The dynamical evolution of the parity splitting energy in nuclei with octupole deformation

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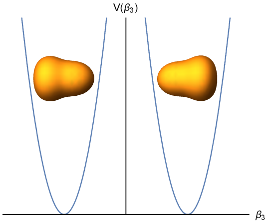
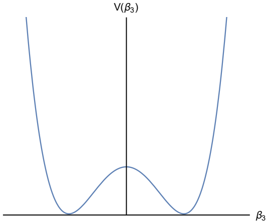
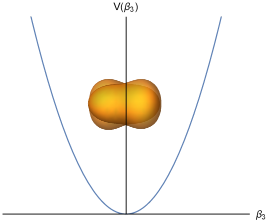
Dynamics of octupole deformation and its spectral signatures



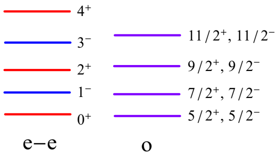


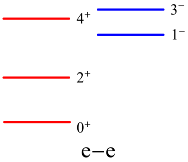


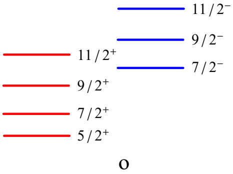
Stable (static, rigid) β_3

Intermediary dynamical regime (tunneling)

Dynamic β_3 (oscillations)







- Enhanced $E1$ and $E3$ transitions.

Quadrupole-octupole collective Hamiltonian

Quadrupole ($\beta_2 = \alpha_{20}$) and octupole ($\beta_3 = \alpha_{30}$) deformation variables limited to axial symmetry

$\Longrightarrow$

$$\hat{H}_{QO} = -\sum_{\lambda=2,3} \frac{\hbar^2}{2B_\lambda} \frac{1}{\beta_\lambda^3} \frac{\partial}{\partial \beta_\lambda} \beta_\lambda^3 \frac{\partial}{\partial \beta_\lambda} + \frac{\hbar^2 \hat{L}^2}{6(B_2\beta_2^2 + 2B_3\beta_3^2)} + U(\beta_2, \beta_3)$$

Solutions $\Phi^{\pm}_{LMK}(\beta_2, \beta_3, \theta) = (\beta_2\beta_3)^{-3/2} \Psi^{\pm}_L(\beta_2, \beta_3) \sqrt{\frac{2L+1}{32\pi^2}} \left[\mathcal{D}^L_{KM}(\Omega) \pm (-1)^L \mathcal{D}^L_{-KM}(\Omega)\right]$.

$$\text{Notations:} \quad B = \frac{B_2 + B_3}{2}, \quad \epsilon = \frac{2B}{\hbar^2} E, \quad u = \frac{2B}{\hbar^2} U$$

$$\text{Change of variables:} \quad \tilde{\beta} = \sqrt{\frac{B_2\beta_2^2 + B_3\beta_3^2}{B}}, \quad \tan\phi = \frac{\beta_3}{\beta_2}\sqrt{\frac{B_2}{B_3}}$$

$$\phi = \begin{cases} 0, & \text{Pure quadrupole deformation } (\beta_3 = 0) \\ \pm\pi/2, & \text{Pure octupole deformation } (\beta_2 = 0) \end{cases}$$

$\Downarrow$ Integration over Euler angles θ for $K = 0$

$$\left[-\frac{\partial^2}{\partial\tilde{\beta}^2} - \frac{1}{\tilde{\beta}}\frac{\partial}{\partial\tilde{\beta}} + \frac{L(L+1)}{3\tilde{\beta}^2(1+\sin^2\phi)} - \frac{1}{\tilde{\beta}^2}\frac{\partial^2}{\partial\phi^2} + u(\tilde{\beta},\phi) + \frac{3}{\tilde{\beta}^2\sin^2 2\phi} - \epsilon_L\right]\Psi^{\pm}_L(\tilde{\beta},\phi) = 0$$

[V. Yu. Denisov, A. Ya. Dzyublik, Nucl. Phys. A **589** (1995) 17]

$\Downarrow$

- Separable and ϕ-independent potential $u(\tilde{\beta}, \phi) = u(\tilde{\beta})$.
- Factorized wave-function $\Psi_L^{\pm}(\tilde{\beta}, \phi) = \psi_L^{\pm}(\tilde{\beta})\chi_L^{\pm}(\phi)$.
- Separation constant $W_L^{\pm}$.

