# DEVELOPMENT OF **BDSIM** TO MODEL COOLING SYSTEMS

UK Muon Collider Meeting
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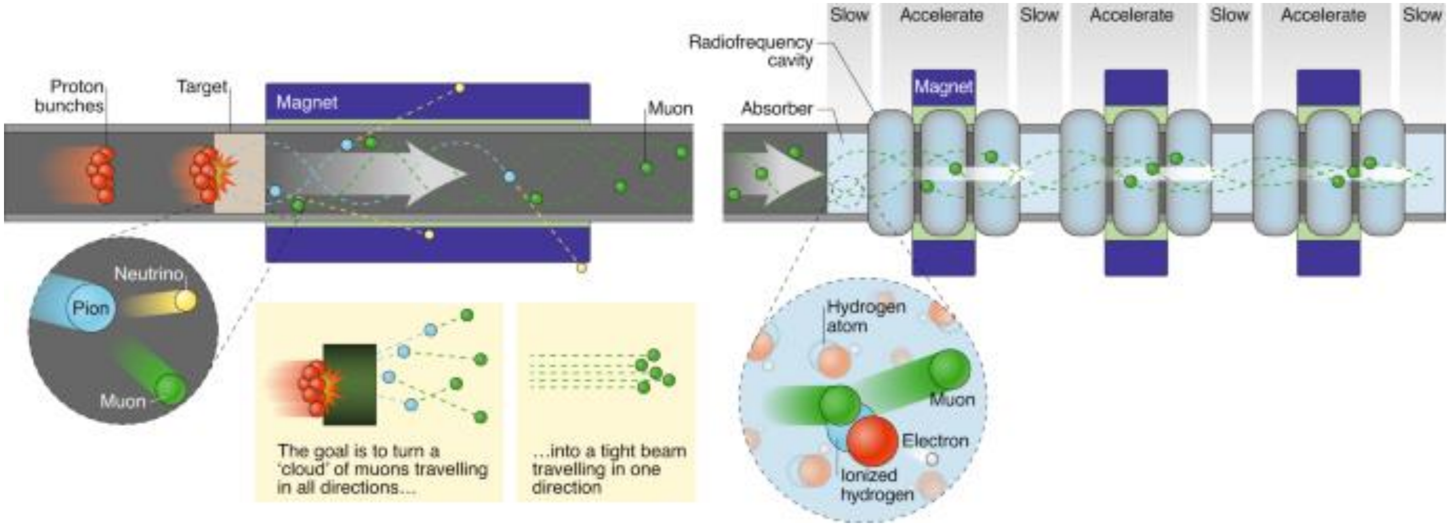
# Ionization Cooling



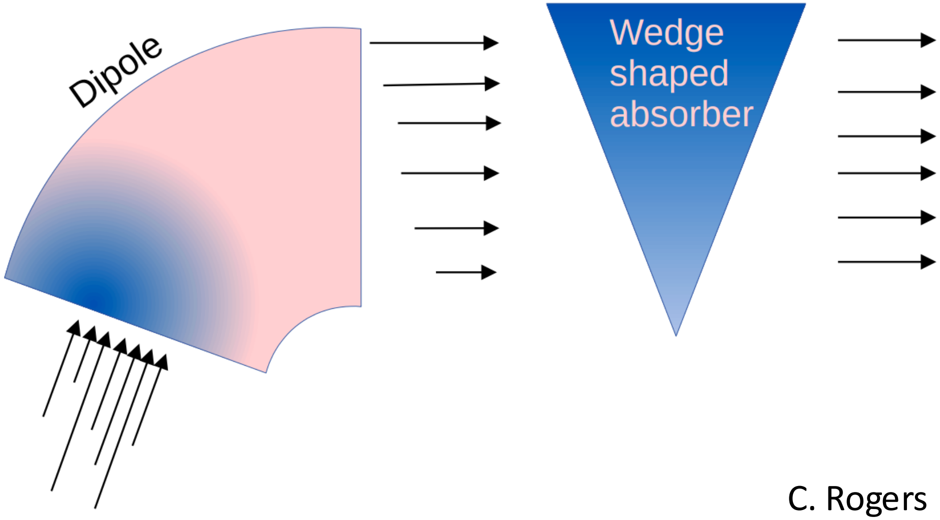
The goal is to turn a 'cloud' of muons travelling in all directions…

…into a tight beam travelling in one direction

- Employing a proton driver for production results in a beam with a large emittance. Traditional methods of cooling such as synchrotron damping are unfeasible!
- Beam loses energy in absorber material
  - Momentum reduced in all directions
  - Longitudinal momentum restored using RF cavities
  - Outcome: a beam with a reduced angular divergence, more parallel
- Multiple Coulomb scattering has an antagonising effect; can be mitigated
  - Tight focusing at absorber
  - Low-Z absorber materials (liquid hydrogen, lithium hydride)
- The beam reaches equilibrium emittance when the two effects cancel out.

# Emittance exchange

- 6D cooling required for the Muon Collider
  - Must also cool the beam longitudinally
  - Can be achieved through emittance exchange



C. Rogers

**Input**: low transverse emittance, high longitudinal emittance → **Dipole**: induces an energy-position correlation; beam wider → **Wedge absorber**: removes energy-position correlation and momentum spread → **Output**: high transverse emittance, low longitudinal emittance

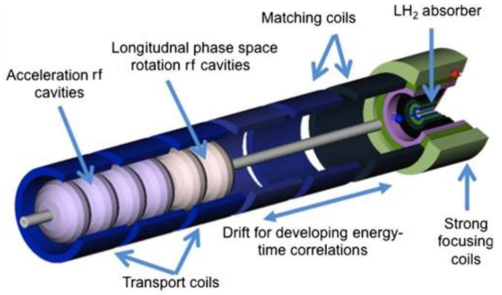
# Muon Collider Cooling Scheme

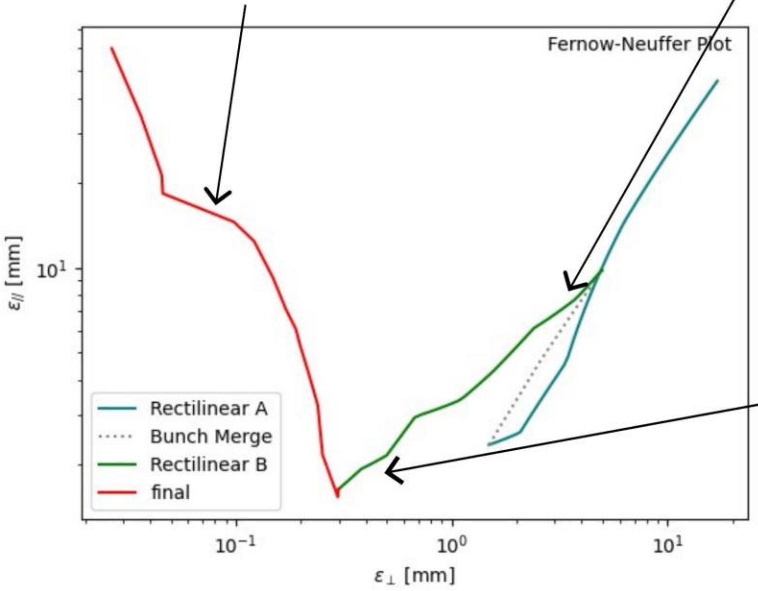
Challenges:

