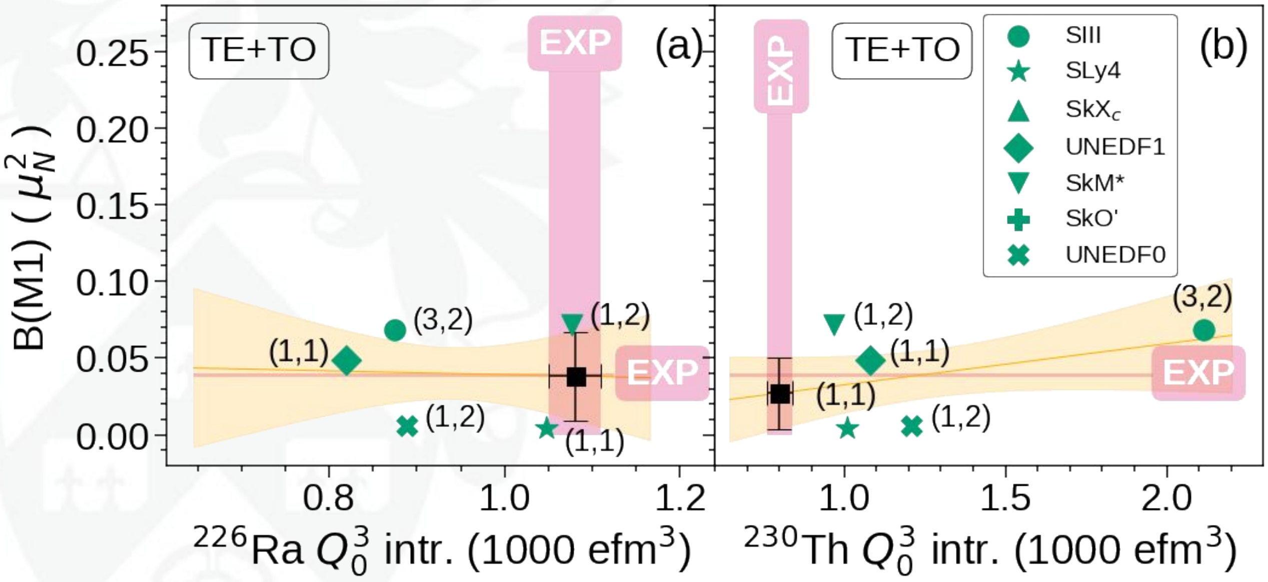
Phys. Rev. Lett. 121 232501

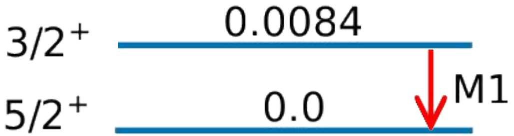


$$\hat{S}_0 = \frac{e}{10}\sqrt{\frac{4\pi}{3}}\sum_p \left(r_p^3 - \frac{5}{3}\overline{r_{\text{ch}}^2} r_p \right) Y_{10}(\Omega_p) \sim \hat{Q}_{30} = e\sum_p r_p^3 Y_{30}(\Omega_p)$$

M1 transition strength

arXiv:2602.02429



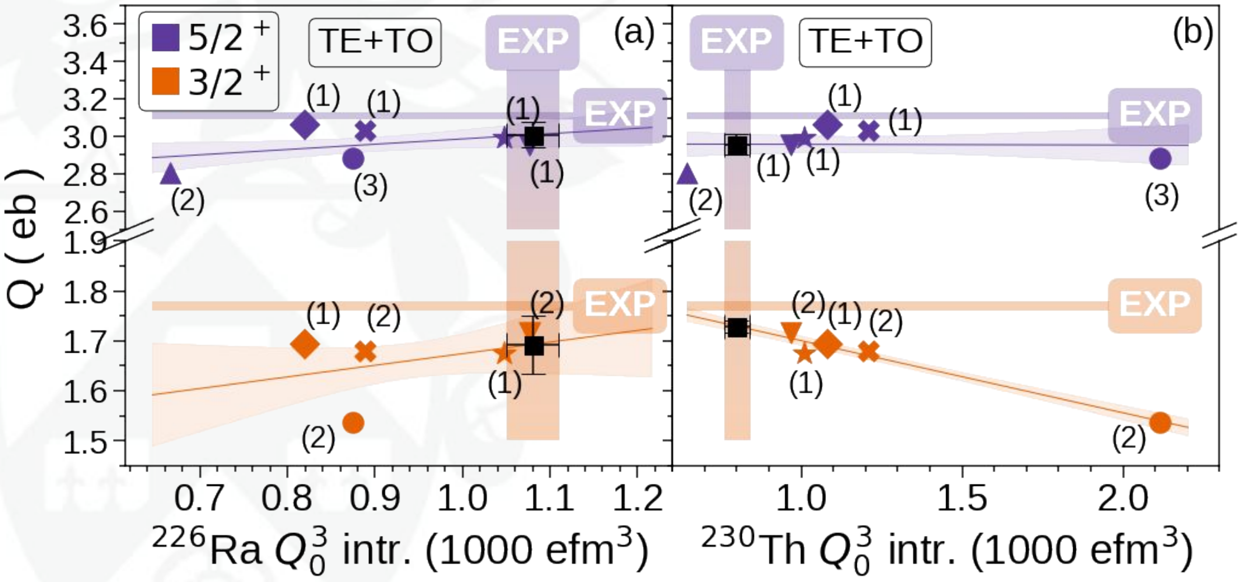


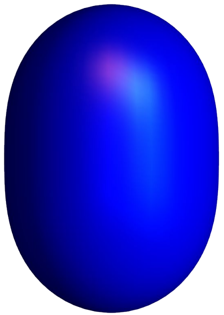
	$3/2^+ \rightarrow 5/2^+$ (μ_N^2)
^{226}Ra reg.	0.04(3)
^{230}Th reg.	0.03(2)
Average	0.04(3)
Exp.	**0.0388(12)**

$$B(M1; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f || \hat{M}_1 || I_i \rangle|^2$$