Radial-like equation: $$\left[-\frac{\partial^2}{\partial\tilde{\beta}^2} - \frac{1}{\tilde{\beta}}\frac{\partial}{\partial\tilde{\beta}} + \frac{W_L^{\pm}}{\tilde{\beta}^2} + u(\tilde{\beta})\right]\psi_L^{\pm}(\tilde{\beta}) = \epsilon_L \psi_L^{\pm}(\tilde{\beta})$$

$$u(\tilde{\beta}) = \frac{w_0}{\tilde{\beta}^2} + u_0(\tilde{\beta}) \begin{cases} \text{Harmonic oscillator potential (HO):} & u_0(\tilde{\beta}) = \tilde{\beta}^2 \\ \text{Infinite square well potential (ISW):} & u_0(\tilde{\beta}) = \begin{cases} 0, & \tilde{\beta} \leq 1 \\ \infty, & \tilde{\beta} > 1 \end{cases} \end{cases}$$

Angular equation: $$\left[-\frac{\partial^2}{\partial\phi^2} + u_L(\phi)\right]\chi_L^{\pm}(\phi) = W_L^{\pm}\chi_L^{\pm}(\phi)$$

Intrinsic potential: $$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1+\sin^2\phi)}$$

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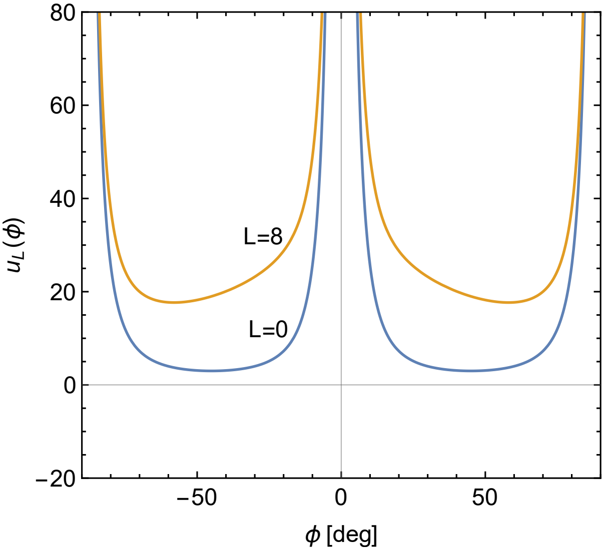
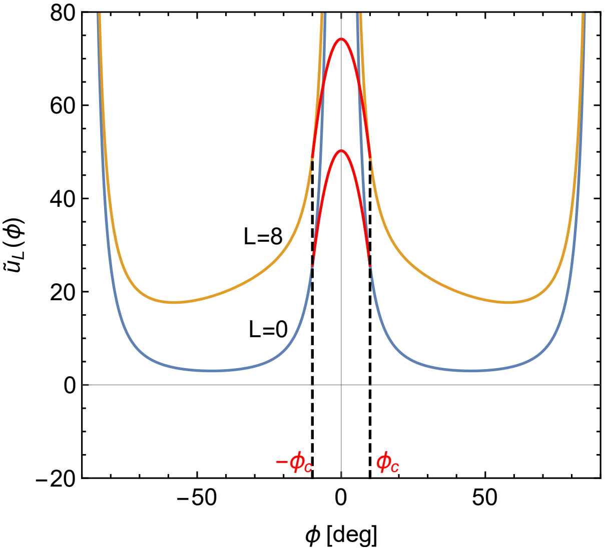


Intrinsic potential

$$u_L(\phi) = \frac{3}{\sin^2 2\phi} + \frac{L(L+1)}{3(1+\sin^2\phi)}$$

$$v(\phi) = -a\phi^2 + b$$



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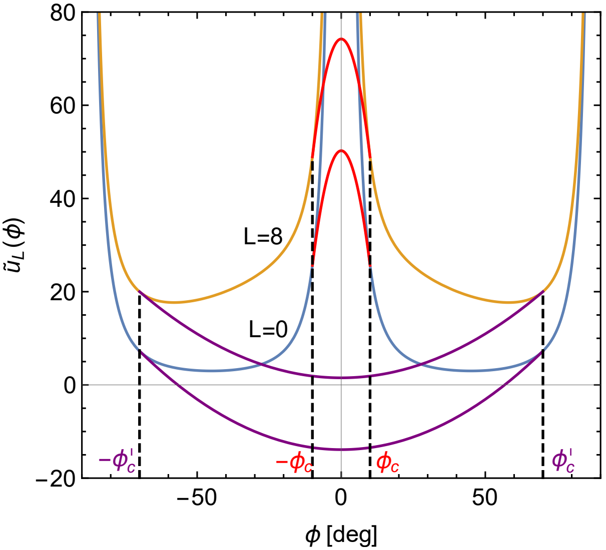
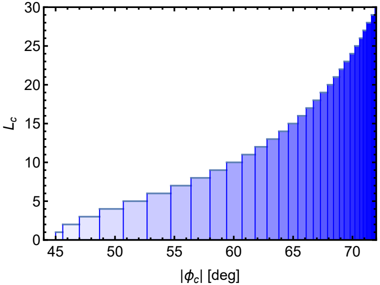
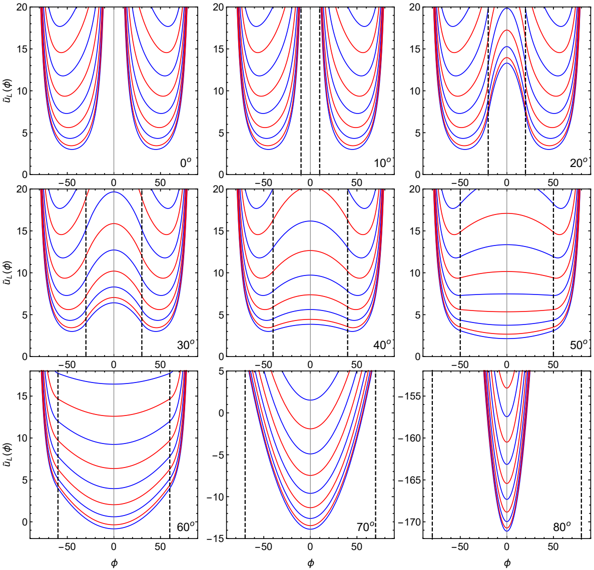


