Impact of the meson-exchange currents on magnetic dipole moments in odd near doubly magic nuclei

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Collaborators: R. Han, B. C. Backes, G. Danneaux, J. Dobaczewski, W. Jiang, and M. Kortelainen

Nuclear Physics Conference 2025
April 24, 2025

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1

Nuclear density functional theory

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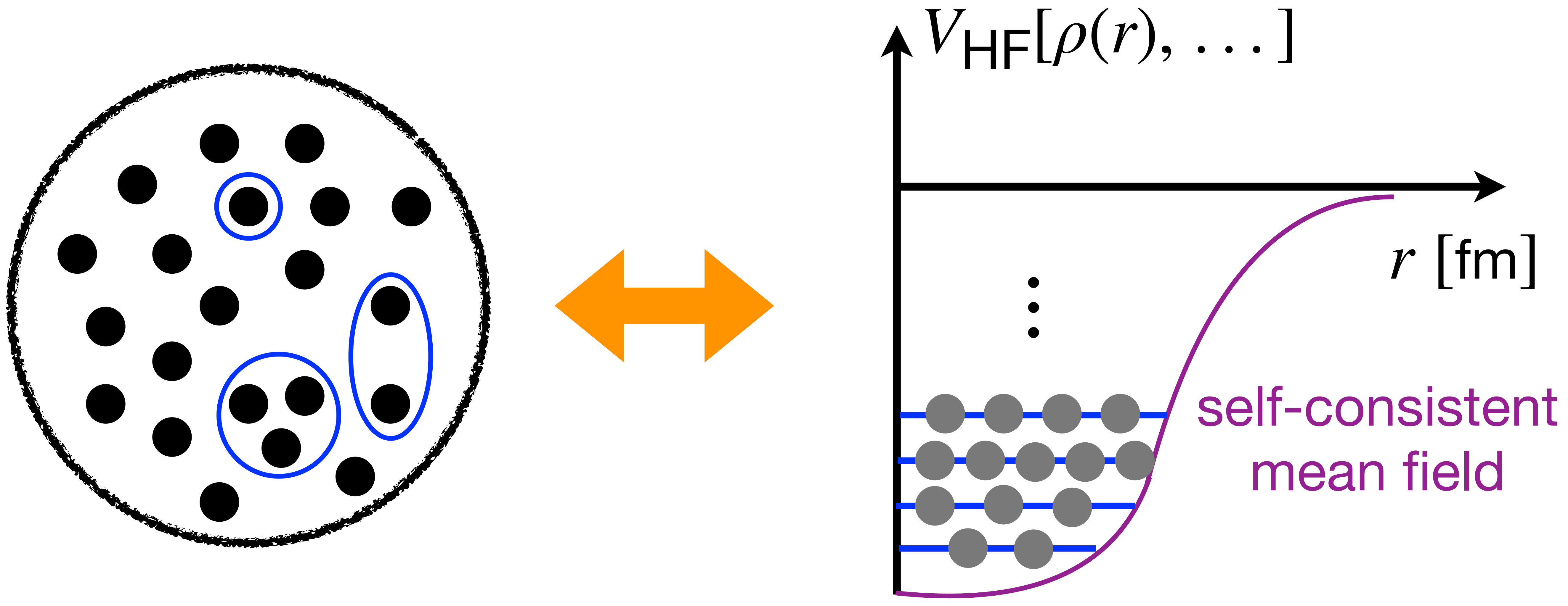

