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Nuclear breathing mode analyzed in the multi-reference density functional theory

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Nuclear breathing mode

Nature of the isoscalar giant monopole resonance (ISGMR)

- fundamental collective nuclear excitation
- isoscalar monopole oscillation: $T = 0$, $L = 0$
- nuclear radial expansion and contraction (*like a "breathing" motion*)

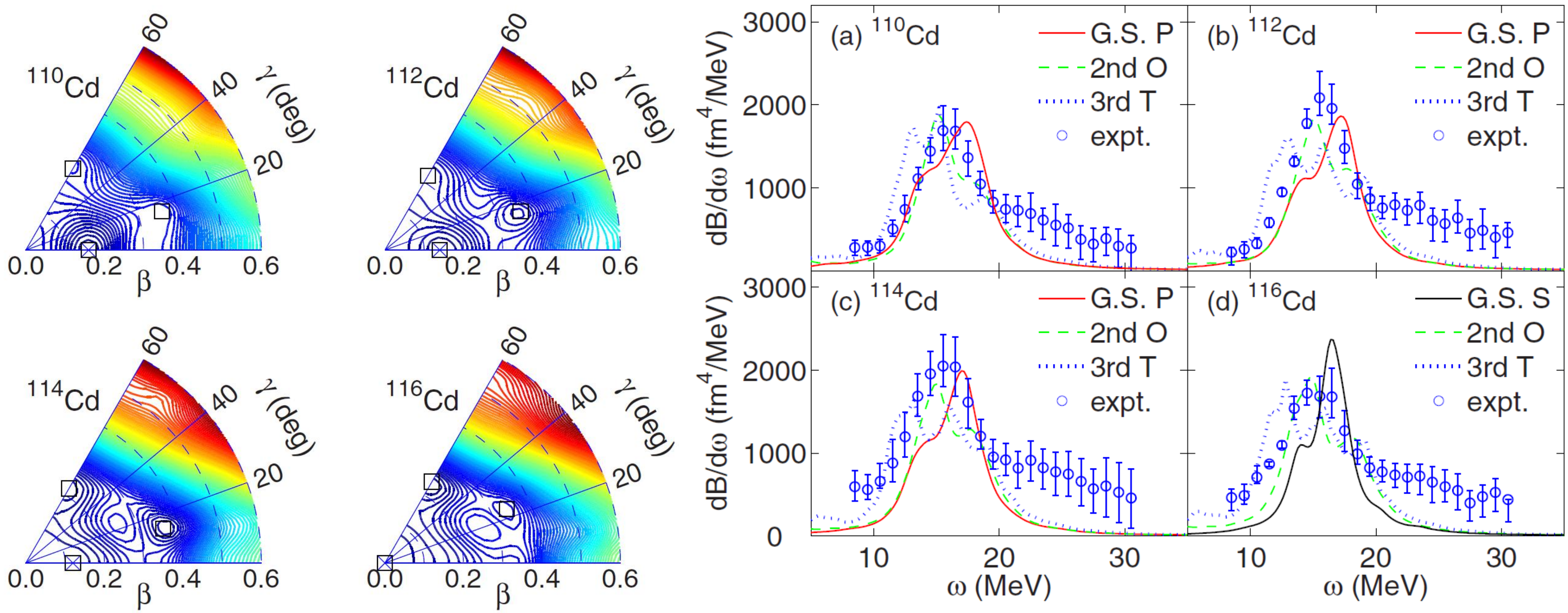
Theoretical and experimental significance

- The oscillation frequency relates to nuclear compressibility, providing critical insights into the nuclear equation of state.

Nuclear structure has a significant influence on the breathing mode.

Breathing modes built on nuclei in various shapes

- Effects of the deformed isomeric states on ISGMR for cadmium isotopes



X. Sun et al., PRC 100, 054605 (2019)

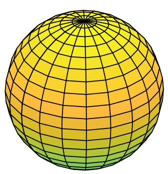
- Including γ-soft freedom improves the agreement between calculations and experimental data of the ISGMRs for Cd isotopes.

