

The dynamical evolution of the parity splitting energy in nuclei with octupole deformation

Radu Budaca

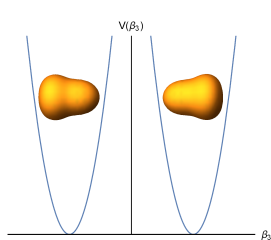
IFIN-HH, Bucharest-Magurele, România

IOP Nuclear Physics Conference, April 2026, Brighton, UK.

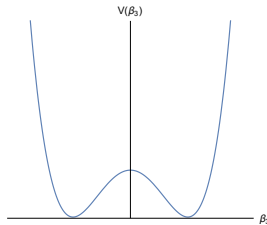
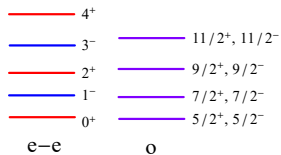
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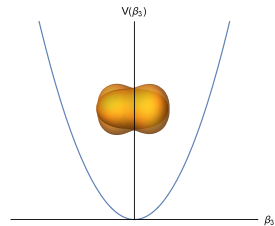
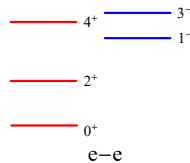
Dynamics of octupole deformation and its spectral signatures



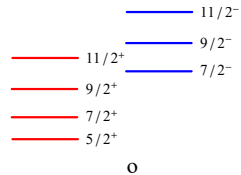
Stable (static, rigid) β_3



Intermediary dynamical regime (tunneling)



Dynamic β_3 (oscillations)



- Enhanced $E1$ and $E3$ transitions.

Quadrupole ($\beta_2 = \alpha_{20}$) and octupole ($\beta_3 = \alpha_{30}$) deformation variables limited to axial symmetry



$$\hat{H}_{QO} = - \sum_{\lambda=2,3} \frac{\hbar^2}{2B_\lambda} \frac{1}{\beta_\lambda^3} \frac{\partial}{\partial \beta_\lambda} \beta_\lambda^3 \frac{\partial}{\partial \beta_\lambda} + \frac{\hbar^2 \hat{L}^2}{6(B_2 \beta_2^2 + 2B_3 \beta_3^2)} + U(\beta_2, \beta_3)$$

Solutions $\Phi_{LMK}^\pm(\beta_2, \beta_3, \theta) = (\beta_2 \beta_3)^{-3/2} \Psi_L^\pm(\beta_2, \beta_3) \sqrt{\frac{2L+1}{32\pi^2}} \left[\mathcal{D}_{KM}^L(\Omega) \pm (-1)^L \mathcal{D}_{-KM}^L(\Omega) \right]$.

Notations: $B = \frac{B_2 + B_3}{2}$, $\epsilon = \frac{2B}{\hbar^2} E$, $u = \frac{2B}{\hbar^2} U$

Change of variables: $\tilde{\beta} = \sqrt{\frac{B_2 \beta_2^2 + B_3 \beta_3^2}{B}}$, $\tan \phi = \frac{\beta_3}{\beta_2} \sqrt{\frac{B_2}{B_3}}$

$$\phi = \begin{cases} 0, & \text{Pure quadrupole deformation } (\beta_3 = 0) \\ \pm \pi/2, & \text{Pure octupole deformation } (\beta_2 = 0) \end{cases}$$

⇓ Integration over Euler angles θ for $K = 0$

$$\left[-\frac{\partial^2}{\partial \tilde{\beta}^2} - \frac{1}{\tilde{\beta}} \frac{\partial}{\partial \tilde{\beta}} + \frac{L(L+1)}{3\tilde{\beta}^2(1 + \sin^2 \phi)} - \frac{1}{\tilde{\beta}^2} \frac{\partial^2}{\partial \phi^2} + u(\tilde{\beta}, \phi) + \frac{3}{\tilde{\beta}^2 \sin^2 2\phi} - \epsilon_L \right] \Psi_L^\pm(\tilde{\beta}, \phi) = 0$$



- Separable and ϕ -independent potential $u(\tilde{\beta}, \phi) = u(\tilde{\beta})$.
- Factorized wave-function $\Psi_L^\pm(\tilde{\beta}, \phi) = \psi_L^\pm(\tilde{\beta})\chi_L^\pm(\phi)$.
- Separation constant $W_L^\pm$.

Radial-like equation:
$$\left[-\frac{\partial^2}{\partial \tilde{\beta}^2} - \frac{1}{\tilde{\beta}} \frac{\partial}{\partial \tilde{\beta}} + \frac{W_L^\pm}{\tilde{\beta}^2} + u(\tilde{\beta}) \right] \psi_L^\pm(\tilde{\beta}) = \epsilon_L \psi_L^\pm(\tilde{\beta})$$

$$u(\tilde{\beta}) = \frac{w_0}{\tilde{\beta}^2} + u_0(\tilde{\beta}) \begin{cases} \text{Harmonic oscillator potential (HO):} & u_0(\tilde{\beta}) = \tilde{\beta}^2 \\ \text{Infinite square well potential (ISW):} & u_0(\tilde{\beta}) = \begin{cases} 0, & \tilde{\beta} \leq 1 \\ \infty, & \tilde{\beta} > 1 \end{cases} \end{cases}$$

