Recent developments for microscopic optical potentials for nucleon-nucleus elastic scattering

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What is the Optical Model?

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\,\Psi(\boldsymbol{r}) + \int d\boldsymbol{r}'\;U(\boldsymbol{r},\boldsymbol{r}')\Psi(\boldsymbol{r}') = E\,\Psi(\boldsymbol{r})$$

<u>Non-local complex potential</u>

- Many-body problem is computational intractable - approximate two-body problem with a potential between projectile and target nucleus.
- Real and imaginary parts - real part describes elastic scattering, imaginary part describes all other inelastic processes.
- In general optical potentials are energy-dependent, nonlocal and complex.

How are they computed?

Two different approaches…

Microscopic:

- Derived from fundamental theory.
- After the choice of nucleon-nucleon (NN) potential, microscopic optical potentials generally rely on few or no free parameters.
- Suitable for extrapolating to situations where there is little to no experimental data available.

Phenomenological:

- Models use free parameters which are fixed to reproduce existing experimental data.
- They are very accurate to the available data.
- But lack predictive power to explore regions where there is little to no experimental data.

We will focus on one type of ***microscopic*** optical potential…

Microscopic optical potentials from multiple scattering theory

Spectator expansion of the optical potential

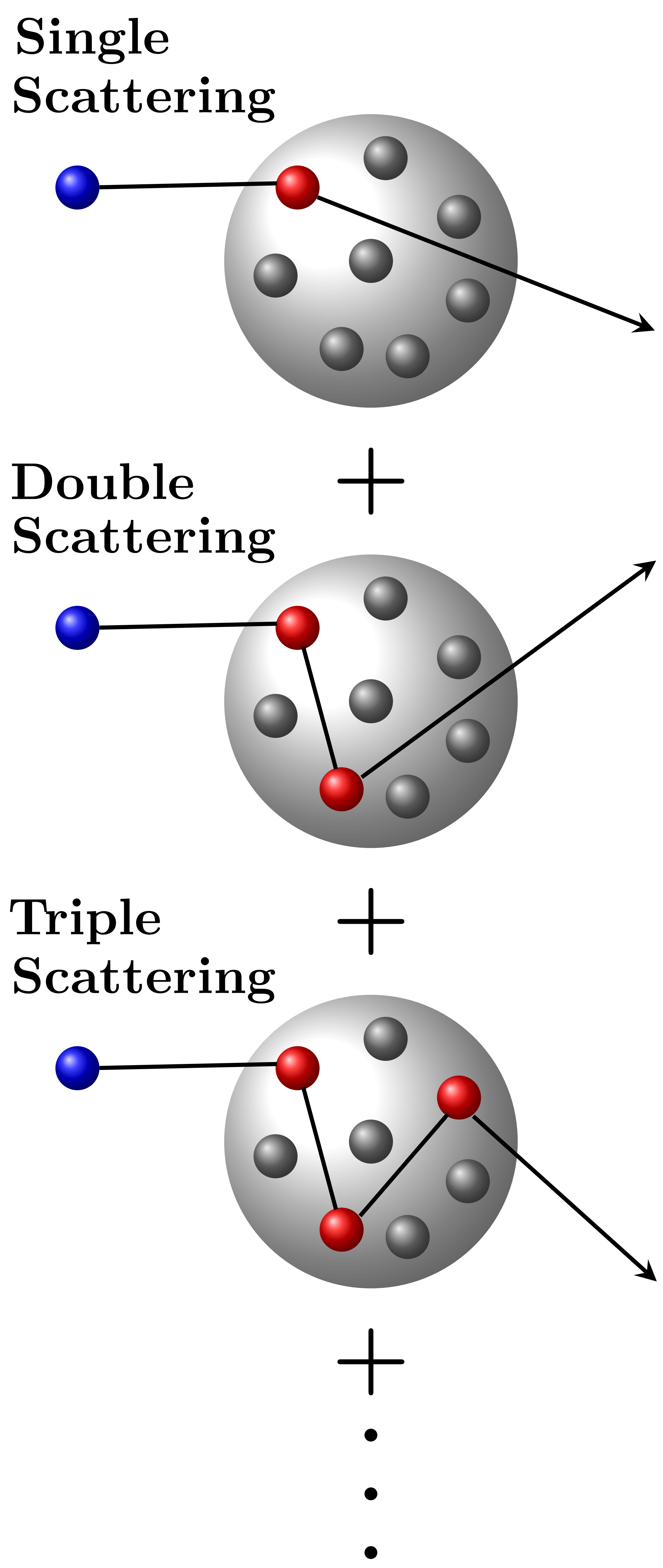
[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U = \sum_{i}^{A} \tau_i$$

