Answers to optics questions

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Collaboration meeting with CSNS

25 July 2025

Science and Technology Facilities Council
ISIS Neutron and Muon Source

Q1. Lattice structure design considerations

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Space charge effects and cell tune

- Fix a cell tune below 0.25, not a ring tune, not a fractional part of ring tune.
  - Avoid envelope instability excited by space charge effects.
- However, this may not be a hard limit in an FFA because of a fast acceleration.

[1] Ingo Hofmann, *et al*, “Space-charge structural instabilities and resonances in high-intensity beams”, Phys. Rev. Lett. **115**, 204802, 2015.
[2] Yao-Shuo Yuan, *et al*, “Modeling of second order space charge driven coherent sum and difference instabilities”, Phys. Rev. Accel. Beams **20**, 104201, 2017.

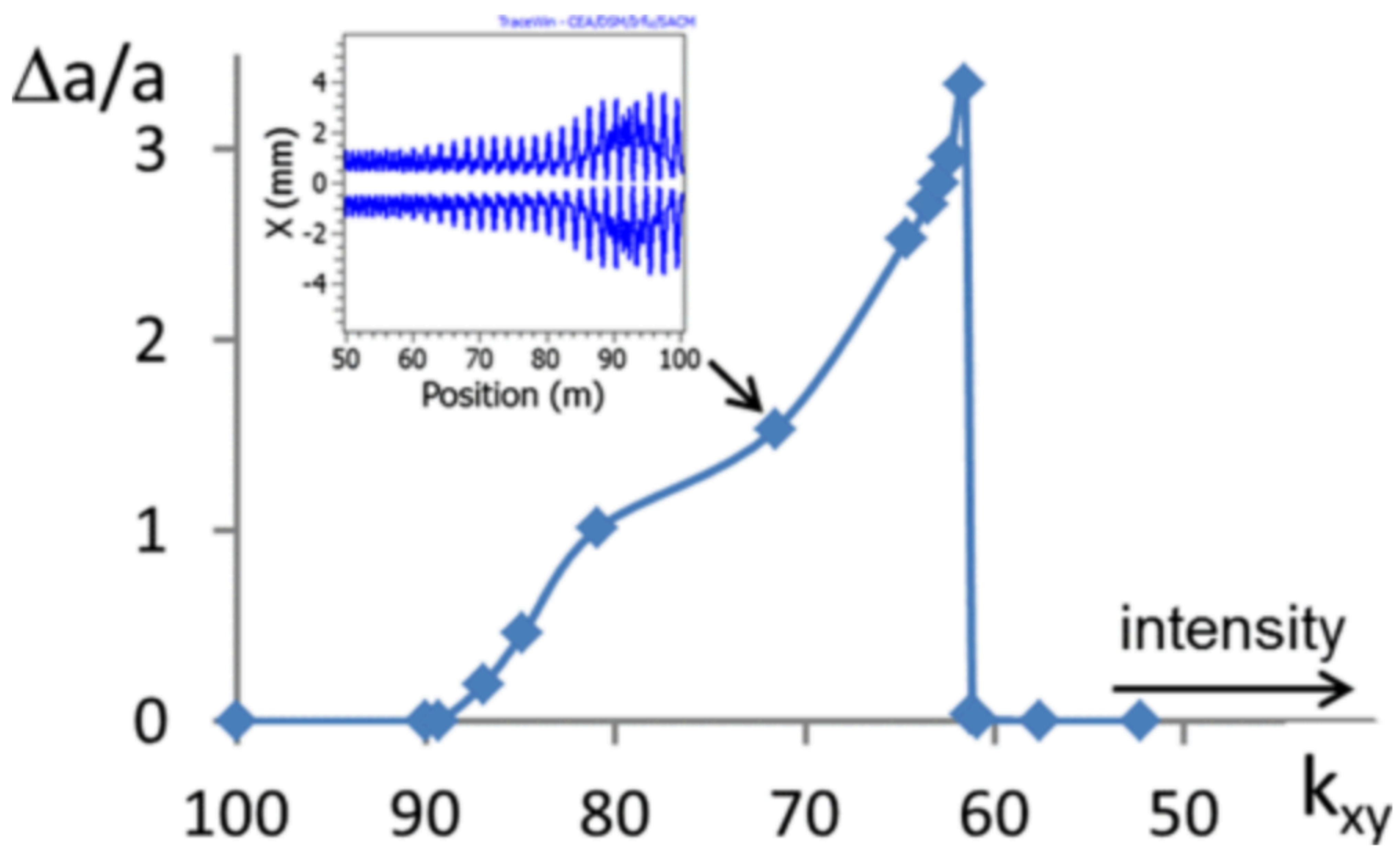

Illustration from [1]

- Bare tune without space charge is set at 100 deg (or q=0.278).
- x-axis is the depressed tune with space charge and y-axis is the amplitude growth.

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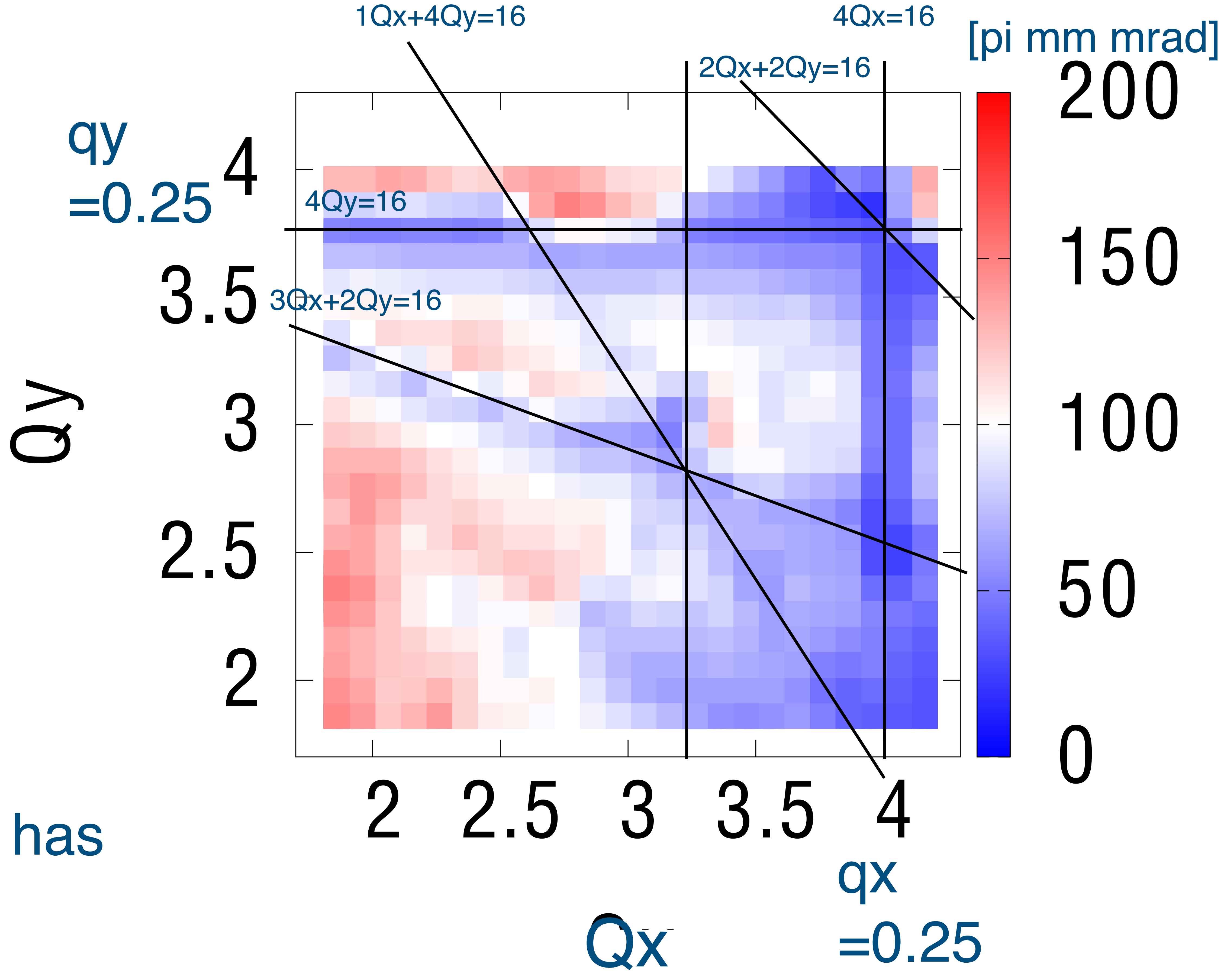
Aperture requirement

From FFA workshop 2022

For high intensity operation of FFA, we need large physical aperture and dynamic aperture larger than physical aperture to reduce space charge effects.

- SNS, J-PARC have ~500 pi mm mrad (geometrical).

- How do we enlarger dynamic aperture?
- How dynamic aperture depends on the k-value?
- Is there any difference in terms of dynamic aperture between radial sector and spiral sector?



It is possible to design a variety of FFA (first order) optics: scaling, nonscaling, novel idea, etc,

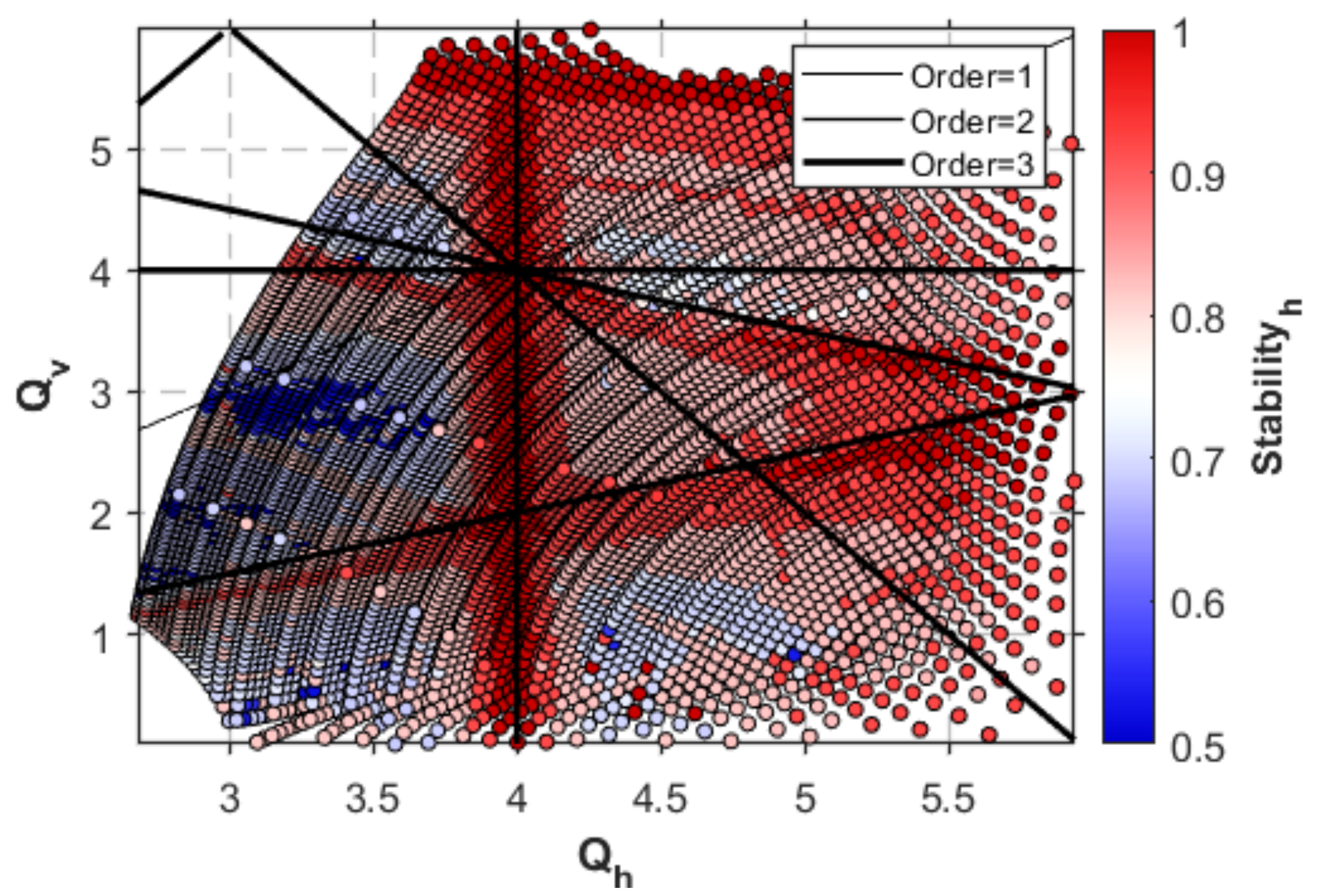
- That is not enough!
- As an accelerator for user operation, nonlinear optics has to be understood well.