- Rectilinear cooling
  - Integration of magnets, absorbers and RF cavities in a compact lattice
  - High RF gradients in strong magnetic fields
- Final cooling
  - Very high magnetic fields required (~ 40+ T)
  - Management of longitudinal emittance growth
  - Liquid hydrogen absorber in the presence of high beam currents

H. K. Sayed, R. B. Palmer, D. Neuffer

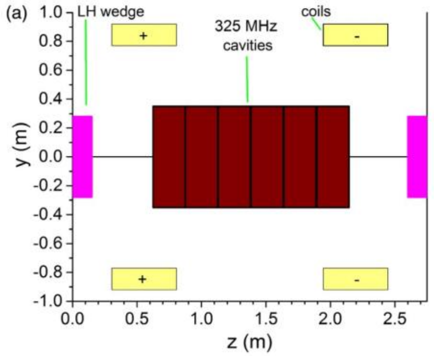
Final cooling



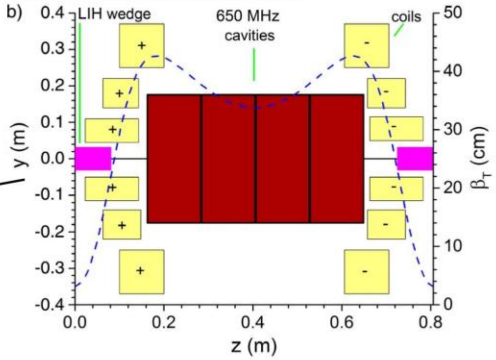


D. Stratakis, R. B. Palmer

Rectilinear B (Stage B1)



Rectilinear B (Stage B8)



# Context

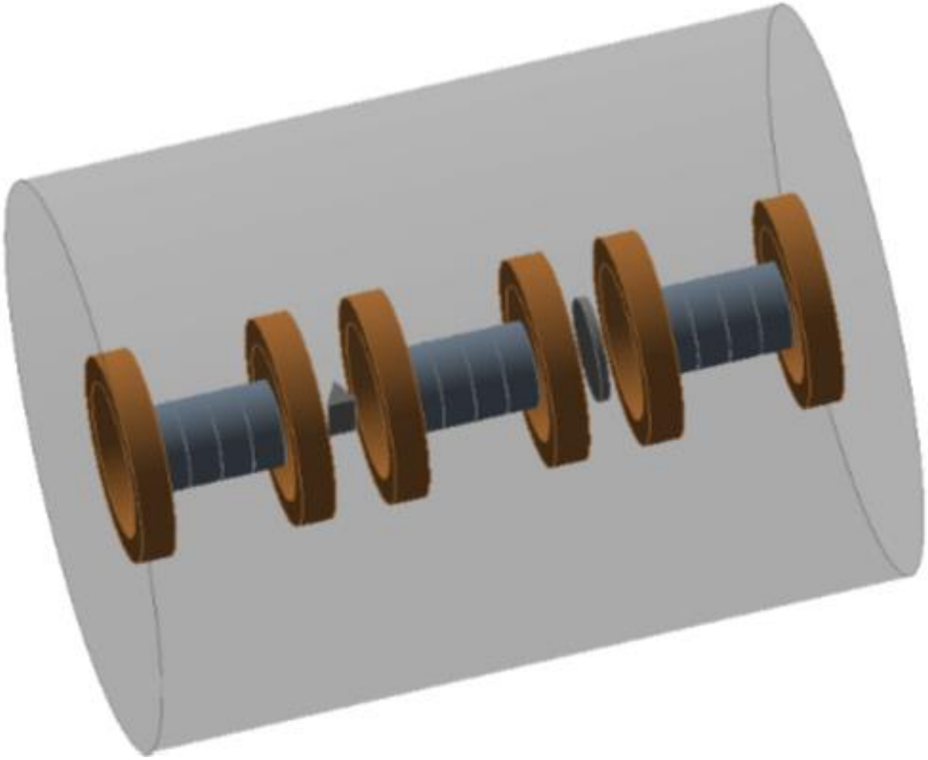
- **Beam Delivery Simulation (BDSIM)**, originally developed for modeling accelerator lattices with beam-intersecting devices, has been used in studies for next-generation collider candidates like FCC-hh, CLIC, and ILC.
- With its modern Geant4-based framework and active development, BDSIM has emerged as a strong candidate for muon cooling simulations.
- An actively maintained and flexible alternative to existing codes such as G4Beamline, ICOOL, RF-Track.
- This work extends BDSIM's capabilities for ionisation cooling by integrating physical models of magnets, RF cavities, absorbers into a dedicated muon cooling beamline element.
- This implementation was started by Laurie Nevay, and subsequently taken over by Paul, Chris and myself.

# Implementation

- A new BDSIM element, **muoncooler**, has been implemented to support the simulation of 6D muon cooling systems.
- This element allows the definition of an entire cooling lattice within BDSIM, and it can include:
  - Solenoids
  - Dipoles (field only, no physical magnet)
  - RF cavities
  - Absorbers (wedge or cylinder shaped)
- Fields from each cooling cell component (magnets, RFs) are summed to compute a 6-vector electromagnetic field spanning the element volume.
  - Fringe effects from all magnets are accurately accounted for across the lattice.

# Visualisation

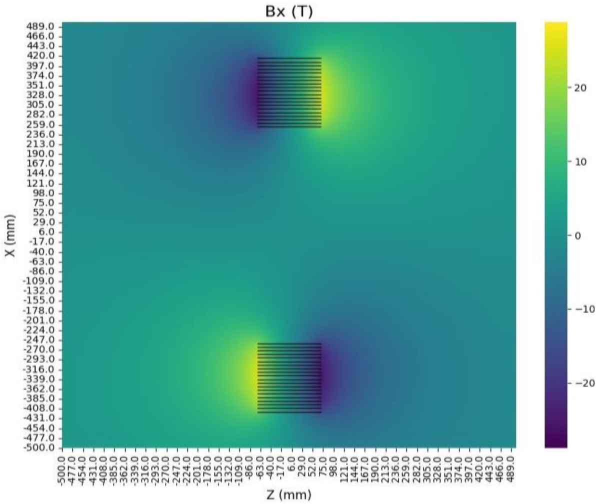
- 3D rendering of an example BDSIM cooling channel
  - Two types of absorbers exhibited
  - No physical dipole magnet, only field