Electric quadrupole moment Q

arXiv:2602.02429



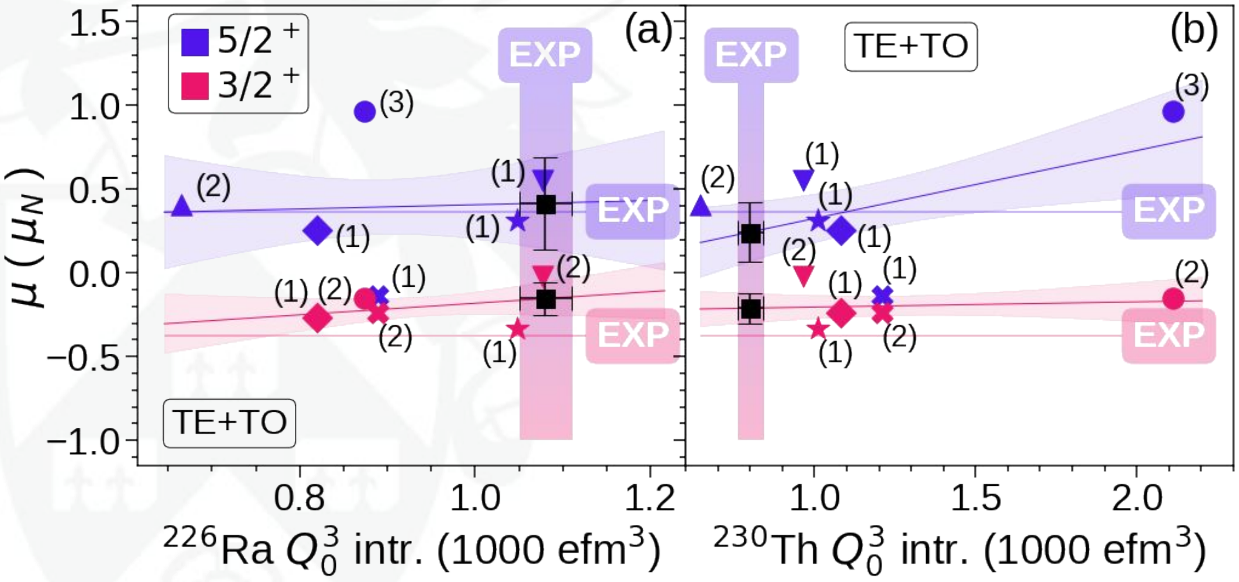


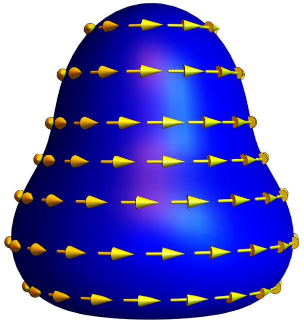
	$5/2^+$ (eb)	$3/2^+$ (eb)
^{226}Ra reg.	3.01(7)	1.69(6)
^{230}Th reg.	2.96(6)	1.729(12)
Average	2.96(9)	1.66(6)
Exp.	**3.11(2)**	**1.77(1)**

$$Q = \sqrt{\frac{16\pi}{5}}\langle II|\hat{Q}_{20}|II\rangle$$

Magnetic dipole moment μ

arXiv:2602.02429





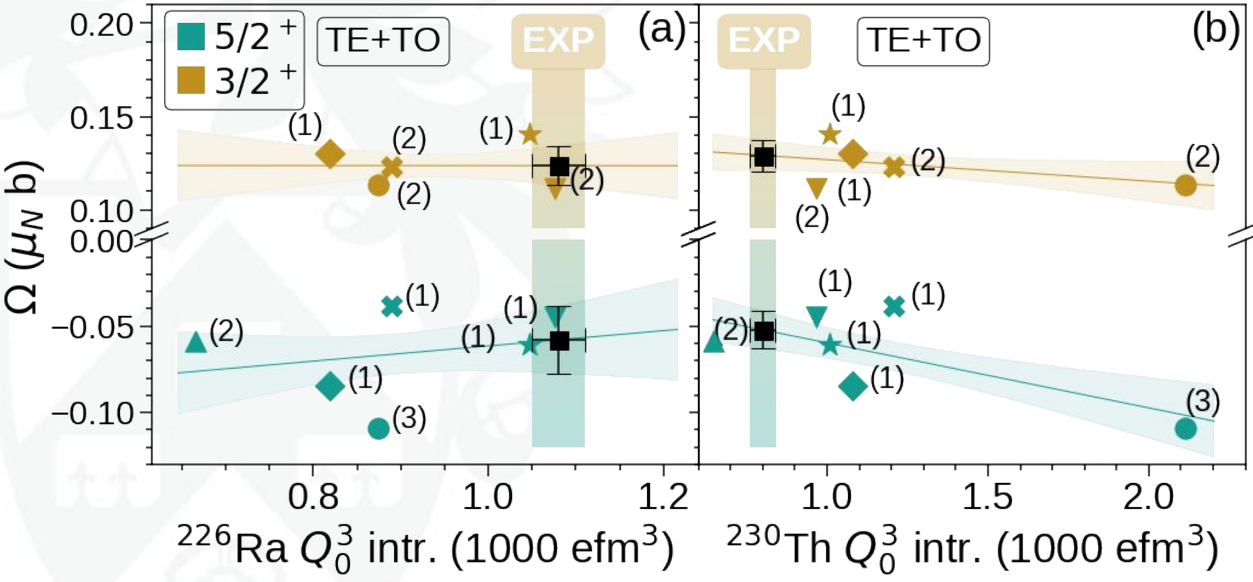
	$5/2^+$ (μ_N)	$3/2^+$ (μ_N)
^{226}Ra reg.	0.4(3)	−0.15(10)
^{230}Th reg.	0.2(2)	−0.21(9)
Average	0.4(3)	-0.20(11)
Exp.	**0.366(6)**	**−0.378(8)**

