Laura Stephenson
Dan Watts

Quantum Entanglement of Positron Annihilation Gammas

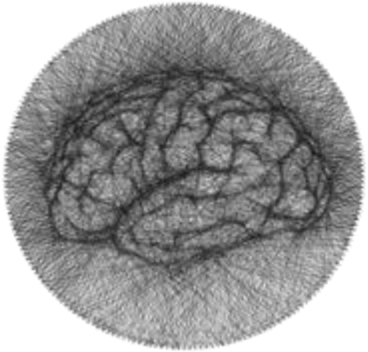
Overview

- Positron Emission Tomography (PET) Imaging
- Quantum Entangled (QE) γ
- Witnessing Entanglement with Compton Scattering (CS)
- 3 γ PET

PET Imaging

Principals

- Leading modality of Cancer and Alzheimer's diagnosis - **Functional imaging**
- Patient is injected with Positron (e^+) emitting biologically labelled radioactive isotope
- Positron thermolises and rapidly annihilates with electron in patient $e^+ + e^- \rightarrow 2\,\gamma$
- Photons escape patient to detector

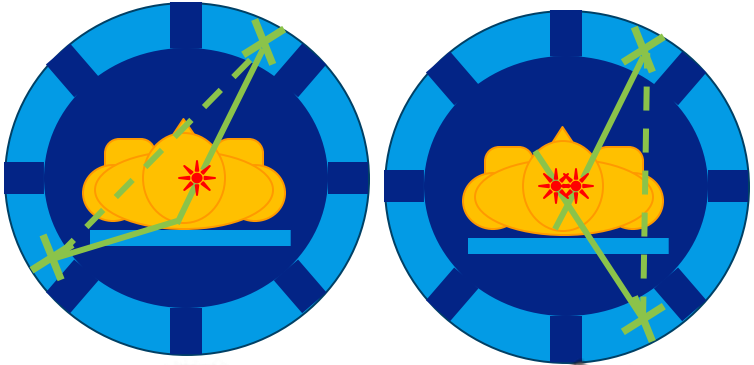


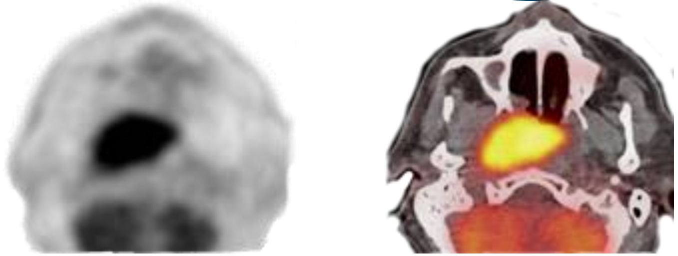
https://simplevibesstudio.com/products/anatomical-brain-algorithmic-string-art-diy

PET Imaging

Challenges

- Scatter and random coincidences introduce errors ≈80% of annihilations discarded!
- Attenuation coefficients used to deconvolve scatter - requires CT (<100 keV→511 keV extrapolation), movement artifacts, CPU time…
- PET scan gives no anatomical information - relies on combined CT
- PET scan no sensitivity to annihilation environment (eg O2, pH, tissue type)





Varoquaux et al., ‘Functional Imaging of Head and Neck Squamous Cell Carcinoma with Diffusion-Weighted MRI and FDG PET/CT’.

Quantum Entangled γ Photons

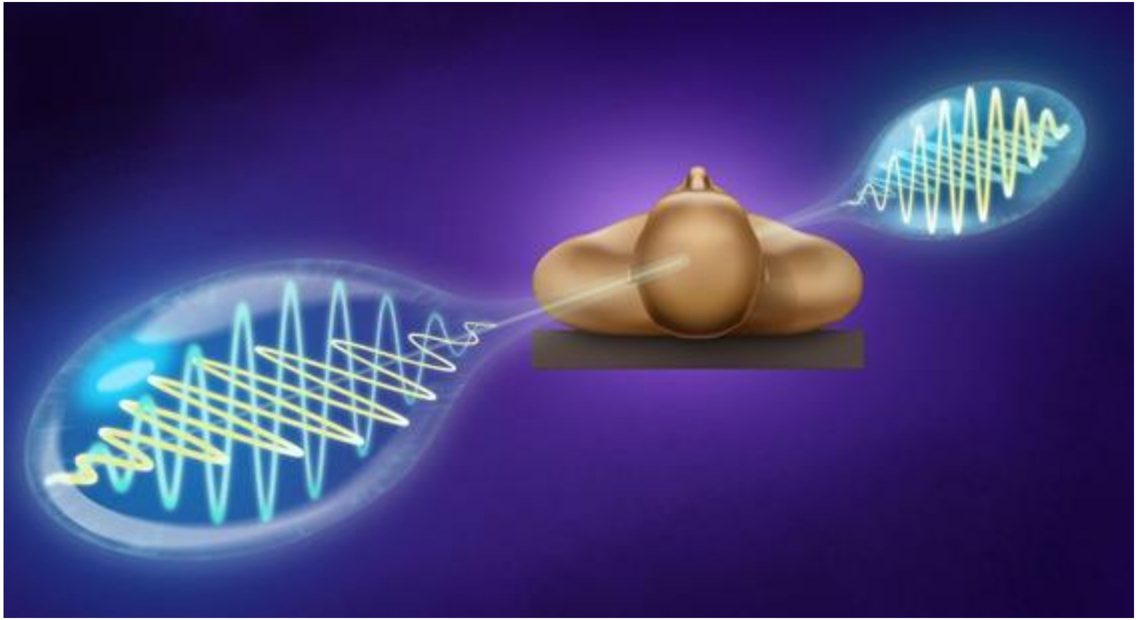
QE in PET Imaging

Photons are back-to-back and orthogonally polarised

Entangled direction (+ , -) and polarisation (x , y)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_{+}\,|y\rangle_{-} + |y\rangle_{-}\,|x\rangle_{+}\right)$$