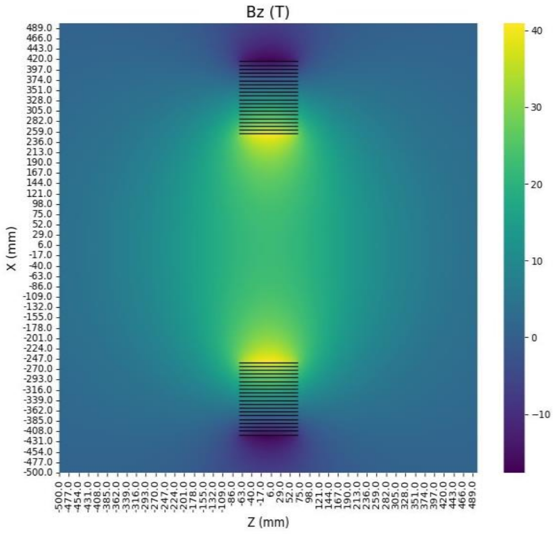


# Solenoid Models

- Two dedicated solenoid field models have been written:
    - Sheet Model (cylindrical current carrying sheet)
    - Block Model
- **Sheets** are modelled following the treatment outlined by Derby et al.
- A bounding box optimisation to speed up computation has been implemented.
- **Solenoid Blocks** are implemented by layering sheets one on top of another for a more realistic 3D representation.

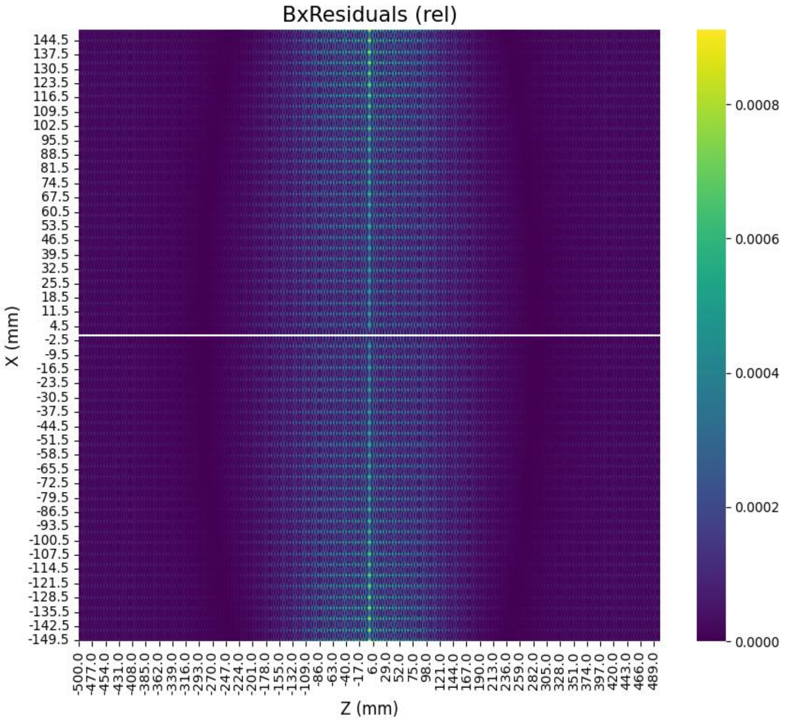
| Parameter | Unit | Magnitude |
|---|---|---|
| Coil inner radius | mm | 250.0 |
| Coil radial thickness | mm | 169.3 |
| Coil outer radius | mm | 419.3 |
| Coil length | mm | 140.0 |
| Current density | A/mm$^2$ | 500.0 |

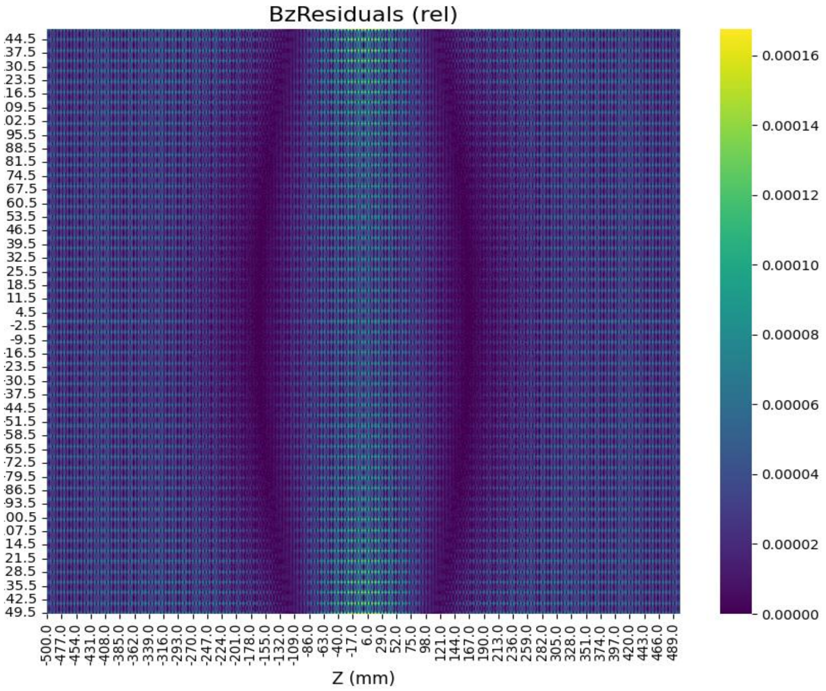




# Solenoid Validation

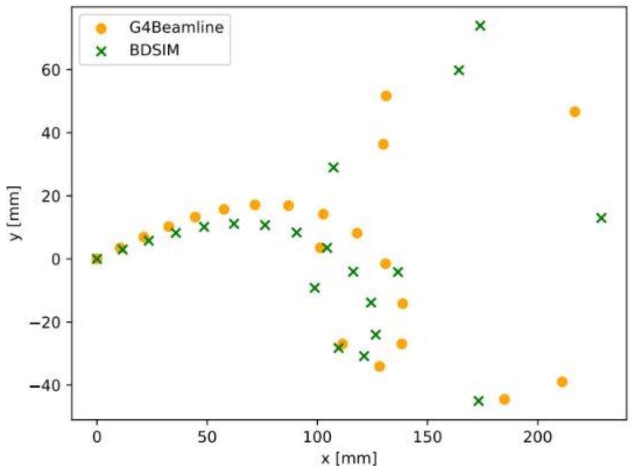
- The BDSIM and G4BL fields were compared and showed good agreement.