$$+ \sum_{i,j\neq i}^{A} \tau_{ij}$$

$$+ \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{ijk}$$

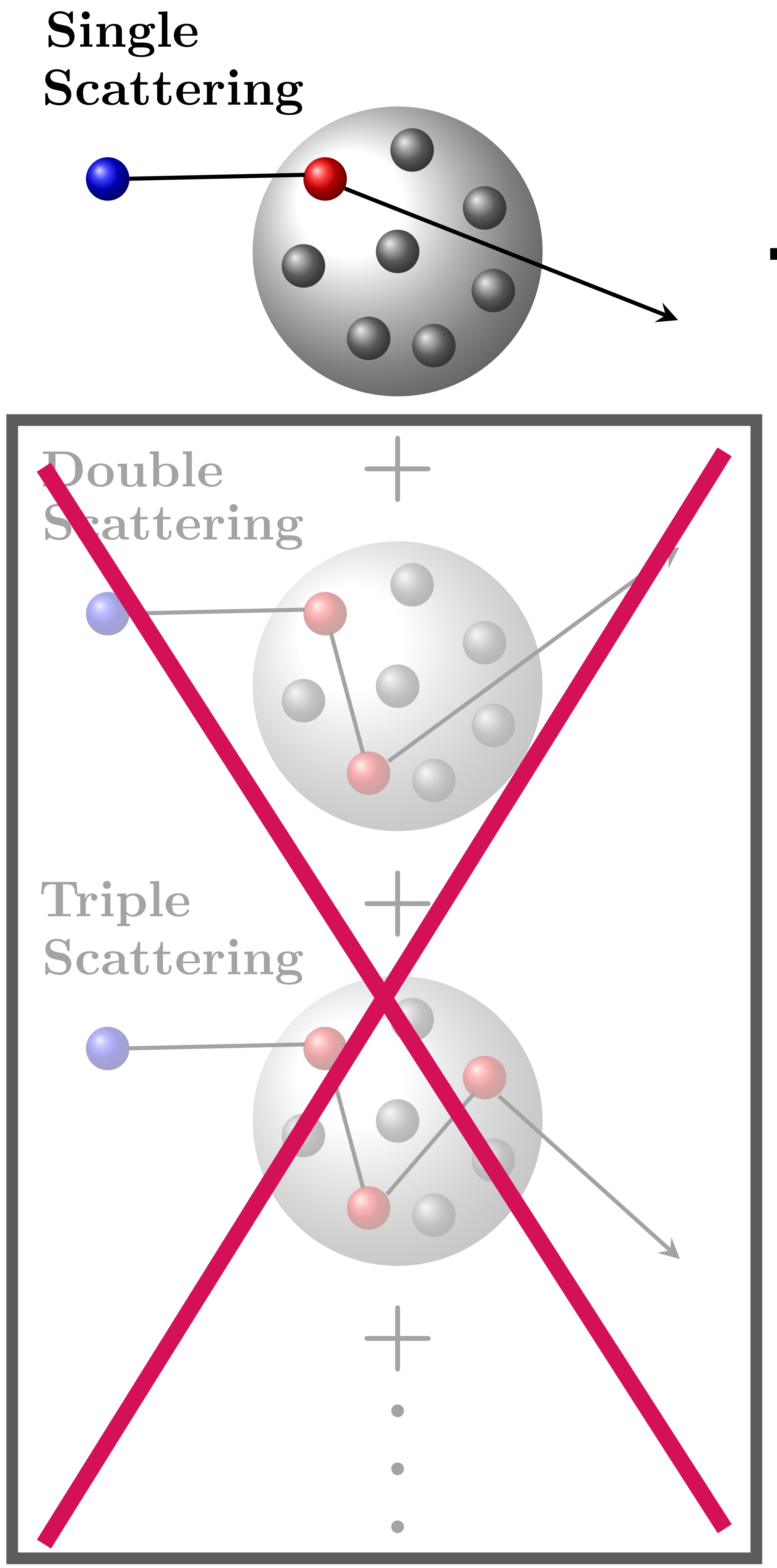
$$+ \dots$$



Microscopic optical potentials from multiple scattering theory

First order approximation

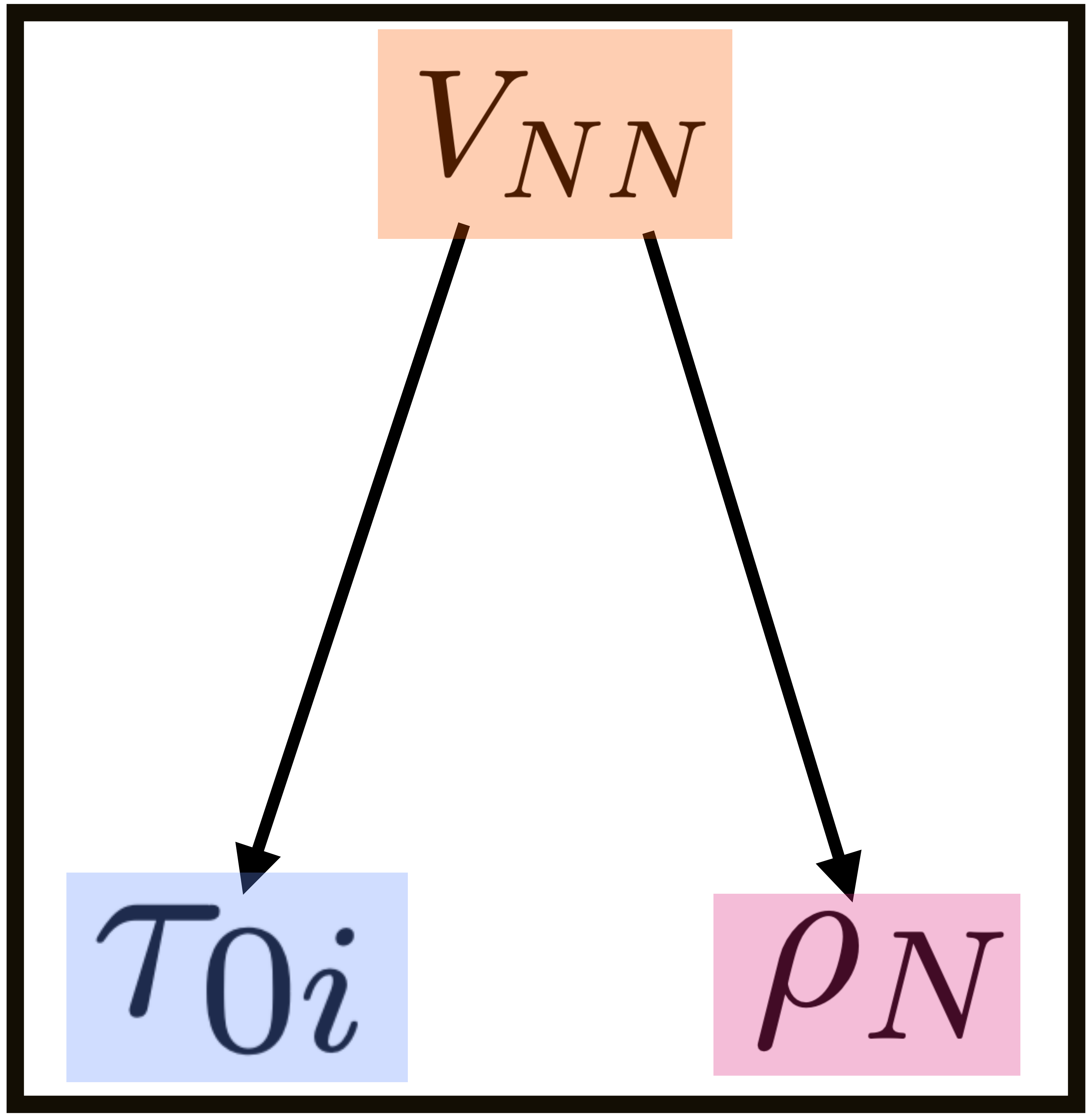
[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]



$$U \approx \sum_{i}^{A} \tau_{0i}$$

- Final step is the approximation of this many-body operator by the two-body free NN t matrix - this is the ***impulse approximation.***
- It is a high-energy approximation and specifically these type of optical potentials are valid for energies above ***100 MeV.***

Nucleon-nucleon potential



- NN potential, V_{NN}, is needed to compute the optical potential - the NN potential enters into the optical potential through the **free NN t matrix** and the **nuclear density.**
- The NN potentials used in this approach are obtained from chiral effective field theory.