Option 1: FD structure→ Increase period number

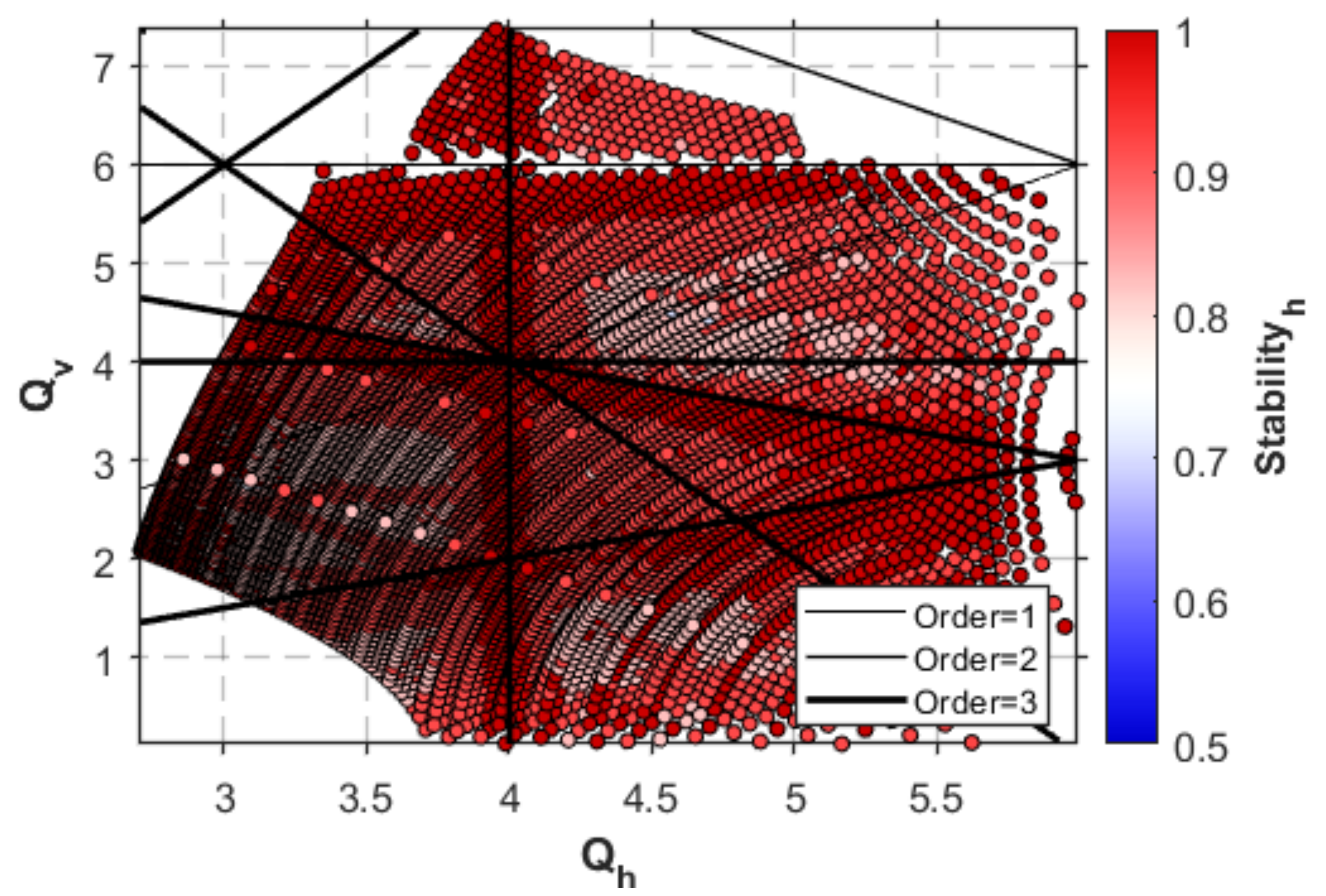
From Bin Wu’s presentation on Monday

- **Full Parameters scan for FD structure under 12m radius conditions**
  - The magnet spiral angle is closely related to lattice stability (12 cells as an example)

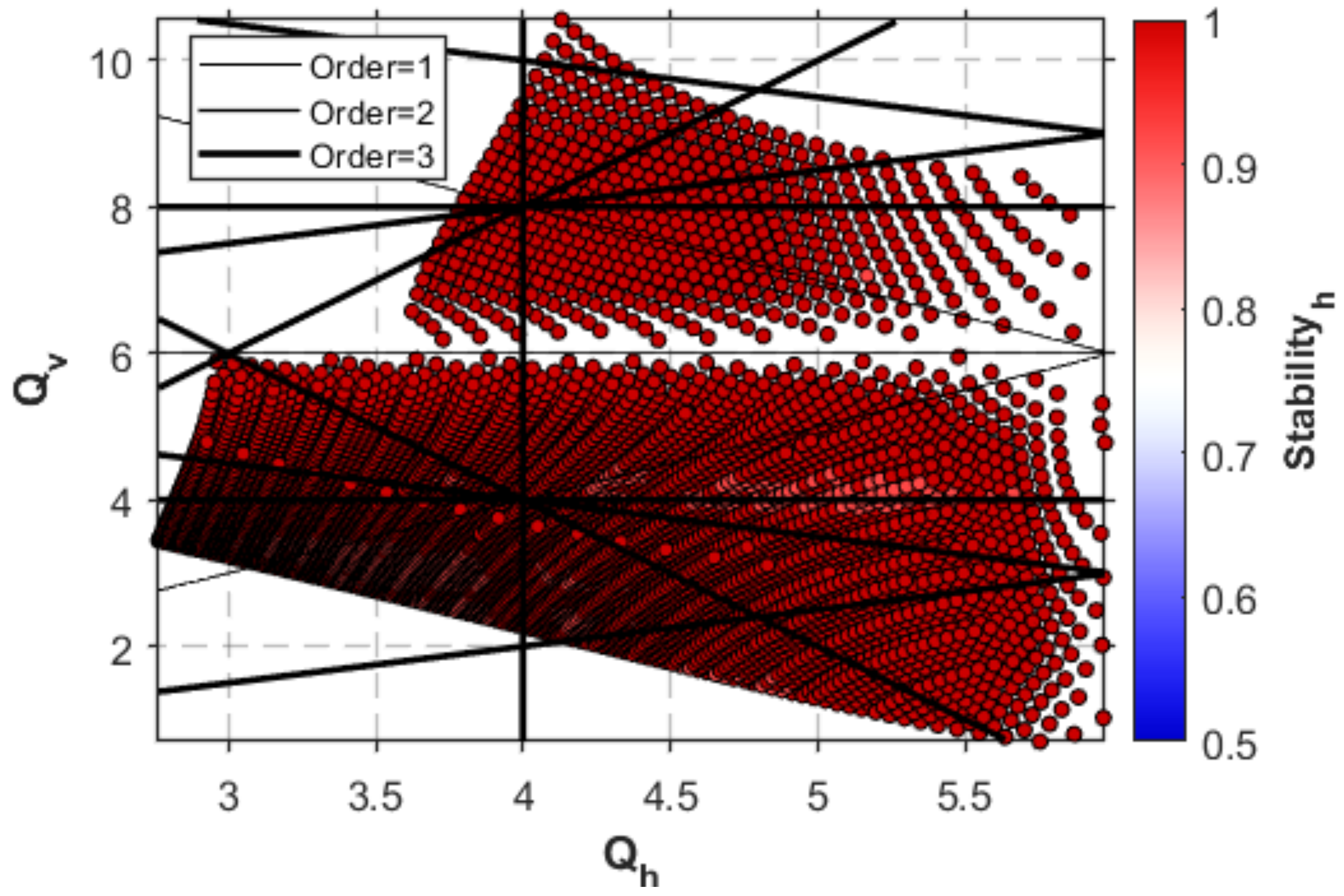
Spiral angle=30°



Spiral angle=40°



Spiral angle=50°



- **Spiral angle trade-off:** Higher spiral angle improves vertical tune but reduces beam stability （Consistent with Dr.M. Topp-Mugglestone's research conclusions） . Maximum spiral angle should not exceed 45°.
- **3rd-order resonance dominance:** Particle stability is more sensitive to 3rd-order resonances than 1st/ 2nd order.

4-th, 5-th … orders are also important.

Resonance structure

- Consider ring tune space including non structure integer and half integer resonances.
- Q=3.5 for 14 cell, Q=3.75 for 15 cell, Q=4 for 16 cell are particularly strong 4-th order resonance.
- A large stopband below qh,v=0.25, especially in the vertical direction, due to systematic octuploe and amplitude dependent tune shift, that is always positive.

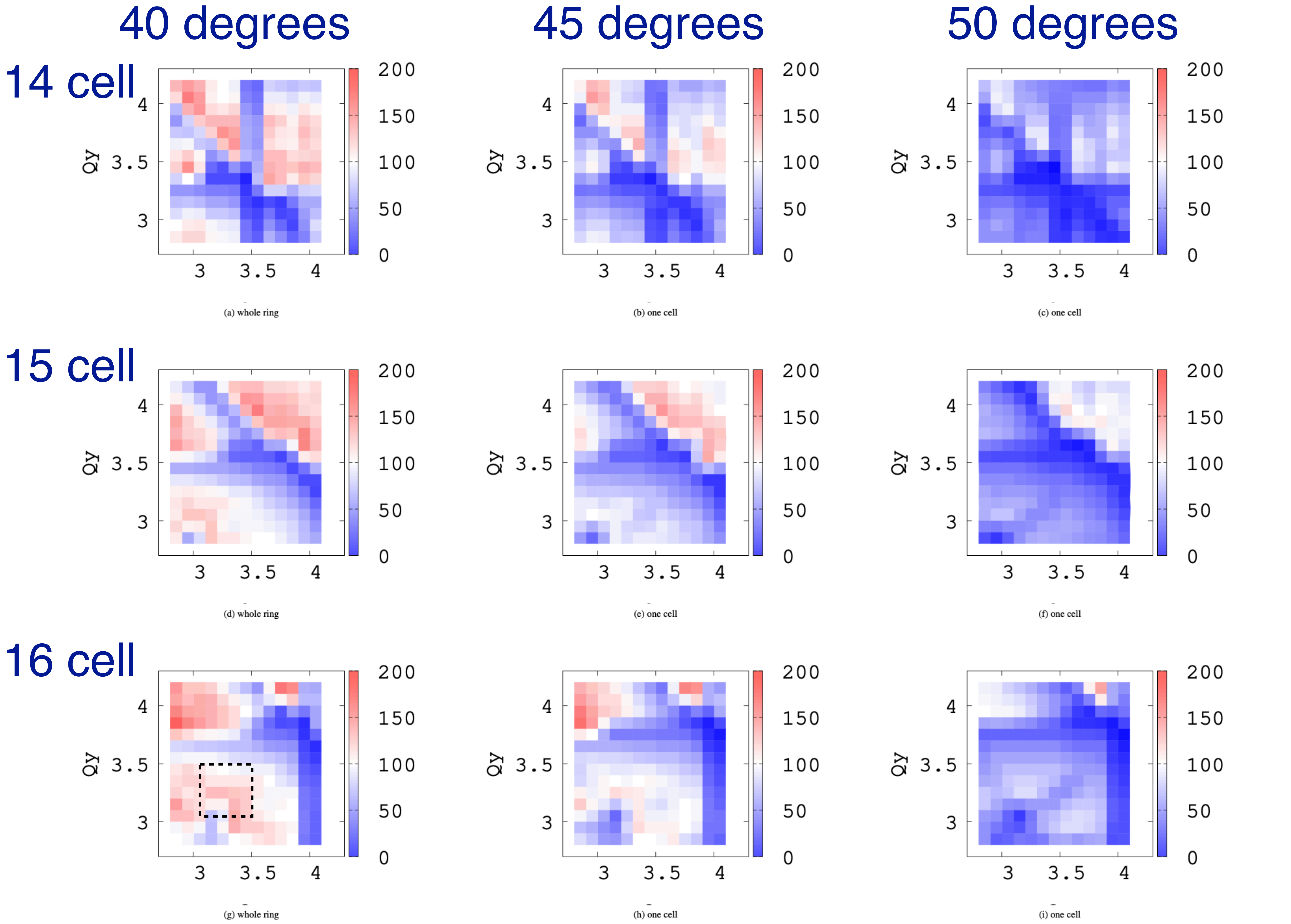


Figure 2.12: Dynamic aperture of 14 to 16 cell lattice. 1st row left: 14 cell with spiral angle of 40 degree, centre: 14 cell with spiral angle of 45 degree, right: 14 cell with spiral angle of 50 degree, 2nd row left: 15 cell with spiral angle of 40 degree, centre: 15 cell with spiral angle of 45 degree, right: 15 cell with spiral angle of 50 degree, 3rd row left: 16 cell with spiral angle of 40 degree, centre: 16 cell with spiral angle of 45 degree, right: 16 cell with spiral angle of 50 degree