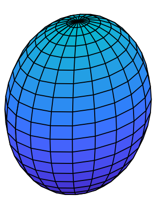
Large amplitude motion & small amplitude vibration

- Breathing modes built on different single-reference states

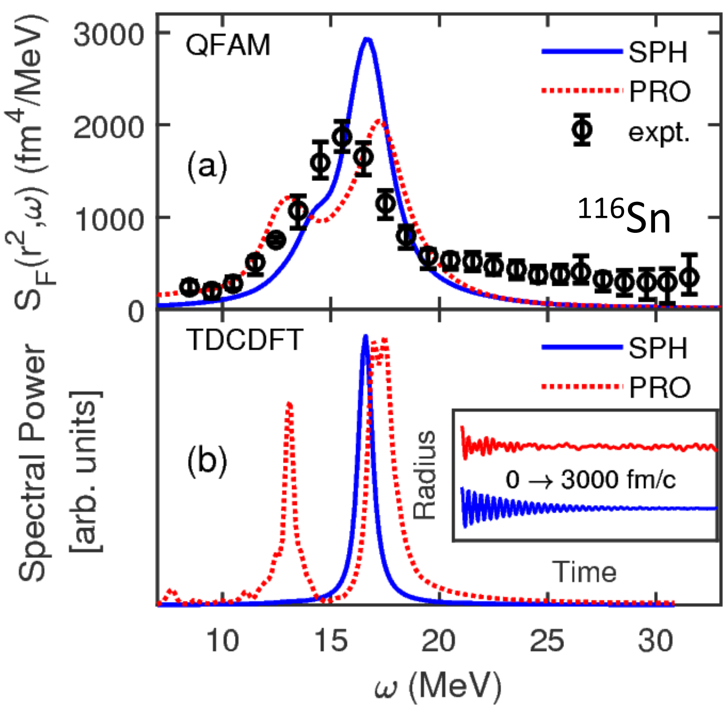
  - Quasiparticle finite amplitude method (QFAM)
  - Time-dependent covariant density function theory (TDCDFT)

Spherical (SPH)
Ground state

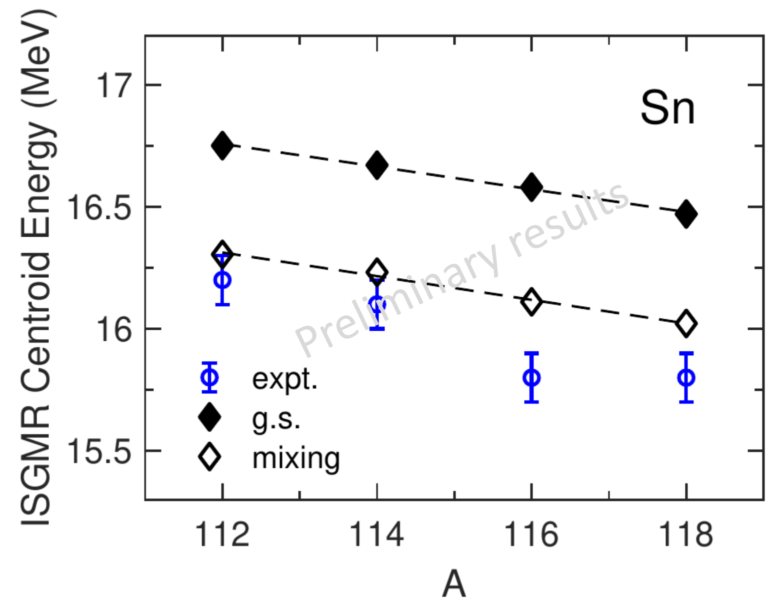




Prolate (PRO)
Shape isomer



X. Sun and Z. Ren, TBP



○ “Naïve” shape mixing

$$\sum_i |g(q_i)|^2 \beta_2^i = \beta_2^{expt.}$$

Multi-reference density functional theory (MRDFT)