The nuclear densities, ρ_N, are computed via the **No-Core Shell Model** (NCSM) that uses a harmonic oscillator (HO) basis (provided by Michael Gennari).

$$\tau_{0i} = v_{0i} + v_{0i}\, g_{0i}\, \tau_{0i}$$

The t matrix is an object which encodes the initial and final states of a scattering event through an interaction potential and allows you to compute cross sections.

Key takeaway - The reaction and structure inputs are generated from the same NN interaction.

The folding integral

$$U \approx \sum_{i}^{A} \tau_{0i}$$

Work done in momentum space -> absorption naturally included.

Algebra…

$$U(\boldsymbol{q}, \boldsymbol{K}; E) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{NN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

Møller factor - kinematical factor that imposes Lorentz invariance. Currently neglected.

Free nucleon-nucleon (NN) t matrix

Nonlocal one-body densities

Problem… In order to compute the integral over the t matrix and the density one needs to preform a 3D numerical interpolation of both of these objects - this is computationally expensive, on the order of hundreds of minutes even for simplest cases.

Can this be simplified?

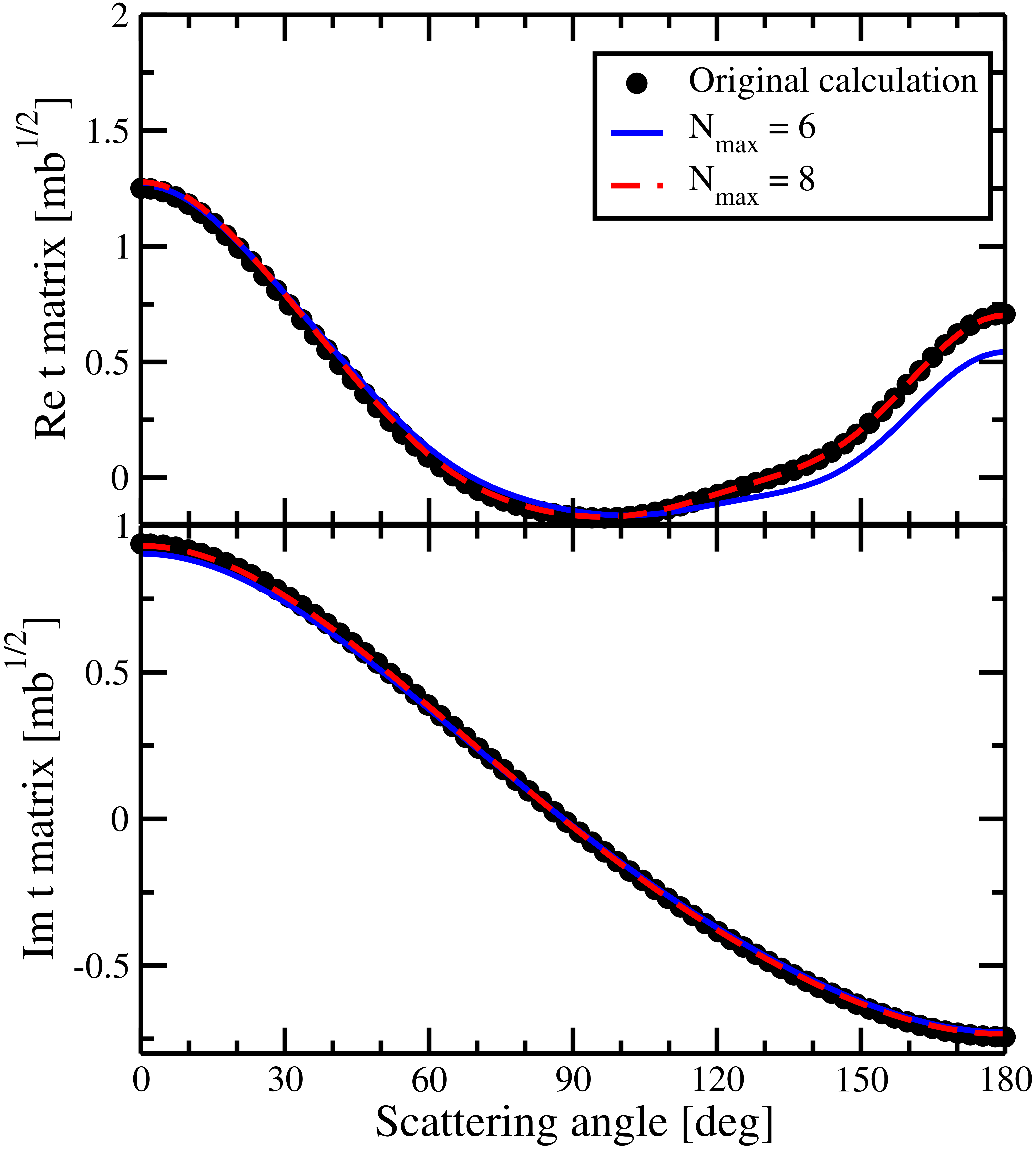
Our approach

Nuclear densities from the NCSM are computed in a HO basis and this basis has some advantages:

- Known coordinate transformation properties and there are codes to compute the transformation brackets for these.

How can we use this?

- Also expand free NN t matrix on HO basis.
- Exploit the known coordinate transformation properties to simplify the calculation of the folding integral.
- Instead compute coefficients of HO expansion.



How does this look for the simplest case?

$$q = k' - k, K = \frac{1}{2}(k' + k)$$

$$U(\boldsymbol{q}, \boldsymbol{K}; E) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{NN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}) \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