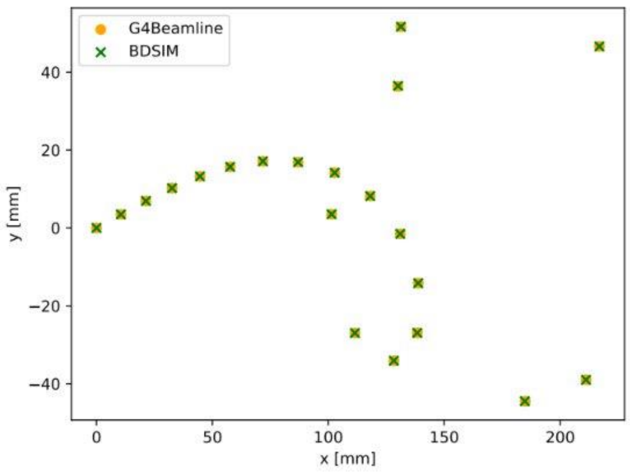


# Solenoid Tracking Validation

- 200 MeV/c muons were initialized along a grid (0 – 200 mm, 10 mm steps) on the *x*-axis and tracked through a single solenoid, starting 0.5 m upstream of the solenoid.
- BDSIM and G4Beamline positions were compared 0.5 m downstream, showing very good agreement.



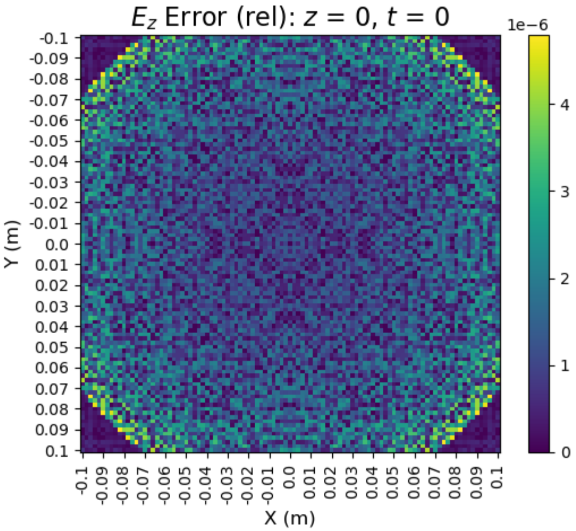
Before

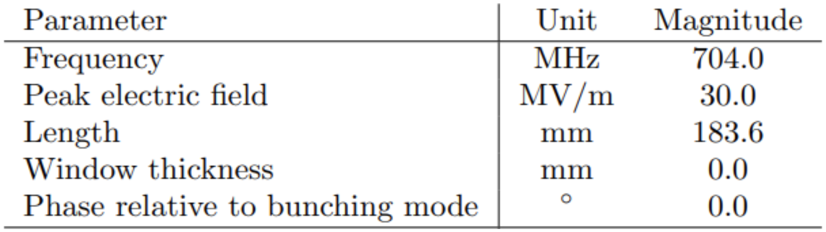


After

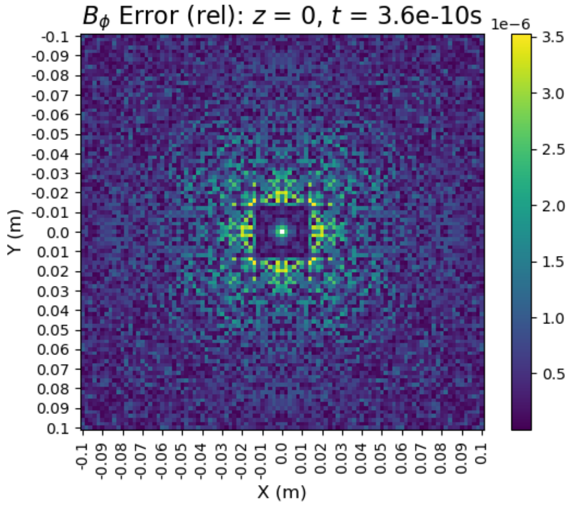
# RF Cavity Checks

- Standard BDSIM RF cavities modelled as a pillbox cavity.
- For the comparisons shown here, the RF cavity parameters are shown below; the field was queried at t = 0 and one quarter period.
- These were compared to theoretical distributions of the same generated in Python as a sanity check and showed good agreement.





| Parameter | Unit | Magnitude |
|---|---|---|
| Frequency | MHz | 704.0 |
| Peak electric field | MV/m | 30.0 |
| Length | mm | 183.6 |
| Window thickness | mm | 0.0 |
| Phase relative to bunching mode | ° | 0.0 |



# Dipole

- Two dipole field models implemented:
  - Hard edge
    - ☐ Constant $B_y$ within the bore of the magnet (defined by length and aperture radius parameters)
  - Enge-type fringe fields
    - ☐ Following the treatment in B. D. Muratori, et al
    - ☐ Currently only one Enge polynomial term included, fields of the form:

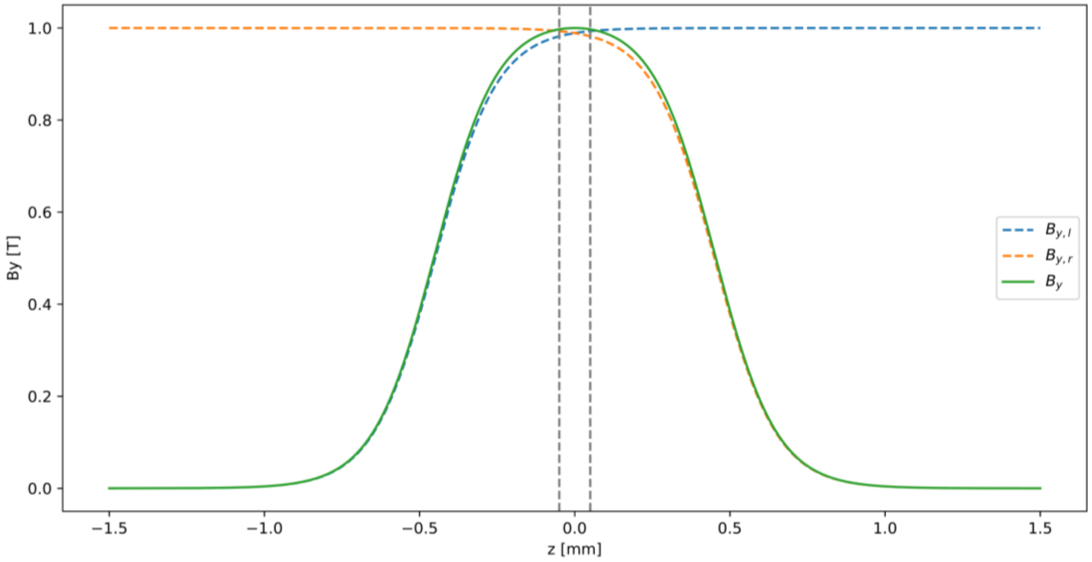
$$B_y = \frac{1 + e^{\alpha z}\cos\alpha y}{1 + 2e^{\alpha z}\cos\alpha y + e^{2\alpha z}}$$

$$B_z = \frac{-e^{\alpha z}\sin\alpha y}{1 + 2e^{\alpha z}\cos\alpha y + e^{2\alpha z}}$$

where $\alpha = a_1/D$, D - magnet aperture, $a_1$ - coefficient of the first order Enge term