1

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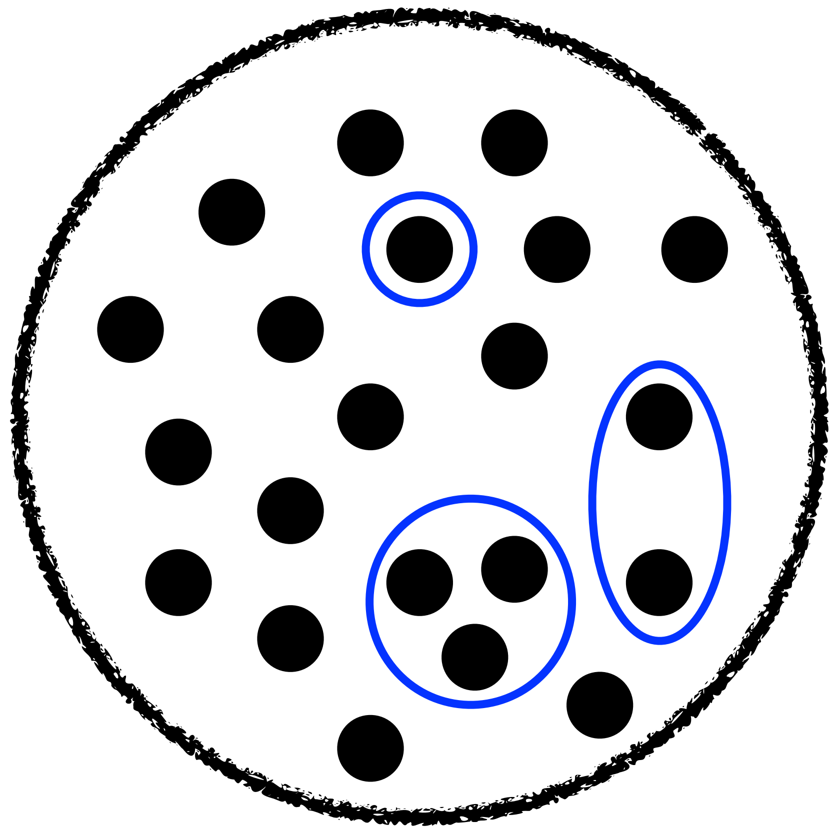
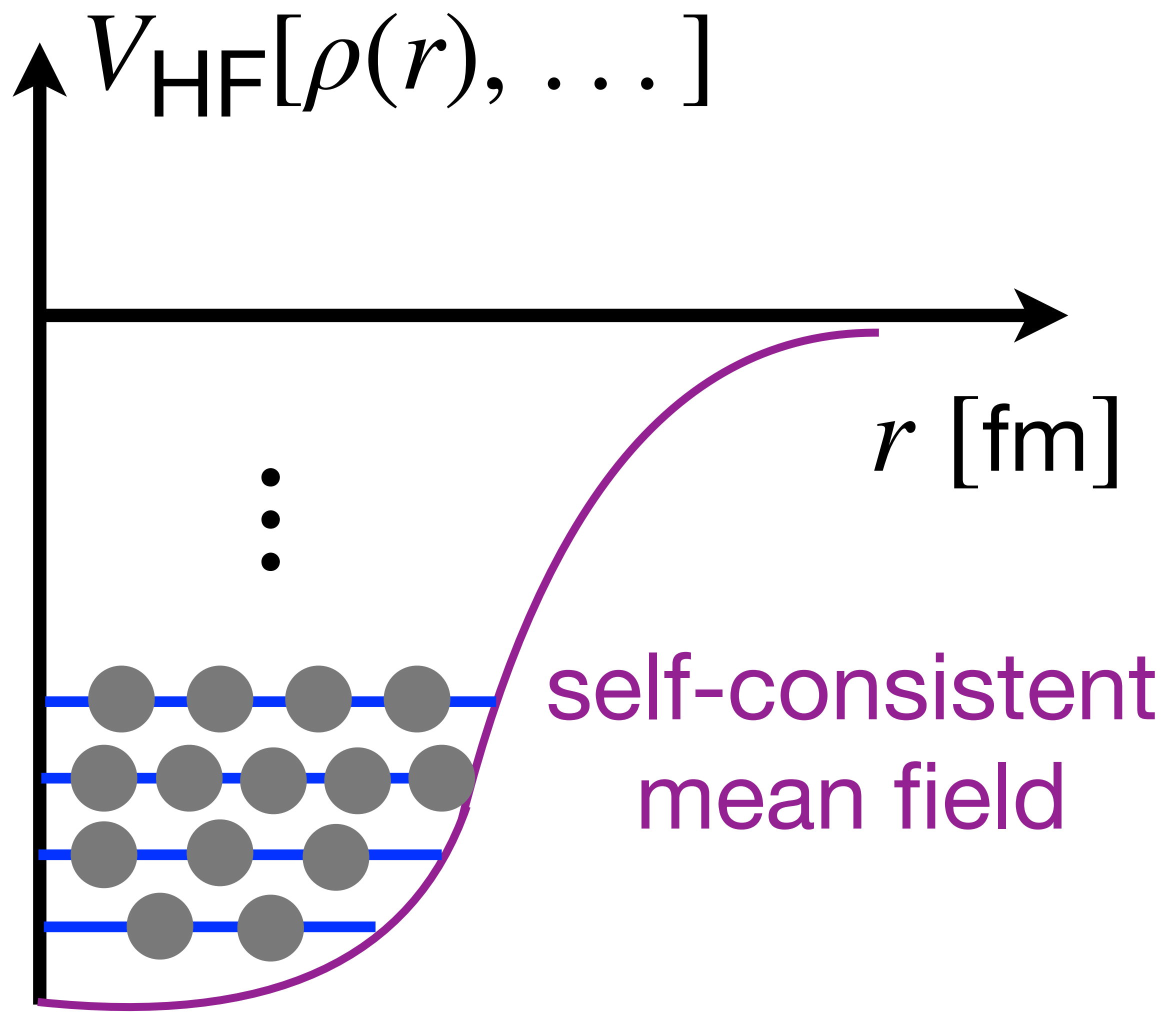