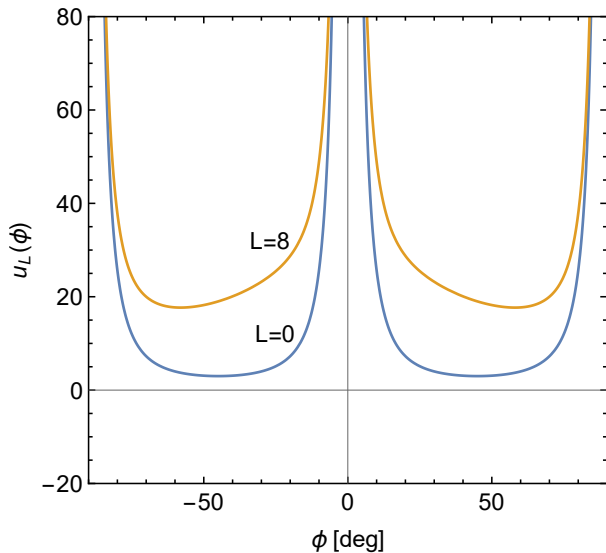
Angular equation:
$$\left[-\frac{\partial^2}{\partial \phi^2} + u_L(\phi) \right] \chi_L^\pm(\phi) = W_L^\pm \chi_L^\pm(\phi)$$

Intrinsic potential:
$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1 + \sin^2 \phi)}$$

Intrinsic ϕ potential

Intrinsic potential

$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1 + \sin^2 \phi)}$$

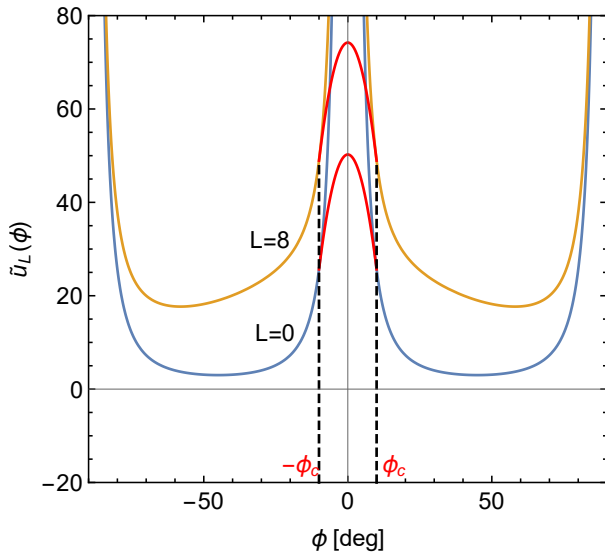


Intrinsic ϕ potential

Intrinsic potential

$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1 + \sin^2 \phi)}$$

$$v(\phi) = -a\phi^2 + b$$

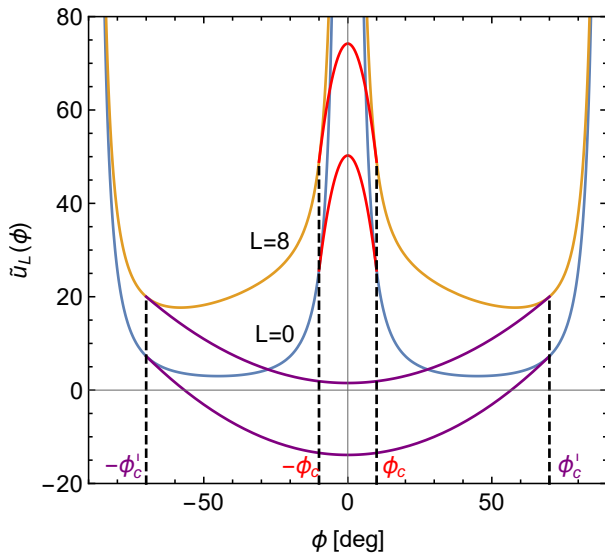


Intrinsic ϕ potential

Intrinsic potential

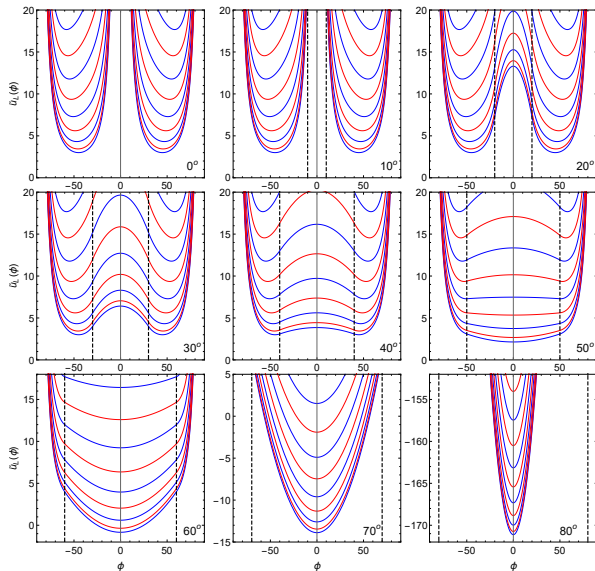
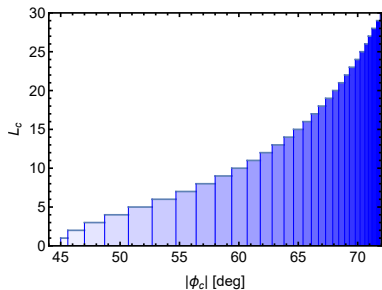
$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1 + \sin^2 \phi)}$$

$$v(\phi) = -a\phi^2 + b$$



The modified potential:

$$\tilde{u}_L(\phi) = \begin{cases} v_L(\phi), & |\phi| < |\phi_c| \\ u_L(\phi), & |\phi| \geq |\phi_c| \end{cases}$$