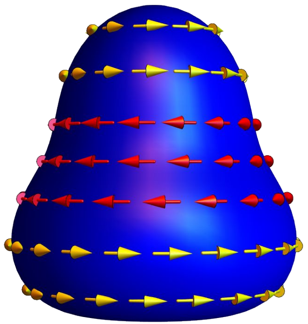
$\mu_{Schmidt}(5/2) = [1.366, -1.913]\ \mu_N$
$\mu_{Schmidt}(3/2) = [1.148, -1.913]\ \mu_N$

$$\mu = \sqrt{\frac{4\pi}{3}} \langle II|\hat{M}_{10}|II\rangle$$

Magnetic octupole moment Ω

arXiv:2602.02429





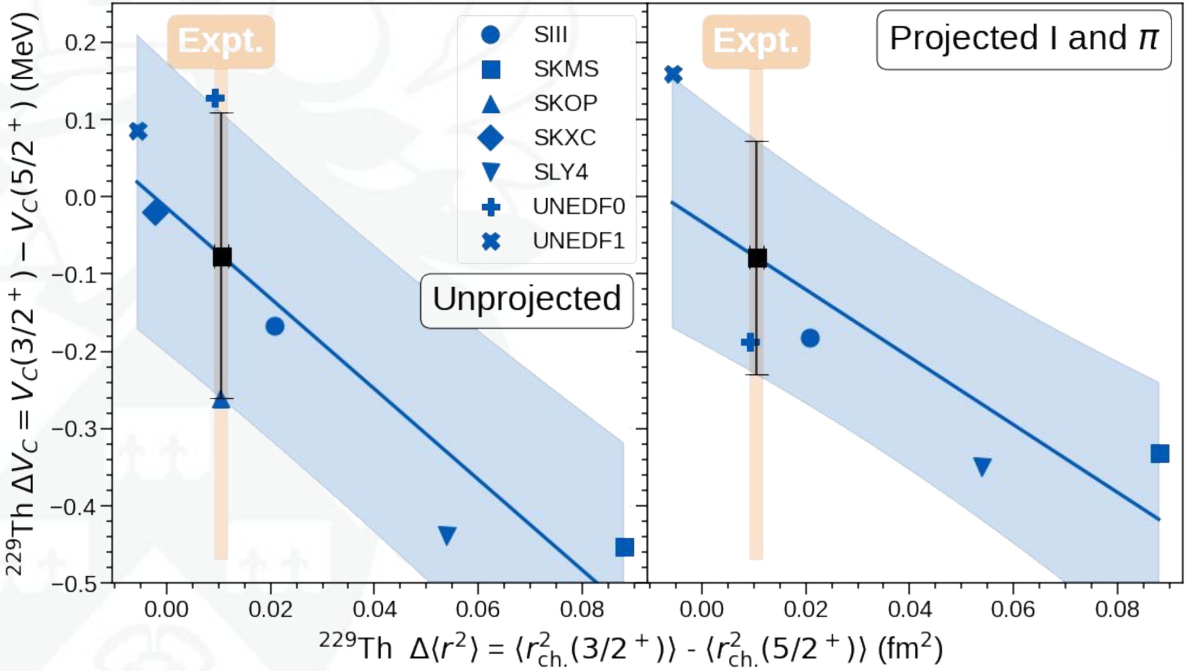
	5/2⁺ (μ_Nb)	3/2⁺ (μ_Nb)
^{226}Ra reg.	−0.06(2)	0.128(11)
^{230}Th reg.	−0.052(11)	0.129(8)
Average	-0.07(2)	0.124(11)
Exp.	**?**	**?**

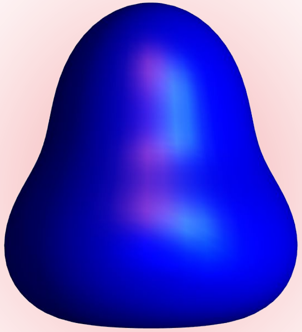
$\langle r^2\rangle = \frac{3}{5} R^2 A^{2/3}$

$\Omega_{\mathrm{Schwartz}}(5/2) = [-0.530,\ 0.177]\ \mu_N b$
$\Omega_{\mathrm{Schwartz}}(3/2) = [-0.371,\ 0.053]\ \mu_N b$

$$\Omega = -\sqrt{\frac{4\pi}{7}}\langle II|\hat{M}_{30}|II\rangle$$

Coulomb energies





	ΔV_c (MeV)
Projected	-0.08(18)
Unprojected	-0.08(15)

$$\frac{\dot{\nu}}{\nu} = \frac{\Delta V_C}{E_\gamma}\frac{\dot{\alpha}}{\alpha}$$

$$E^{\text{dir.}}_{\text{Coul.}}[\rho] = \frac{e^2}{2}\int dV \int dV' \frac{\rho_\pi(\boldsymbol{r})\rho_\pi(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} + E^{\text{exc. Slat.}}_{\text{Coul.}}[\rho] = -\frac{3e^2}{4}\sqrt[3]{\frac{3}{\pi}}\int dV \rho_\pi^{4/3}(\boldsymbol{r})$$

Conclusions

1. We performed state-of-the-art DFT calculations to obtain nuclear observables that **compare favourably with experimental data**.

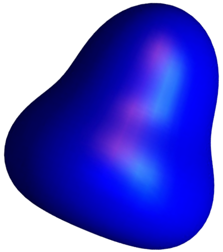
2. We addressed three fundamental questions in describing these observables: **the role of configuration interaction**, **time-odd core polarization**, and **octupole correlations** within a fully symmetry conserving approach.

3. The need to fine-tune **functional's octupole polarizability** within global adjustments to experimental data is the most essential way to improve our predictive power.

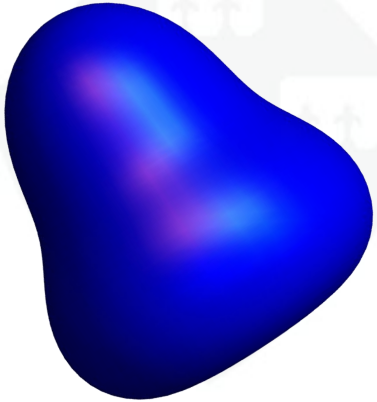
4. A more rigorous method to **reduce the spread of the models** implemented is required.

Acknowledgements

We thank Herlik Wibowo for fruitful discussions and Pierre Becker and Alessandro Pastore for their early involvement in the project. COLFUTURO financially supported ARG. This work was partially supported by the STFC Grant Nos. ST/V001035/1 and ST/Y000285/1, and by a Leverhulme Trust Research Project Grant. This project was partly undertaken on the Viking Cluster, a high-performance compute facility provided by the University of York. We are grateful for computational support from the University of York High Performance Computing service, Viking, and the Research Computing team.



Thank you!