Trial s.p. wave-function $\{ |\varphi_i^{\mathrm{trial}}\rangle \}$

↓

Calculate the densities $\{\rho, \ldots\}$

↓

Calculate the mean field $V_{\mathrm{HF}}[\rho, \ldots]$

↓

Solve the HF equation

$$\left[\frac{p^2}{2m} + V_{\mathrm{HF}}[\rho, \ldots]\right] |\varphi_i\rangle = \varepsilon_i |\varphi_i\rangle$$

↓

New s.p. wave-function $\{ |\varphi_i^{\mathrm{new}}\rangle \}$ (→ back to "Calculate the densities")

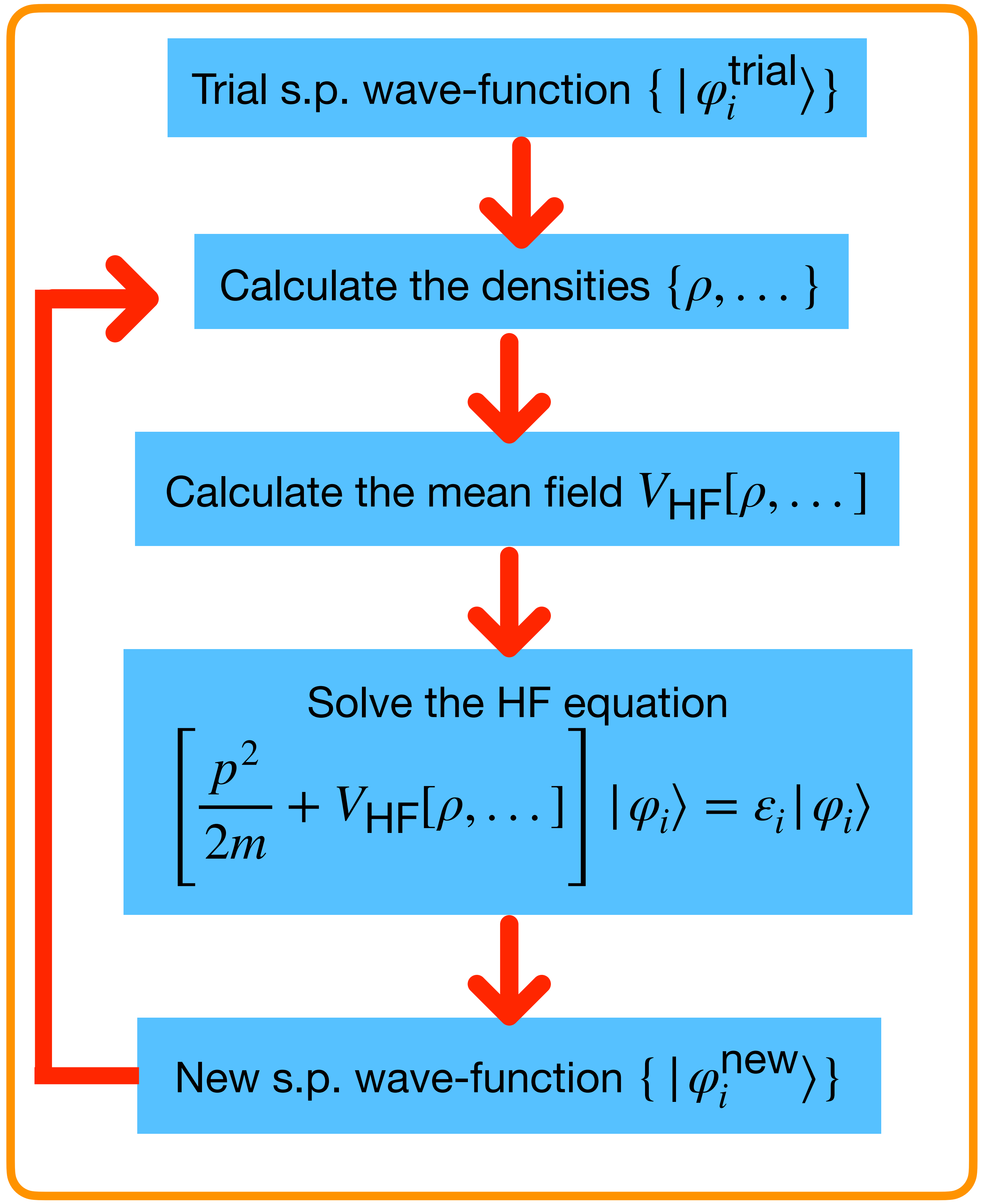
2