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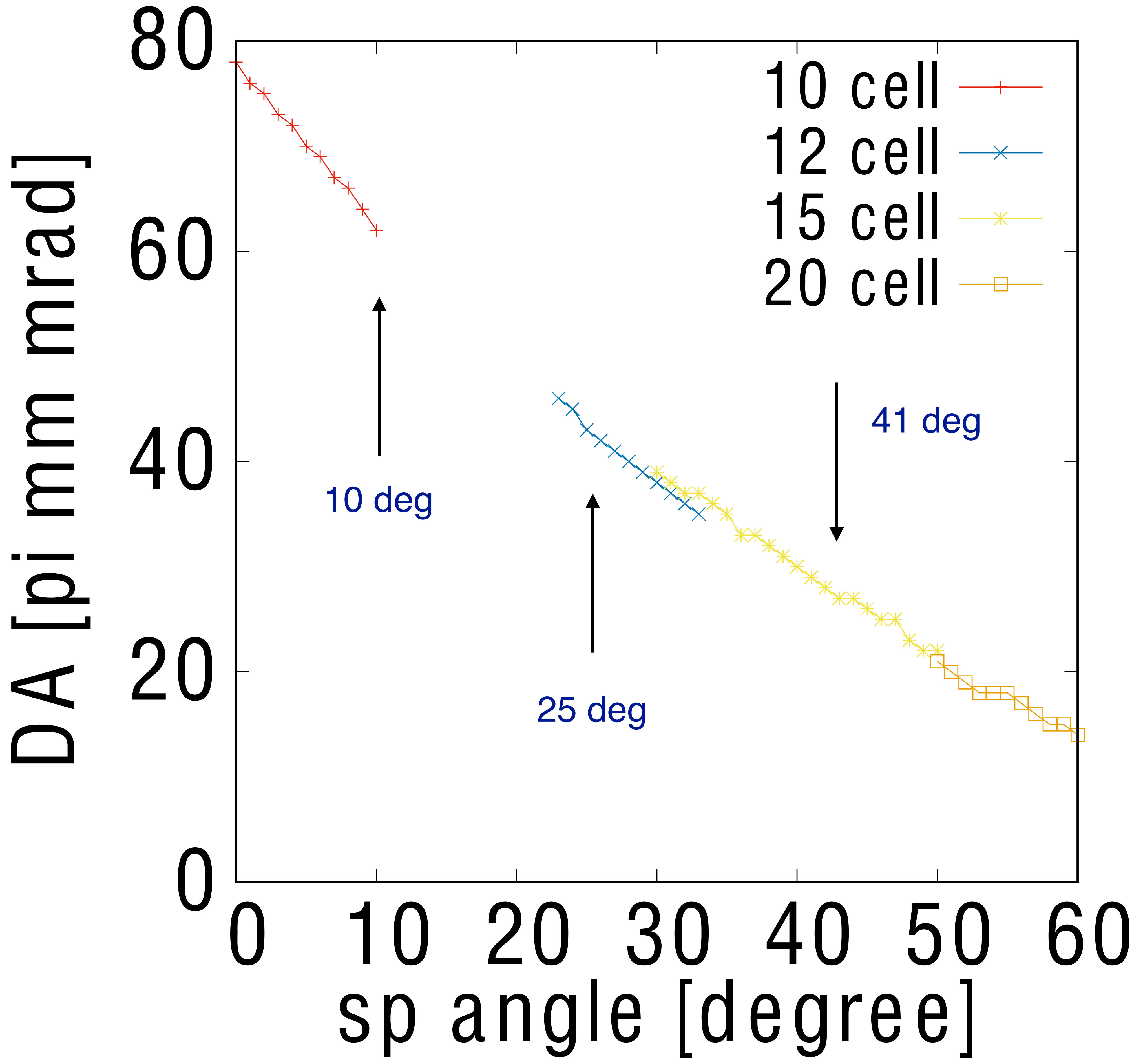
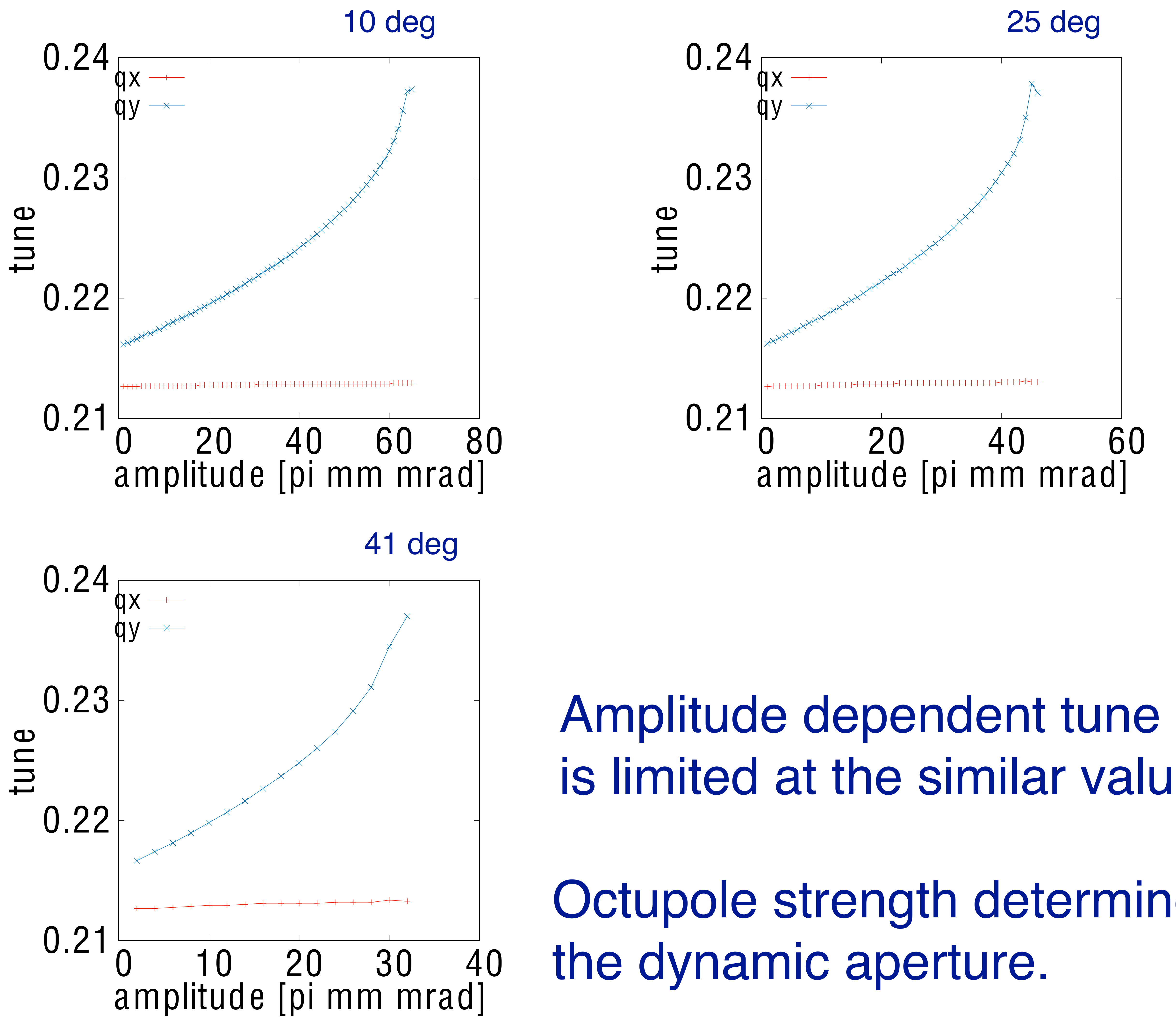
Dynamic aperture for different number of cell lattices

From FFA workshop 2022





Amplitude dependent tune shift is limited at the similar value.

Octupole strength determines the dynamic aperture.

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Procedure to choose the number of cells

- q (cell tune) < 0.25 to avoid space charge instability.
- Avoid nonlinear structure resonances (up to 5 th order at least).
- Enough dynamic aperture. A small number of cell is preferable.
- Reasonable orbit excursion. A large number of cell is preferable.

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Q2. Spiral angle effects on octupole components and vertical tune

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Octupole in the fringe fields

From FFA workshop 2022

Proceedings of EPAC 2002, Paris, France

STUDY OF ACCEPTANCE OF FFAG ACCELERATOR

M. Aiba, S. Machida, Y. Mori, A. Mutoh, J. Nakano, C. Ohmori, I. Sakai, Y. Sato, M. Sugaya,
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A.Yamazaki, Y. Yuasa (KEK), K. Koba (FNAL)

Abstract

The aim of this study is to establish the generalized procedure to design a FFAG accelerator having large transverse acceptance. Due to the large momentum and transverse acceptance, it is considered that the FFAG accelerator is quite appropriate for a phase rotator or a secondary particle accelerator [1]. Some analytical ways and tracking simulations were performed to study the problem of non-linear motion in FFAG accelerator.

enhances the higher order components of the guiding field, which causes the tune shift. Here, B_z show the vertical component of the field, B_0 is the field strength at the radius r_0 and k is the k value. In this study, positive k value is assumed.

$$B_z = B_0\left(\frac{r}{r_0}\right)^k = B_0 + B_0\frac{k}{r_0}x + B_0\frac{k(k-1)}{2!r_0}x^2 + \cdots \quad (1)$$

(*Taylor Expansion around* $r_0, r = r_0 + x$)