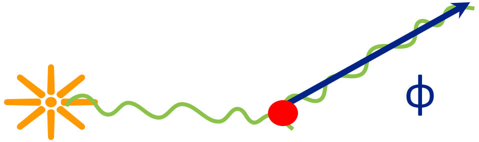
Any effect on one photon is immediately felt on the other



Witnessing Entanglement with CS

Polarisation dependant CS

Compton scattering (CS) γ + e -> $\gamma^{/}$ + e$^{/}$

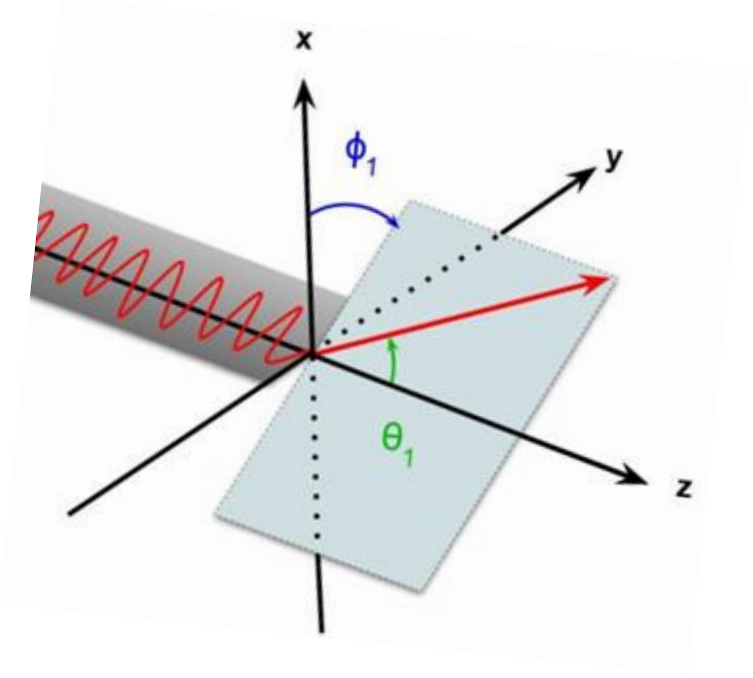


ϕ is the plane of scattering which relates to polarization

$$\boxed{I = \frac{e^4}{r^2 m^2 c^4} I_0 \frac{\sin^2\phi}{[1+\alpha(1-\cos\theta)]^3}}$$

CS described by Klein Nishina is proportional to sin^{2}ϕ
Therefore CS depends on γ polarization→ entangled!

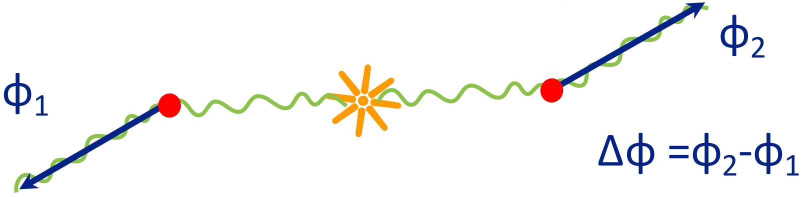
YAZAKI, 'How the Klein–Nishina Formula Was Derived'.



Witnessing Entanglement with CS

Double CS

Double CS occurs when both photons scatter

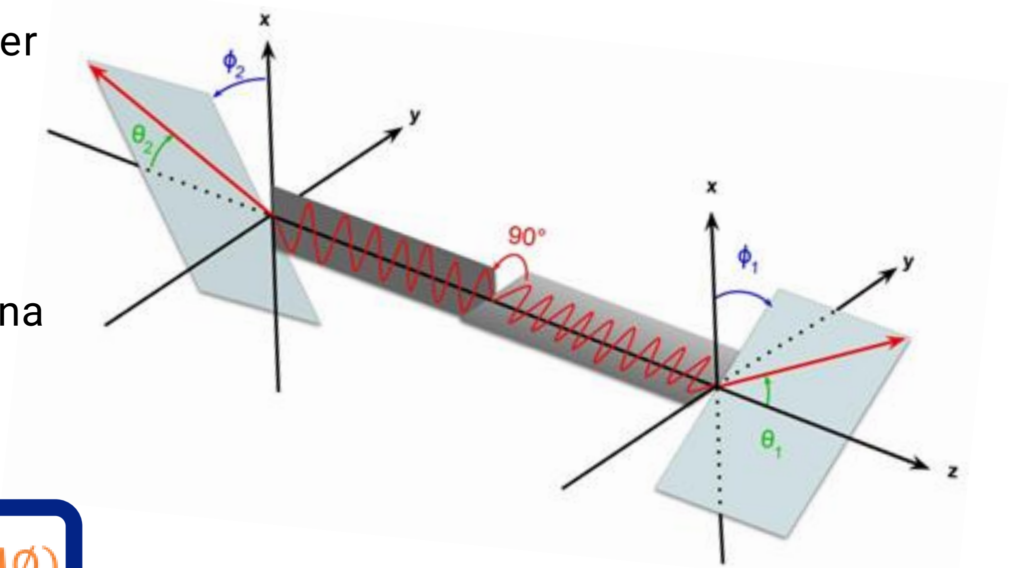


Incorporating wavefunction into Klein Nishina

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_{+}\,|y\rangle_{-} + |y\rangle_{-}\,|x\rangle_{+}\right)$$

$$\downarrow$$

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{r_0^{\,4}}{16}\left(K_a(\theta_1\,\theta_2) - K_b(\theta_1\,\theta_2)\cos(2\Delta\phi)\right)$$