Nuclear DFT methodology

2

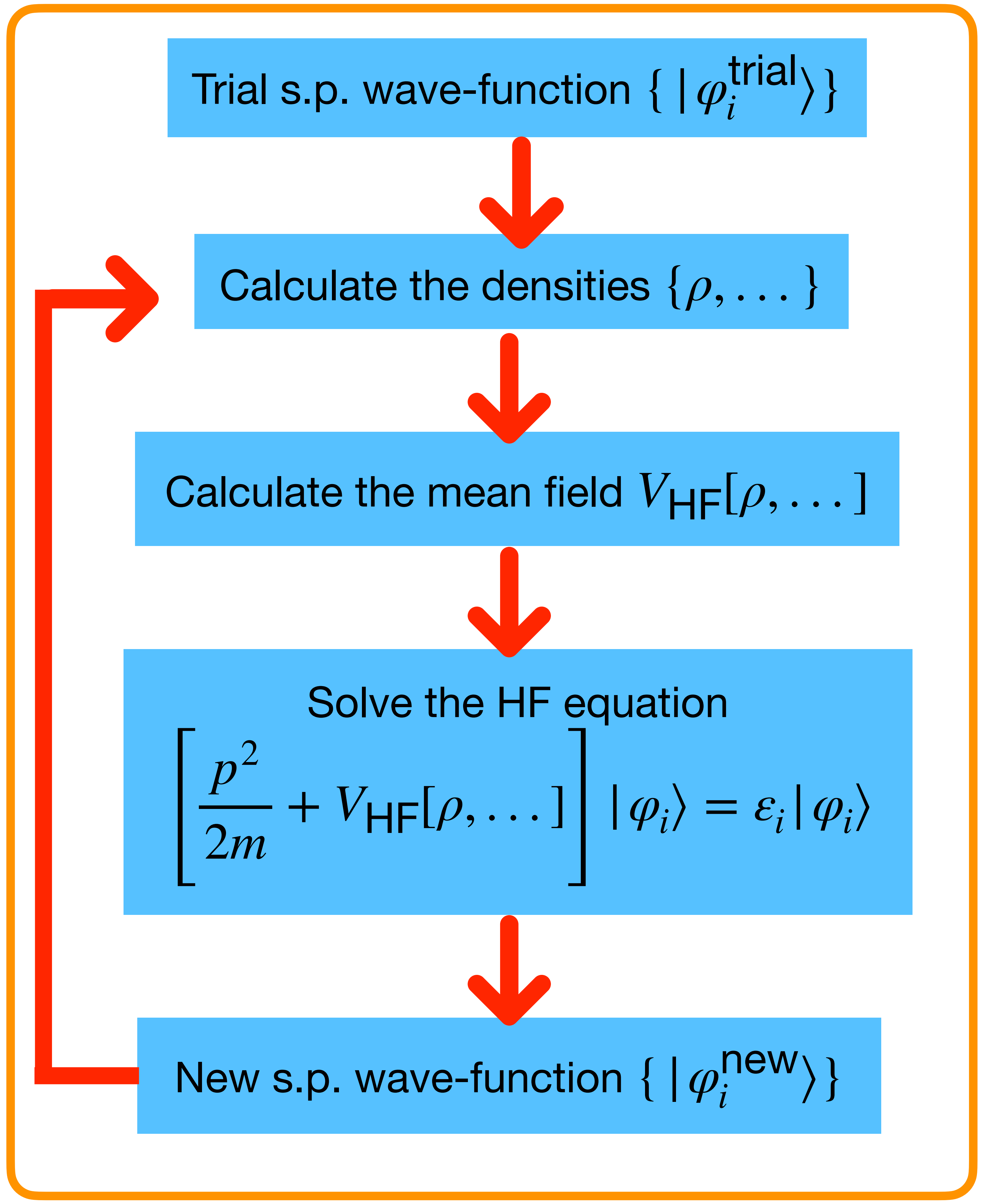
Nuclear DFT methodology

Self-consistency

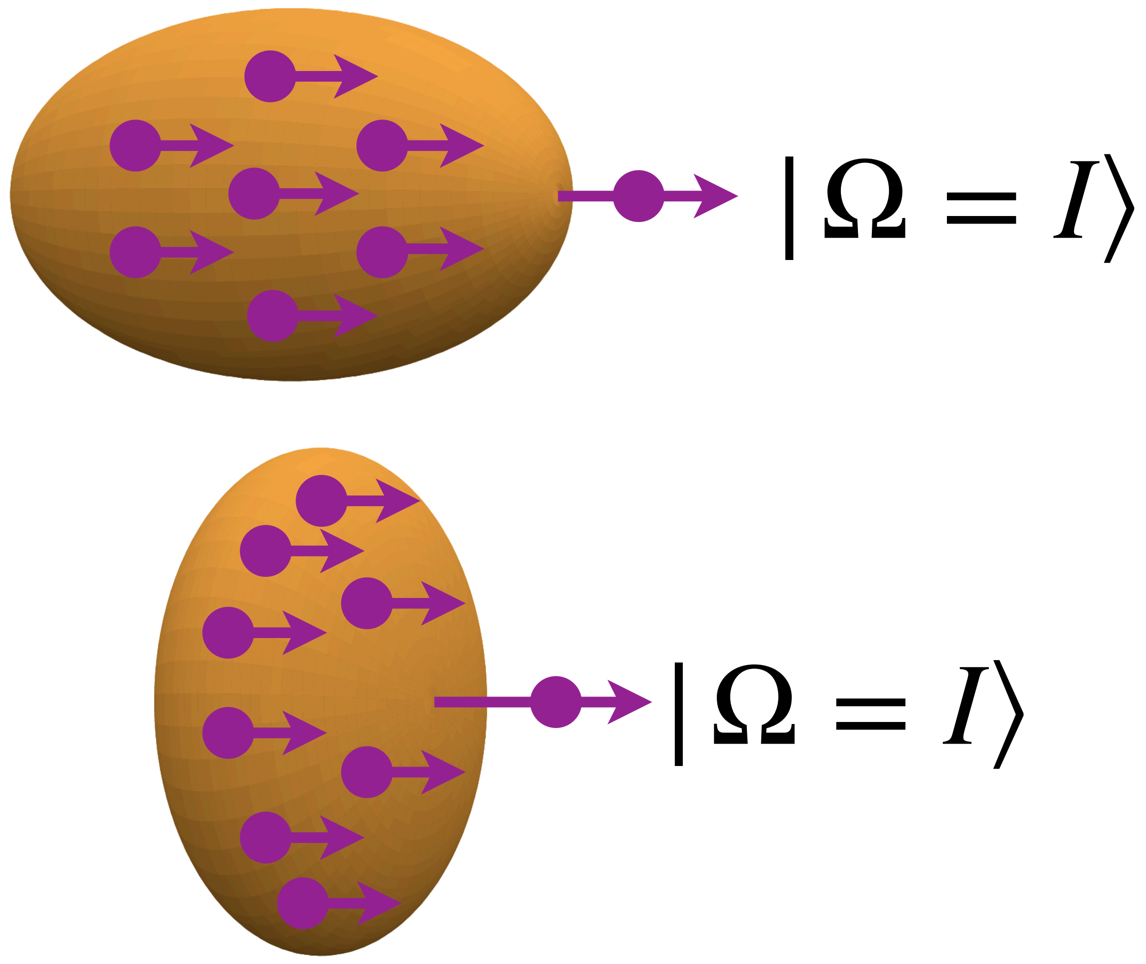


Nuclear DFT methodology

Self-consistency



Spin-core polarization



Landau parameter g_0'