$$orbit\ excursion = r_{inj}\left(A^{\frac{1}{k+1}} - 1\right) \quad (2)$$

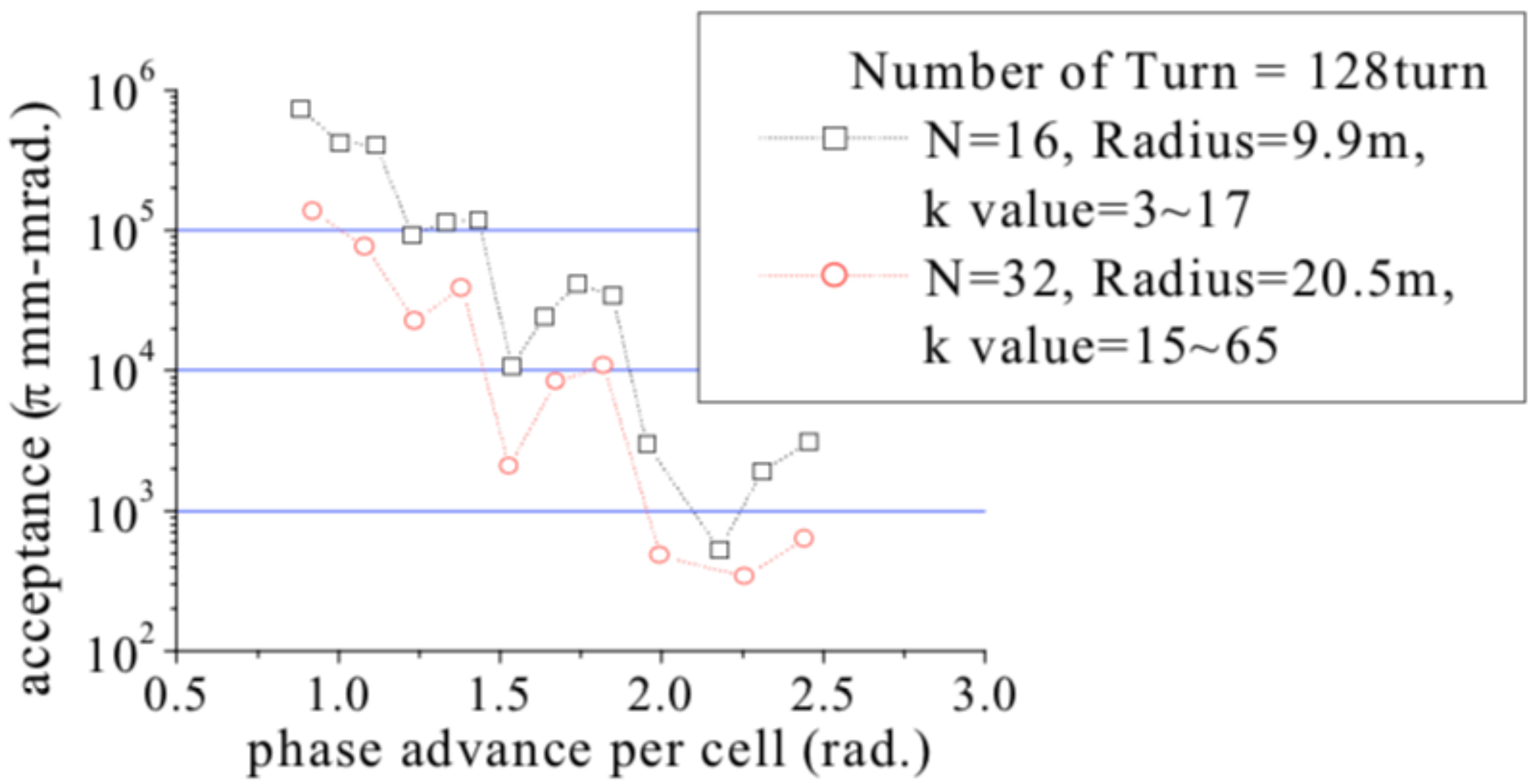


FIGURE 1. Acceptance vs. Phase Advance

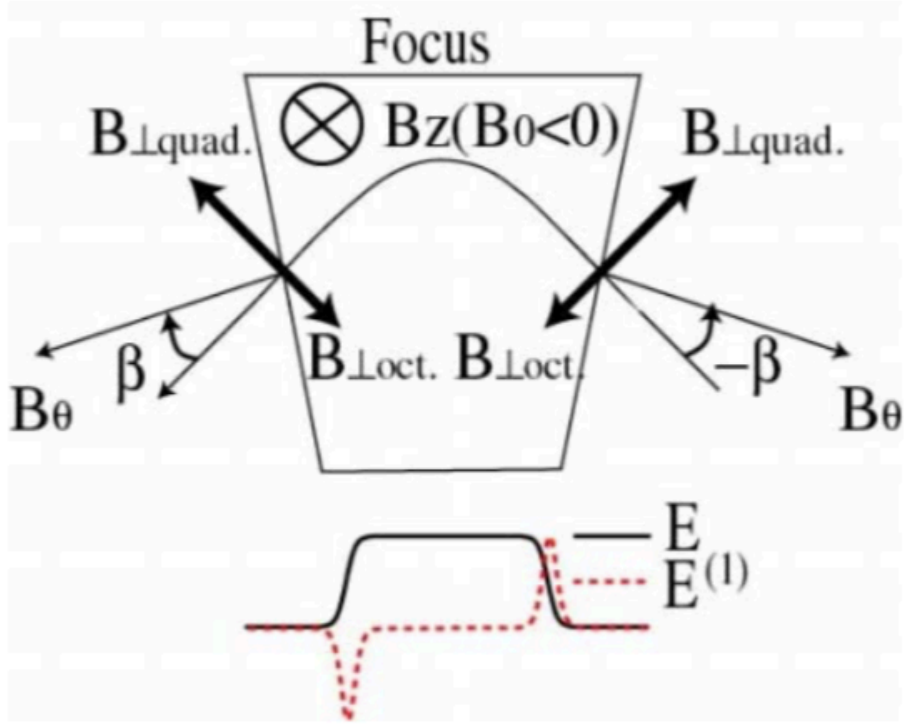


FIGURE 3. Octupole Component in Edge Focus

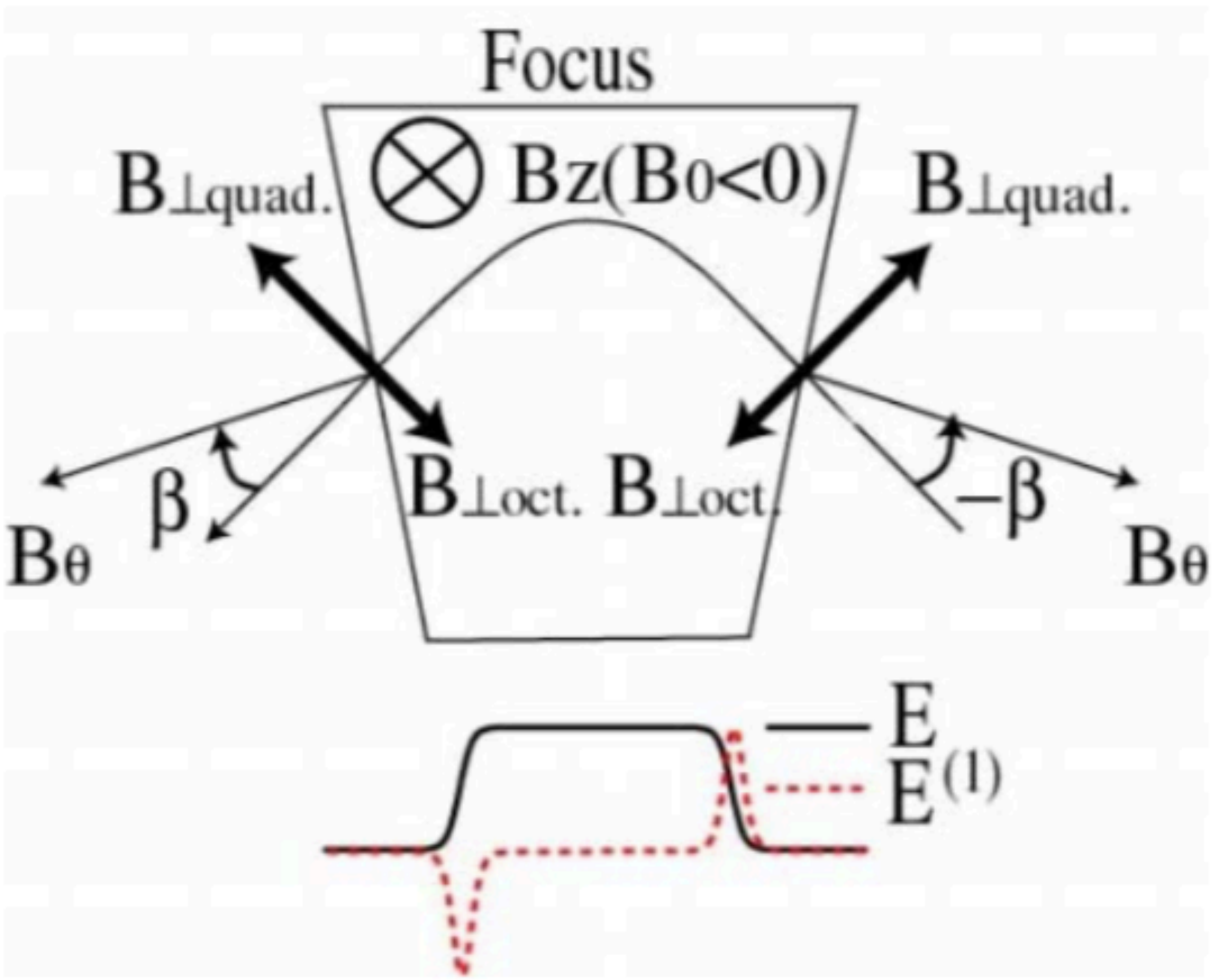
$$B_{\perp oct} = -\frac{1}{3!}\left(E^{(1)}k^2 + E^{(3)}\right)\frac{B_0}{r_0^3}z^3 \sin\beta$$

$$\cong -\frac{E^{(1)}k^2}{3!}\frac{B_0}{r_0^3}z^3 \sin\beta = O(s)z^3$$

- E(1) is the first derivative of fringe field extent with azimuthal direction.
- E(3) is the third derivative of …

- DA decreases with phase advance.
  - Higher k-value means stronger nonlinearity.
- Dips appear when a systematic resonance occurs.
  - All order of multipoles exist.
- **Amplitude dependent tune shift due to octupoles is the primary source of the DA limit.**
  - **Tune gets to a nearby systematic resonance.**

Octupole in the fringe fields



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- E(1) is the first derivative of fringe field extent with azimuthal direction.
- E(3) is the third derivative of …

Tune shift with octupole B4 and amplitude Jx,y.

$$\Delta Q_y = \frac{q}{p}\frac{3B_4}{8\pi}\left(\beta_y^2 J_y - 2\beta_y\beta_x J_x\right)$$

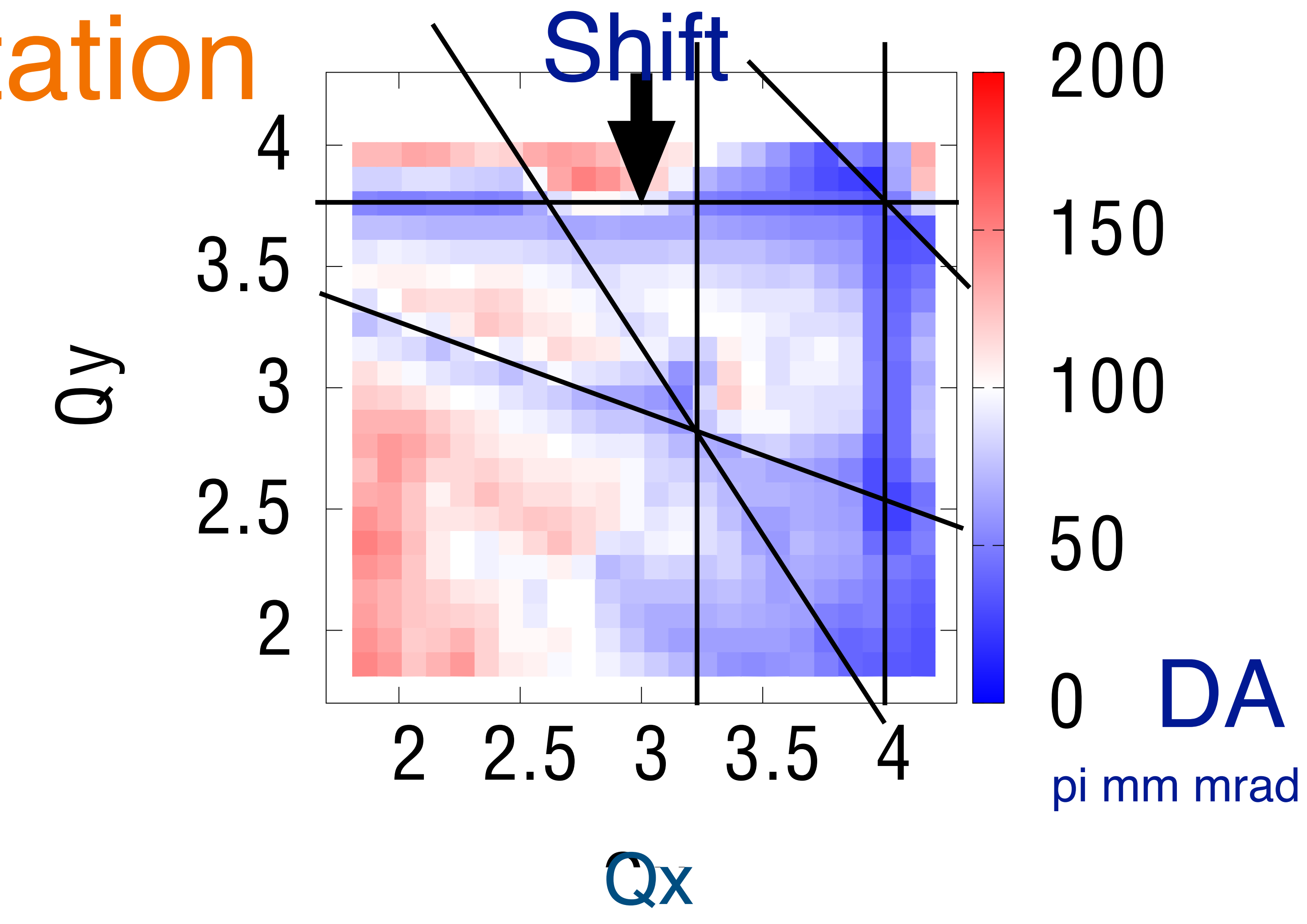
Steeper fringe field gradient increases octupole component.

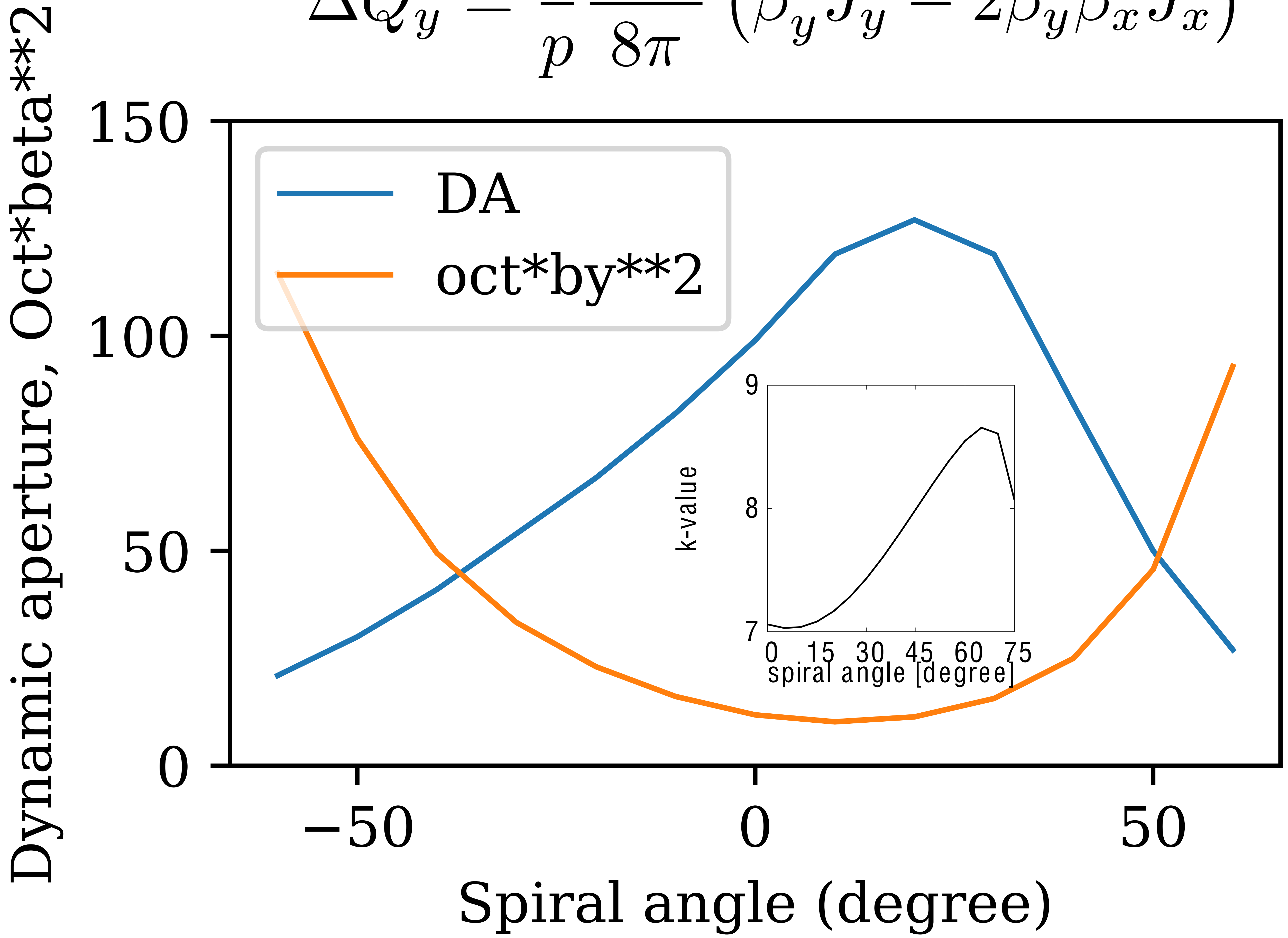
Dynamic aperture of 16 cell lattice

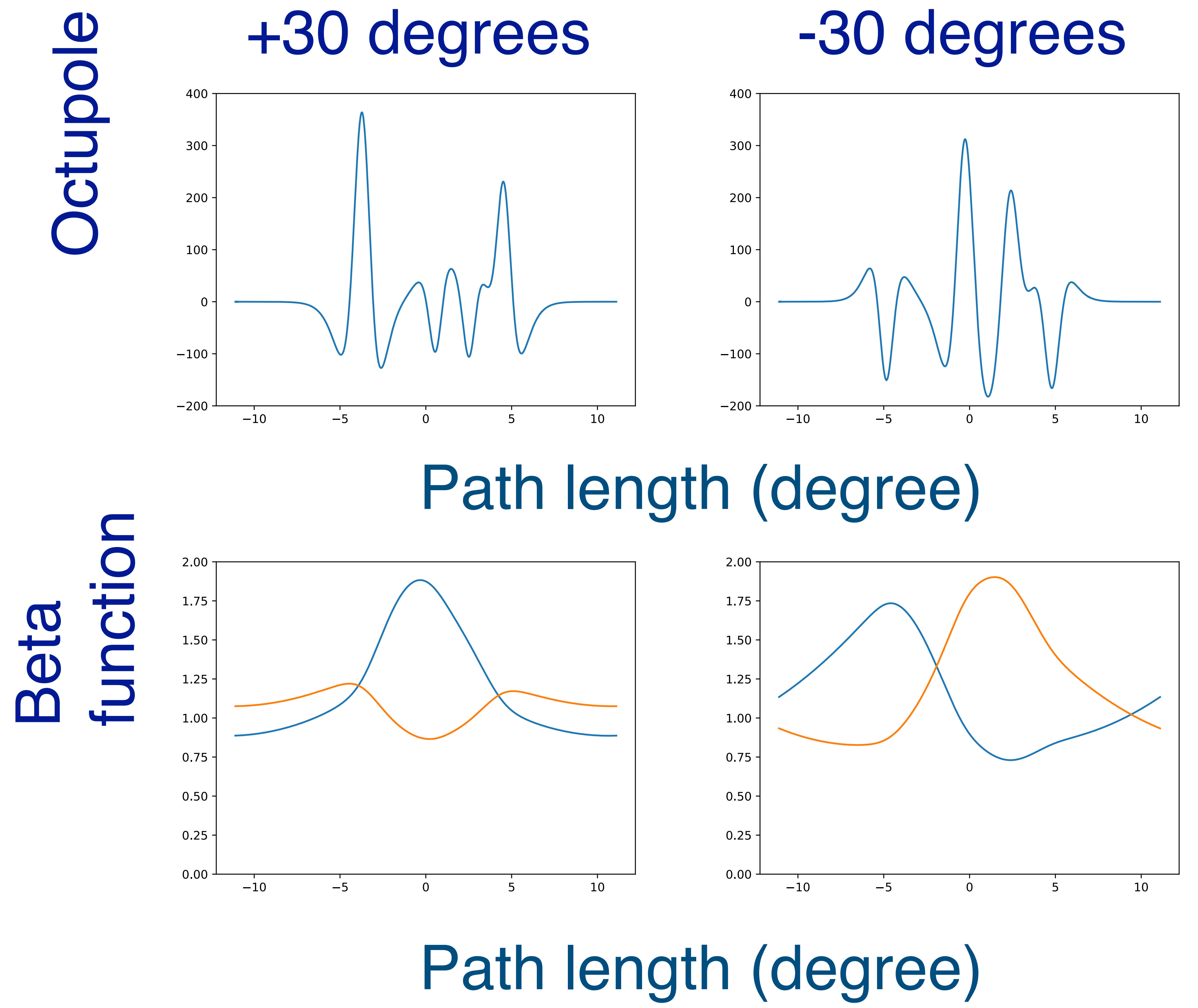
From presentation on Monday

- Dynamic aperture (DA) is limited by amplitude dependent tune shift.
- Octupole component with beta functions explains asymmetry of DA with spiral angle.