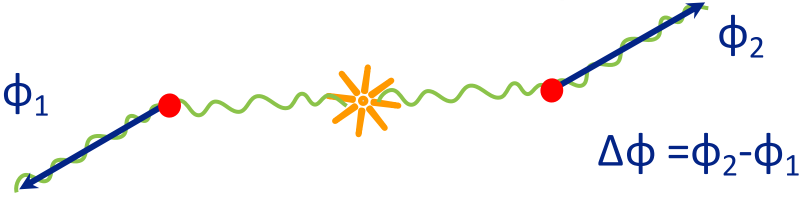


Watts et al., ‘Photon Quantum Entanglement in the MeV Regime and Its Application in PET Imaging’.

Witnessing Entanglement with CS

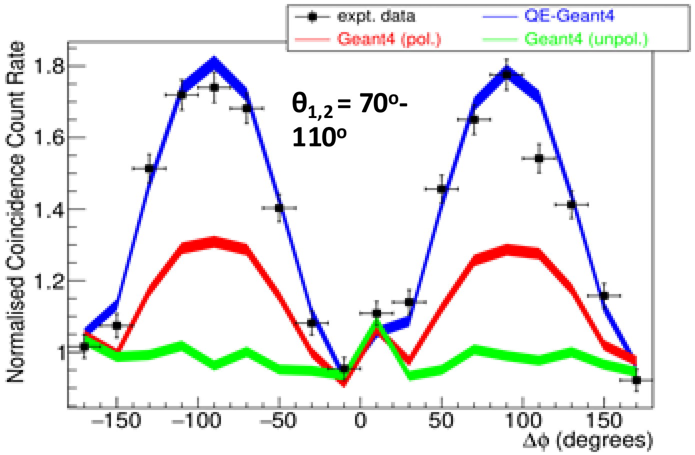
cos(2Δϕ) Witness

Double CS occurs when both photons scatter



Entanglement is observable from the ***magnitude*** of **cos(2Δϕ)** modulation

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{r_0^{\ 4}}{16}\left(K_a(\theta_1\ \theta_2)\ - K_b(\theta_1\ \theta_2)\cos(2\Delta\phi)\right)$$



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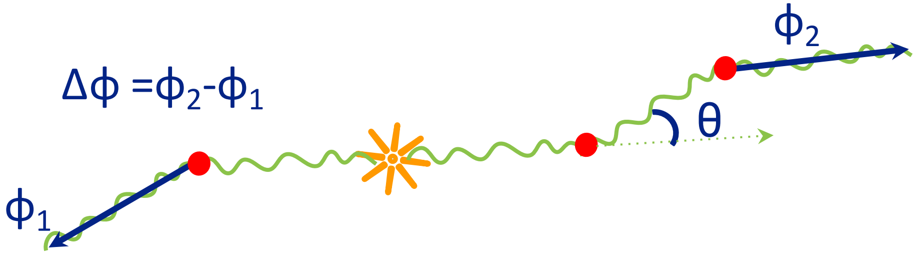
Witnessing Entanglement with CS

Triple Compton Scattering

Photons often scatters in the patient before reaching the detector



Witnessing Entanglement with CS

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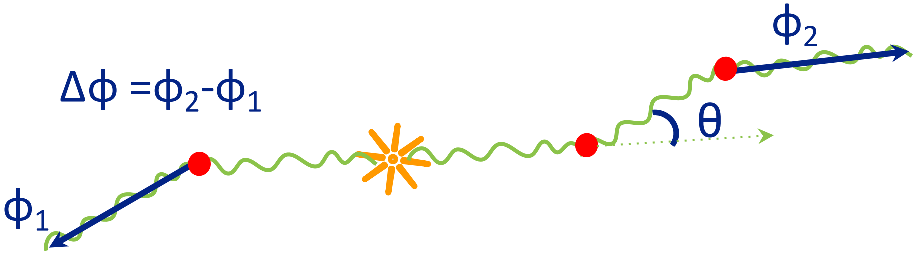
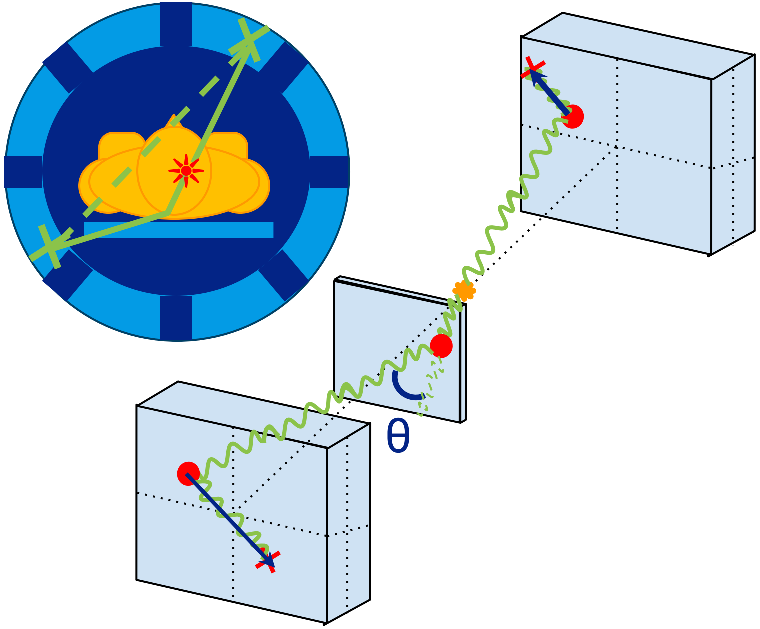
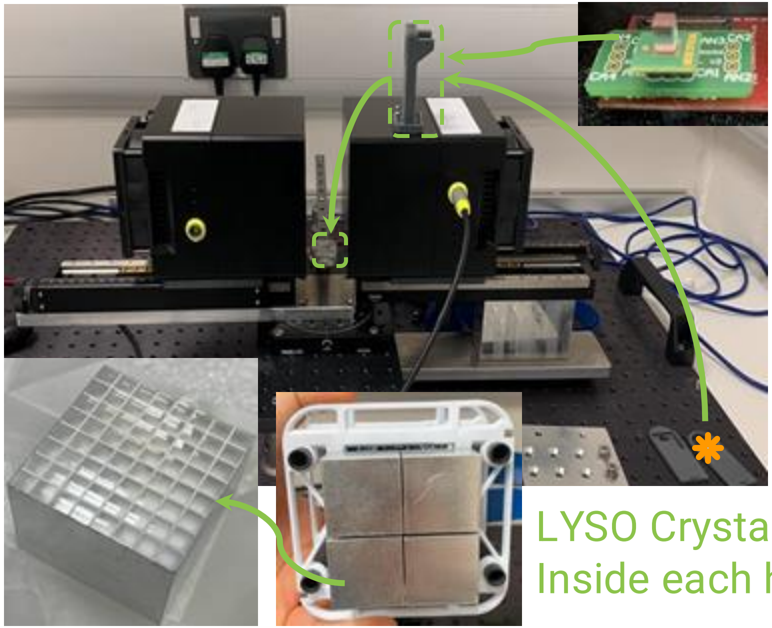