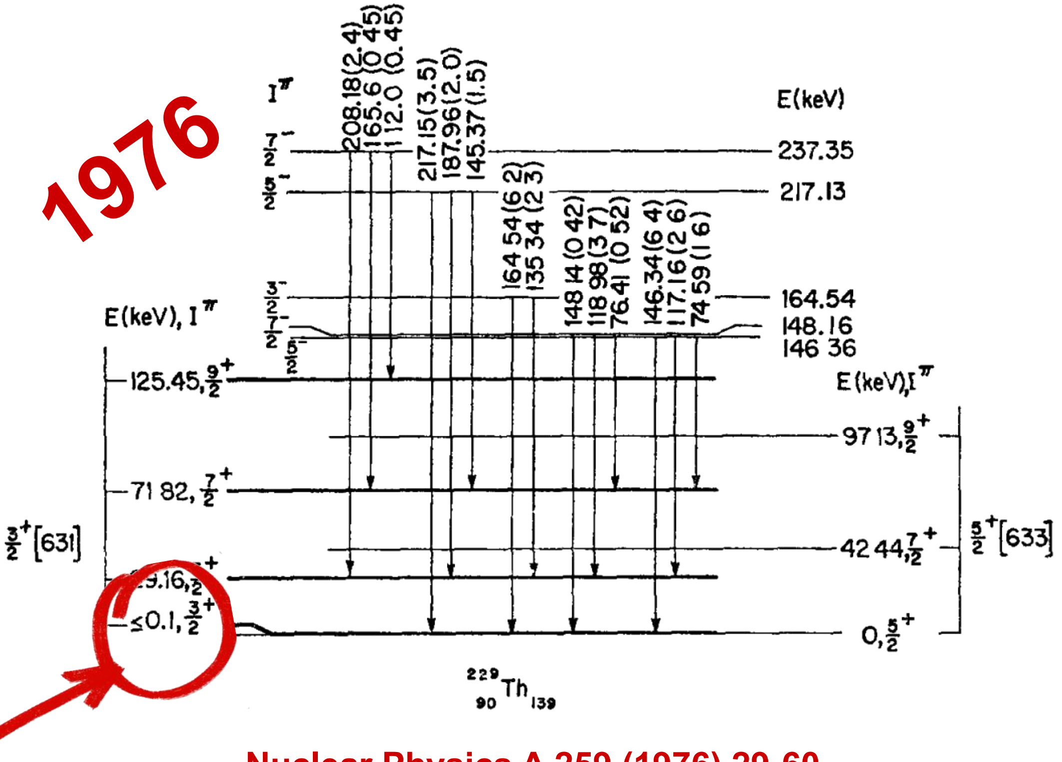


Supplementary slides

The distinctive property of ^{229}Th

$$^{233}\mathrm{U} \rightarrow {}^{229}\mathrm{Th} + {}^{4}\mathrm{He}$$

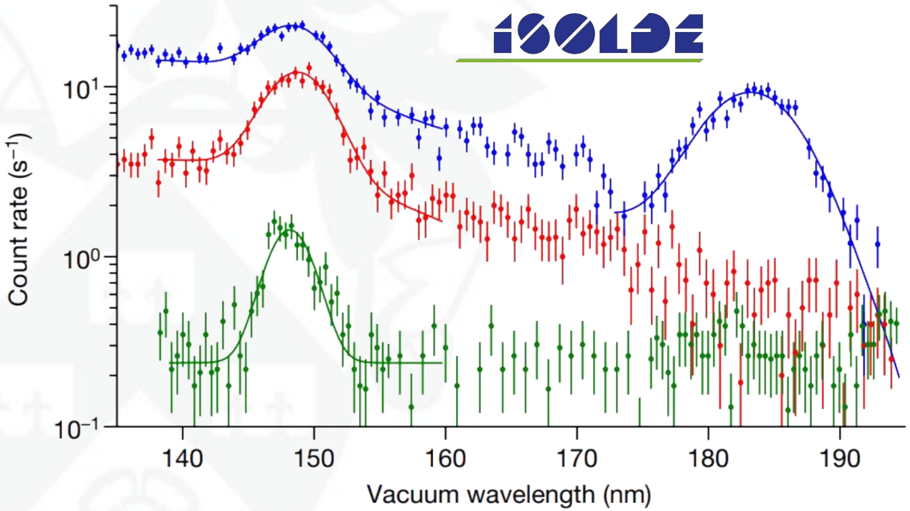
. . . an upper limit of ≈0.1 keV has tentatively been placed on the energy separation of these two states.



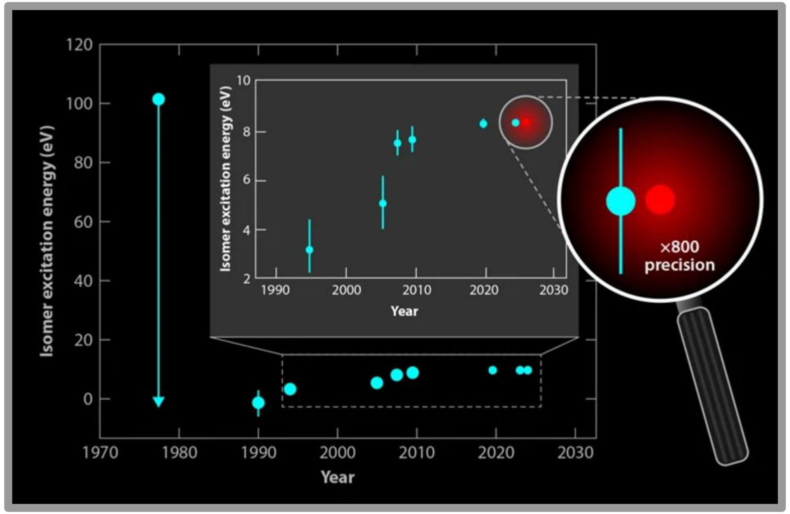
Nuclear Physics A 259 (1976) 29-60

The distinctive property of ^{229}Th

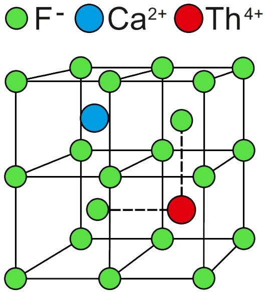
$$^{229}\mathrm{Ac} \rightarrow {}^{229}\mathrm{Th} + e^- + \bar{\nu}_e$$