$$g_0' = N_0 \left(2C_1^s + 2C_1^T (3\pi^2\rho_0/2)^{2/3}\right)$$

$$\frac{1}{N_0} \approx 150 \frac{m}{m^*} \text{ MeV} \cdot \text{fm}^3$$

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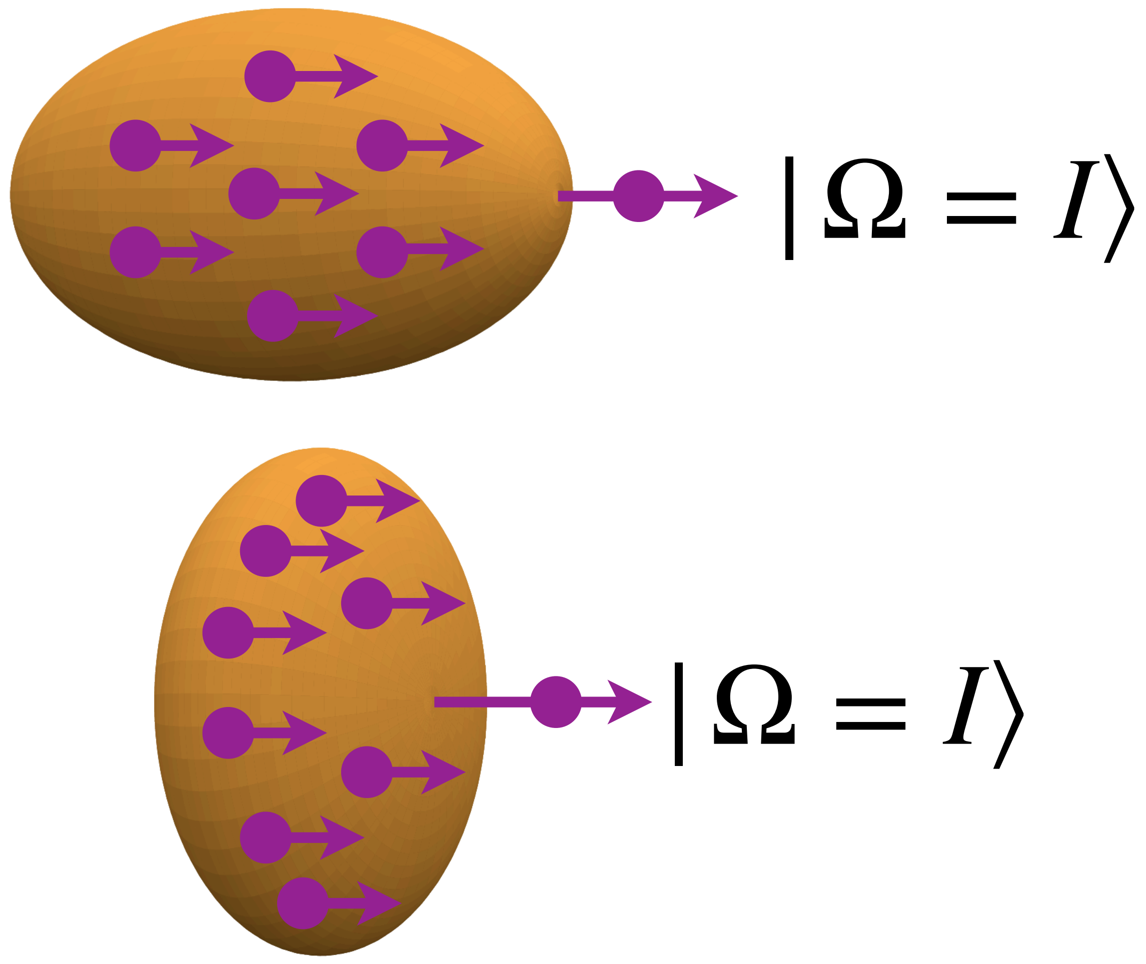