Experimentally determined if entanglement is preserved in triple CS using segmented LYSO Calorimeters



Witnessing Entanglement with CS

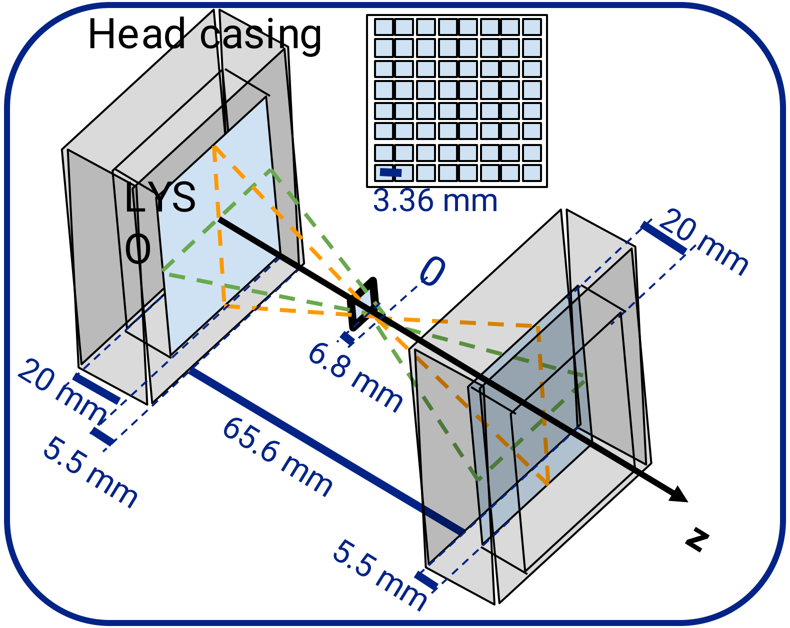
Experimental Setup



Scatterer LYSO crystal

Source location

LYSO Crystals Inside each head



Witnessing Entanglement with CS

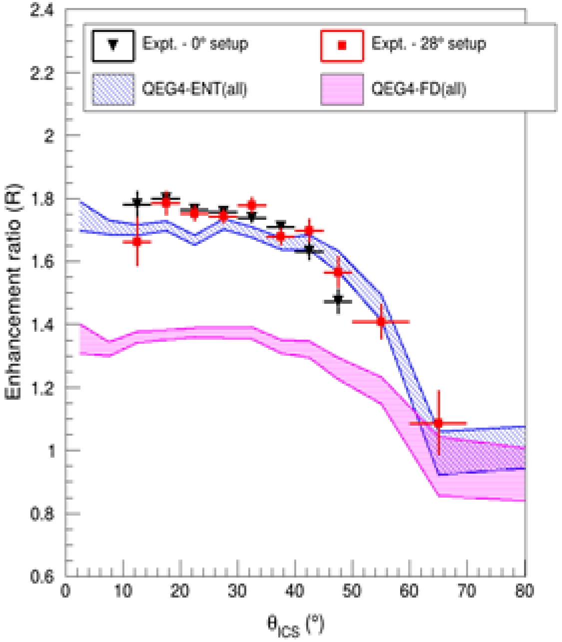
Small Angle results

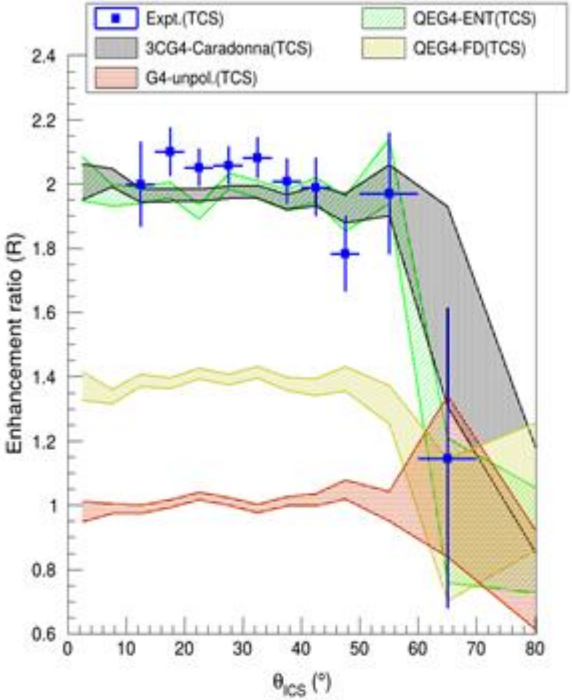
Events **with** (left) and **without** (right) multiple scattering