The modified potential:

$$\tilde{u}_L(\phi) = \begin{cases} v_L(\phi), & |\phi| < |\phi_c| \\ u_L(\phi), & |\phi| \geq |\phi_c| \end{cases}$$





- Total angular momentum $\vec{J} = \vec{L} + \vec{j}$
- Strong coupling hypothesis $\Rightarrow$ collective degrees of freedom are dominant
- Intrinsic angular momentum projection $K = \Omega$
- Coriolis interaction

$$L(L+1) \;\rightarrow\; J(J+1) - K^2 + \pi a \delta_{K\frac{1}{2}} (-)^{J+\frac{1}{2}} \left(J + \frac{1}{2}\right)$$

a - decoupling parameter (adjustable).

$\pi = \pi_p \pi_c$ - total parity defined by the parity of the odd nucleon and of the deformed core.

- Particle-rotor wave function

$$\Phi^{\pi}_{JMK}(\beta_2, \beta_3, \theta) = (\beta_2\beta_3)^{-3/2} \Psi^{\pi_c}_{JK}(\beta_2, \beta_3) \sqrt{\frac{2J+1}{16\pi^2}} \left[D^J_{KM}(\theta) f^{\pi_p}_K + \pi_c (-1)^{J+K} D^J_{-KM}(\theta) f^{\pi_p}_{-K} \right]$$

- Even-even nuclei ($K=0$ bands, HO&ISW)

Heavy nuclei ^{220,222}Rn, $^{222-228}$Ra, $^{224-234}$Th, $^{230-240}$U, $^{236-240}$Pu

Medium nuclei $^{142-148}$Ba, ^{146,148}Ce, $^{148-152}$Nd, $^{150-154}$Sm, $^{148-160}$Gd, $^{164-170}$Er

RB, P. Buganu, A.I. Budaca, Phys. Rev. C **106** (2022) 014311
RB, P. Buganu, A.I. Budaca, Eur. Phys. J. A **59** (2023) 242
RB, A.I. Budaca, P. Buganu, Phys. Scr. **99** (2024) 035309
RB, EPL **151** (2025) 34002
RB, S. Pascu, Phys. Rev. C **113** (2026) 014326

- Odd-A nuclei (HO) [RB, At. Data Nucl. Data Tables **161** (2025) 101692; Eur. Phys. J. A **62** (2026) 32]

Heavy mass

$K=1/2$ ^{219,221}Fr, ^{225}Ra, ^{227}Th, ^{237}U, ^{239}Pu, $^{229-233}$Pa

$K=3/2$ ^{223}Ra, ^{225}Th, ^{225,227}Ac

$K=5/2$ ^{221}Ra, ^{231}Th, ^{233}U, $^{239-243}$Am

Medium mass

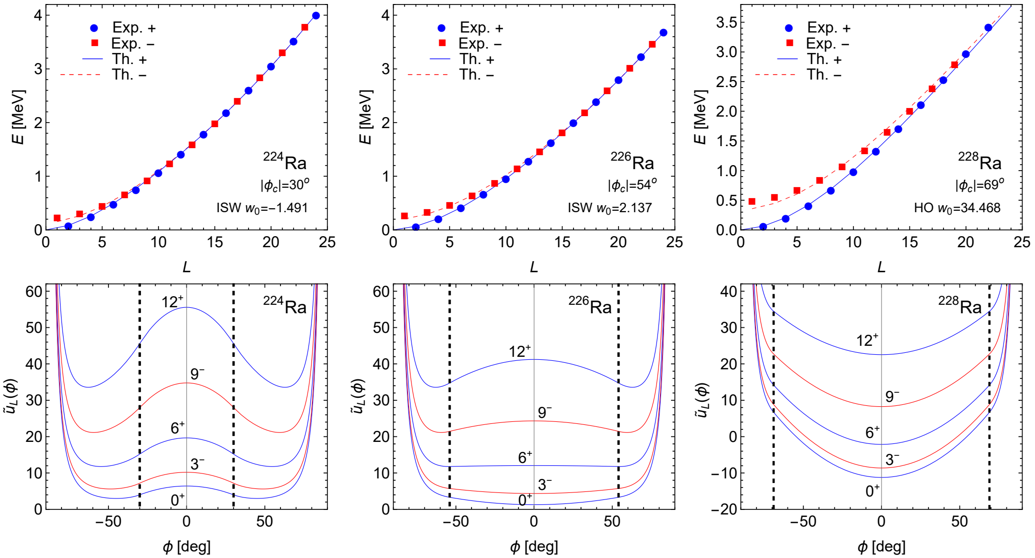
$K=3/2$ ^{151}Ce, ^{153}Nd

$K=5/2$ $^{155-159}$Eu

- The parameters fitted from experimental energy levels are used for predictions on $\underline{E0, E1, E2, E3}$ transition probabilities.