# Dipole

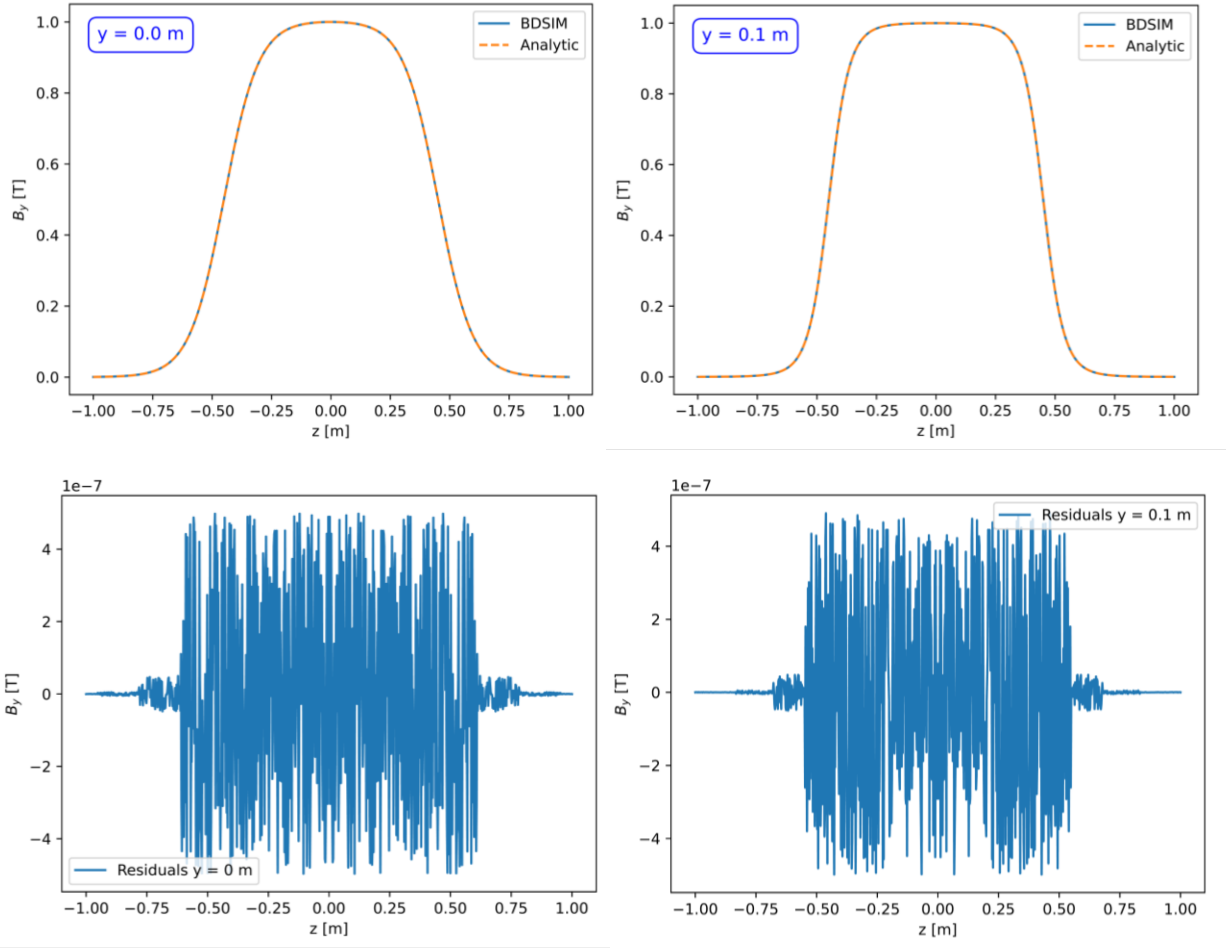
- Dipole field modelled as the sum of the left (entry) and right (exit) fringe fields



# Dipole – $B_y$ sanity check

- Compared BDSIM output with output of the analytic expression

| Parameter | Unit | Value |
|---|---|---|
| Aperture radius | m | 0.2 |
| Magnet length | m | 0.1 |
| Field strength | T | 1 |
| Enge coefficient | - | 5.5 |



# Dipole – $B_z$ sanity check

- Compared BDSIM output with output of the analytic expression

| **Parameter** | **Unit** | **Value** |
|---|---|---|
| Aperture radius | m | 0.2 |
| Magnet length | m | 0.1 |
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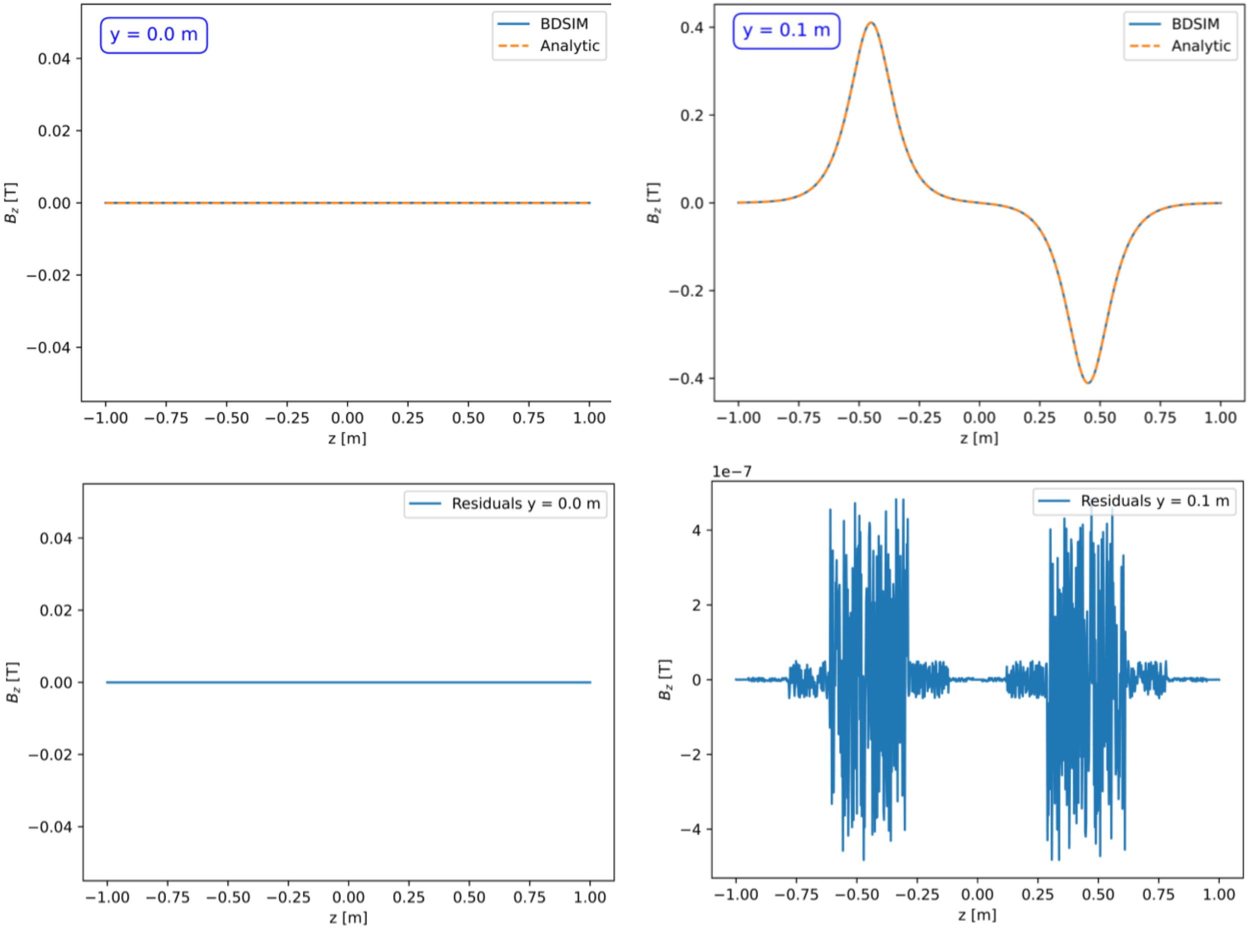