Only consider central terms + more algebra…

$$U(\boldsymbol{q}, \boldsymbol{K}; E) = \frac{1}{\hat{J}_f} \sum_{\substack{\Lambda\Omega \\ N_1 L_1 N_2 L_2}} (J_i M_i \Lambda\Omega | J_f M_f)\, \mathcal{U}^{\Lambda}_{N_1 L_1 N_2 L_2} \left[\varphi^*_{N_1 L_1}(\alpha \boldsymbol{q}) \otimes \varphi^*_{N_2 L_2}(\epsilon \boldsymbol{K})\right]^{\Lambda}_{\Omega}$$

Clebsch-Gordon coefficient involving initial and final spin of the target nuclei

Expansion coefficient, hiding 6j symbols, HO transformation brackets, coefficients for t matrix and density

Coupled HO wave functions. Alpha, epsilon are just constants, artefact of HO coordinate transformations

General case in coordinate space

- Goal was to derive an expression for the optical potential that included all the contributions e.g. central, spin-orbit and tensor.
- We found that Fourier transforming to coordinate space led to simplifications in the expressions.
- Many phenomenological optical potentials are constructed in coordinate space.

$$U^{I'I}_{S'M'_S SM_S}(\boldsymbol{R}_+, \boldsymbol{R}_-; E) = (-1)^{I+I'-S'-S}\hat{S} \sum_{\substack{\lambda\mu\Omega_a K_2 \\ N_1 L_1 N_2 L_2}} (SM_S\lambda\mu|S'M'_S) \begin{Bmatrix} \Omega_a & K_2 & \lambda \\ \frac{1}{2} & I & S \\ \frac{1}{2} & I' & S' \end{Bmatrix}$$

$$R_+ = \frac{r' + r}{2}, R_- = r' - r$$

$$\times\, \mathcal{U}^{p\Omega_a K_2 \lambda}_{N_1 L_1 N_2 L_2}(E)\,(-1)^{-\mu}\left[\psi_{N_1 L_1}(c_1 \boldsymbol{R}_+) \otimes \psi_{N_2 L_2}(c_2 \boldsymbol{R}_-)\right]^{\lambda}_{-\mu}$$

λ encodes the different contributions from the central, spin-orbit and tensor interactions.

$I(I')$ is initial (final) spin of target, $S(S')$ is the initial (final) spin of projectile-target nucleon pair (called “channel spin”).

Equation also allows for the inclusion of spin-polarised densities from the NCSM.