Well above classical limit (pink:left & yellow:right)

Demonstrated that Photonic QE at MeV scale is rather robust!

Watts et al., 'Photon Quantum Entanglement in the MeV Regime and Its Application in PET Imaging'.





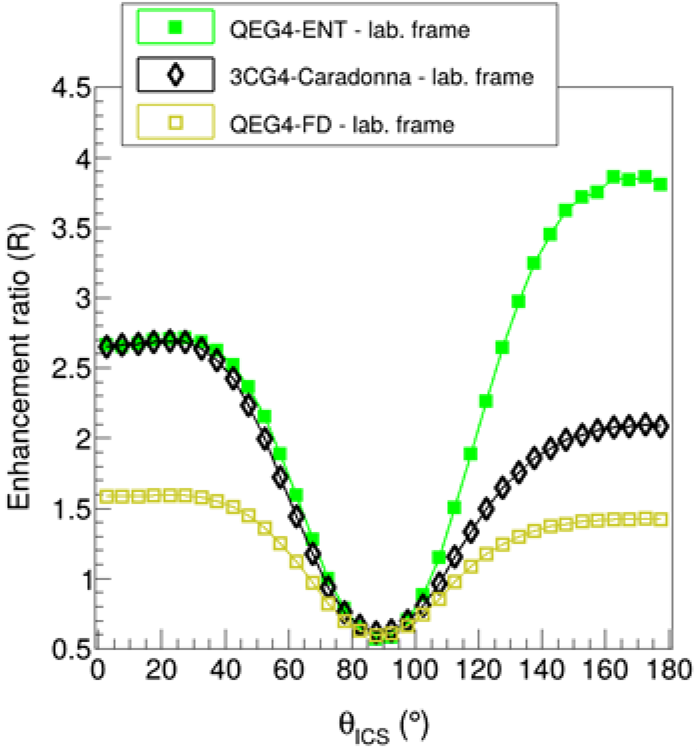
Witnessing Entanglement with CS

Expanding Theory

Predict rise in entanglement ratio for larger scattering angles- due to non perfect detector

Large angle scattering breaks the degeneracy between the two theories.

Investigating Backscattering will inform us if entanglement is **fully** maintained in TCS or not



Witnessing Entanglement with CS

Backscattering

No need for new experiment!

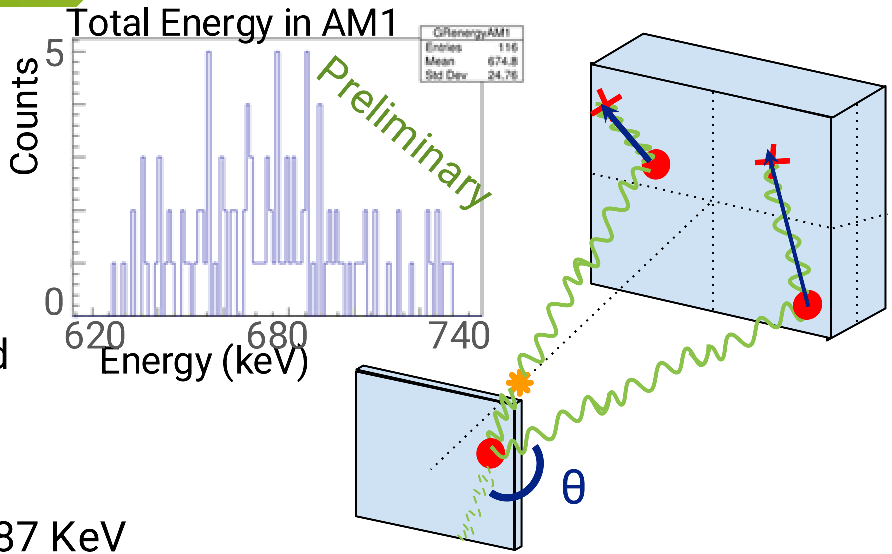
Events with 4 hits (ϕ_1& ϕ_2) in the same head

1st photon deposit ≈511 KeV in head

2nd photon deposits ≈335 KeV in Scatterer and

≈176 KeV in head

Observe a Gaussian peak in head energy of ≈687 KeV



Three γ PET

UNIVERSITY of York

Theory

Prior to annihilation the Positron can form Positronium (Ps)

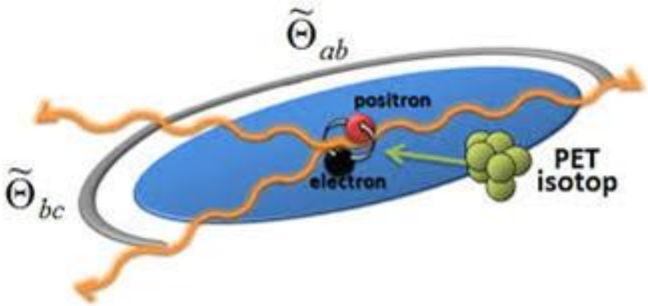
→ singlet (para-Ps) or triplet(ortho-Ps) spin states

Ground state Decay of para-Ps $e^+ + e^- \rightarrow 2\gamma$

Ground state Decay of ortho-Ps $e^+ + e^- \rightarrow 3\gamma$

The decay rates are sensitive to the medium such as O2 concentration, electron density, pH….