- Total angular momentum $\vec{J} = \vec{L} + \vec{j}$
- Strong coupling hypothesis $\Rightarrow$ collective degrees of freedom are dominant
- Intrinsic angular momentum projection $K = \Omega$
- Coriolis interaction

$$L(L+1) \rightarrow J(J+1) - K^2 + \pi a \delta_{K\frac{1}{2}} (-)^{J+\frac{1}{2}} \left(J + \frac{1}{2} \right)$$

a - decoupling parameter (adjustable).

$\pi = \pi_p \pi_c$ - total parity defined by the parity of the odd nucleon and of the deformed core.

- Particle-rotor wave function

$$\Phi_{JMK}^{\pi}(\beta_2, \beta_3, \theta) = (\beta_2 \beta_3)^{-3/2} \Psi_{JK}^{\pi_c}(\beta_2, \beta_3) \sqrt{\frac{2J+1}{16\pi^2}} \left[D_{KM}^J(\theta) f_K^{\pi_p} + \pi_c (-1)^{J+K} D_{-KM}^J(\theta) f_{-K}^{\pi_p} \right]$$

- Even-even nuclei ($K = 0$ bands, HO&ISW)

Heavy nuclei $^{220,222}\text{Rn}$, $^{222-228}\text{Ra}$, $^{224-234}\text{Th}$,
 $^{230-240}\text{U}$, $^{236-240}\text{Pu}$

Medium nuclei $^{142-148}\text{Ba}$, $^{146,148}\text{Ce}$, $^{148-152}\text{Nd}$,
 $^{150-154}\text{Sm}$, $^{148-160}\text{Gd}$, $^{164-170}\text{Er}$

RB, P. Baganu, A.I. Budaca, Phys. Rev. C **106** (2022) 014311

RB, P. Baganu, A.I. Budaca, Eur. Phys. J. A **59** (2023) 242

RB, A.I. Budaca, P. Baganu, Phys. Scr. **99** (2024) 035309

RB, EPL **151** (2025) 34002

RB, S. Pascu, Phys. Rev. C **113** (2026) 014326

- Odd- A nuclei (HO) [RB, At. Data Nucl. Data Tables **161** (2025) 101692; Eur. Phys. J. A **62** (2026) 32]

Heavy mass

$K = 1/2$ $^{219,221}\text{Fr}$, ^{225}Ra , ^{227}Th , ^{237}U , ^{239}Pu ,
 $^{229-233}\text{Pa}$

$K = 3/2$ ^{223}Ra , ^{225}Th , $^{225,227}\text{Ac}$

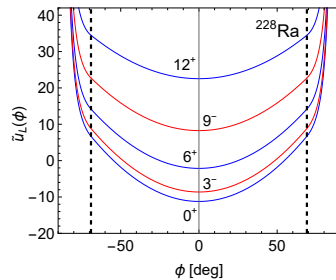
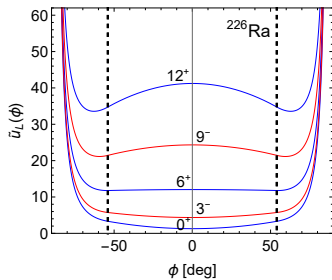
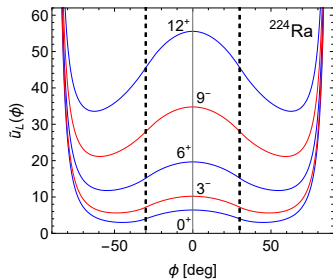
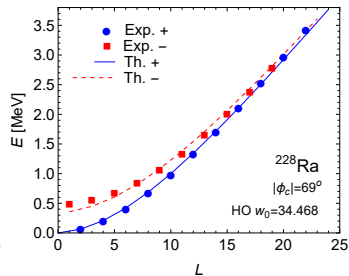
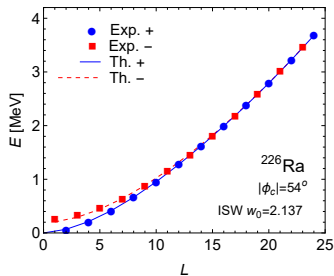
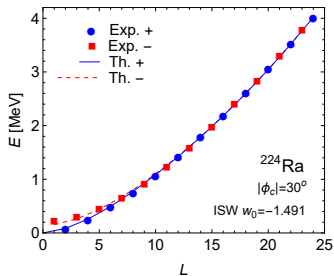
$K = 5/2$ ^{221}Ra , ^{231}Th , ^{233}U , $^{239-243}\text{Am}$

Medium mass

$K = 3/2$ ^{151}Ce , ^{153}Nd

$K = 5/2$ $^{155-159}\text{Eu}$

- The parameters fitted from experimental energy levels are used for predictions on $E0$, $E1$, $E2$, $E3$ transition probabilities.