- Multi-reference state is the superposition of various single-reference states:

$$|\Psi\rangle = \int d\boldsymbol{q} f(\boldsymbol{q}) |\Phi(\boldsymbol{q})\rangle$$

- The weight function $f(\boldsymbol{q})$ is determined from the variational principle

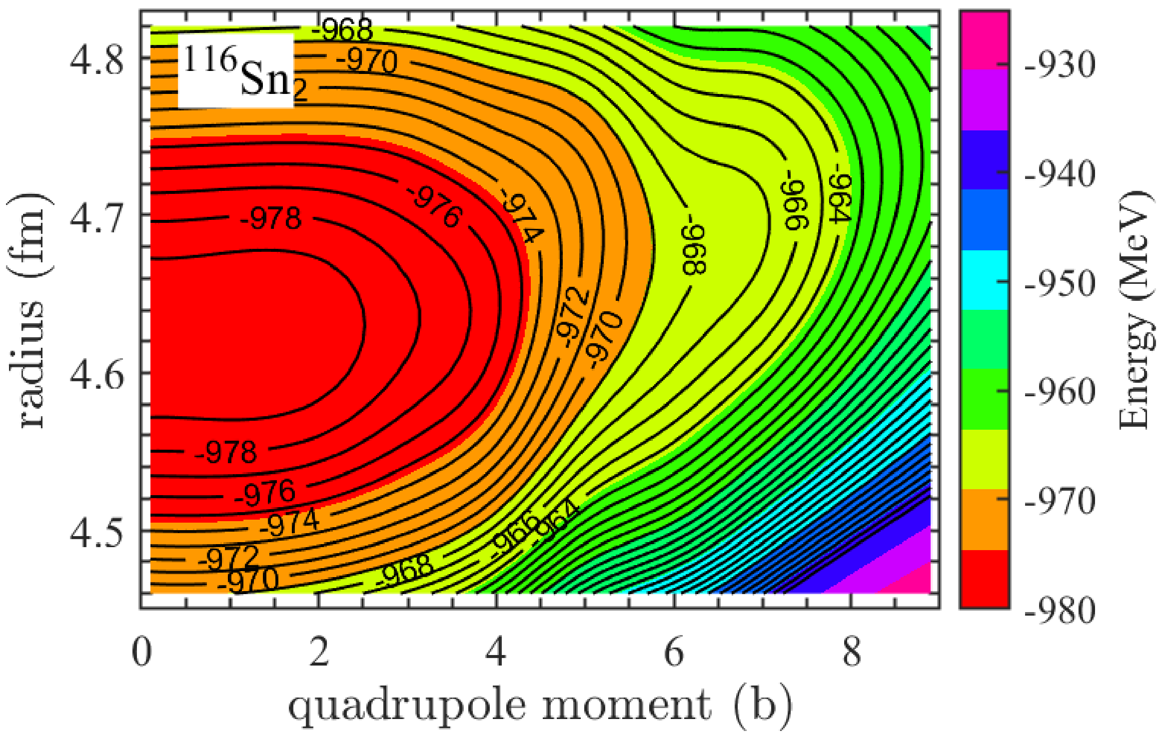
$$\delta \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0 \rightarrow \int d\boldsymbol{q}' \underbrace{\langle \Phi(\boldsymbol{q}) | \hat{H} | \Phi(\boldsymbol{q}') \rangle}_{\mathcal{H}} f_k(\boldsymbol{q}') = E_k \int d\boldsymbol{q}' \underbrace{\langle \Phi(\boldsymbol{q}) | \Phi(\boldsymbol{q}') \rangle}_{\mathcal{N}} f_k(\boldsymbol{q}')$$

- Transition amplitude (with symmetry restoration)