→Ratio 2:3 γ photons could infer annihilation medium!

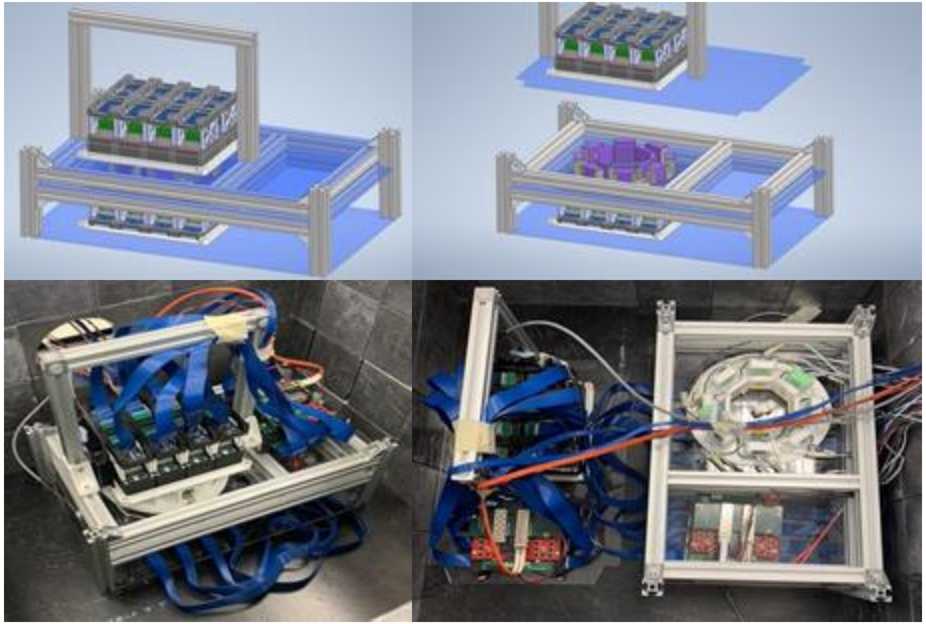


Hiesmayr and Moskal, ‘Genuine Multipartite Entanglement in the 3-Photon Decay of Positronium’.

Three γ PET

Experimental setup

- York’s expanded LYSO segmented calorimeter array in z plane
- Tokyo’s GaGG ring in x-y plane acting as compton camera
- Measure example in nature of a genuinely fully entangled multiparticle system



Uenomachi, Shimazoe, and Takahashi, ‘Double Photon Coincidence Crosstalk Reduction Method for Multi-Nuclide Compton Imaging’.

Future of PET Imaging

Applying QE

- Scatter and random coincidences introduce errors ≈80% of annihilations discarded!

- Attenuation coefficients used to deconvolve scatter-

- PET scan gives no anatomical information -

- PET Scan no sensitivity to annihilation environment (eg O2, pH)

https://leedstestobjects.com/wp-content/uploads/NU4-PET-IQ-product-specifications.pdf?x54702

Future of PET Imaging

Applying QE

- Scatter and random coincidences introduce errors ≈80% of annihilations discarded!
- Recycle scattered data to build map with AI
- Filter randoms based on QE witness

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Future of PET Imaging

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- PET scan gives no anatomical information -
- Build Scatter map and anatomical information with AI

- PET Scan no sensitivity to annihilation environment (eg O2, pH)
- Use 2:3 γ ratio to identify medium

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Thank you for listening!
Any Questions?

Extra details

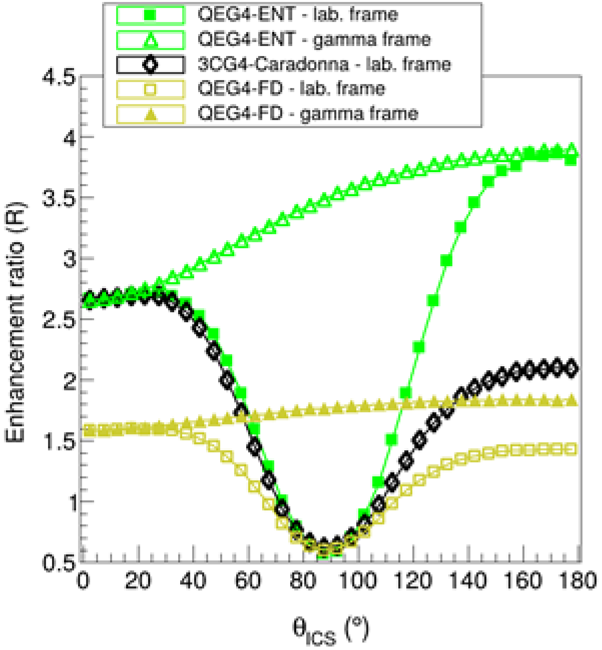
Perfect Detector

1st theory (York) – implemented in G4 (3CG4-Caradonna)
Stokes Muller matrix formalism based on quantum field theory, includes decoherence effects

Also developed a model (QEG4-Ent)
-> final CS according to “partial polarization ansatz “of Snyder et. al. (assuming no decoherence)

Decoherence (at least its effect on R) -> small

TCS brings in measurement frame effects
“Photon frame” - phi relative to γ poln. –> only accessible in simulation.