Fine spectral structure of positive and negative parity states

$$\delta E(L) = E(L) - \frac{1}{2} [E(L-1) + E(L+1)],$$

- Simple mathematical modeling

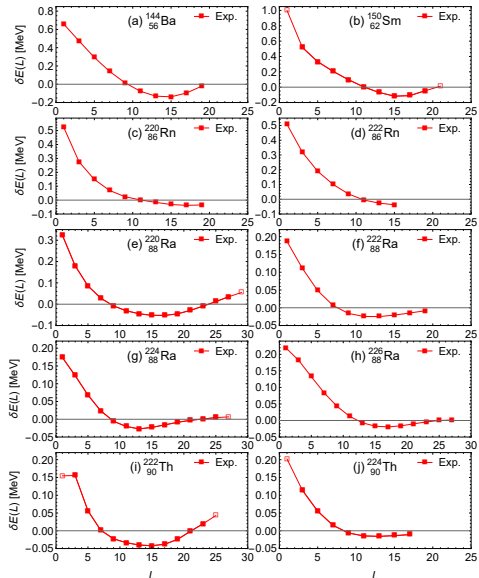
$$E_+(L) = AL(L+1) + B[L(L+1)]^2 + \dots$$

$$E_-(L) = E_0 + A'L(L+1) + B'[L(L+1)]^2 + \dots$$

- Phenomenological interpretation

$$E_n(I) = \Omega_3 \left(n + \frac{1}{2} \right) + \frac{(I - nj)^2}{2\mathcal{J}}$$

n octupole phonon carries $j \approx 3\hbar$ units of angular momentum. [S. Frauendorf, Phys. Rev. C **77** (2008) 021304(R)]



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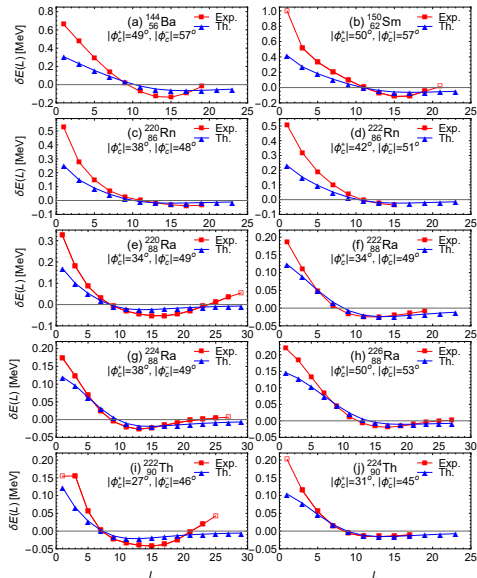
$$E_-(L) = E_0 + A'L(L+1) + B'[L(L+1)]^2 + \dots$$

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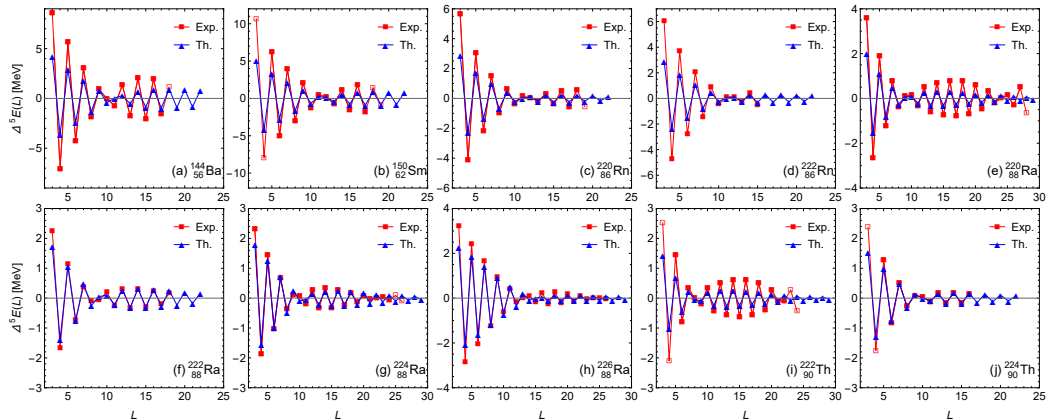
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- Present approach $|\phi_c|^+ < |\phi_c^-|$