Differential cross sections for spin zero targets

[**MS potential** - D.G.Madland. OECD/NEA Spec. Mtg. Nucleon-Nucleus Opt.Mod.to 200 MeV,Paris (1997) p.129.]

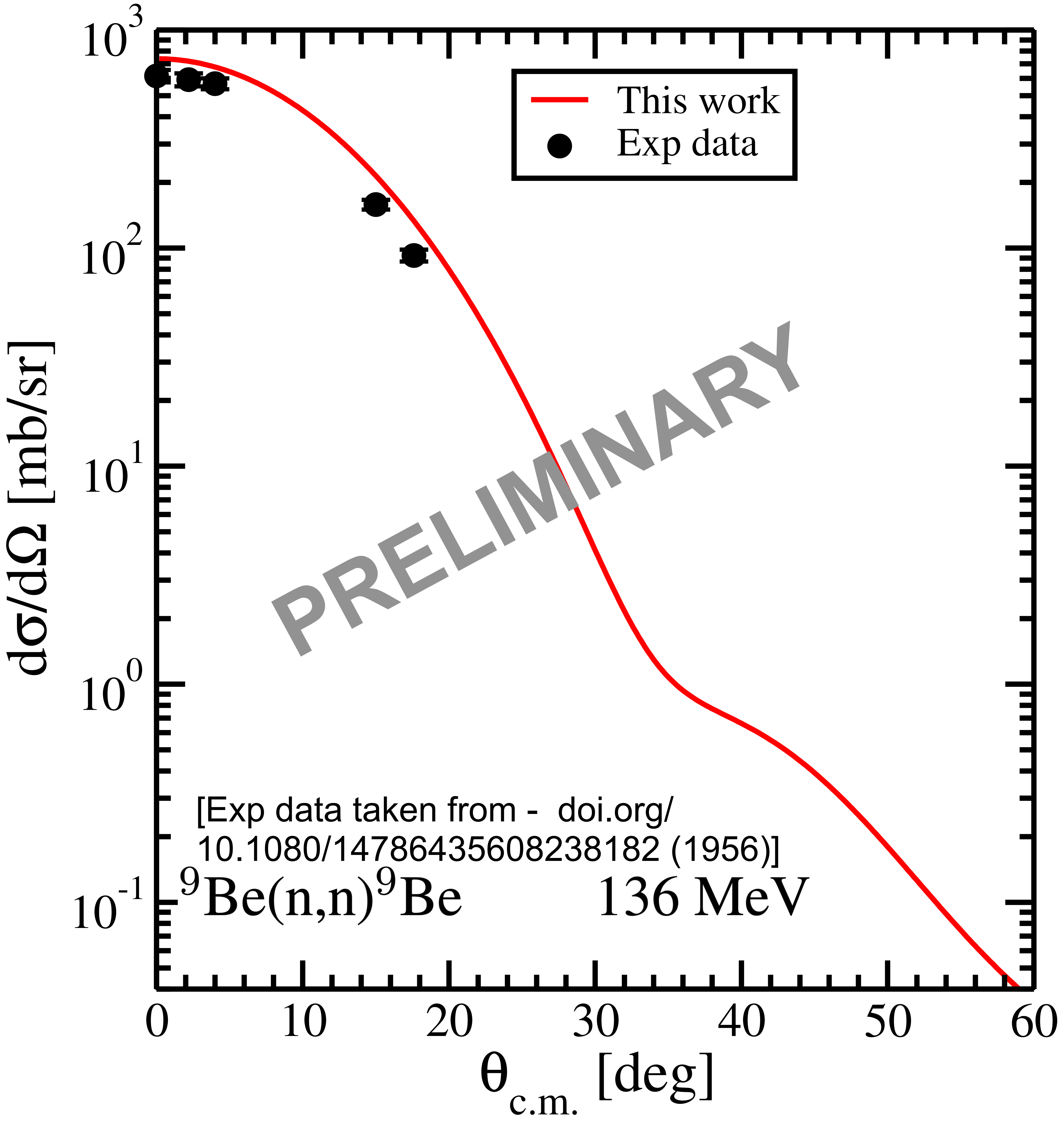
Differential cross sections for spin zero and non zero spin targets