$$\Delta Q_y = \frac{q}{p}\frac{3B_4}{8\pi}\left(\beta_y^2 J_y - 2\beta_y\beta_x J_x\right)$$







Q3, Q10. Lattice type comparison: FD vs FDF performance

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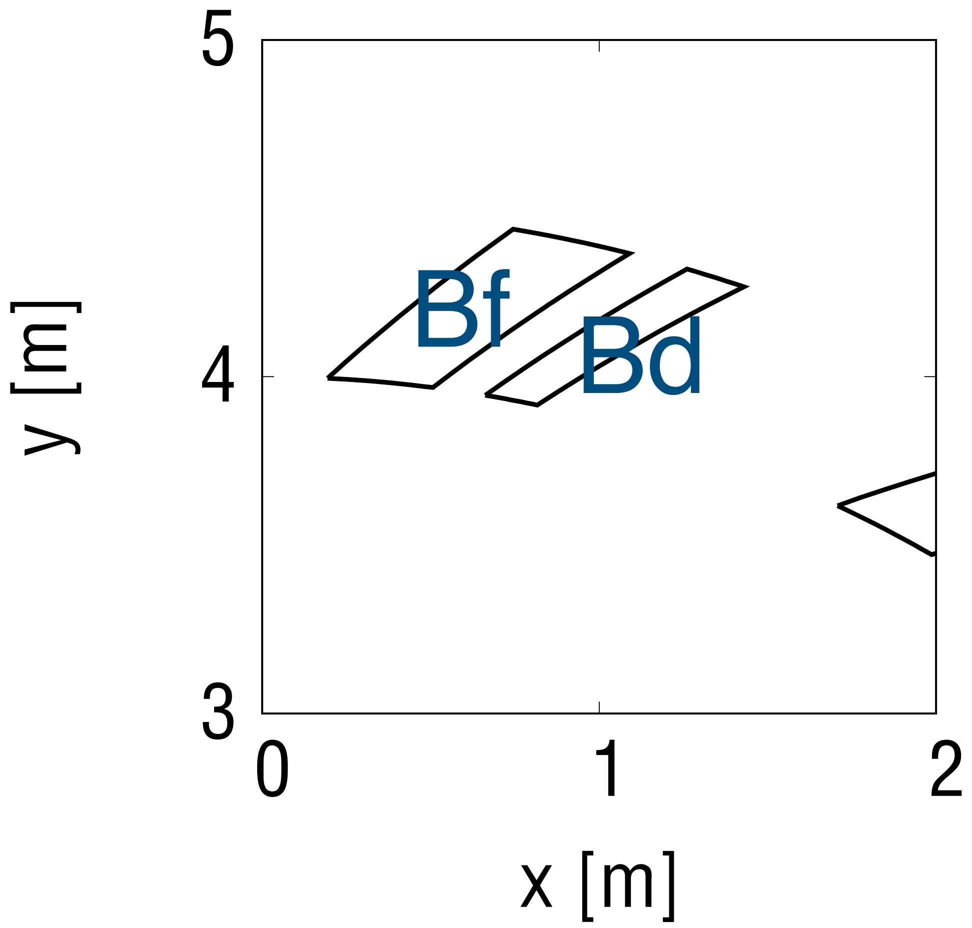
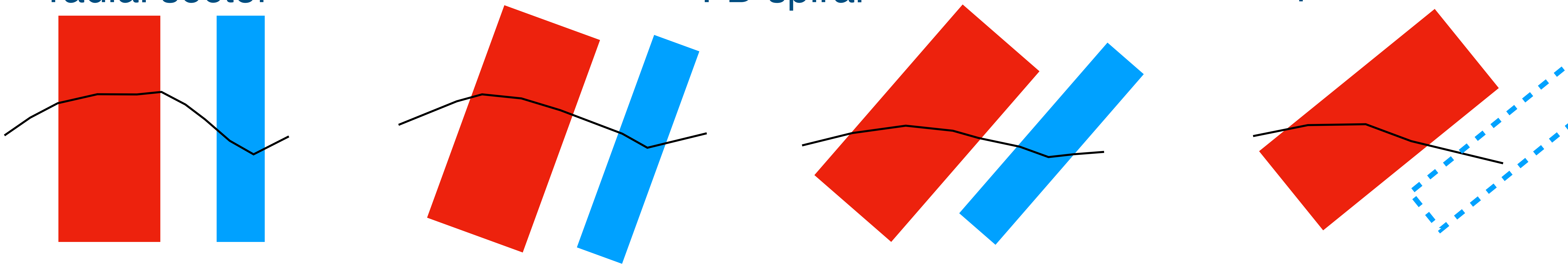
DF(FD)spiral FFA

From FFA workshop 2022



radial sector | FD spiral | | spiral sector

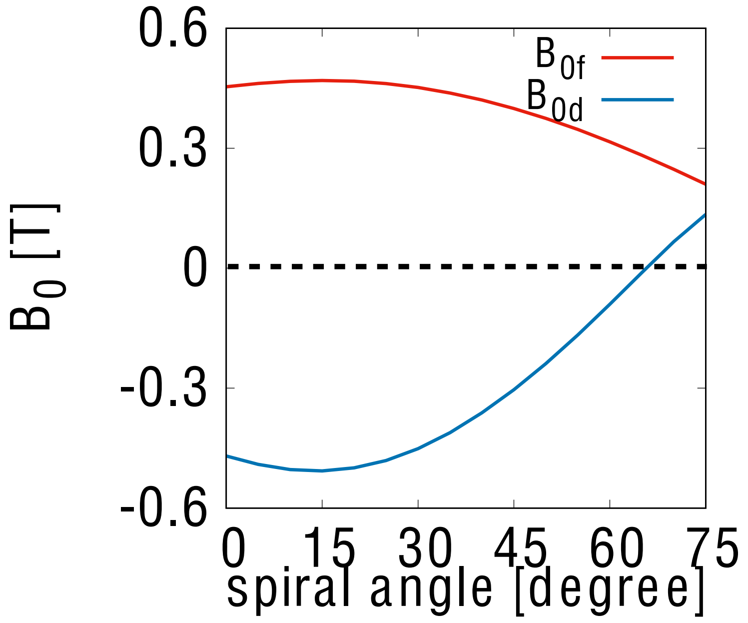


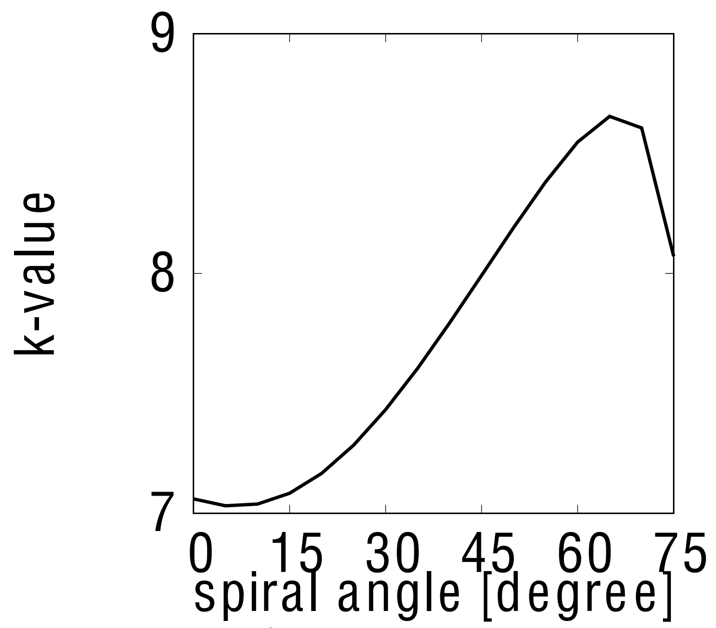
Bf (DFD) Bd (DDD) (entrance, body, exit)

Bf (DFO) Bd (ODD)

Bf (DFF) Bd (FDD)

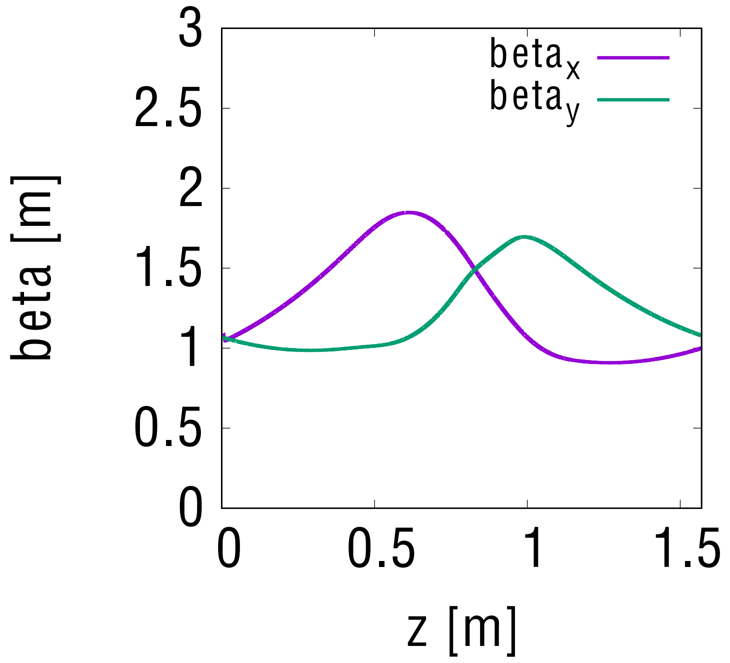
Bf (DFF)

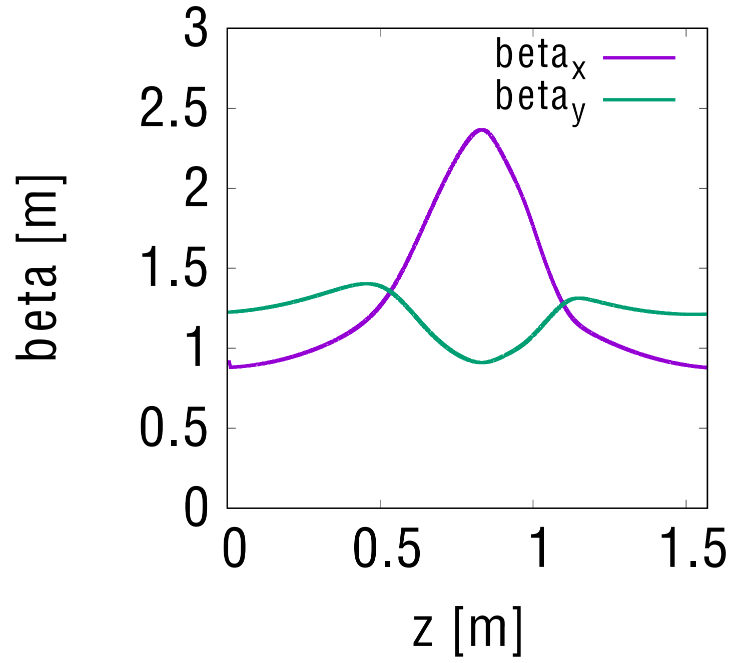


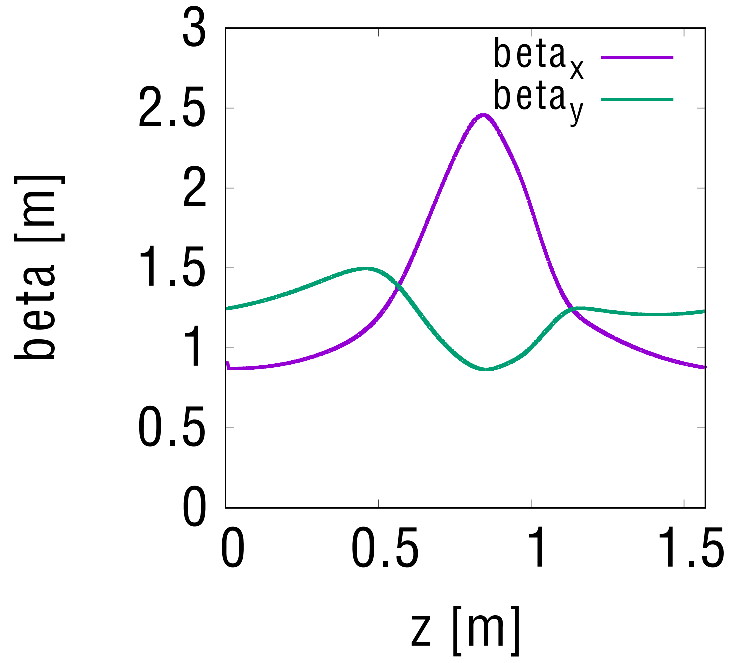


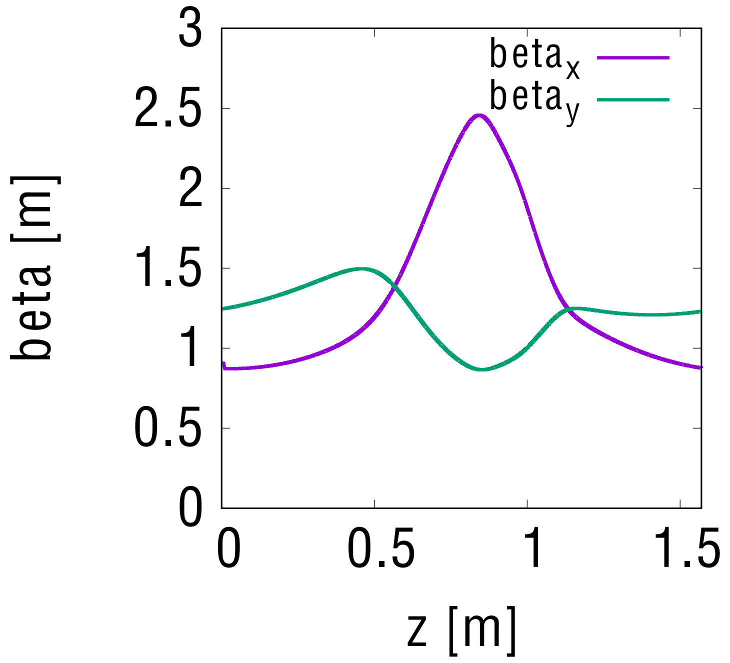
FD | DFD | DFD | DF

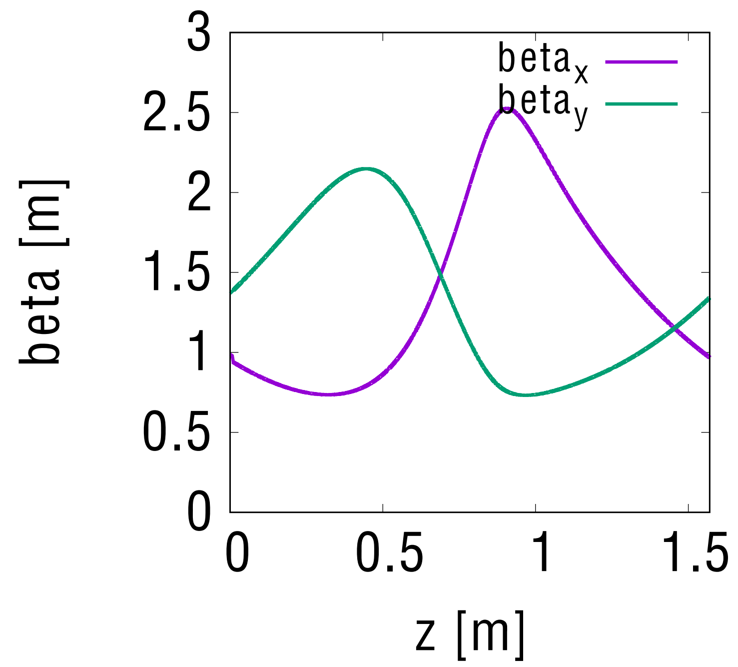
Spiral angle=0 deg | 35 deg | 40 deg | 45 deg | 65 deg