Even-even Ra nuclei

RB, P. Buganu, A.I. Budaca, Phys. Rev. C **106** (2022) 014311



Fine spectral structure of positive and negative parity states

$$\delta E(L) = E(L) - \frac{1}{2}\left[E(L-1) + E(L+1)\right],$$

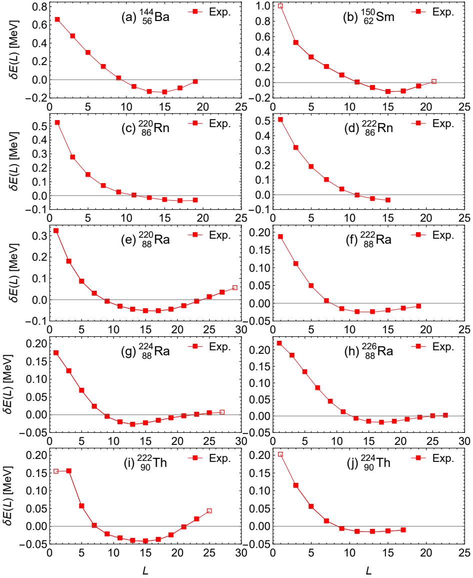
- Simple mathematical modeling

$$\begin{aligned} E_+(L) &= AL(L+1) + B\left[L(L+1)\right]^2 + \ldots \\ E_-(L) &= E_0 + A'L(L+1) + B'\left[L(L+1)\right]^2 + \ldots \end{aligned}$$

- Phenomenological interpretation

$$E_n(I) = \Omega_3\left(n + \frac{1}{2}\right) + \frac{(I - nj)^2}{2\mathcal{J}}$$

n octupole phonon carries $j \approx 3\hbar$ units of angular momentum. [S. Frauendorf, Phys. Rev. C **77** (2008) 021304(R)]



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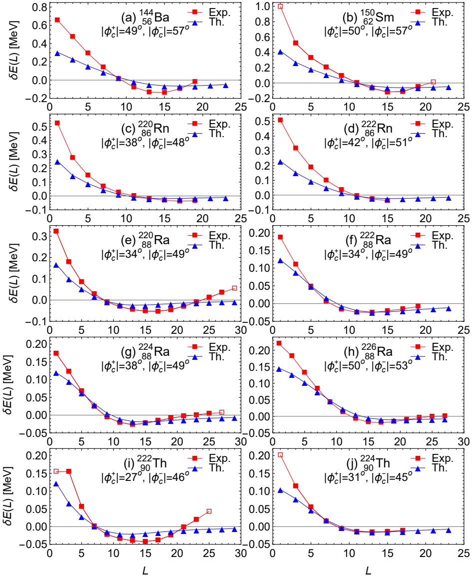
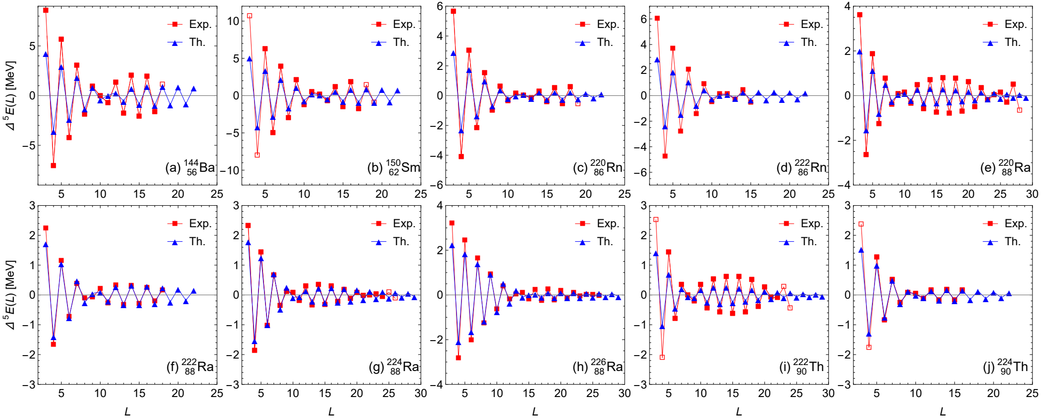
- **Present approach** $|\phi_c|^+ < |\phi_c^-|$