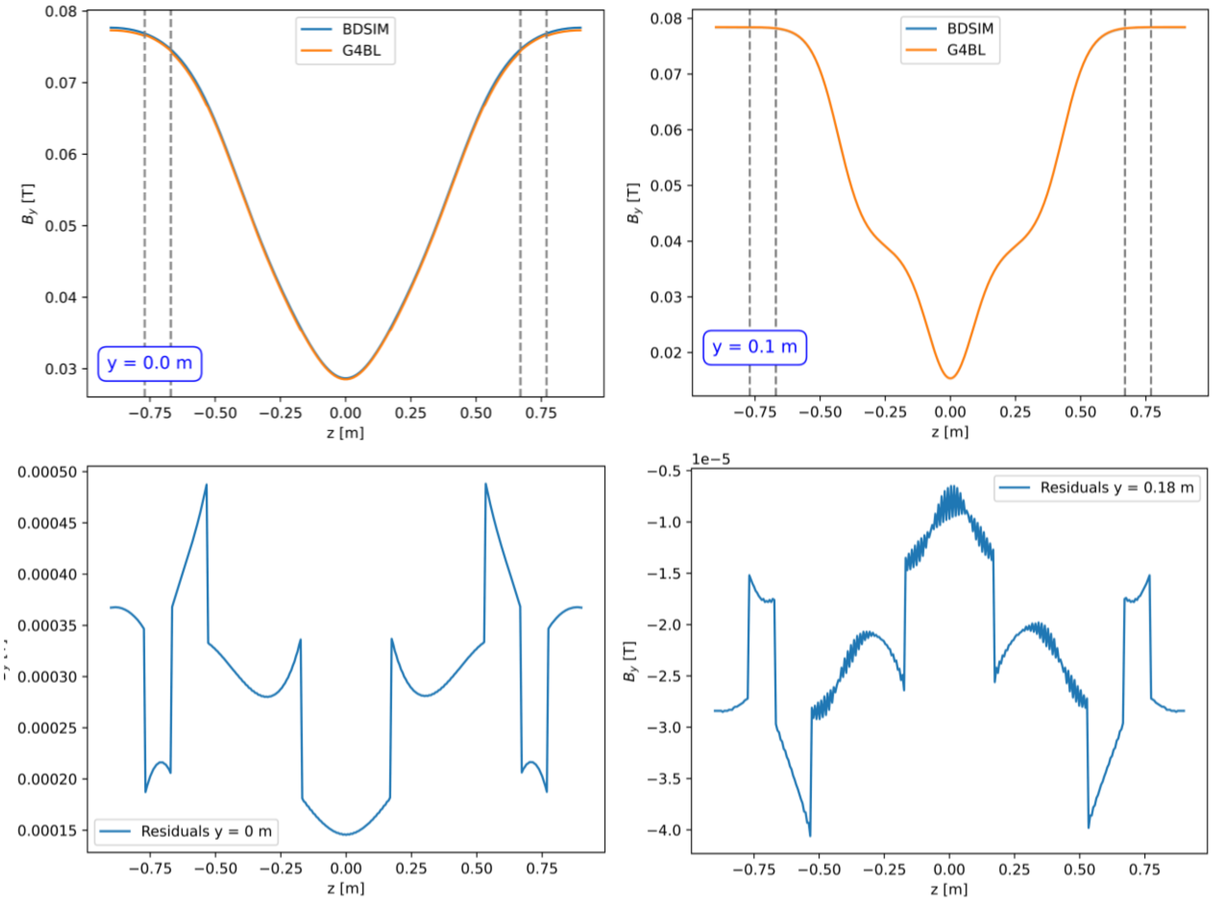
# Dipole – G4BL comparison

- Compared BDSIM and GB4L dipole field outputs for the Rectilinear A Stage 1 cell

| Parameter | Unit | Value |
|---|---|---|
| Aperture radius | m | 0.3 |
| Magnet length | m | 0.1 |
| Field strength | T | 0.03918 |
| Enge coefficient | - | 5.5 |

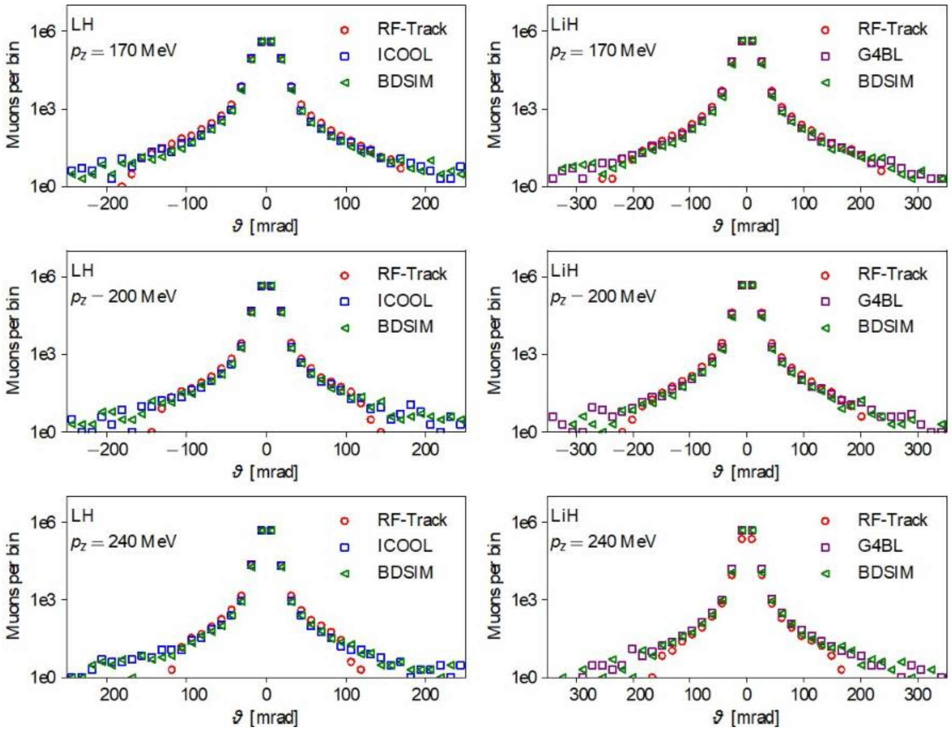
Main source of discrepancy arises from the G4BL definition: left and right fringe fields are 'attached' to a central constant field.