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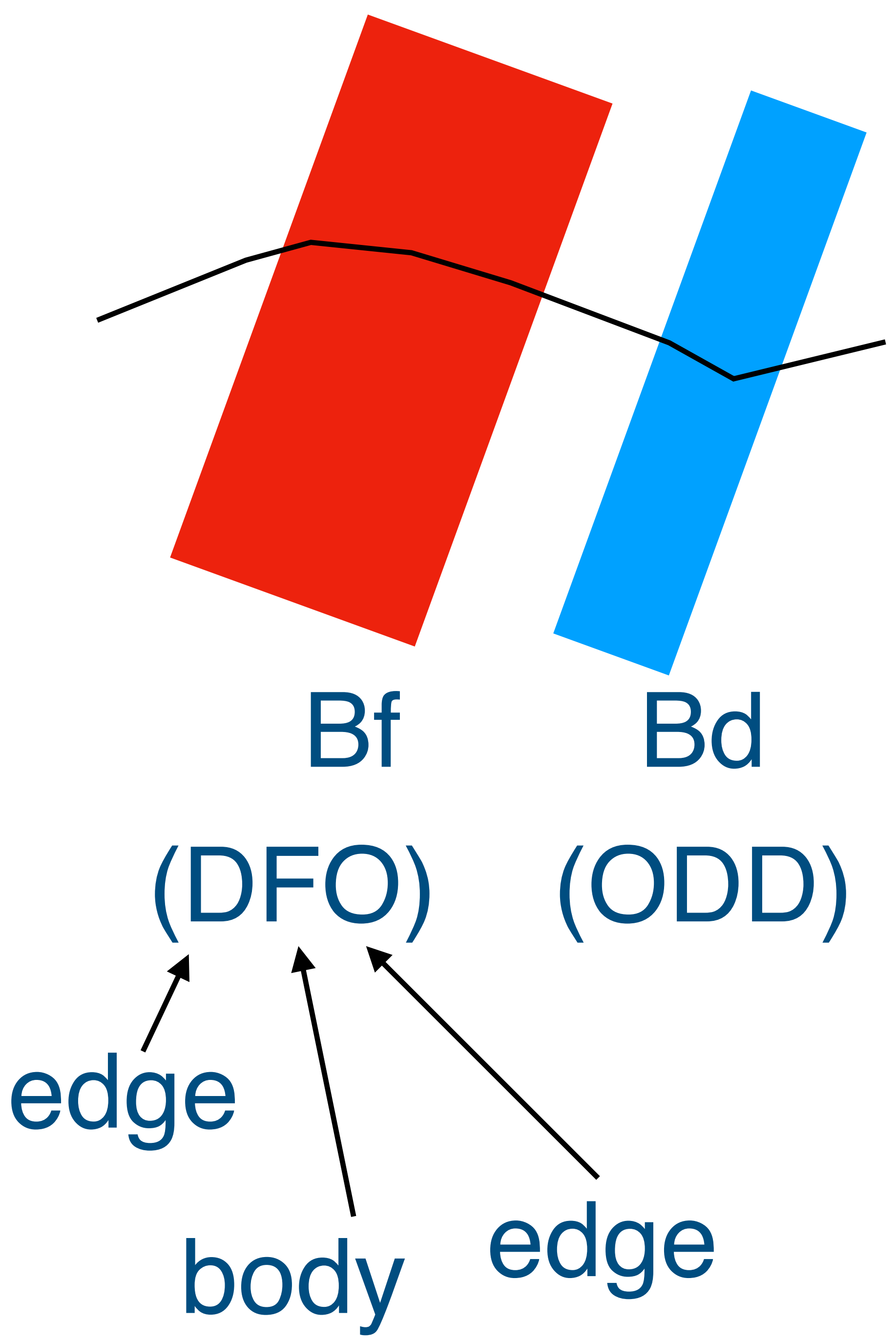
cell tune = (0.213125, 0.213125)