"Beat" pattern of the $\Delta L = 1$ staggering

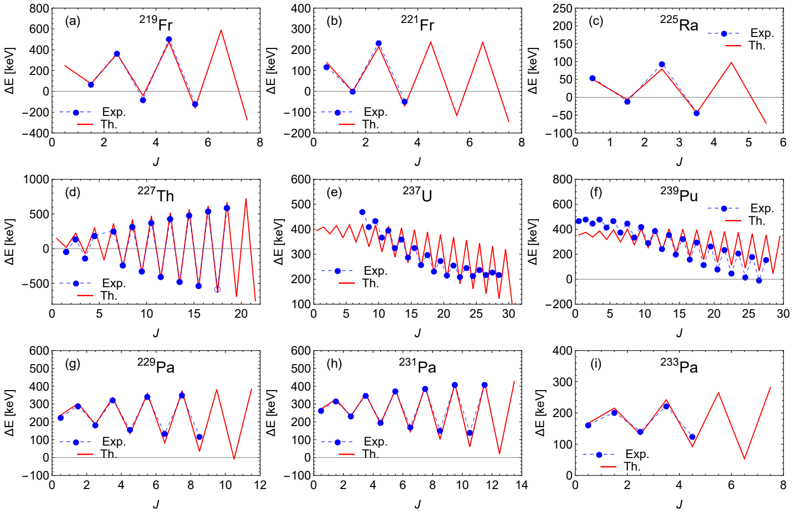
$$\Delta^5 E(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2), \qquad \Delta E(L) = E(L) - E(L-1)$$



[RB, EPL **151** (2025) 34002]

Energy difference between $K = 1/2$ bands in odd mass nuclei within strong coupling

$$\Delta E = E(J^{\pm}) - E(J^{\mp})$$

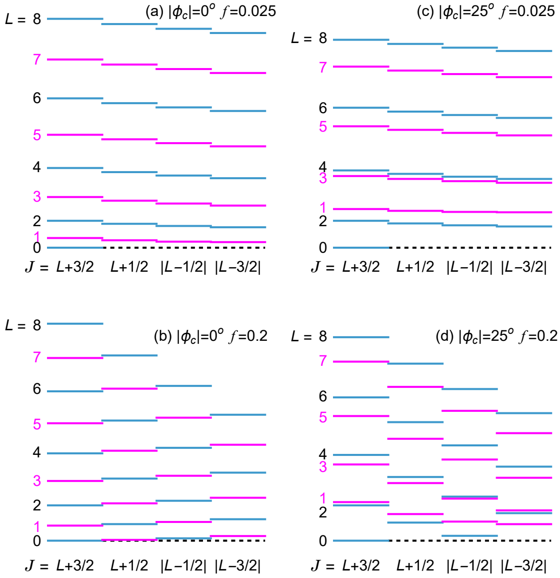


[RB, At. Data Nucl. Data Tables **161** (2025) 101692]

Adjustable spin-spin particle-core interaction

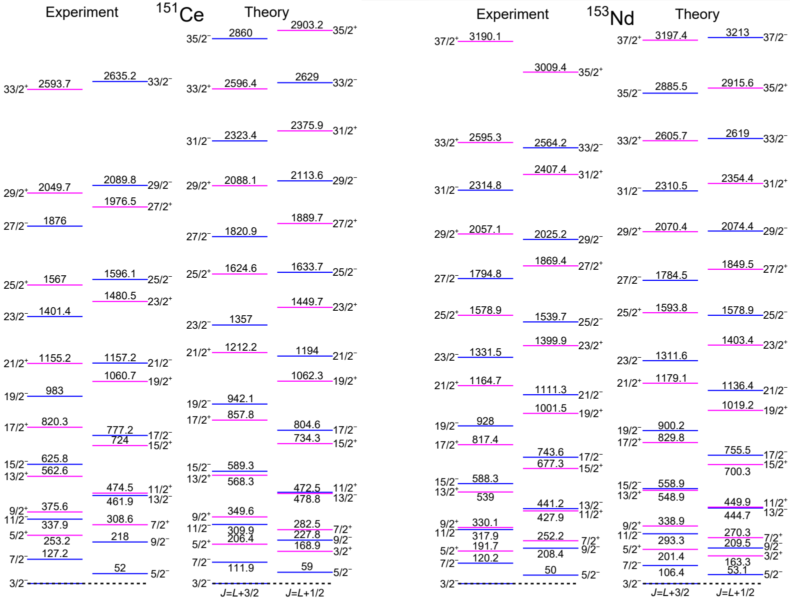
$$\hat{H} = \hat{H}_{QO} + \frac{\hbar^2 f}{B\tilde{\beta}^2}\hat{\vec{L}} \cdot \hat{\vec{j}}$$

- j - total spin of valence nucleons
- m_j - laboratory frame projections
- $J = |L + m_j|$ - total angular momentum
- $J = j$ - ground state



Intermediate particle-core coupling - experimental realization ($N = 93$ nuclei)