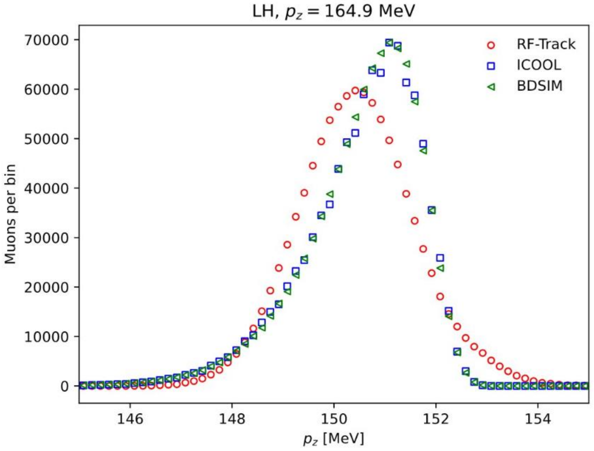
The fringes do not fully converge giving rise to small discontinuities in the field, seen in the residuals



# Absorber Validations

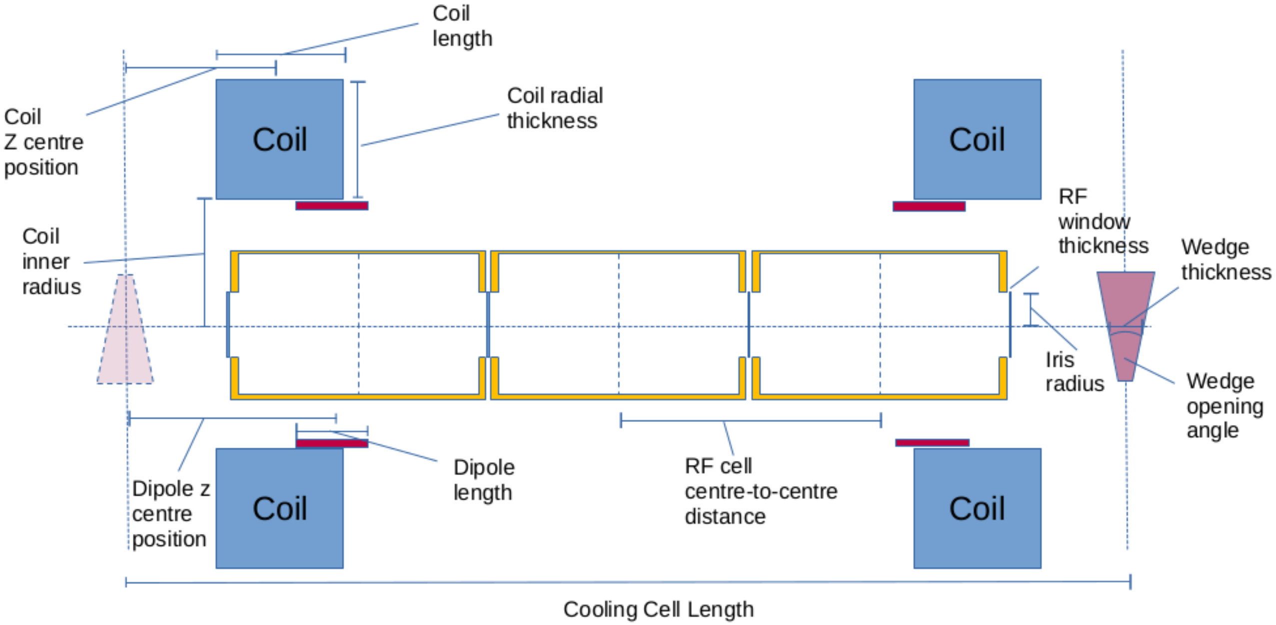
- Adapted from Bernd Stechauner's talk at the Fermilab workshop.
- BDSIM shows good agreement with G4 based codes like G4BL and ICOOL, but shows deviations from RF track, due to the energy straggling model assumed in RF track.





# Cooling Lattice Simulations

- 4D cooling was simulated using the solenoid lattice from an earlier Demonstrator cooling cell design (2022-11-01-release). Results presented at IPAC24.
- Here we're going to show the first attempt at simulating a 6D cooling lattice based on the **2024-05-24-release**, documented in the deliverable D8.1.



| Parameter | Unit | Value | Tol |
|---|---|---|---|
| Cooling Cell Length | mm | 800 | |
| **Beam Physics** | | | |
| Momentum | MeV/c | 200 | |
| Twiss beta function | mm | 107 | |
| Dispersion in X | mm | 38.5 | |
| Dispersion in Y | mm | 20.3 | |
| Beam Pipe Radius | mm | 81.6 | |
| **Solenoid Parameters** | | | |
| | Unit | Value | Tol |
| B0 | T | 8.75 | 0.25 |
| B0.5 | T | 0 | 0.02 |
| B1 | T | 1.25 | 0.025 |
| B2 | T | 0 | 0.5 |
| **Coil Geometry** | | | |
| Inner Radius | mm | 250 | |
| Length | mm | 140 | |
| Radial Thickness | mm | 169.3 | |
| Z Centre Position | mm | 100.7 | |
| Current Density | A/mm$^2$ | 500 | |
| **RF Cavity** | | | |
| Center-to-centre distance | mm | 188.6 | |
| Gradient E0 | MV/m | 30 | |
| Iris Radius | mm | 81.6 | |
| Number of RF Cells | | 3 | |
| Frequency | GHz | 0.704 | |
| Synchronous Phase | degree | 20 | |
| Window Thickness | mm | 0.1 | |
| **Wedge** | | | |
| Material | | LiH | |
| Opening Angle | degree | 10 | |
| Thickness | mm | 20 | |
| Alignment | | Horizontal | |
| **Dipole** | | | |
| Length | mm | 100 | |
| Polarity | | + - - + | |
| Field | T | 0.2 | |
| Z Centre Position | mm | 160 | |
| Field Direction | | Vertical | |