Extra details

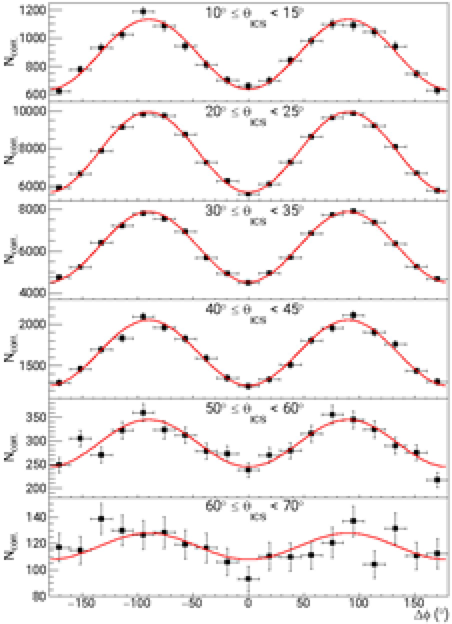
R Entanglement witness

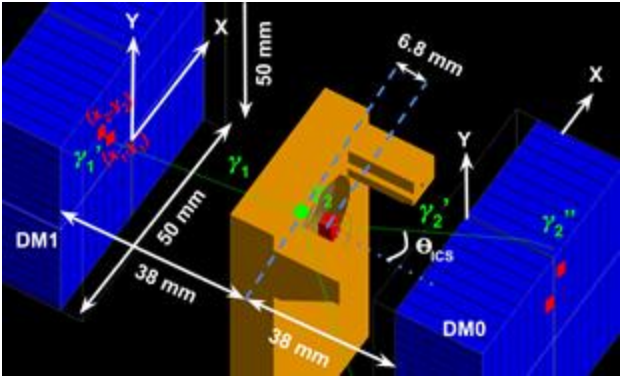
Δφ distributions for different intermediate CS angles
Event mixing to remove detector acceptance

Fit Δφ distributions with:
Acos(2Δφ) +B

“Enhancement” (R) is:
R = (B-A) / (B+A)

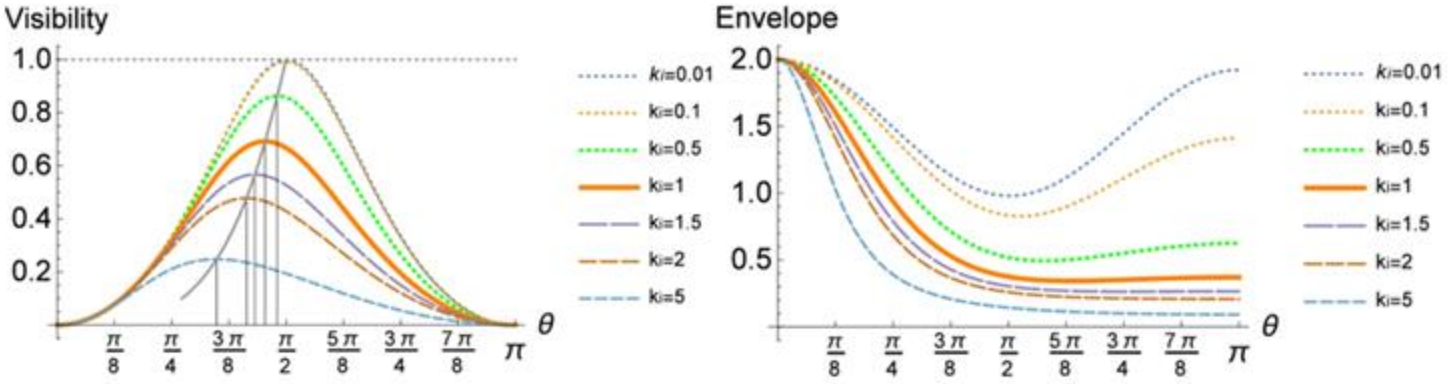
R - Quantitative measure of correlation between the final CS of the two γ





Watts et al., ‘Photon Quantum Entanglement in the MeV Regime and Its Application in PET Imaging’.

Theta effect on visibility



The plots (left graphic) show the visibility $\mathscr{V}(\bar{\Theta}, k_i)$ in dependence of Compton scattering angle $\bar{\Theta}$ for different initial energies k_i. For a $k_i=1\equiv 511\,keV$, e.g. photons emerging from a positronium decay, the maximum value of the visibility reaches its maximum of 0.69 at $\bar{\Theta}=81{,}67°$. The vertical lines mark the angles for which the visibility maximizes. The plots (right graphic) show the envelope function $\mathcal{F}(\bar{\Theta}, k_i)$ in dependence of $\bar{\Theta}$ and k_i. Thus there is a tradeoff between statistic (high value of $\mathcal{F}(\bar{\Theta}, k_i)$) and the maximum of the visibility $\mathscr{V}(\bar{\Theta}, k_i)$.

Hiesmayr and Moskal, ‘Witnessing Entanglement In Compton Scattering Processes Via Mutually Unbiased Bases’.