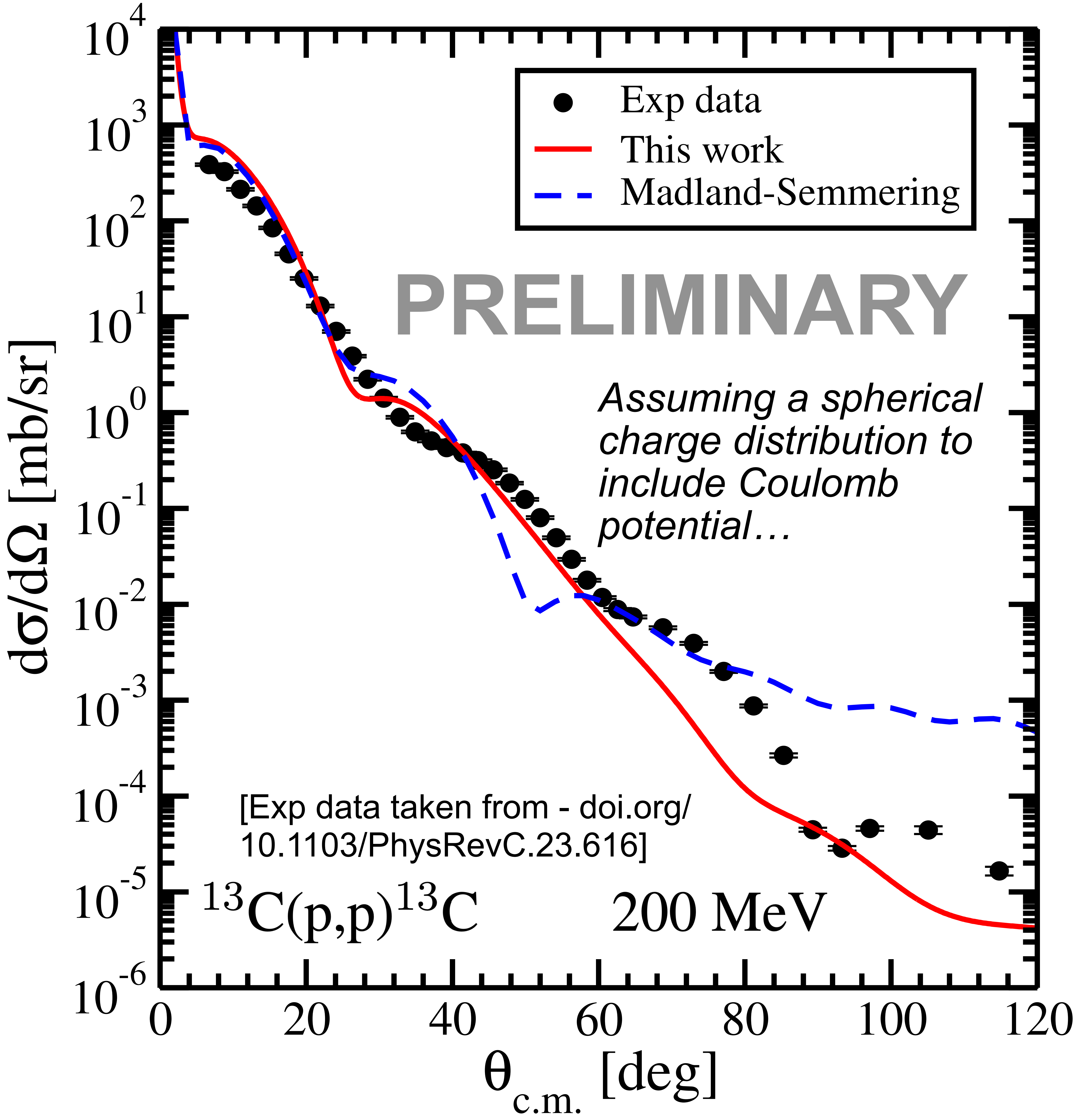
Conclusions

- We have derived an expression for a microscopic optical potential in coordinate space which allows us to go beyond the central term.
- Expansion of t matrix in HO basis leads to a microscopic optical potential with reduced computational time, order of minutes as opposed to hours.
- Investigations of systematics needed e.g. convergence of HO basis used for t matrix.
- Still need to include the Møller factor + full treatment of the Coulomb potential.
- Perform calculations with other available NN potentials.