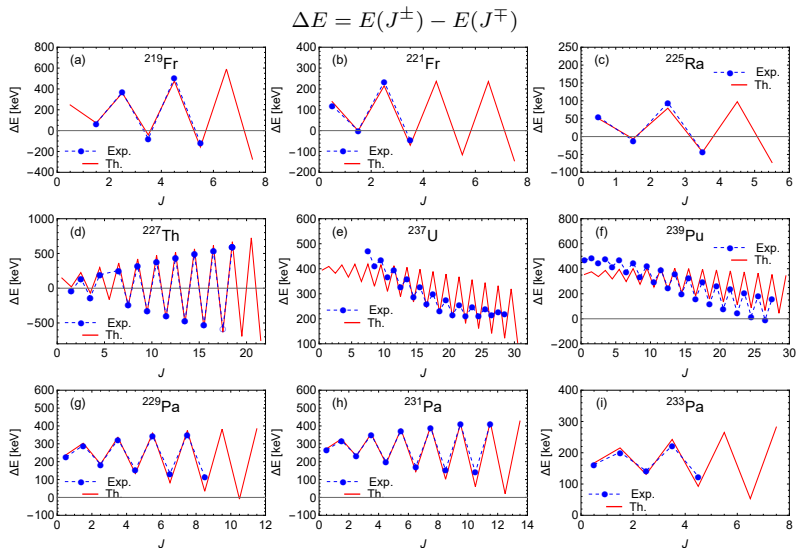


"Beat" pattern of the $\Delta L = 1$ staggering

$$\Delta^5 E(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2), \quad \Delta E(L) = E(L) - E(L-1)$$



Energy difference between $K = 1/2$ bands in odd mass nuclei within strong coupling



Adjustable spin-spin particle-core interaction

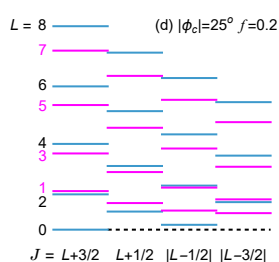
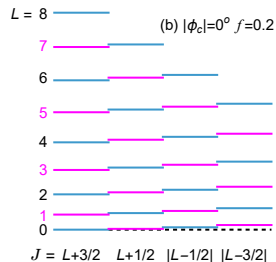
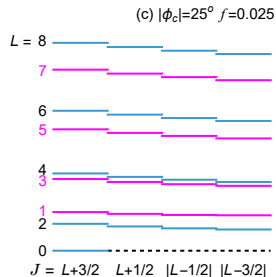
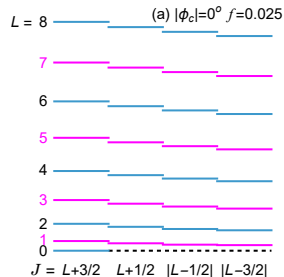
$$\hat{H} = \hat{H}_{QO} + \frac{\hbar^2 f}{B\tilde{\beta}^2} \hat{\vec{L}} \cdot \hat{\vec{j}}$$

j - total spin of valence nucleons

m_j - laboratory frame projections

$J = |L + m_j|$ - total angular momentum

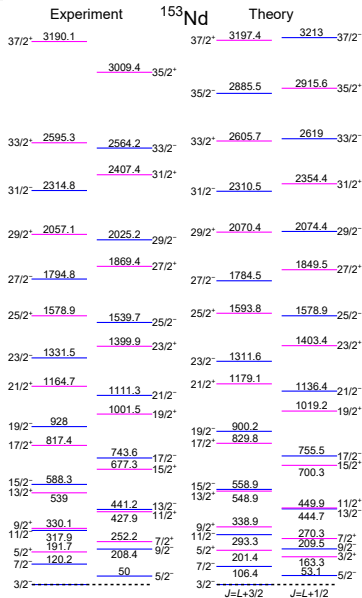
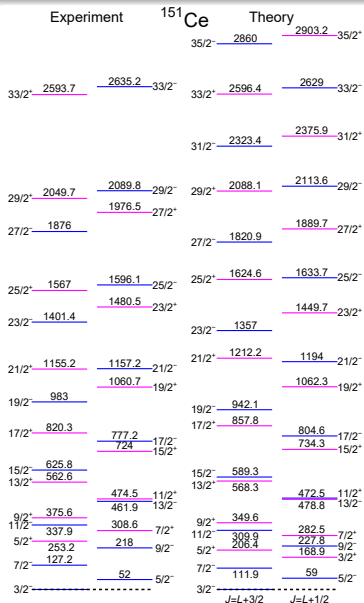
$J = j$ - ground state



Intermediate particle-core coupling - experimental realization ($N = 93$ nuclei)

$$|\phi_c| = 47^\circ$$

$$f = 0.17$$

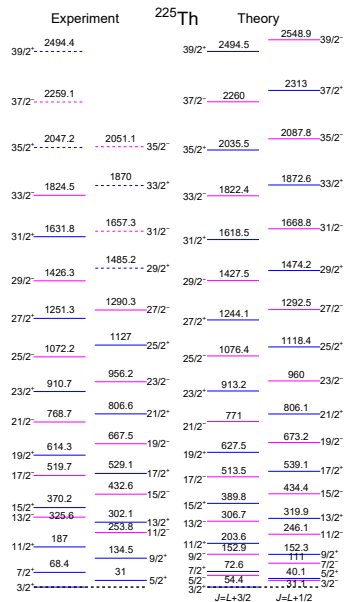
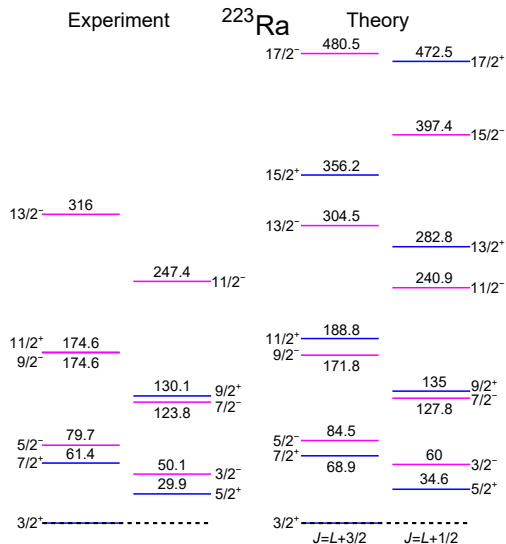