Nature 617 (2023) 706–710



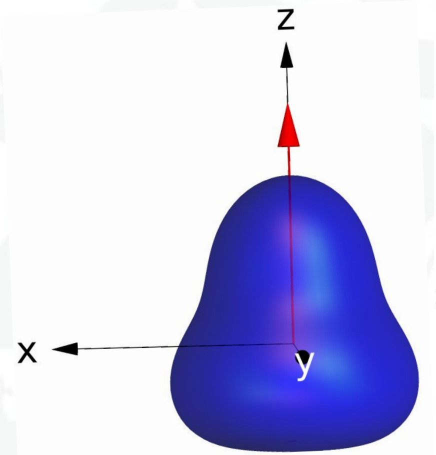
Physics 17, 71 (2024)

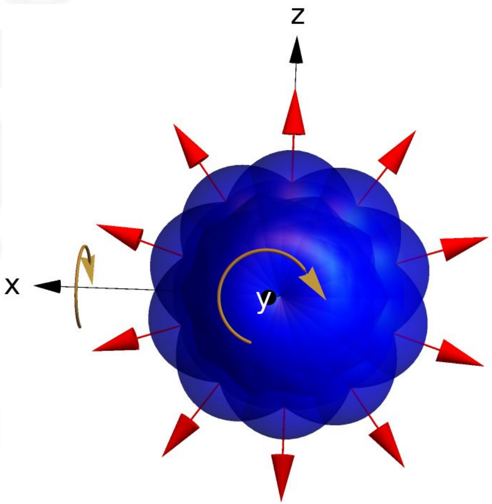


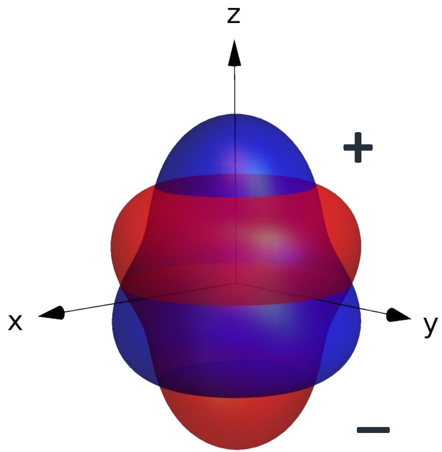
Physical Review Letters 132, 182501 (2024)

Angular momentum and parity projection

$$\hat{P}^J_{MK} = \frac{2J+1}{16\pi^2}\int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \, \sin(\beta) \int_0^{4\pi} d\gamma \, D^{J*}_{MK}(\alpha,\beta,\gamma)\hat{R}(\alpha,\beta,\gamma)$$



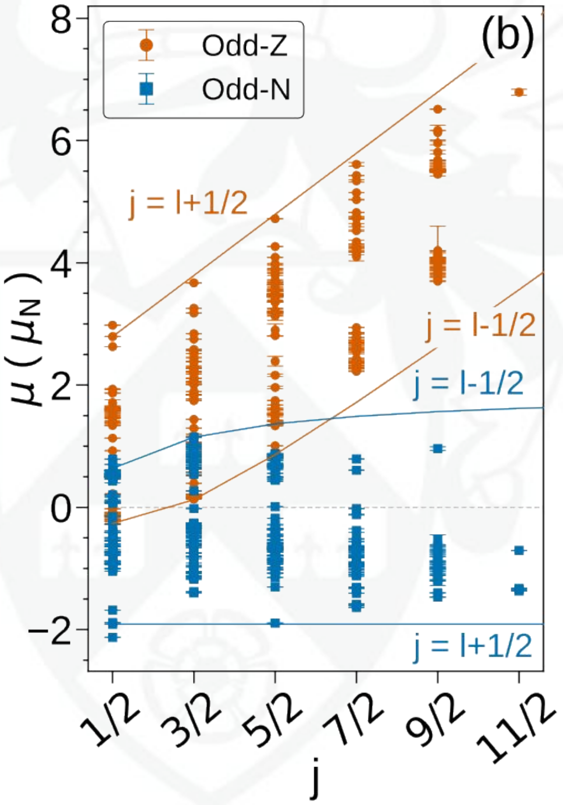


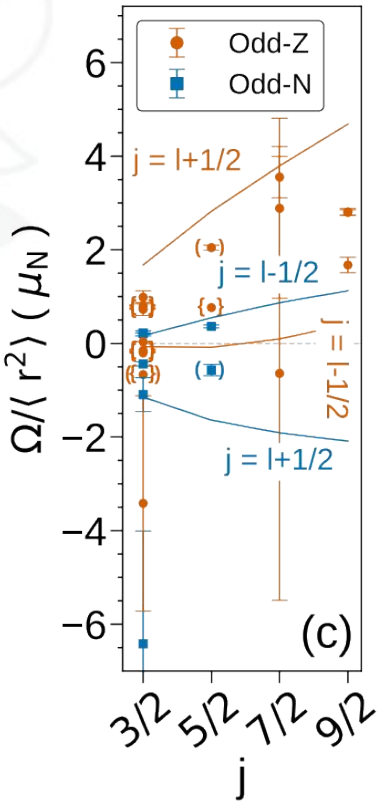


No particle-number projection is performed

$$\hat{P}^p = \frac{1}{2}(1 + pe^{-i\pi\hat{N}_-})$$

Schmidt and Schwartz lines





$$\mu_{\text{s.p.}}^{(\tau)} = \begin{cases} \mu_N \left[\left(j - \frac{1}{2}\right) g_l^{(\tau)} + \frac{1}{2} g_s^{(\tau)}\right], & j = l + 1/2, \\ \mu_N \frac{j}{j+1} \left[\left(j + \frac{3}{2}\right) g_l^{(\tau)} - \frac{1}{2} g_s^{(\tau)}\right], & j = l - 1/2. \end{cases}$$

$$\Omega_{\text{s.p.}}^{(\tau)} = +\mu_N \frac{3}{2} \frac{(2j-1)}{(2j+4)(2j+2)} \langle r^2 \rangle$$

$$\times \begin{cases} (j+2)\left[(j - \frac{3}{2}) g_l^{(\tau)} + g_s^{(\tau)}\right], & j = l + \frac{1}{2} \\ (j-1)\left[(j + \frac{5}{2}) g_l^{(\tau)} - g_s^{(\tau)}\right], & j = l - \frac{1}{2} \end{cases}$$

$$g_l^{(p)} = 1, \quad g_l^{(n)} = 0,$$

$$g_s^{(p)} = 2(1+\kappa_p) = 5.587, \quad g_s^{(n)} = 2\kappa_n = -3.826$$

arXiv:2511.04632

Hellmann-Feynman theorem and α(t)