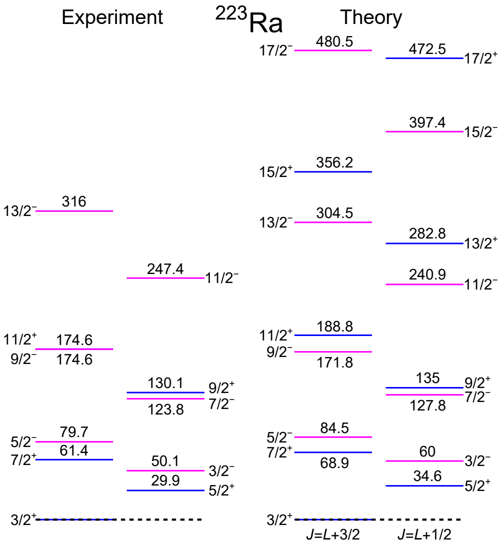
$$|\phi_c| = 47^\circ$$
$$f = 0.17$$

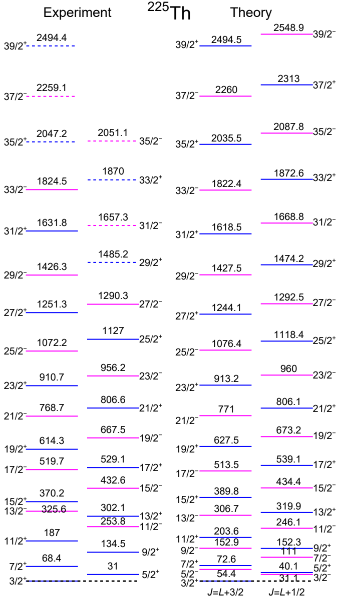


$$|\phi_c| = 50^\circ$$
$$f = 0.19$$

Intermediate particle-core coupling - experimental realization ($N = 135$ nuclei)

$|\phi_c| = 28^\circ \quad f = 0.19$



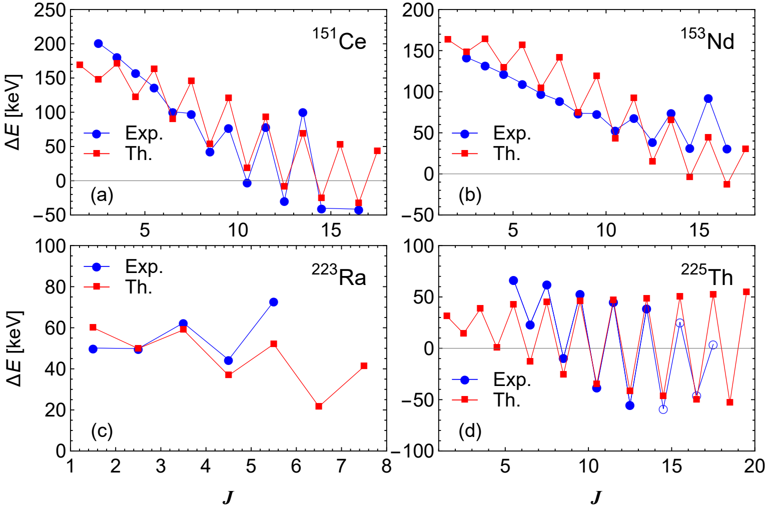


$|\phi_c| = 16^\circ$

$f = 0.15$

Parity splitting energy staggering

$$\Delta E = E(J^{\pm}) - E(J^{\mp})$$



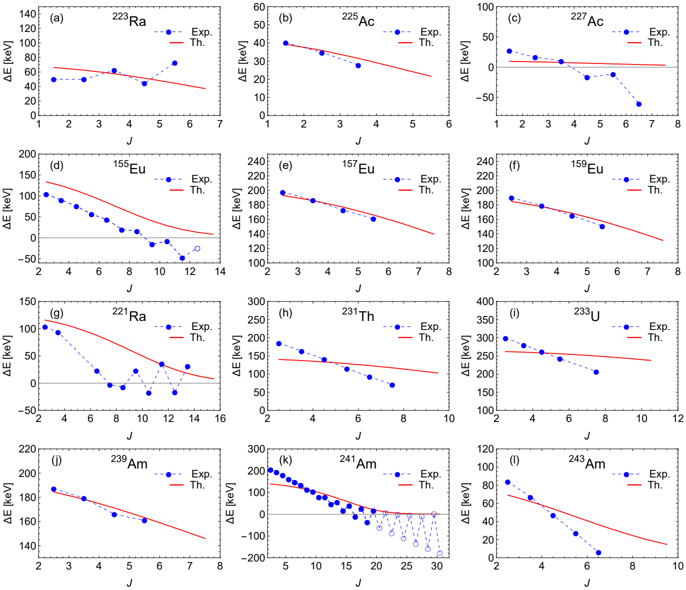
[RB, Eur. Phys. J. A **62** (2026) 32]

Conclusions & Perspectives

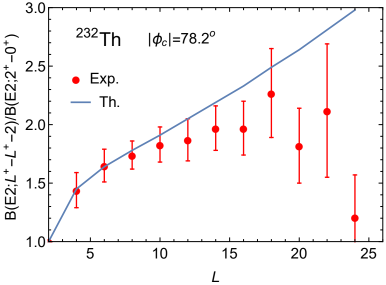
- An axially symmetric geometric model is employed for the description of the fine spectroscopic features of nuclei with strong octupole correlations.
- The positive parity states of even-even pear-shaped nuclei, have an octupole deformation which is more stable to quantum fluctuations, in comparison to the negative parity states.
- The combined effect of the parity splitting mechanism and of the particle-core interaction generates a diverse relative distribution of positive and negative parity states in odd A nuclei.
- New specific spectroscopic patterns are proposed as signatures for octupole deformation in odd mass nuclei.
- Future projects:

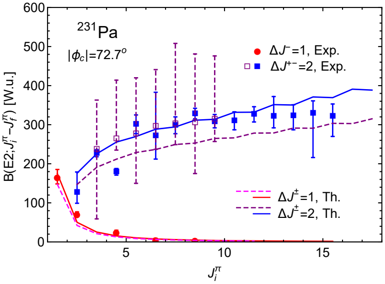
  Applications to nuclei from the $(N \sim 56,\ Z \sim 34)$ region.
  Interpretation of the excited states (two octupole phonon states).
  Inclusion of triaxiality (rigid, soft) $\Rightarrow$ description of the γ and g bands.