$$|\phi_c| = 50^\circ$$

$$f = 0.19$$

Intermediate particle-core coupling - experimental realization ($N = 135$ nuclei)

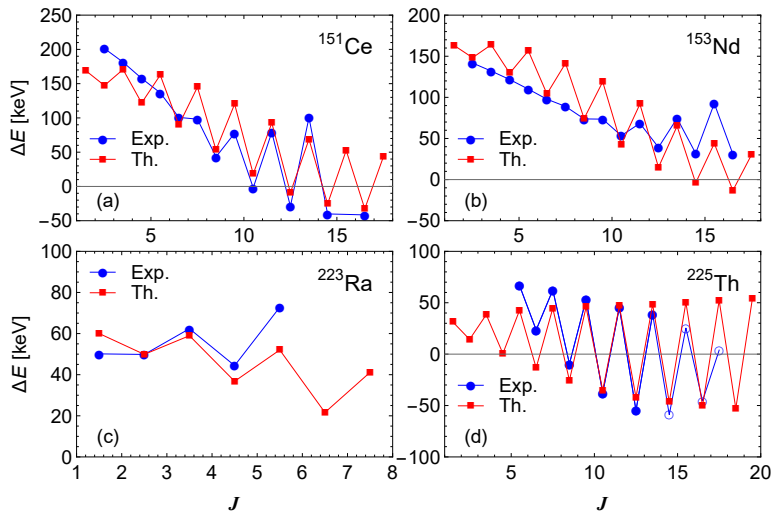
$$|\phi_c| = 28^\circ \quad f = 0.19$$



$$|\phi_c| = 16^\circ$$

$$f = 0.15$$

$$\Delta E = E(J^\pm) - E(J^\mp)$$



- An axially symmetric geometric model is employed for the description of the fine spectroscopic features of nuclei with strong octupole correlations.
- The positive parity states of even-even pear-shaped nuclei, have an octupole deformation which is more stable to quantum fluctuations, in comparison to the negative parity states.
- The combined effect of the parity splitting mechanism and of the particle-core interaction generates a diverse relative distribution of positive and negative parity states in odd A nuclei.
- New specific spectroscopic patterns are proposed as signatures for octupole deformation in odd mass nuclei.

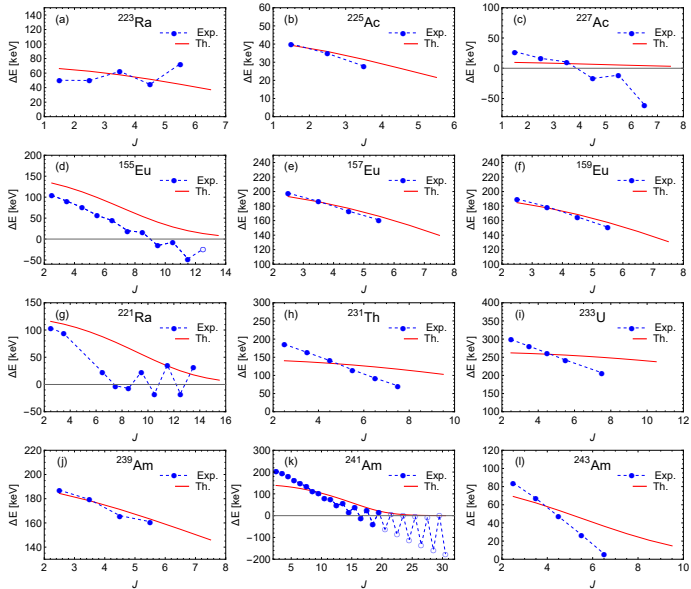
- Future projects:

Applications to nuclei from the ($N \sim 56$, $Z \sim 34$) region.