The HF-Theorem

$$\frac{\partial E(\lambda)}{\partial \lambda} = \left\langle \Psi(\lambda) \left| \frac{\partial H(\lambda)}{\partial \lambda} \right| \Psi(\lambda) \right\rangle$$

Application to the excited and ground states of ^{229}Th

$$\frac{\partial (E_{3/2^+} - E_{5/2^+})}{\partial t} = \left\langle 3/2^+ \left| \frac{\partial V_C}{\partial t} \right| 3/2^+ \right\rangle - \left\langle 5/2^+ \left| \frac{\partial V_C}{\partial t} \right| 5/2^+ \right\rangle$$

$$= \Big[\langle 3/2^+ | V_C | 3/2^+ \rangle - \langle 5/2^+ | V_C | 5/2^+ \rangle \Big] \frac{\dot{\alpha}}{\alpha}$$

$$\dot{\nu} = \frac{\Delta V_C}{h} \frac{\dot{\alpha}}{\alpha} \quad \Rightarrow \quad \frac{\dot{\nu}}{\nu} = \frac{\Delta V_C}{E_\gamma} \frac{\dot{\alpha}}{\alpha}$$

Physics Letters B 650 4 (2007)

Multipole operators

General formulas

$$\hat{Q}_{\lambda\mu} = e\sum_{i=1}^{Z} Q_{\lambda\mu}(\boldsymbol{r}_i)$$

$$\hat{M}_{\lambda\mu} = \mu_N\sqrt{\lambda(2\lambda+1)}\sum_{\tau=n,p}\sum_{i=1}^{N(\tau)}\left[Q_{\lambda-1}(\boldsymbol{r}_i)\otimes\left(g_s^{(\tau)}\hat{S}_{1,i}^{(\tau)}+g_l^{(\tau)}\frac{2}{\lambda+1}\hat{L}_{1,i}^{(\tau)}\right)\right]_{\lambda\mu}$$

Electric quadrupole, magnetic dipole and magnetic octupole

$$\hat{Q}_{20} = e\sqrt{\frac{16\pi}{5}}\sum_{i=1}^{Z} r_i^2 Y_{20}(\theta_i,\phi_i) = e\sum_{i=1}^{Z}(2z_i^2-x_i^2-y_i^2)$$

$$\hat{\mu}_z = \sqrt{\frac{4\pi}{3}}\hat{M}_{10} = \mu_N\sum_{\tau=n,p}\left[g_s^{(\tau)}\hat{S}_z^{(\tau)}+g_l^{(\tau)}\hat{L}_z^{(\tau)}\right]$$

$$\hat{M}_{30} = -3\mu_N\sqrt{\frac{7}{16\pi}}\sum_{\tau=n,p}\sum_{i=1}^{N(\tau)}\Big[xz\left(g_s^{(\tau)}\hat{S}_{i,x}^{(\tau)}+g_l^{(\tau)}\hat{L}_{i,x}^{(\tau)}\right)+yz\left(g_s^{(\tau)}\hat{S}_{i,y}^{(\tau)}+g_l^{(\tau)}\hat{L}_{i,y}^{(\tau)}\right)$$
$$+(2z^2-x^2-y^2)\left(g_s^{(\tau)}\hat{S}_{i,z}^{(\tau)}+g_l^{(\tau)}\hat{L}_{i,z}^{(\tau)}\right)\Big].$$

arXiv:2511.04632

The Schiff moment

Definition

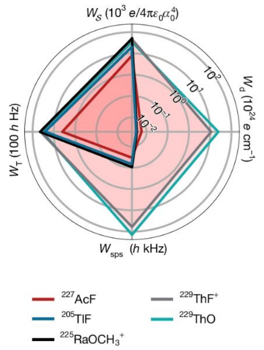
$$\hat{S}_0 = \frac{e}{10}\sqrt{\frac{4\pi}{3}}\sum_p \left(r_p^3 - \frac{5}{3}\overline{r_{\rm ch}^2}\, r_p\right) Y_{10}(\Omega_p)$$

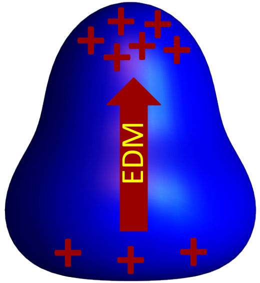
Observable

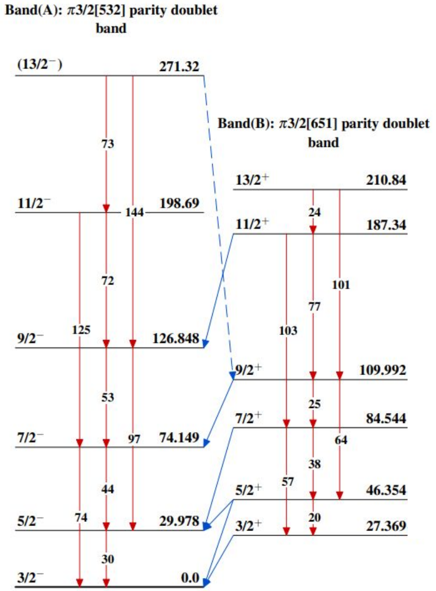
$$S_{\rm lab} \approx -2\,{\rm Re}\left\{\frac{\langle\Psi_0|\hat{S}_0|\bar{\Psi}_0\rangle\langle\bar{\Psi}_0|\hat{V}_{\rm P,T}|\Psi_0\rangle}{\Delta E}\right\}$$