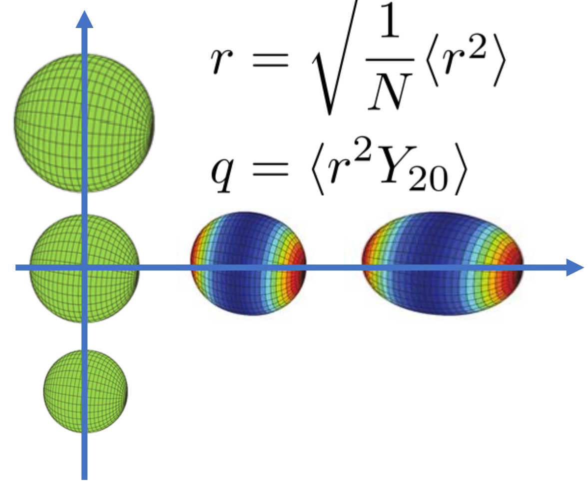
$$\begin{aligned} B(\hat{T}_\lambda; I_{k_i} \rightarrow I_{k_f}) &= \frac{1}{2I_i + 1} |\langle \Psi; I_{k_f} || \hat{T}_\lambda || \Psi; I_{k_i} \rangle|^2 \\ &= \frac{1}{2I_i + 1} \left| \sum_{q',q} f_{k_f}^{I_f *}(q') \langle I_f q' || \hat{T}_\lambda || I_i q \rangle f_{k_i}^{I_i}(q) \right|^2 \end{aligned}$$

MRDFT potential energy surface of ^{116}Sn

- Constrained Hartree-Fock energy calculated with SkM* functional
- Numerical details: 16 HO shells



- 2D collective space



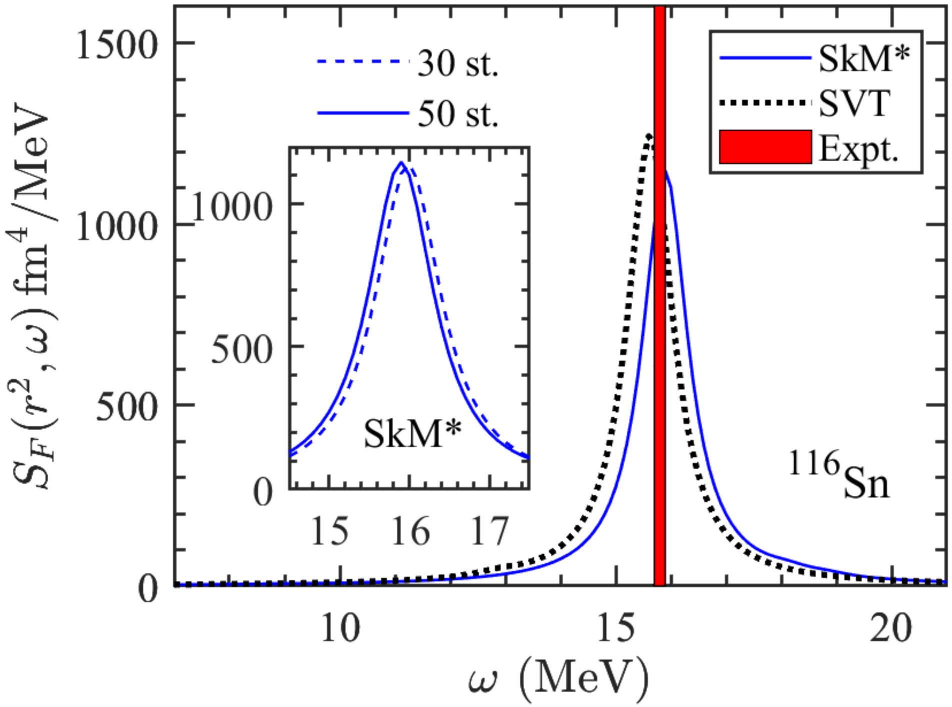
$$r = \sqrt{\frac{1}{N}\langle r^2 \rangle}$$

$$q = \langle r^2 Y_{20} \rangle$$

- Spherical shape is predicted by the HF energy.