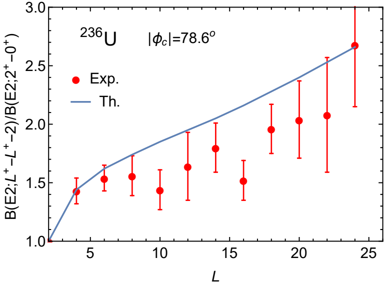
Numerical applications - Energy difference for $K \neq 1/2$ bands of odd mass nuclei

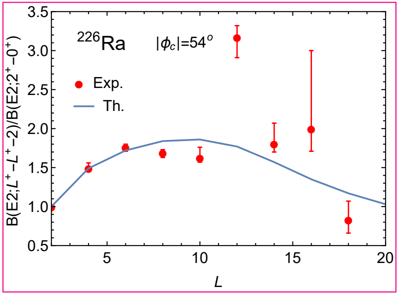


Numerical applications - $E2$ transitions









Numerical applications - $E1$ and $E3$ transitions

- $B(E1)$ transition probabilities $\quad \dfrac{B(E1; L^- \to (L+1)^+)}{B(E1; L^- \to (L-1)^+)} > 1$
- $B(E3)$ transition probabilities

^{146}Ba

$$\frac{B(E3; 5^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.52^{+601}_{-85} \text{ Exp.} \\ 1.67 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 7^- \to 4^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.70^{+765}_{-109} \text{ Exp.} \\ 2.31 \text{ Th.} \end{cases}$$

^{148}Nd

$$\frac{B(E3; 5^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.88^{+40}_{-31} \text{ Exp.} \\ 1.91 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 7^- \to 4^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 2.12^{+51}_{-36} \text{ Exp.} \\ 3.03 \text{ Th.} \end{cases}$$

^{224}Ra

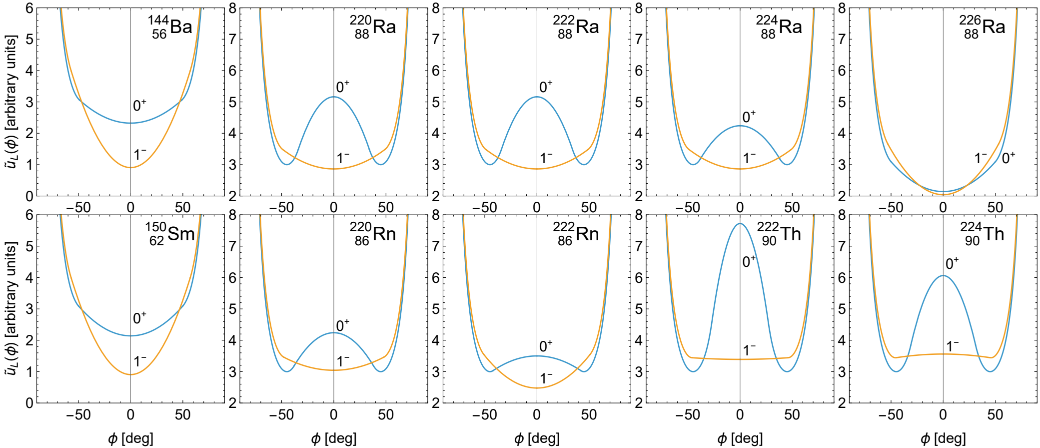
$$\frac{B(E3; 1^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 5(1) \text{ Exp.} \\ 3.21 \text{ Th.} \end{cases}$$

$$\frac{B(E3; 5^- \to 2^+)}{B(E3; 3^- \to 0^+)} = \begin{cases} 1.45(42) \text{ Exp.} \\ 1.68 \text{ Th.} \end{cases}$$

^{236}U

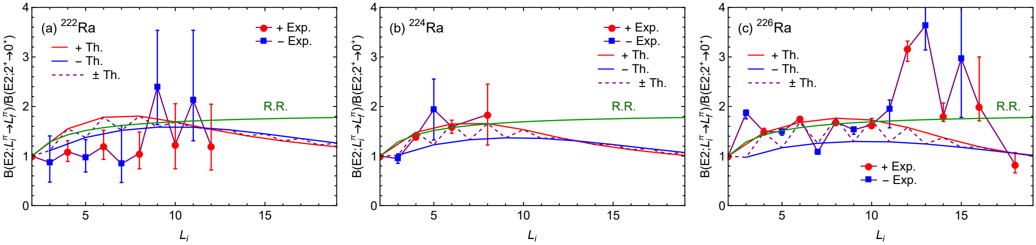
$$\frac{B(E3; 0^+ \to 3^-)}{B(E3; 1^- \to 4^+)} = \begin{cases} 2.58(51) \text{ Exp.} \\ 1.71 \text{ Th.} \end{cases}$$