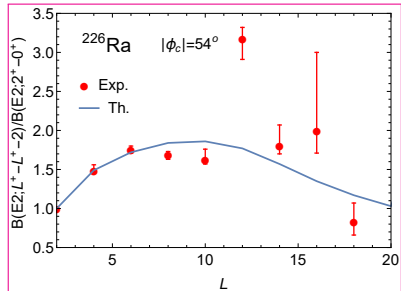
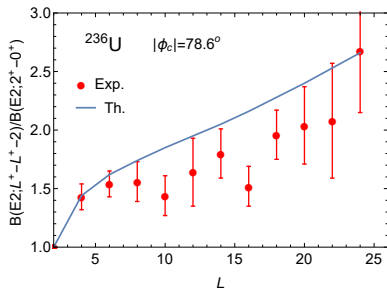
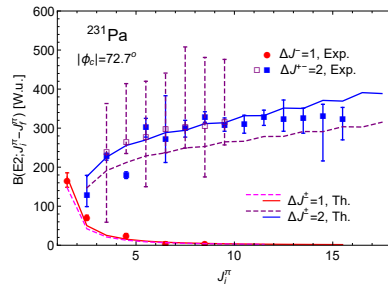
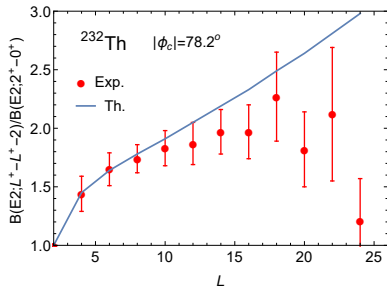
Interpretation of the excited states (two octupole phonon states).

Inclusion of triaxiality (rigid, soft) $\Rightarrow$ description of the γ and g bands.

Numerical applications - Energy difference for $K \neq 1/2$ bands of odd mass nuclei



Numerical applications - $E2$ transitions



- $B(E1)$ transition probabilities $\frac{B(E1; L^- \rightarrow (L+1)^+)}{B(E1; L^- \rightarrow (L-1)^+)} > 1$

- $B(E3)$ transition probabilities

^{146}Ba

$$\frac{B(E3; 5^- \rightarrow 2^+)}{B(E3; 3^- \rightarrow 0^+)} = \begin{cases} 1.52_{-85}^{+601} \text{ Exp.} \\ 1.67 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 7^- \rightarrow 4^+)}{B(E3; 3^- \rightarrow 0^+)} = \begin{cases} 1.70_{-109}^{+765} \text{ Exp.} \\ 2.31 \text{ Th.} \end{cases}$$

^{224}Ra

$$\frac{B(E3; 1^- \rightarrow 2^+)}{B(E3; 3^- \rightarrow 0^+)} = \begin{cases} 5(1) \text{ Exp.} \\ 3.21 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 5^- \rightarrow 2^+)}{B(E3; 3^- \rightarrow 0^+)} = \begin{cases} 1.45(42) \text{ Exp.} \\ 1.68 \text{ Th.} \end{cases}$$

^{148}Nd

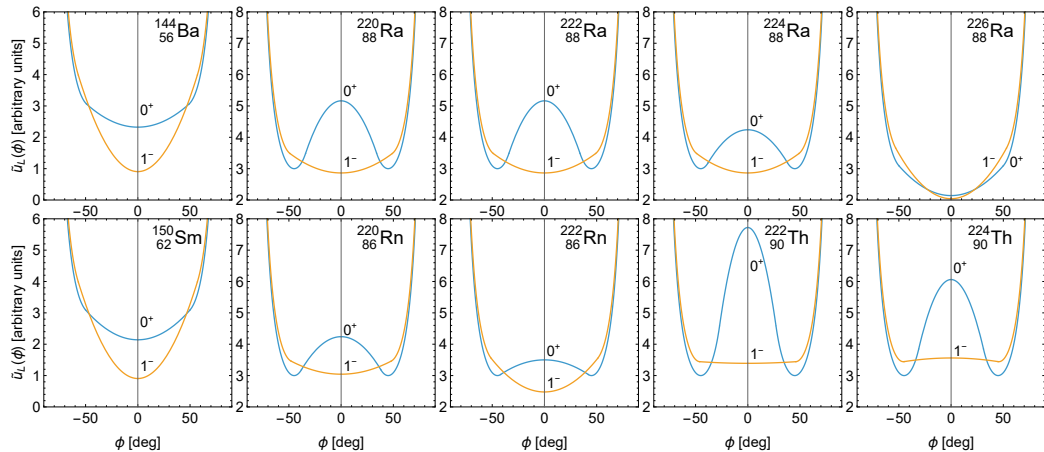
$$\frac{B(E3; 5^- \rightarrow 2^+)}{B(E3; 3^- \rightarrow 0^+)} = \begin{cases} 1.88_{-31}^{+40} \text{ Exp.} \\ 1.91 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 7^- \rightarrow 4^+)}{B(E3; 3^- \rightarrow 0^+)} = \begin{cases} 2.12_{-36}^{+51} \text{ Exp.} \\ 3.03 \text{ Th.} \end{cases}$$

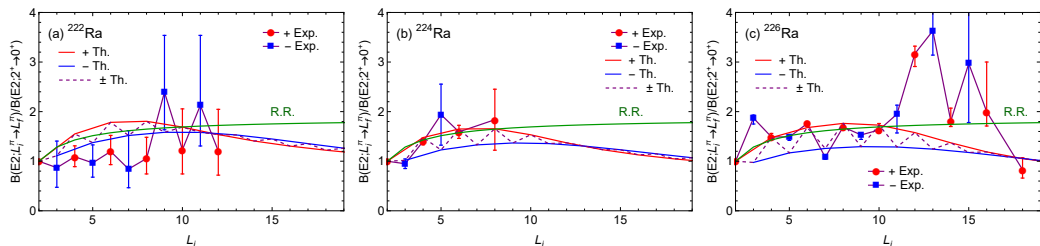
^{236}U

$$\frac{B(E3; 0^+ \rightarrow 3^-)}{B(E3; 1^- \rightarrow 4^+)} = \begin{cases} 2.58(51) \text{ Exp.} \\ 1.71 \text{ Th.} \end{cases}$$