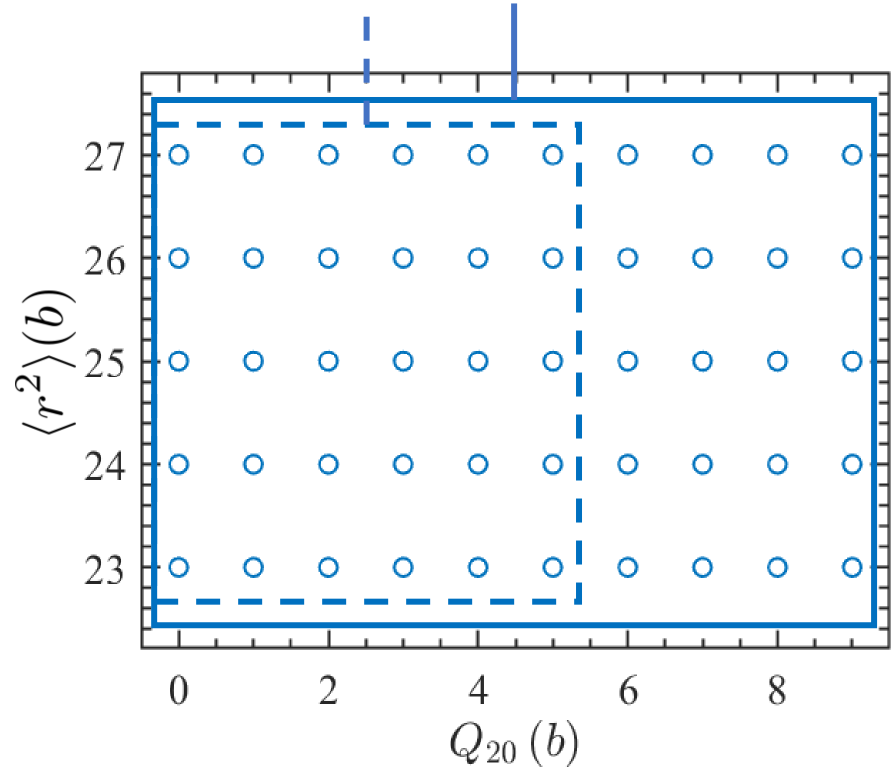
Breathing mode of ^{116}Sn calculated with MRDFT