PT violating potential

$$\hat{V}_{\rm P,T}(\mathbf{r}_1-\mathbf{r}_2) = -\frac{g m_\pi^2}{8\pi m_N}\Big\{(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\mathbf{r}_1-\mathbf{r}_2)\Big[\bar{g}_0\vec{\tau}_1\cdot\vec{\tau}_2 - \frac{\bar{g}_1}{2}(\tau_{1z}+\tau_{2z}) + \bar{g}_2(3\tau_{1z}\tau_{2z}-\vec{\tau}_1\cdot\vec{\tau}_2)\Big]$$
$$-\frac{\bar{g}_1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{r}_1-\mathbf{r}_2)(\tau_{1z}-\tau_{2z})\Big\}\frac{\exp(-m_\pi|\mathbf{r}_1-\mathbf{r}_2|)}{m_\pi|\mathbf{r}_1-\mathbf{r}_2|^2}\left[1+\frac{1}{m_\pi|\mathbf{r}_1-\mathbf{r}_2|}\right]$$
$$+\frac{1}{2m_N^3}[\bar{c}_1+\bar{c}_2\vec{\tau}_1\cdot\vec{\tau}_2](\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{\nabla}\delta^3(\mathbf{r}_1-\mathbf{r}_2),$$







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The reduced transition probability

Fermi's golden rule

$$t_{1/2} = \frac{\ln 2}{T_{fi}}$$

From Nucleons to Nucleus, J. Suhonen

$$T_{fi}^{(\sigma\lambda\mu)} = \frac{2}{\epsilon_0 \hbar} \frac{\lambda + 1}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda+1} \left| \langle \xi_f \, J_f \, m_f | \mathcal{M}_{\sigma\lambda\mu} | \xi_i \, J_i \, m_i \rangle \right|^2$$

Average over initial states, sum over final states and apply Wigner-Eckart theorem

$$T_{fi}^{(\sigma\lambda)} = \frac{1}{2J_i + 1} \sum_{m_i \mu m_f} T_{fi}^{(\sigma\lambda\mu)}$$

$$= \frac{2}{\epsilon_0 \hbar} \frac{\lambda + 1}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda+1} B(\sigma\lambda \, ; \, \xi_i J_i \to \xi_f J_f)$$

Reduced transition probability

$$B(\sigma\lambda \, ; \, \xi_i J_i \to \xi_f J_f) \equiv \frac{1}{2J_i + 1} \left| (\xi_f \, J_f \| \boldsymbol{\mathcal{M}}_{\sigma\lambda} \| \xi_i \, J_i) \right|^2$$

Landau parameters

$$\beta = (3\pi^2/2)^{2/3}$$

$$N_0 = \pi^{-2}\left(\frac{\hbar^2}{2m}\right)^{-1}\frac{m^*}{m}\left(\frac{3\pi^2\rho_{\mathrm{sat.}}}{2}\right)^{1/3}$$

$$x_0 = C_0^s[0]/C_0^s[\rho_{\mathrm{sat}}]$$ **= 1**

$$x_1 = C_1^s[0]/C_1^s[\rho_{\mathrm{sat}}]$$ **= 1**

$$g_0 = N_0\left(2C_0^{ss} + 2C_0^{JJ}\beta\rho_{\mathrm{sat.}}^{2/3}\right)$$ **= 0.4**

$$g_0' = N_0\left(2C_1^{ss} + 2C_1^{JJ}\beta\rho_{\mathrm{sat.}}^{2/3}\right)$$ **= 1.20 (SkXc, SkM*, SIII, UNEDF0),**
1.0 (SkO'),
1.3 (SLy4),
1.7 (UNEDF1)

$$g_1 = -2N_0C_0^{JJ}\beta\rho_{\mathrm{sat.}}^{2/3}$$

$$g_1' = -2N_0C_1^{JJ}\beta\rho_{\mathrm{sat.}}^{2/3}$$

→ **Unconstrained. Values given from the global fit**

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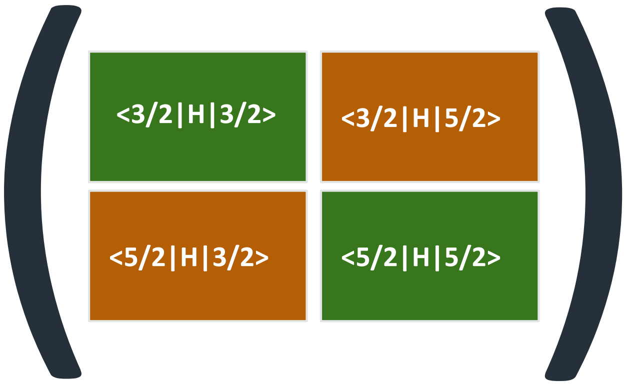
Mixing and matrix elements

$$|\Phi_{I+\Omega}\rangle = \sum_{n'} a_{n'I+\Omega} |\Phi_{n'I+\Omega}\rangle$$

Collective wavefunction

$$\sum_{n'} H^{I+\Omega}_{nn'} a_{n'I+\Omega} = E \sum_{n'} N^{I+\Omega}_{nn'} a_{n'I+\Omega}$$

Hill-Wheeler equation



$$\langle \Phi_{5/2+5/2} | \hat{M} | \Phi_{3/2+3/2} \rangle = \sum_{nn'} a^*_{n5/2+5/2} M^{\frac{5}{2}\frac{3}{2}}_{nn'} a_{n'3/2+3/2}$$

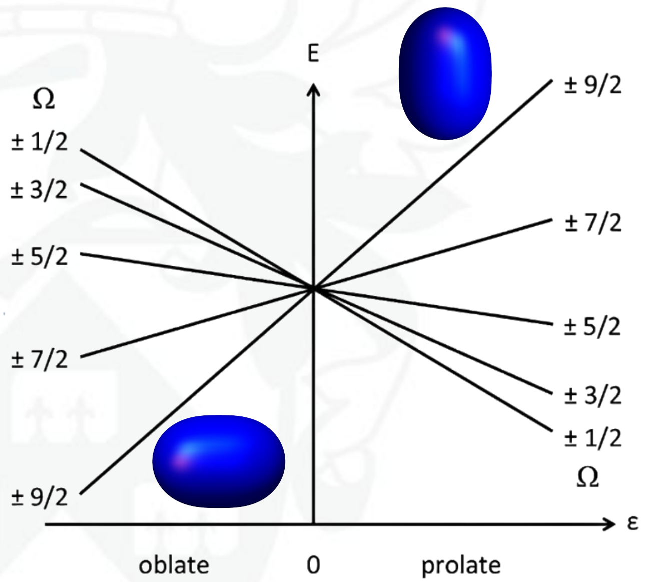
$$\mathrm{B(M1} : 3/2^+_1 \rightarrow 5/2^+_1) = \frac{1}{4} |\langle \Phi_{5/2+5/2} | \hat{M} | \Phi_{3/2+3/2} \rangle|^2$$