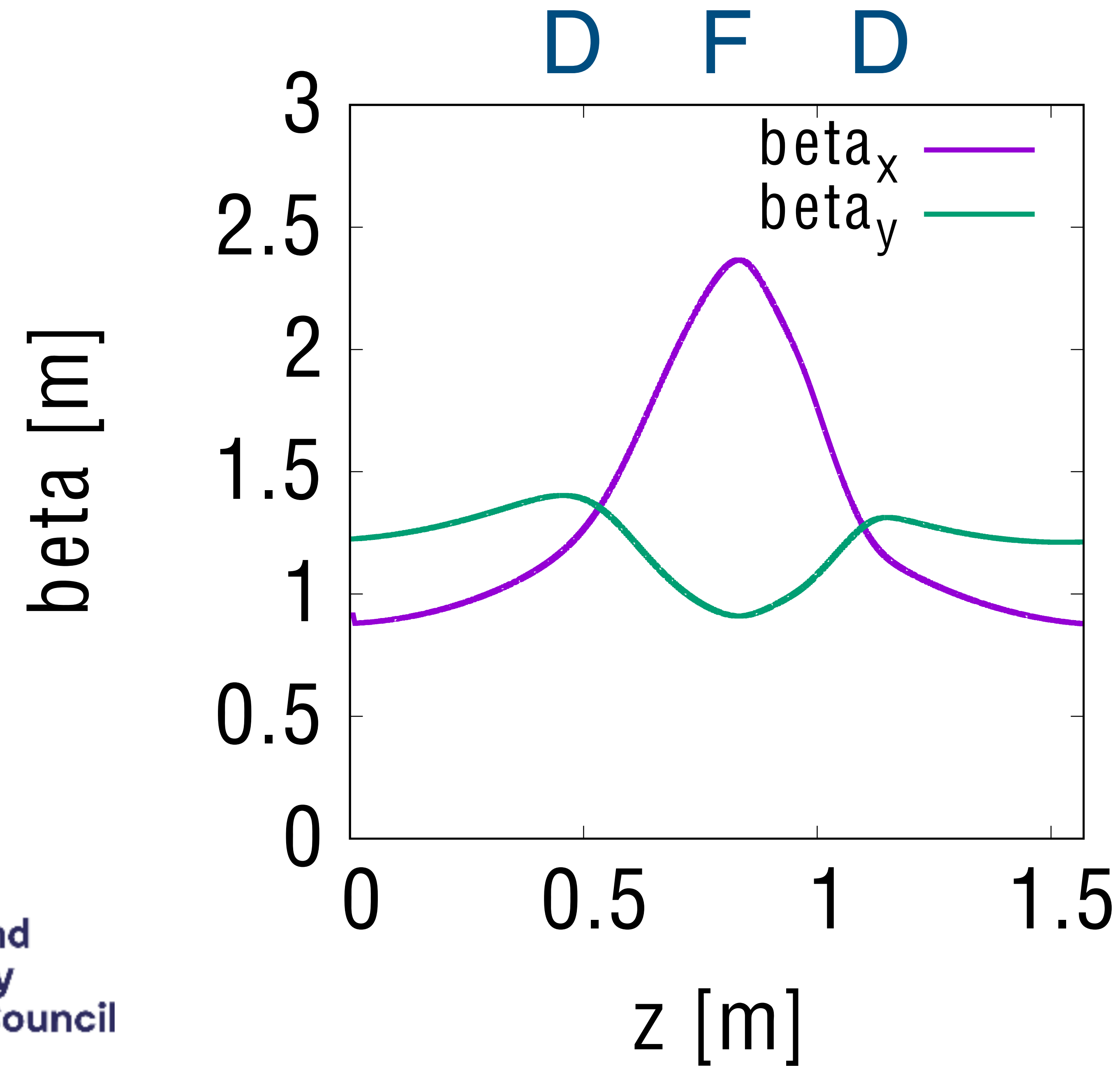
FD doublet spiral vs FDF triplet spiral

optics

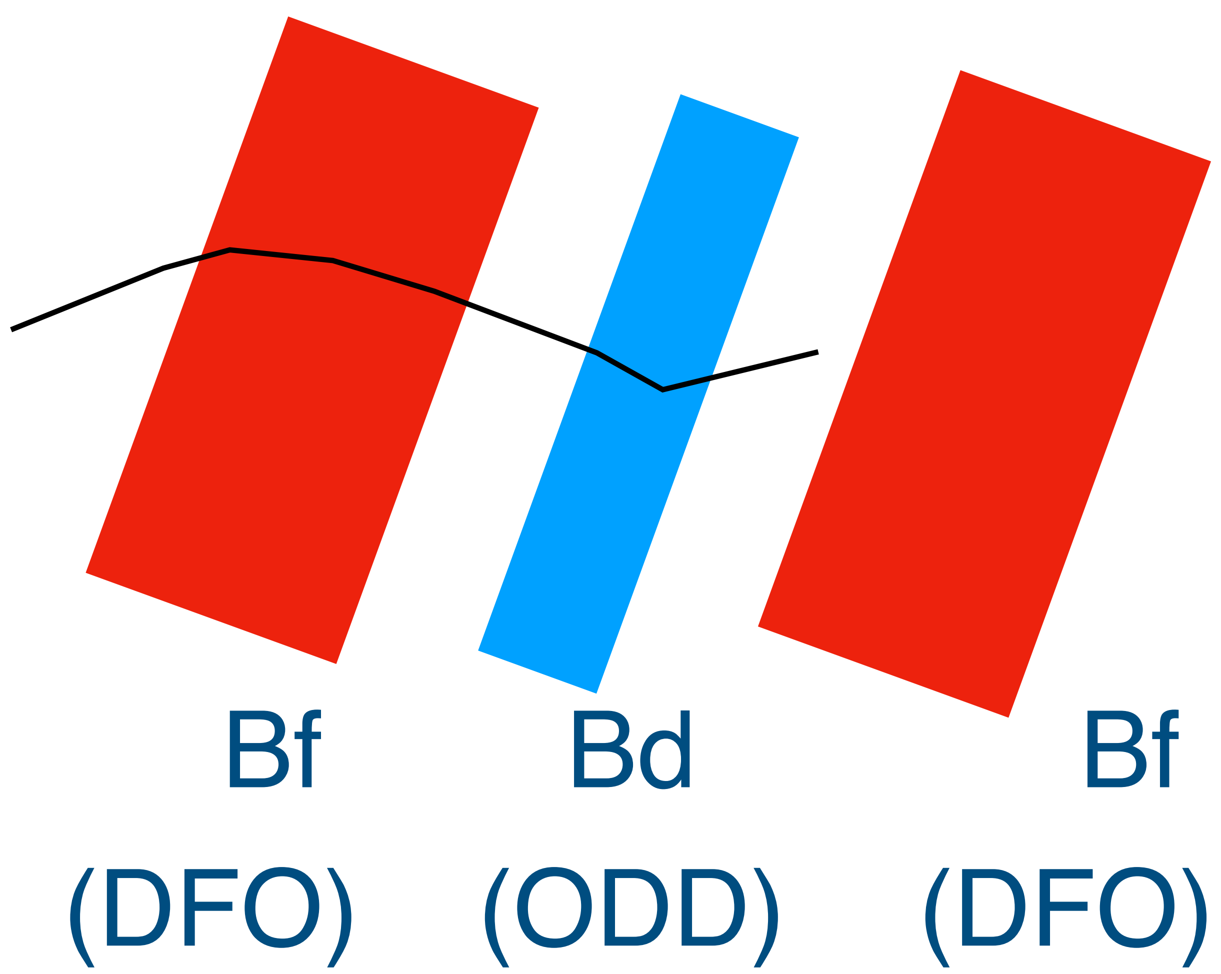
FD



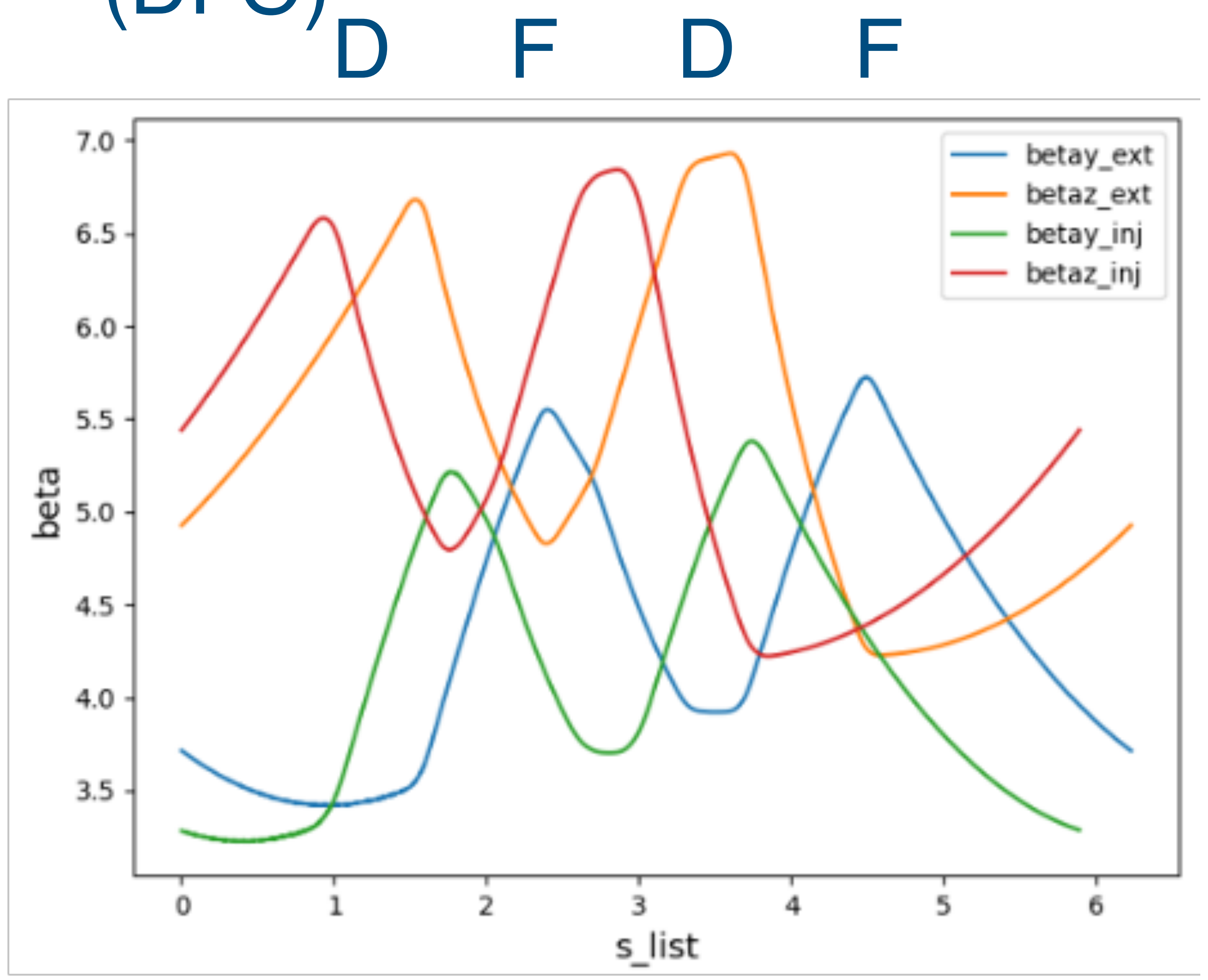
From optics point of view, FD doublet with spiral angle looks triplet DFD, not doublet.



FDF



FDF triplet with spiral angle should be DFDF.



From Bin Wu's presentation on Monday

Dynamic aperture of 16 cell lattice

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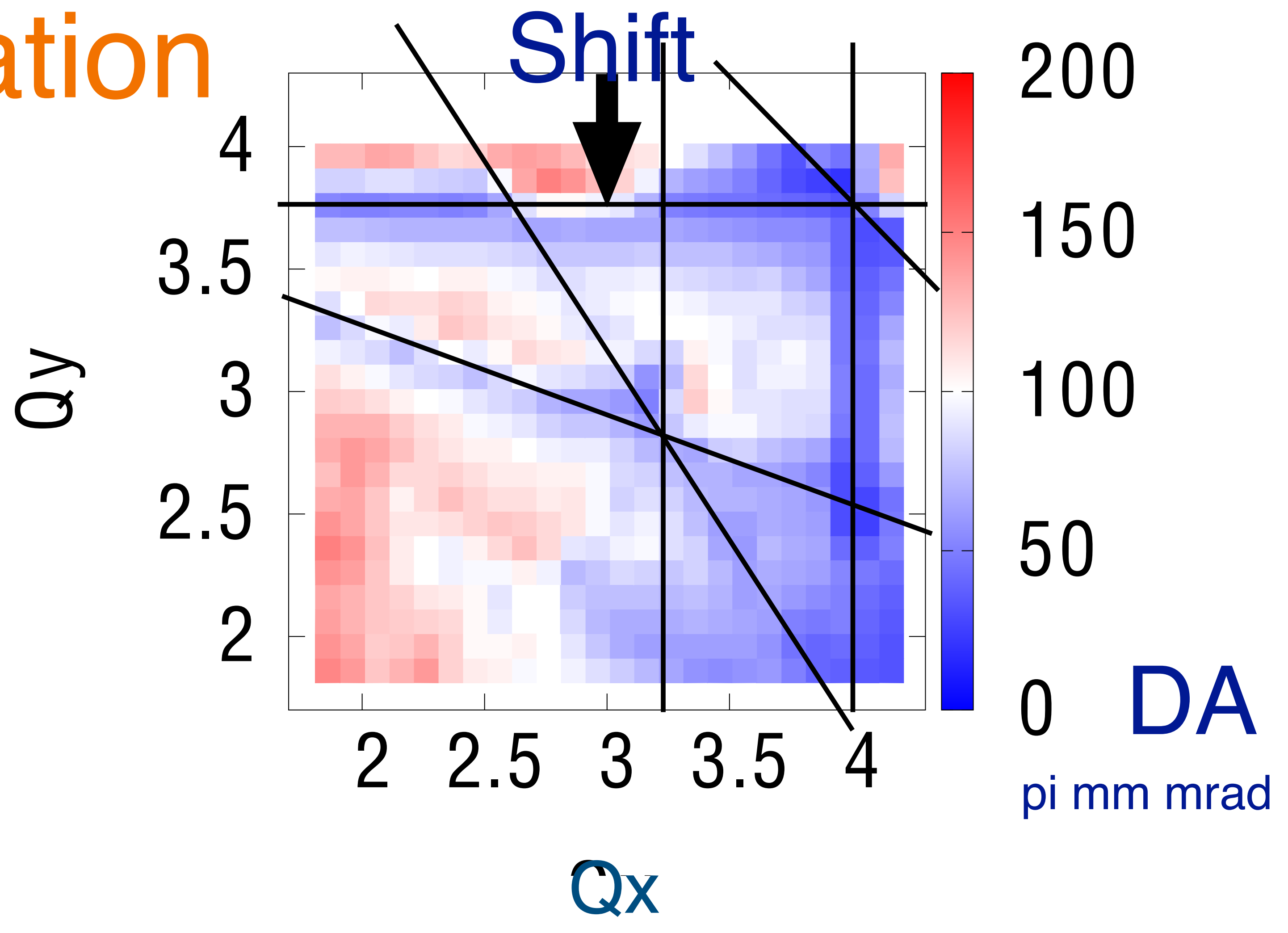

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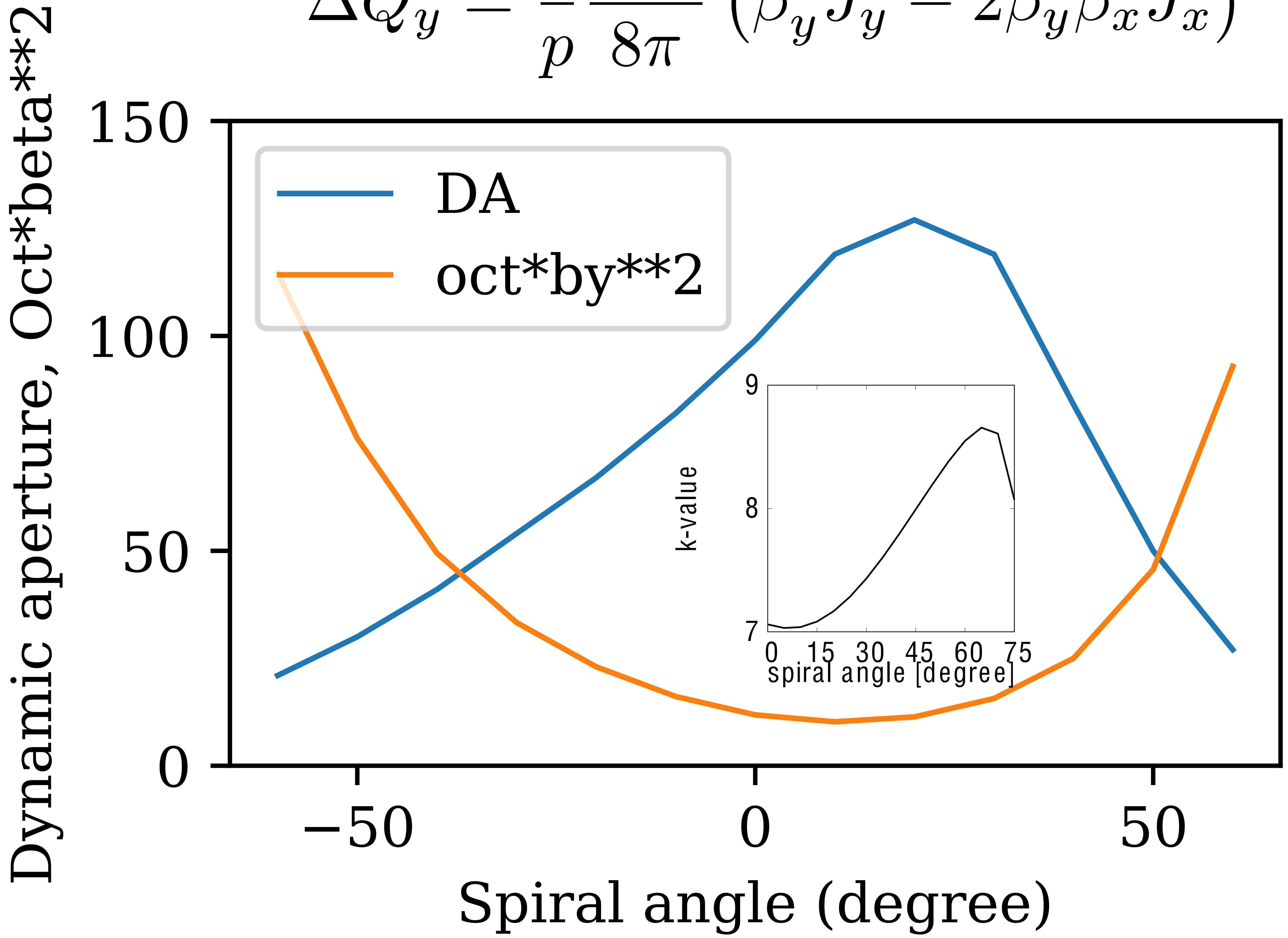
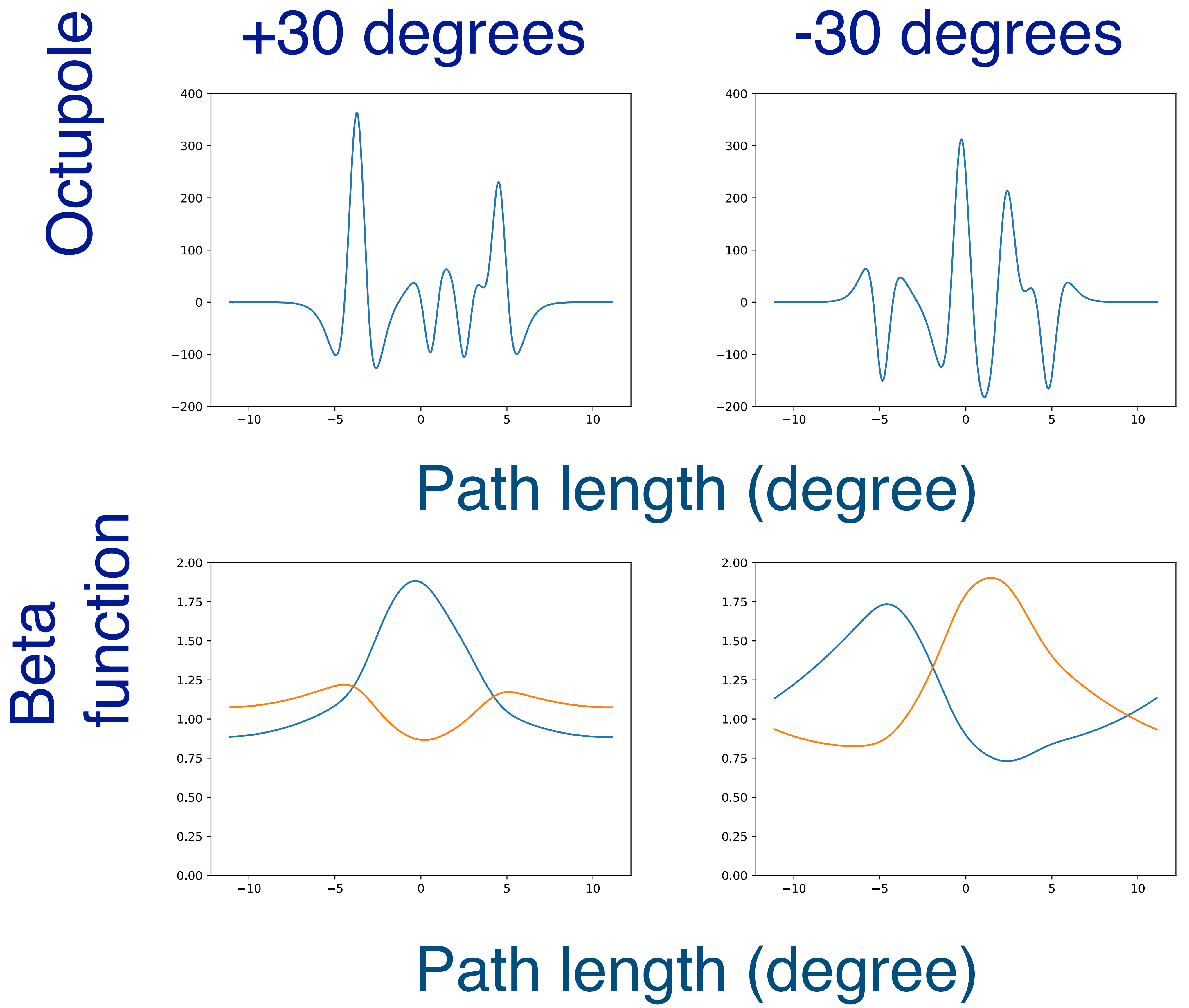