ISGMR strength function
calculated with SkM* / SVT functional



MRDFT configuration space

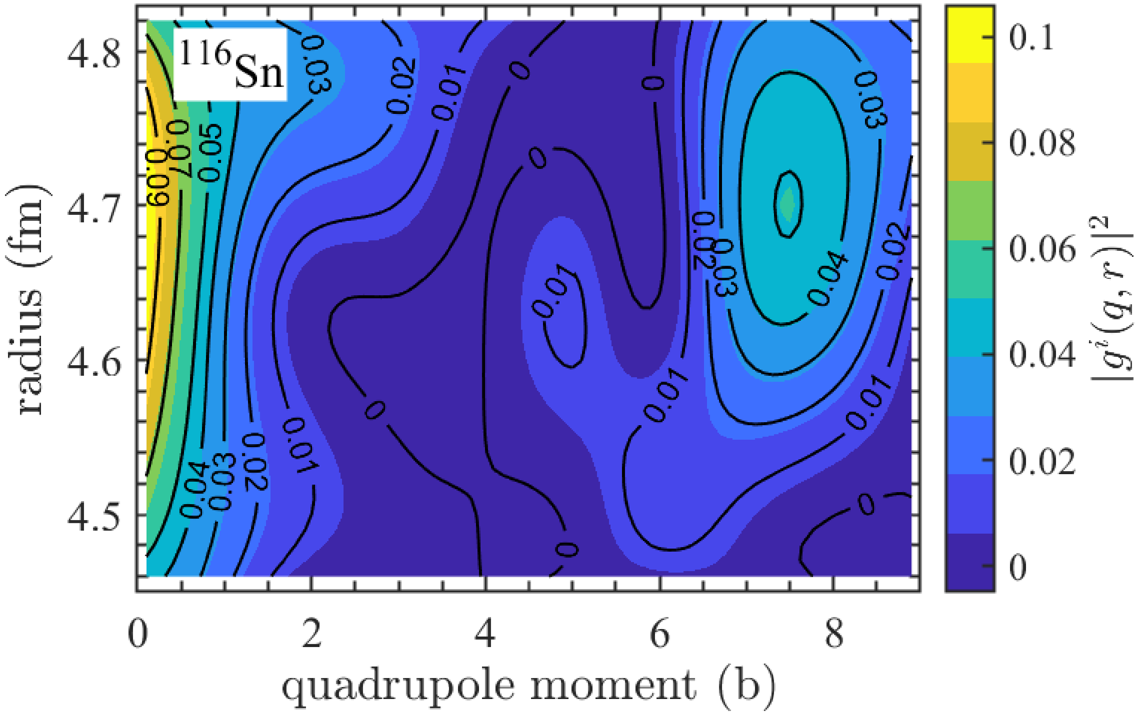
30 states / 50 states



- Energy of the ISGMR calculated with MRDFT agrees with the experiment.
- The admixture of prolate shapes **slightly** soften the breathing mode.

Spherical and prolate shape mixing

- Collective wavefunction of the breathing mode



$$\mathcal{G} \equiv \mathcal{N}^{1/2}\mathcal{F}$$

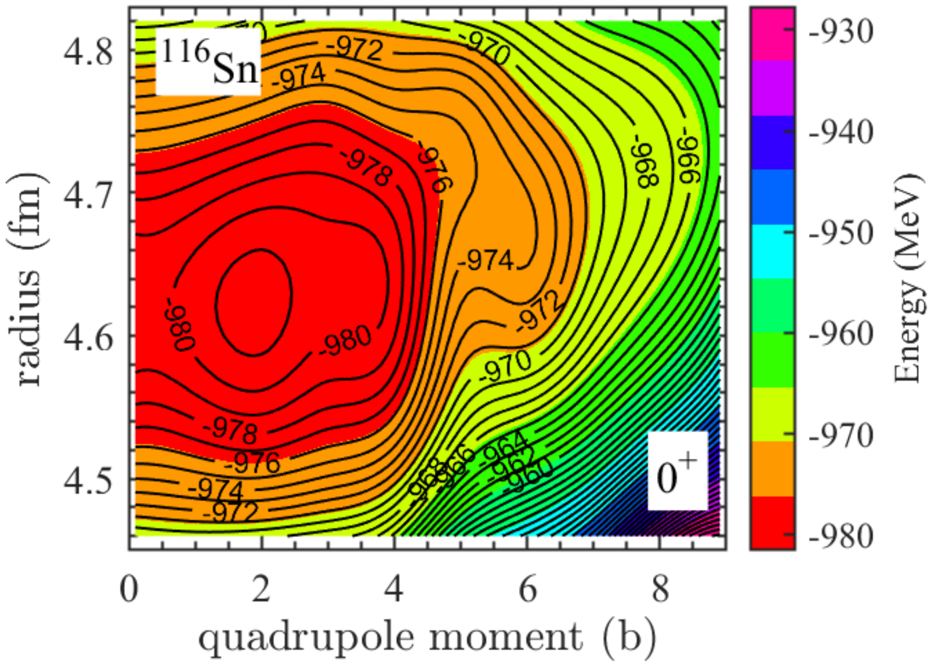
- Collective wavefunction concentrates at $q \sim 0$ b;
- A secondary bump appears at $q \sim 7$ b;

- The breathing mode is dominated by the vibrations of near-spherical shapes.
- The *possible* shape isomer also makes some contribution.