Parity specific ϕ -potential



Transition probabilities in the parity specific model



Nucleus		$\frac{B(E1, 1^- \rightarrow 2^+)}{B(E1, 1^- \rightarrow 0^+)}$	$\frac{B(E1, 3^- \rightarrow 4^+)}{B(E1, 3^- \rightarrow 2^+)}$	$\frac{B(E3, 1^- \rightarrow 2^+)}{B(E3, 3^- \rightarrow 0^+)}$	$\frac{B(E3, 3^- \rightarrow 2^+)}{B(E3, 3^- \rightarrow 0^+)}$	$\frac{B(E3, 5^- \rightarrow 2^+)}{B(E3, 3^- \rightarrow 0^+)}$
$^{220}_{86}\text{Rn}$	Exp.					2.73(155)
	Th.	3.76	1.76	5.22	2.37	2.70
$^{222}_{88}\text{Ra}$	Exp.	1.96^{+94}_{-62}		1.32(77)	0.63(71)	1.60(44)
	Th.	2.32	1.63	3.35	1.53	1.76
$^{224}_{88}\text{Ra}$	Exp.			5.00(101)		1.45(49)
	Th.	2.18	1.55	3.20	1.45	1.67
$^{226}_{88}\text{Ra}$	Exp.	1.83(87)	0.86(24)	2.83^{+32}_{-46}	1.13(32)	2.18(20)
	Th.	2.13	1.50	3.14	1.41	1.62

$E1 [10^{-5} \text{ W.u.}]$				$E2 [\text{W.u.}]$				$E3 [\text{W.u.}]$	
$J^p \rightarrow J'^{p'}$	Th	Exp.[42]	Exp.[39]	$J^p \rightarrow J'^{p'}$	Th	Exp.[42]	Exp.[39]	$J^p \rightarrow J'^{p'}$	Th
$\frac{3}{2}^- \rightarrow \frac{5}{2}^+$	59	50(9)	54(6)	$\frac{5}{2}^+ \rightarrow \frac{3}{2}^+$	49		> 123	$\frac{7}{2}^- \rightarrow \frac{3}{2}^+$	66
$\frac{3}{2}^- \rightarrow \frac{3}{2}^+$	46	119(16)	127(7)	$\frac{7}{2}^+ \rightarrow \frac{5}{2}^+$	23	≈70	57(21)	$\frac{9}{2}^- \rightarrow \frac{3}{2}^+$	66
$\frac{5}{2}^- \rightarrow \frac{5}{2}^+$	17		81(19)	$\frac{7}{2}^+ \rightarrow \frac{3}{2}^+$	49	≈44	37(11)	$\frac{3}{2}^- \rightarrow \frac{5}{2}^+$	75
$\frac{5}{2}^- \rightarrow \frac{3}{2}^+$	17		89(7)	$\frac{7}{2}^- \rightarrow \frac{5}{2}^-$	18	110(50)	144(49)	$\frac{5}{2}^- \rightarrow \frac{5}{2}^+$	63
$\frac{7}{2}^- \rightarrow \frac{7}{2}^+$	9	20_{-5}^{+22}	4.7(11)	$\frac{7}{2}^- \rightarrow \frac{3}{2}^-$	27	10(6)	14(7)	$\frac{7}{2}^- \rightarrow \frac{5}{2}^+$	1.6
$\frac{7}{2}^- \rightarrow \frac{5}{2}^+$	51	79(24)	10.4(22)	$\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$	14		34(10)	$\frac{9}{2}^- \rightarrow \frac{5}{2}^+$	36
$\frac{9}{2}^+ \rightarrow \frac{7}{2}^-$	61		47(18)·10 ²	$\frac{9}{2}^+ \rightarrow \frac{5}{2}^+$	52		45(14)	$\frac{7}{2}^+ \rightarrow \frac{3}{2}^-$	53
$\frac{9}{2}^- \rightarrow \frac{9}{2}^+$	5.6		~ 3.4	$\frac{9}{2}^- \rightarrow \frac{7}{2}^-$	10		187(100)	$\frac{9}{2}^+ \rightarrow \frac{3}{2}^-$	55
$\frac{9}{2}^- \rightarrow \frac{7}{2}^+$	60		29(7)	$\frac{9}{2}^- \rightarrow \frac{5}{2}^-$	46		~105	$\frac{5}{2}^- \rightarrow \frac{7}{2}^+$	35
$\frac{11}{2}^- \rightarrow \frac{9}{2}^+$	64		37.2(30)	$\frac{11}{2}^+ \rightarrow \frac{7}{2}^+$	66		280(85)	$\frac{7}{2}^- \rightarrow \frac{7}{2}^+$	48
$\frac{11}{2}^- \rightarrow \frac{11}{2}^+$	3.7			$\frac{11}{2}^- \rightarrow \frac{7}{2}^-$	45		72(31)	$\frac{9}{2}^- \rightarrow \frac{7}{2}^+$	48