Transition calculation

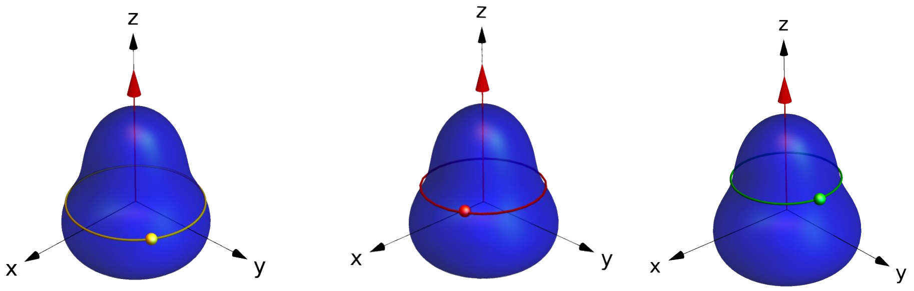
$$\langle \Phi_{I+I} | \hat{O} | \Phi_{I+I} \rangle = \sum_{nn'} a^*_{nI+I} \mathcal{O}^{II}_{nn'} a_{n'I+I}$$

Moments calculation

Nilsson labels and configurations



$$[Nn_z\Lambda]\Omega$$



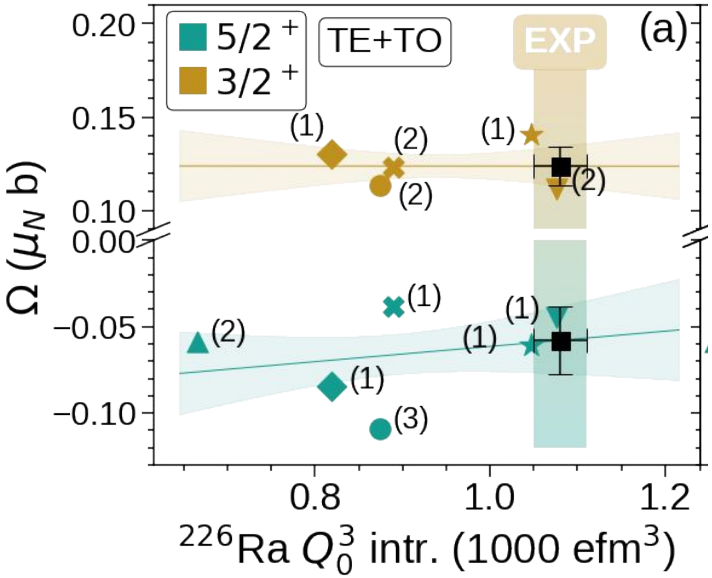
Ω	Configurations
$5/2^{+}$	(642)3/2 (741)3/2 (631)3/2
$3/2^{+}$	(633)5/2 (752)5/2 (622)5/2

Error bars of regressions

$$\Delta\mathcal{O}(\text{est.}) = \sqrt{\Delta\mathcal{O}(\text{the.})^2 + \Delta\mathcal{O}(\text{exp.})^2}$$

$$\Delta\mathcal{O}(\text{exp.}) = b\,\Delta Q_0^3(\text{exp.})$$

$$\Delta\mathcal{O}(\text{the.}) = \sqrt{C_{aa} + 2C_{ab}Q_0^3(\text{exp.}) + C_{bb}\left[Q_0^3(\text{exp.})\right]^2}$$



Skyrme coupling constants

	SkX$_c$ [66]	SkM* [67]	UNEDF1 [68]	SIII [69]	SkO' [70]	SLy4 [71]	UNEDF0 [72]
C_0^ρ	−539.250000	−991.875000	−779.373009	−423.281250	−787.282125	−933.341250	−706.382929
C_1^ρ	283.286000	390.137500	287.722131	268.078125	246.942585	830.051485	240.049522
C_0^s	44.138239	38.905862	30.554818	41.584341	219.237417	44.243392	34.079267
C_1^s	135.000706	116.717587	129.857975	124.753022	101.970111	143.791024	102.237800
C_0^τ	−0.821625	34.687500	−0.989914	44.375000	15.000072	57.129375	−12.917242
C_1^τ	−44.706725	−34.062500	−33.632096	−30.625000	−4.156240	24.656385	−45.189417
C_0^T	−7.389900	0.000000	0.000000	0.000000	−104.093563	0.000000	0.000000
C_1^T	−23.625000	0.000000	0.000000	0.000000	−9.171875	0.000000	0.000000
C_0^j	0.821625	−34.687500	0.989914	−44.375000	−15.000072	−57.129375	12.917242
C_1^j	44.706725	34.062500	33.632096	30.625000	4.156240	−24.656385	45.189417
C_0^J	7.389900	0.000000	0.000000	0.000000	104.093563	0.000000	0.000000
C_1^J	23.625000	0.000000	0.000000	0.000000	9.171875	0.000000	0.000000
$C_0^{\Delta\rho}$	−46.011656	−68.203125	−45.135131	−62.968750	−52.787044	−76.996406	−55.260600
$C_1^{\Delta\rho}$	22.750481	17.109375	−145.382168	17.031250	−32.162034	15.657086	−55.622600
$C_t^{\Delta s}$, $C_t^{\nabla s}$	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
$C_0^{\nabla J} = C_0^{\nabla j}$	−72.850000	−97.500000	−74.026333	−90.000000	−102.450600	−92.250000	−79.530800
$C_1^{\nabla J} = C_1^{\nabla j}$	0.000000	−32.500000	−35.658261	−30.000000	41.444400	−30.750000	45.630200

Local parameters

Skyrme	g_0'	$V_{0,n}$	$V_{0,p}$	$Q_0^3(^{226}\text{Ra})$	$Q_0^3(^{230}\text{Th})$
SkX_c	1.2	139.02	173.63	0.6654	0.6446
SkM*	1.2	181.47	216.25	1.0763	0.9667
UNEDF1	1.7	145.35	169.79	0.8194	1.0797
SIII	1.2	181.14	220.19	0.8742	2.1131
SkO′	1.0	163.82	184.34	0.9482	1.2839
SLy4	1.3	207.76	231.89	1.0475	1.0088
UNEDF0	1.2	130.61	158.39	0.8887	1.2077

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arXiv:2602.02429