Parity specific ϕ-potential



[RB, EPL **151** (2025) 34002]

Transition probabilities in the parity specific model



Nucleus		$\frac{B(E1,1^- \to 2^+)}{B(E1,1^- \to 0^+)}$	$\frac{B(E1,3^- \to 4^+)}{B(E1,3^- \to 2^+)}$	$\frac{B(E3,1^- \to 2^+)}{B(E3,3^- \to 0^+)}$	$\frac{B(E3,3^- \to 2^+)}{B(E3,3^- \to 0^+)}$	$\frac{B(E3,5^- \to 2^+)}{B(E3,3^- \to 0^+)}$
$^{220}_{86}$Rn	Exp.					2.73(155)
	Th.	3.76	1.76	5.22	2.37	2.70
$^{222}_{88}$Ra	Exp.	1.96^{+94}_{-62}		1.32(77)	0.63(71)	1.60(44)
	Th.	2.32	1.63	3.35	1.53	1.76
$^{224}_{88}$Ra	Exp.			5.00(101)		1.45(49)
	Th.	2.18	1.55	3.20	1.45	1.67
$^{226}_{88}$Ra	Exp.	1.83(87)	0.86(24)	2.83^{+32}_{-46}	1.13(32)	2.18(20)
	Th.	2.13	1.50	3.14	1.41	1.62

^{223}Ra transition probabilities

$E1$ [10^{-5} W.u.]				$E2$ [W.u.]				$E3$ [W.u.]	
$J^p \rightarrow J'^{p'}$	Th	Exp.[42]	Exp.[39]	$J^p \rightarrow J'^{p'}$	Th	Exp.[42]	Exp.[39]	$J^p \rightarrow J'^{p'}$	Th
$\frac{3}{2}^- \rightarrow \frac{5}{2}^+$	59	50(9)	54(6)	$\frac{5}{2}^+ \rightarrow \frac{3}{2}^+$	49		>123	$\frac{7}{2}^- \rightarrow \frac{3}{2}^+$	66
$\frac{3}{2}^- \rightarrow \frac{3}{2}^+$	46	119(16)	127(7)	$\frac{7}{2}^+ \rightarrow \frac{5}{2}^+$	23	≈70	57(21)	$\frac{9}{2}^- \rightarrow \frac{3}{2}^+$	66
$\frac{5}{2}^- \rightarrow \frac{5}{2}^+$	17		81(19)	$\frac{7}{2}^+ \rightarrow \frac{3}{2}^+$	49	≈44	37(11)	$\frac{3}{2}^- \rightarrow \frac{5}{2}^+$	75
$\frac{5}{2}^- \rightarrow \frac{3}{2}^+$	17		89(7)	$\frac{7}{2}^- \rightarrow \frac{5}{2}^-$	18	110(50)	144(49)	$\frac{5}{2}^- \rightarrow \frac{5}{2}^+$	63
$\frac{7}{2}^- \rightarrow \frac{7}{2}^+$	9	20^{+22}_{-5}	4.7(11)	$\frac{7}{2}^- \rightarrow \frac{3}{2}^-$	27	10(6)	14(7)	$\frac{7}{2}^- \rightarrow \frac{5}{2}^+$	1.6
$\frac{7}{2}^- \rightarrow \frac{5}{2}^+$	51	79(24)	10.4(22)	$\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$	14		34(10)	$\frac{9}{2}^- \rightarrow \frac{5}{2}^+$	36
$\frac{9}{2}^+ \rightarrow \frac{7}{2}^-$	61		$47(18)\cdot 10^2$	$\frac{9}{2}^+ \rightarrow \frac{5}{2}^+$	52		45(14)	$\frac{7}{2}^+ \rightarrow \frac{3}{2}^-$	53
$\frac{9}{2}^- \rightarrow \frac{9}{2}^+$	5.6		~ 3.4	$\frac{9}{2}^- \rightarrow \frac{7}{2}^-$	10		187(100)	$\frac{9}{2}^+ \rightarrow \frac{3}{2}^-$	55
$\frac{9}{2}^- \rightarrow \frac{7}{2}^+$	60		29(7)	$\frac{9}{2}^- \rightarrow \frac{5}{2}^-$	46		~ 105	$\frac{5}{2}^- \rightarrow \frac{7}{2}^+$	35
$\frac{11}{2}^- \rightarrow \frac{9}{2}^+$	64		37.2(30)	$\frac{11}{2}^+ \rightarrow \frac{7}{2}^+$	66		280(85)	$\frac{7}{2}^- \rightarrow \frac{7}{2}^+$	48
$\frac{11}{2}^- \rightarrow \frac{11}{2}^+$	3.7			$\frac{11}{2}^- \rightarrow \frac{7}{2}^-$	45		72(31)	$\frac{9}{2}^- \rightarrow \frac{7}{2}^+$	48