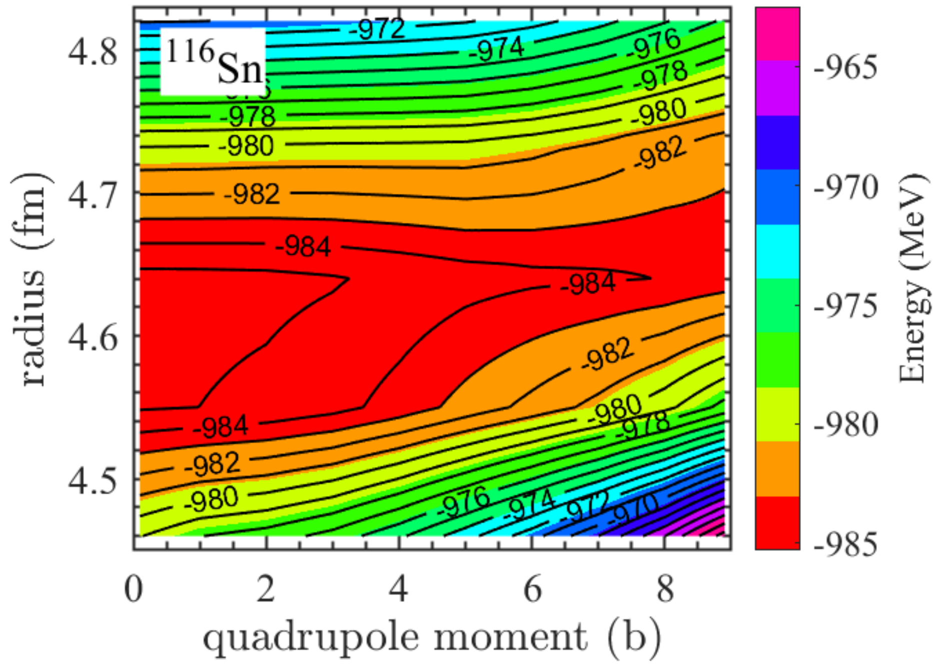
Prospective

- The prolate shape isomer is more significant when symmetry-restoration or pairing interaction are considered.

+ Angular momentum projection



+ Pairing interaction



▪ Will the shape mixing effect be enhanced?

Summary

- MRDFT provides a self-consistent approach for analyzing large-amplitude motion and small-amplitude vibration.
- The energy of the breathing mode is in good agreement with the experiment.
- The shape mixing effects require further investigations, *e.g.*, symmetry restoration, pairing, different functionals…

Thank you for your attention！

ISGMR built on ground state and on shape isomer

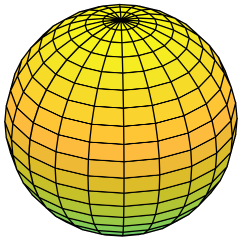
- ✓ Shape isomers usually have lifetimes longer than the typical durations of giant resonances;

  &. shape isomer of ^{116}Sn: 160 ps

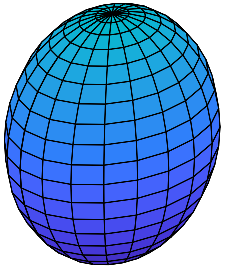
  J. Blachot, Nuclear Data Sheets 111, 717(2010)