Extra details

Removing Randoms

FBP
PET
image

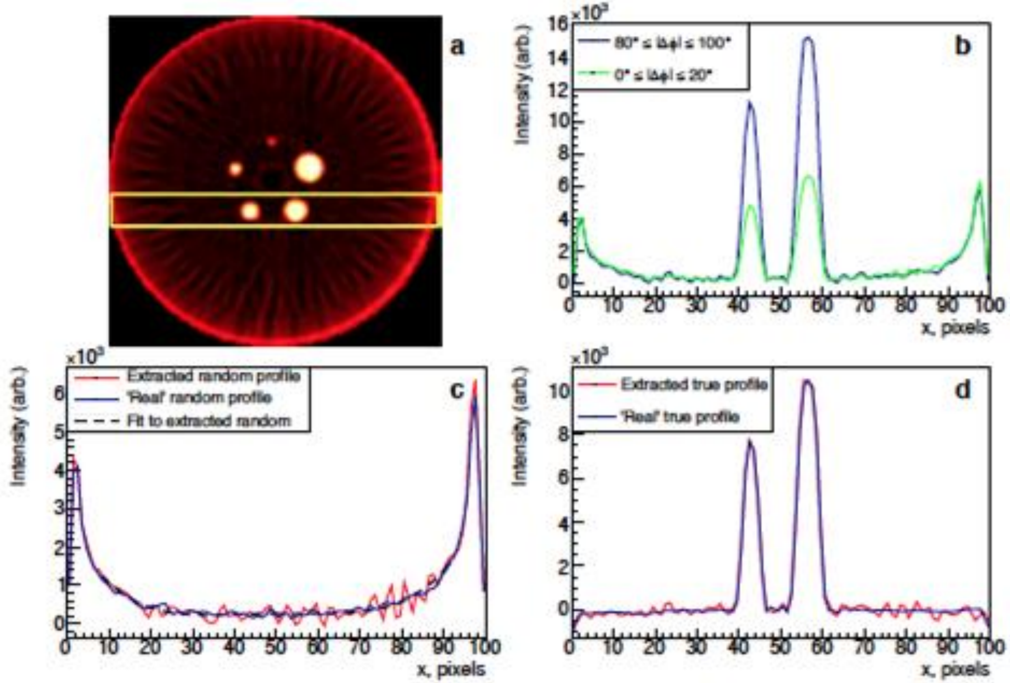


Image slice from ΔΦ around max (min) amplitude

Different weighting:
Spatially resolved determination of random profile

Subtract slices using weight derived from G4 simulation
Isolate image from true events!

Watts et al., 'Photon Quantum Entanglement in the MeV Regime and Its Application in PET Imaging'.

Extra details

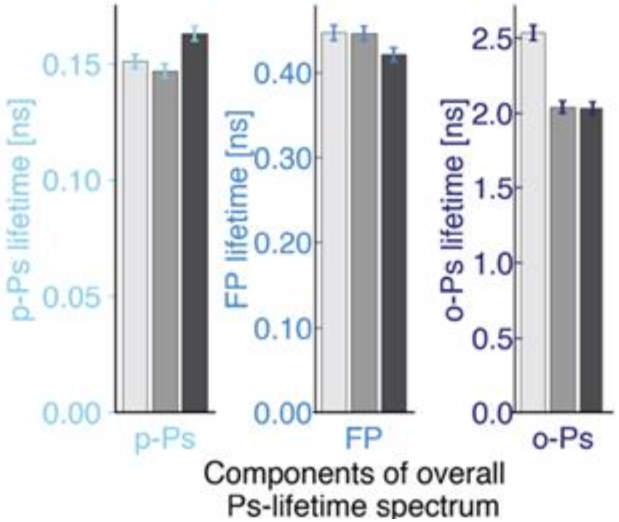
Positronium decay rates

O-Ps lifetime vacuum 142 ns
P-Ps lifetime vacuum 125 ps

light grey is adipose tissue, medium grey is hepatic tissue and dark grey is muscle

70x more events via pick off (2γ) than self annihilation (3γ) in intermolecular voids

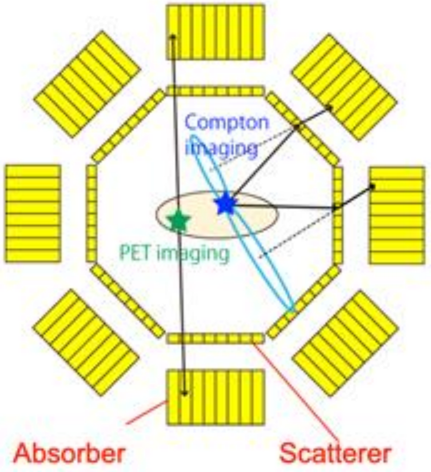
Typical PET 330 MBq F18 half life 109 min

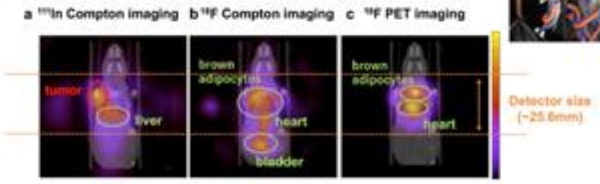


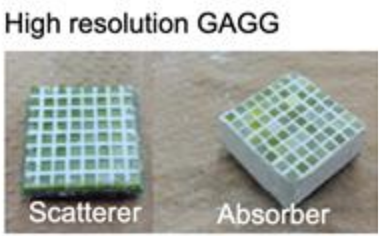
Tao, ‘Positronium Annihilation in Molecular Substances’.
Avachat et al., ‘Ortho-Positronium Lifetime for Soft-Tissue Classification’.
Moskal et al., ‘Positronium Imaging with the Novel Multiphoton PET Scanner’.

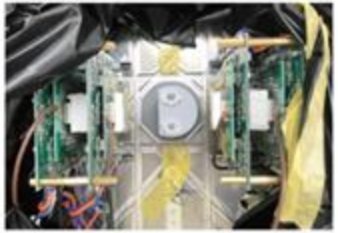
Extra details

GaGG Ring details









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Extra details

References

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