Backup slides

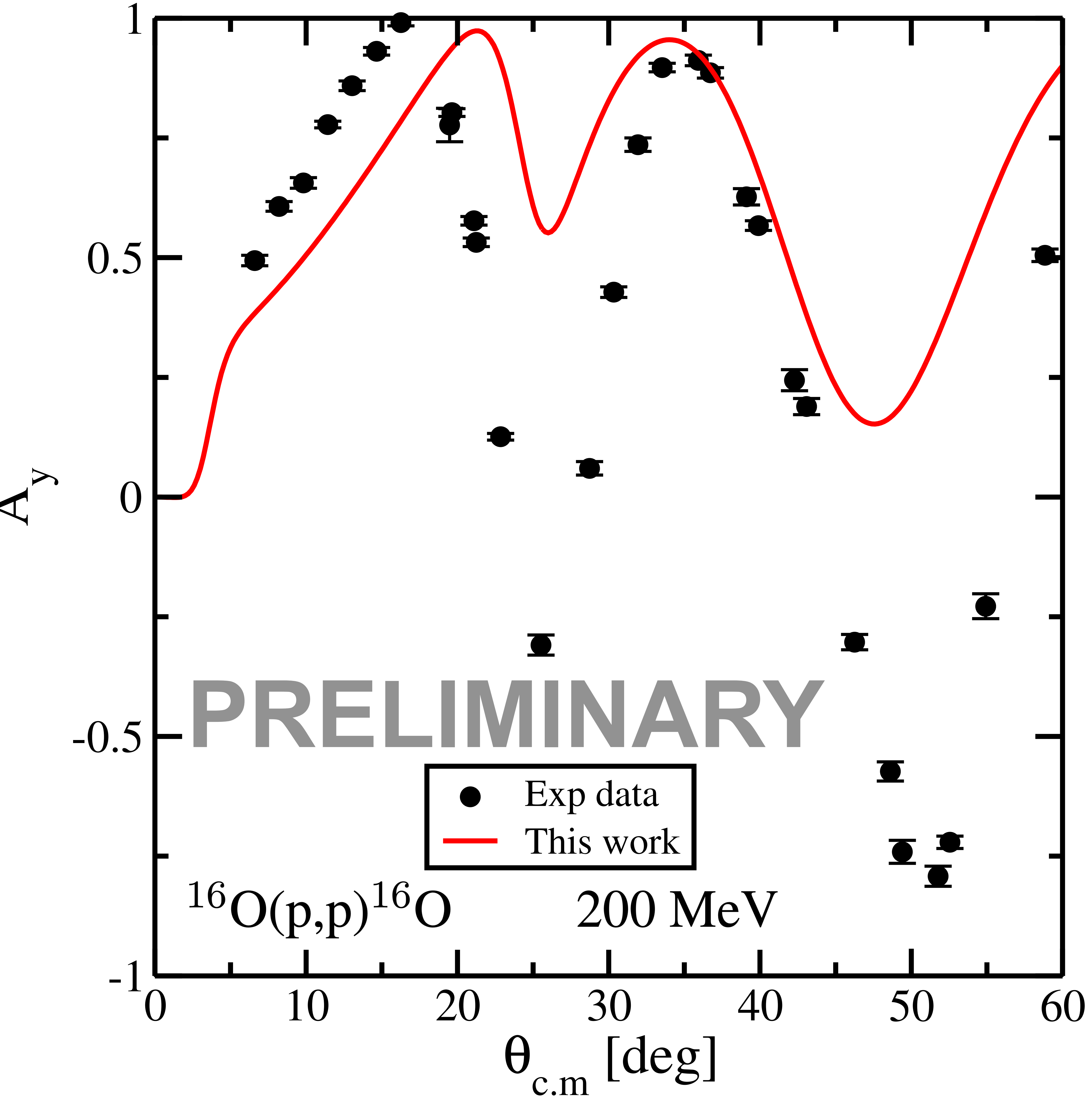
Differential cross sections for non-zero spin targets - *preliminary results*





Assuming a spherical charge distribution to include Coulomb potential…

[Exp data taken from - doi.org/10.1103/PhysRevC.31.1]



[Exp data taken from - doi.org/10.1103/PhysRevC.23.616]