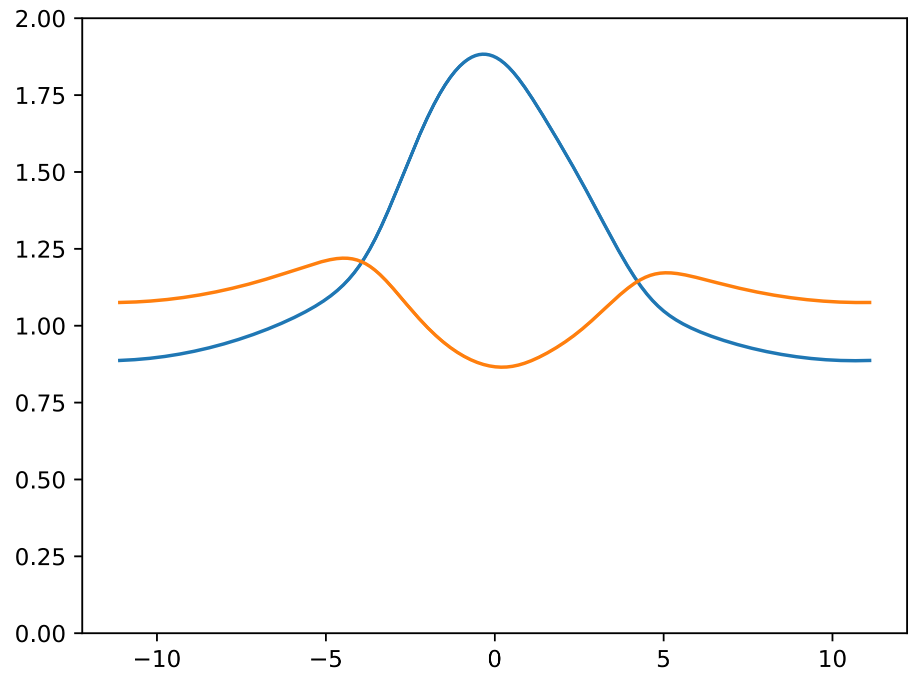
FD doublet spiral vs FDF triplet spiral

octupole and tune shift

$$\Delta Q_y = \frac{q}{p}\frac{3B_4}{8\pi}\left(\beta_y^2 J_y - 2\beta_y\beta_x J_x\right)$$

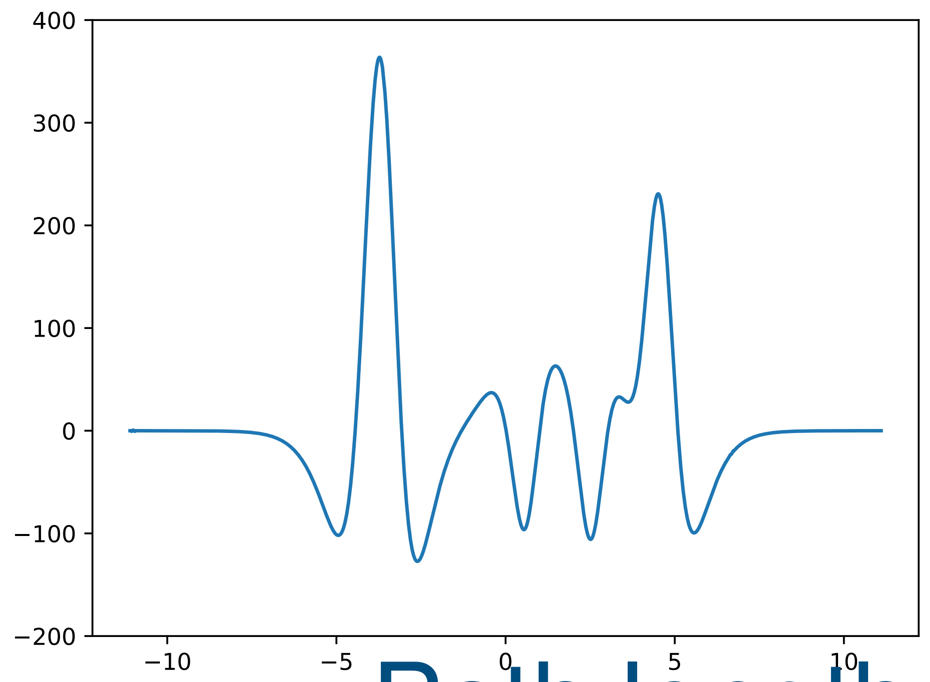
FD

Beta function



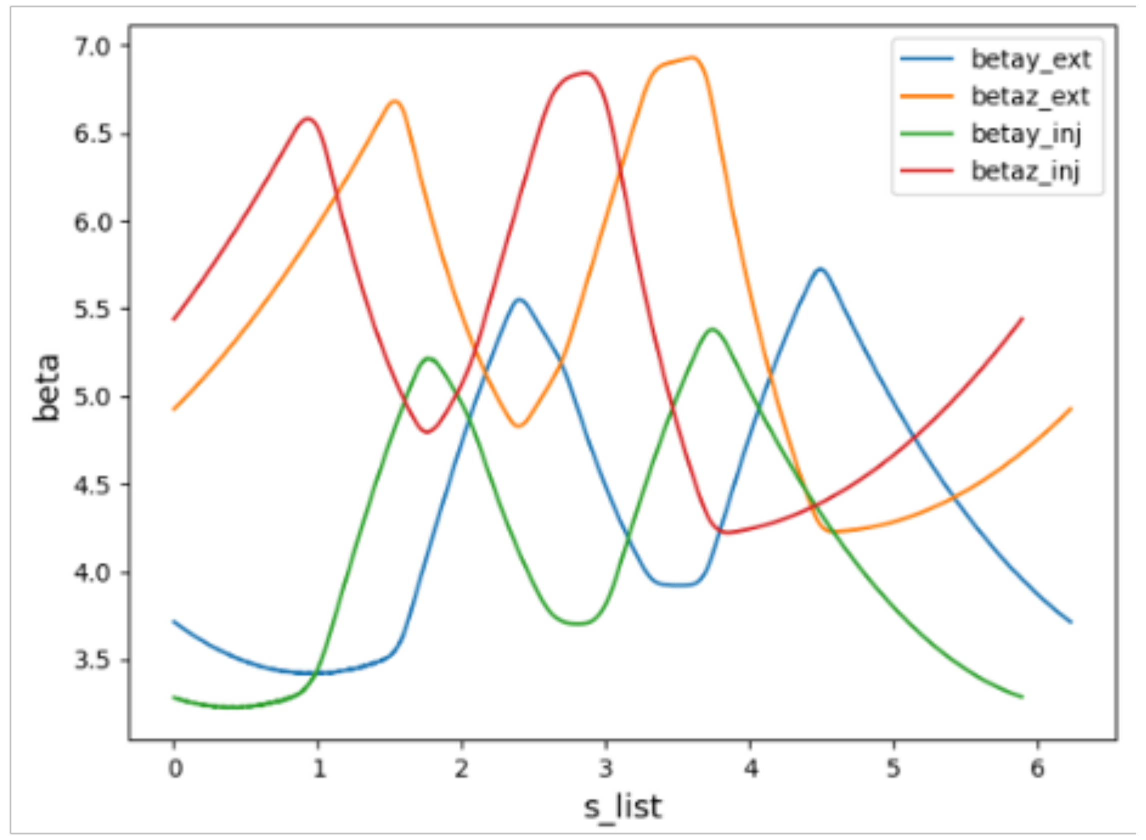
Path length (degree)

Octupole



Path length (degree)

CSNS design → FDF



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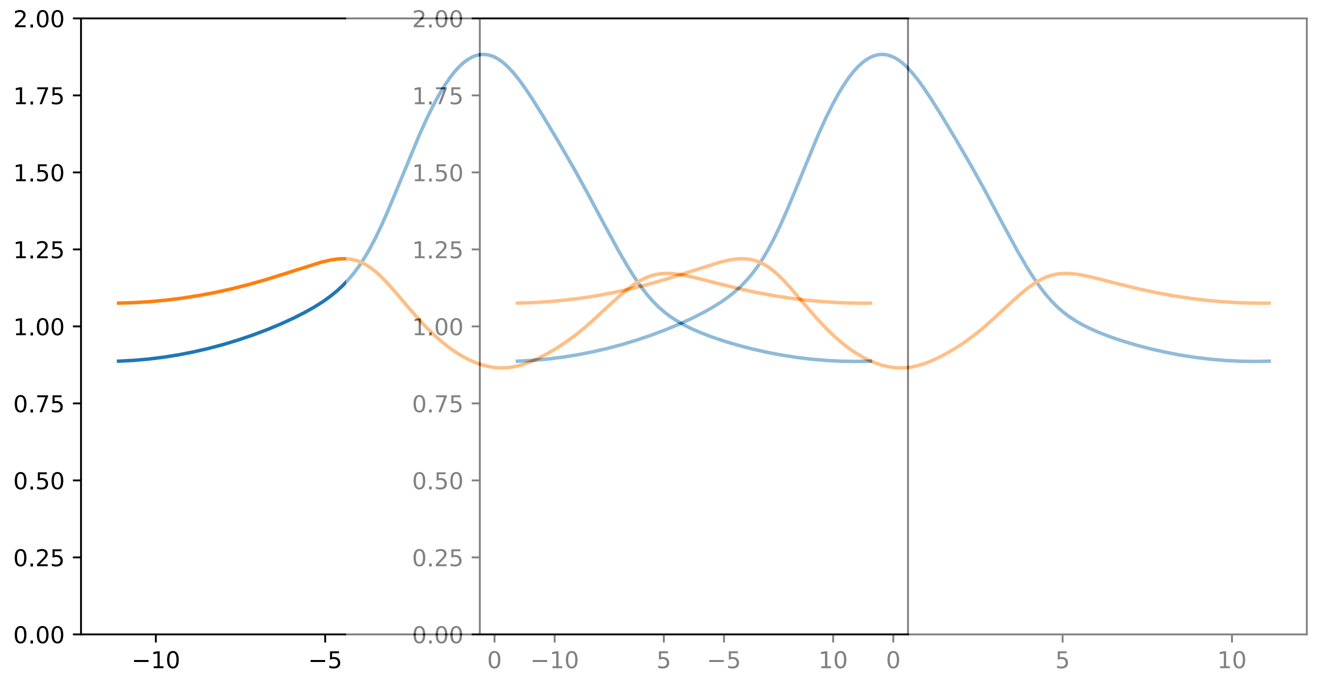
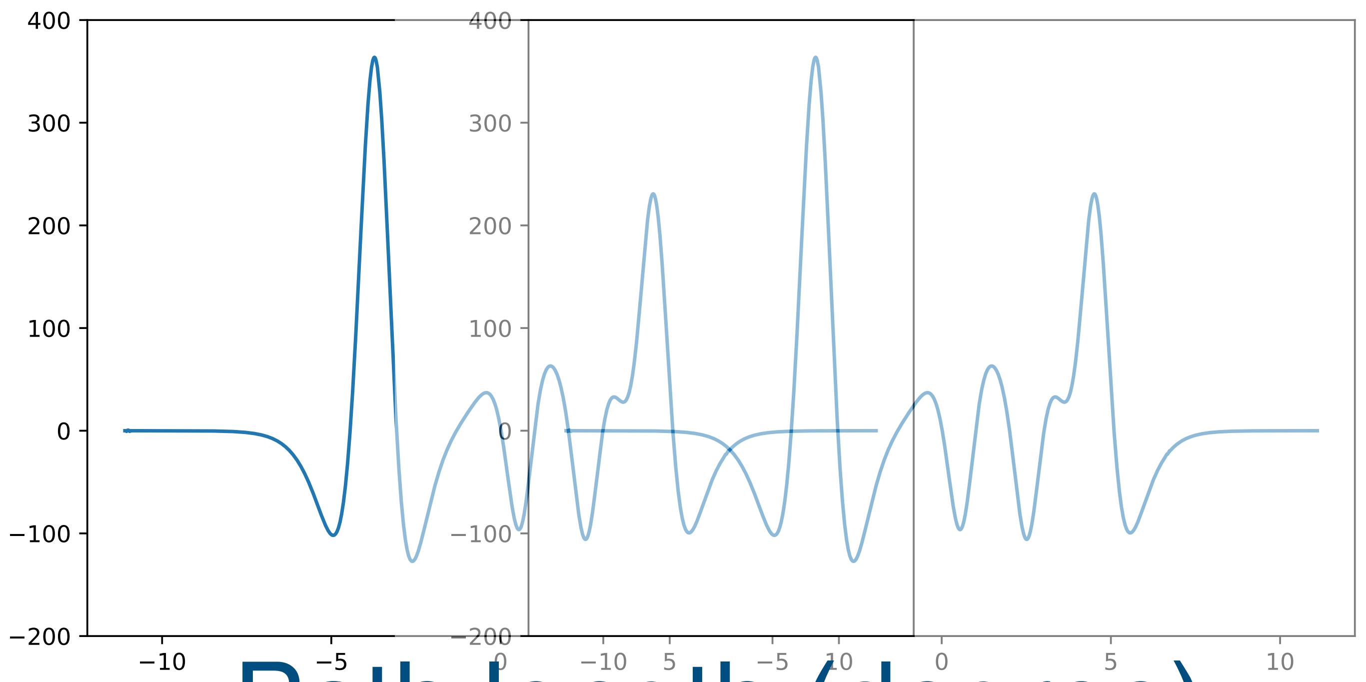


Path length (degree)

Octupole



Path length (degree)

Any cancellations by additional F?

Procedure to choose FD or FDF

- DA is determined by the amplitude dependent tune shift.
- Octuple components strongly depend on the fringe fields.
- Harmonic analysis along the orbit is a tool to evaluate DA.

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Q8. Optical functions of the lattice is very sensitive to the fringe field parameters

Enough flexibility

- FD doublet spiral lattice.
- K-value of the lattice magnet is one of the key parameters for operation.

Q9. Vertical tune shift much greater than horizontal one

EPAC 2002 paper

Proceedings of EPAC 2002, Paris, France

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$(Taylor\ Expansion\ \ around\ \ r_0, r = r_0 + x)$

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Thank you for your attention

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