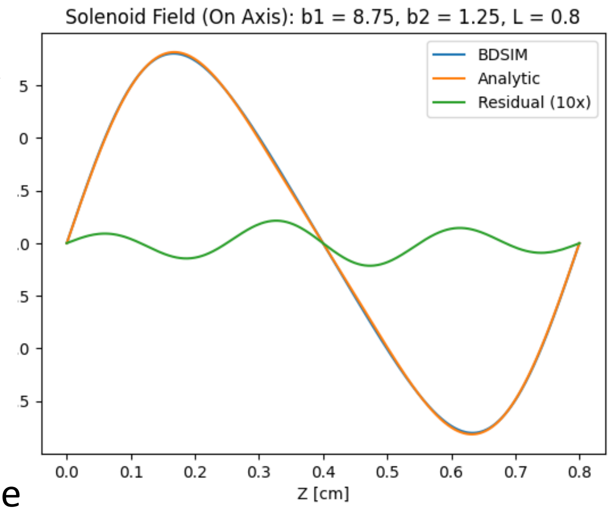
# Beam Optics

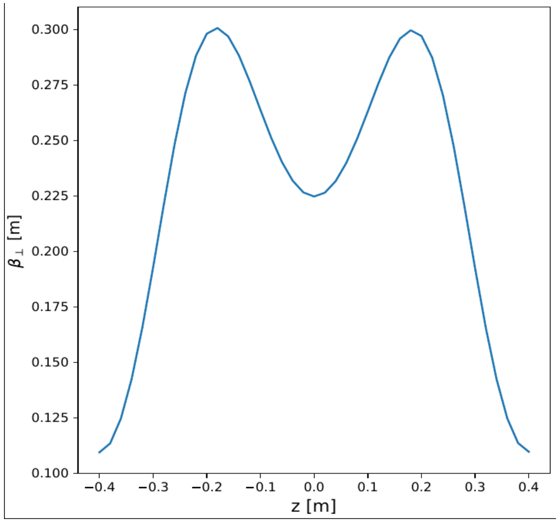
- A solenoid lattice was set up using the solenoid block model.

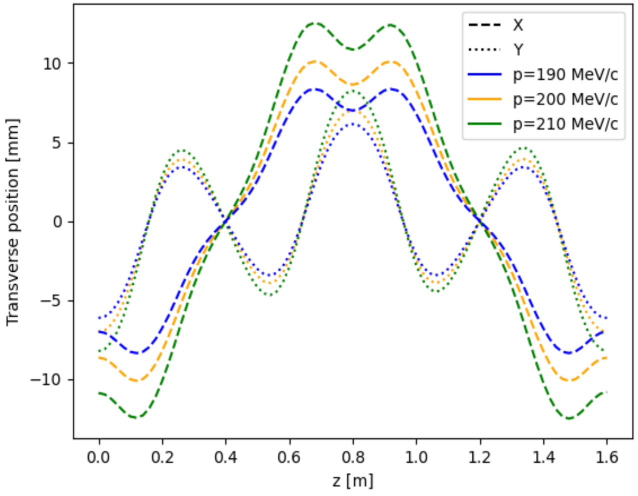
- BDSIM on-axis $B_z$ field profile compared with the ideal field with two harmonic components.

- Optical beta function for a 200 MeV/c beam shows tight focus at the absorbers.

- Dipoles were added to create dispersion; significant *x*-plane dispersion at absorber locations observed.

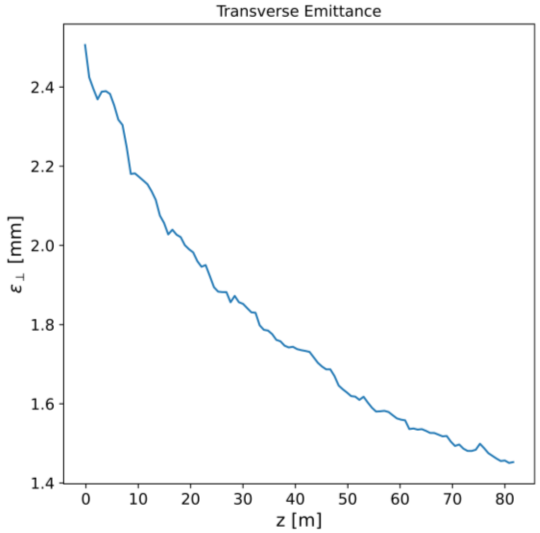


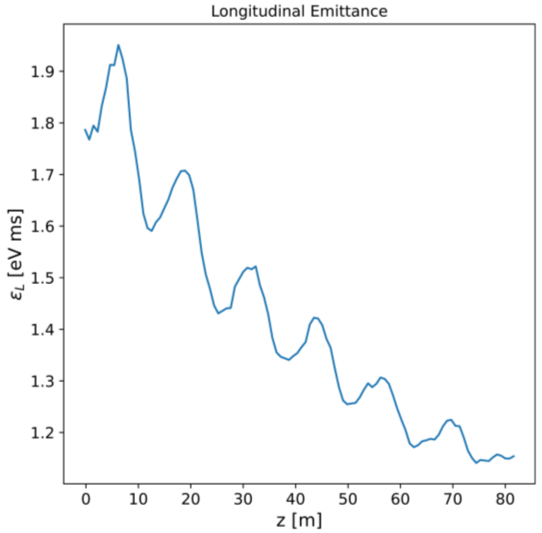


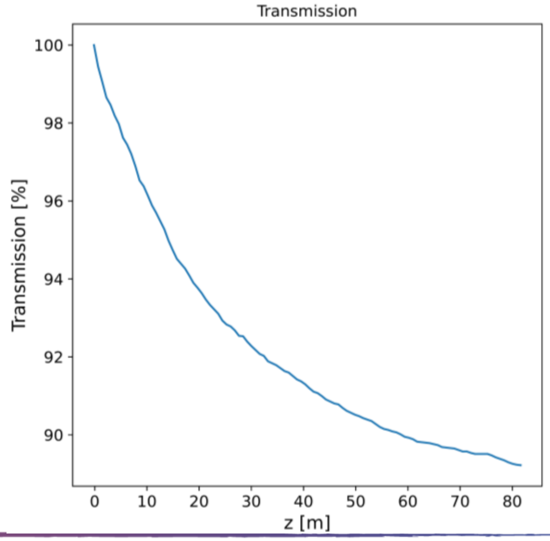


# Cooling performance

- ~ 80 m long channel
- Initial longitudinal mismatch leads to oscillating behaviour in longitudinal emittance
- **Preliminary**: Emittance reduction observed, accompanied by beam losses in the tails
- **Next:** finalise like-for-like cooling benchmarking with the G4BL study done by Chris Rogers on a 200 m lattice


Transverse Emittance


Longitudinal Emittance


Transmission

# Summary and outlook

- A '**muoncooler**' beamline element implemented in BDSIM
    - Code now suitable for ionisation cooling simulations
- Further work
    - Finalise cooling benchmarking against G4BL (potentially RF-Track as well if resources available)
    - Optimise the field map interrogation
    - Add a new RF cavity model, which accounts for non-zero dE_z/dz near the cavity aperture (under development by BDSIM collaborators)
    - Add more Enge polynomial terms, for flexibility in fringe field definition
    - Potential comparisons with field maps generated by dedicated magnet and RF software (CST Studio Suite, Comsol, etc.), to understand limitations

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# Back up