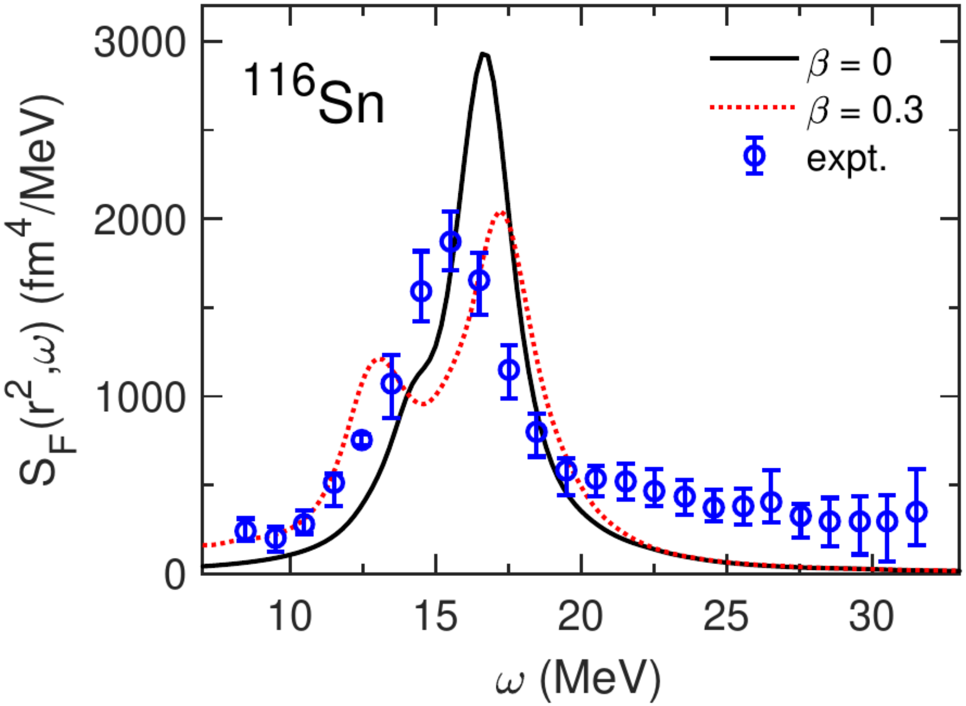
- ✓ When shape isomers are produced, giant resonances could be built on them.

ground state



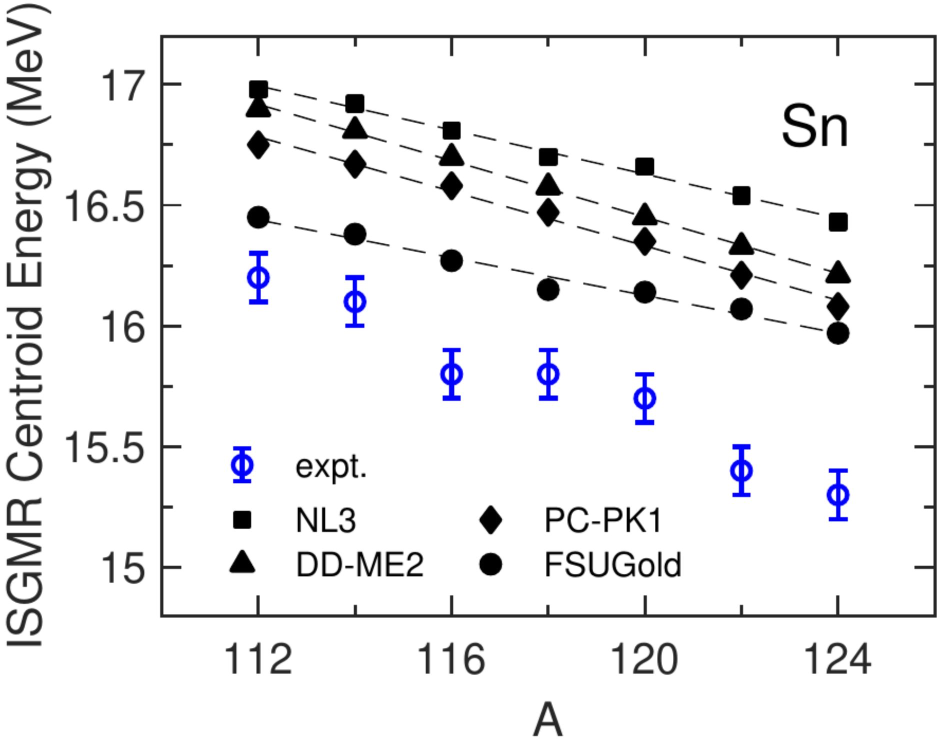
shape isomer





Softness of tin isotopes

- ✓ A long-standing open question: tin isotopes are soft;
- ✓ Theories that predict ISGMR energies for $^{208}\mathrm{Pb}$ and $^{90}\mathrm{Zr}$ properly **always over-estimate** the ISGMR energies for tin isotopes.



expt. T. Li et al., PRL 99, 162503

Is there any nuclear structure effect should be taken into account?