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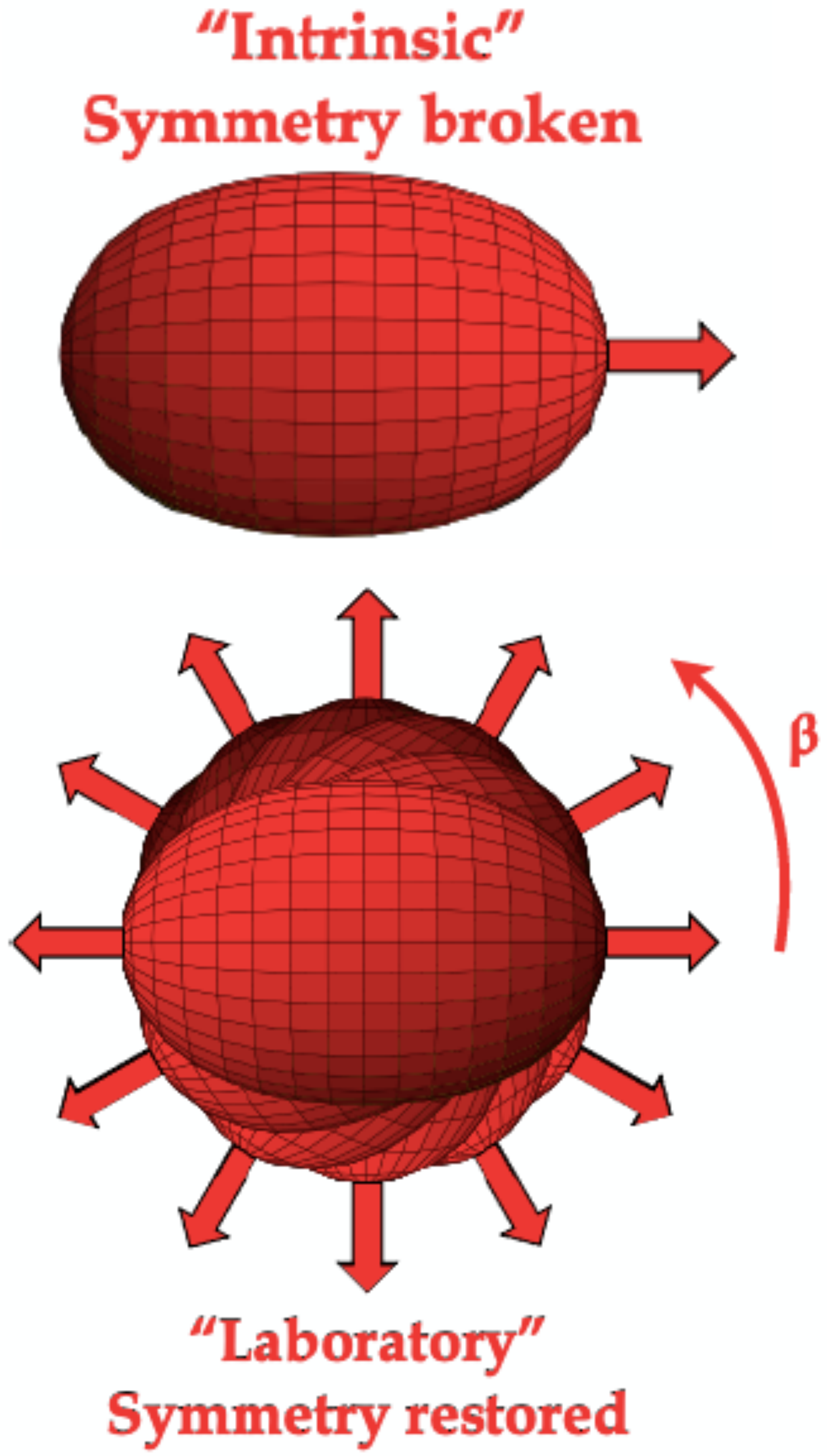
Symmetry restoration



J. A. Sheikh *et al.*, J. Phys. G: Nucl. Part. Phys. **48**, 123001 (2021)

3

Nuclear magnetic dipole moments

3

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Spectroscopic magnetic dipole μ moment is defined as:

$$\mu = \sqrt{\frac{4\pi}{3}} \langle II | \hat{M}_{10} | II \rangle,$$

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*

$|II\rangle$ = angular momentum projected (AMP) states

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$$\hat{M}_{10} = \sqrt{\frac{3}{4\pi}} \mu_N \sum_{i=1}^{A} \left\{ g_s^{(i)} s_{zi} + g_\ell^{(i)} \ell_{zi} \right\},$$

$$g_s^{(i)} = g_s^{(\pi)}(g_s^{(\nu)}) = 5.59(-3.83), \quad g_\ell^{(i)} = g_\ell^{(\pi)}(g_\ell^{(\nu)}) = 1(0)\,.$$

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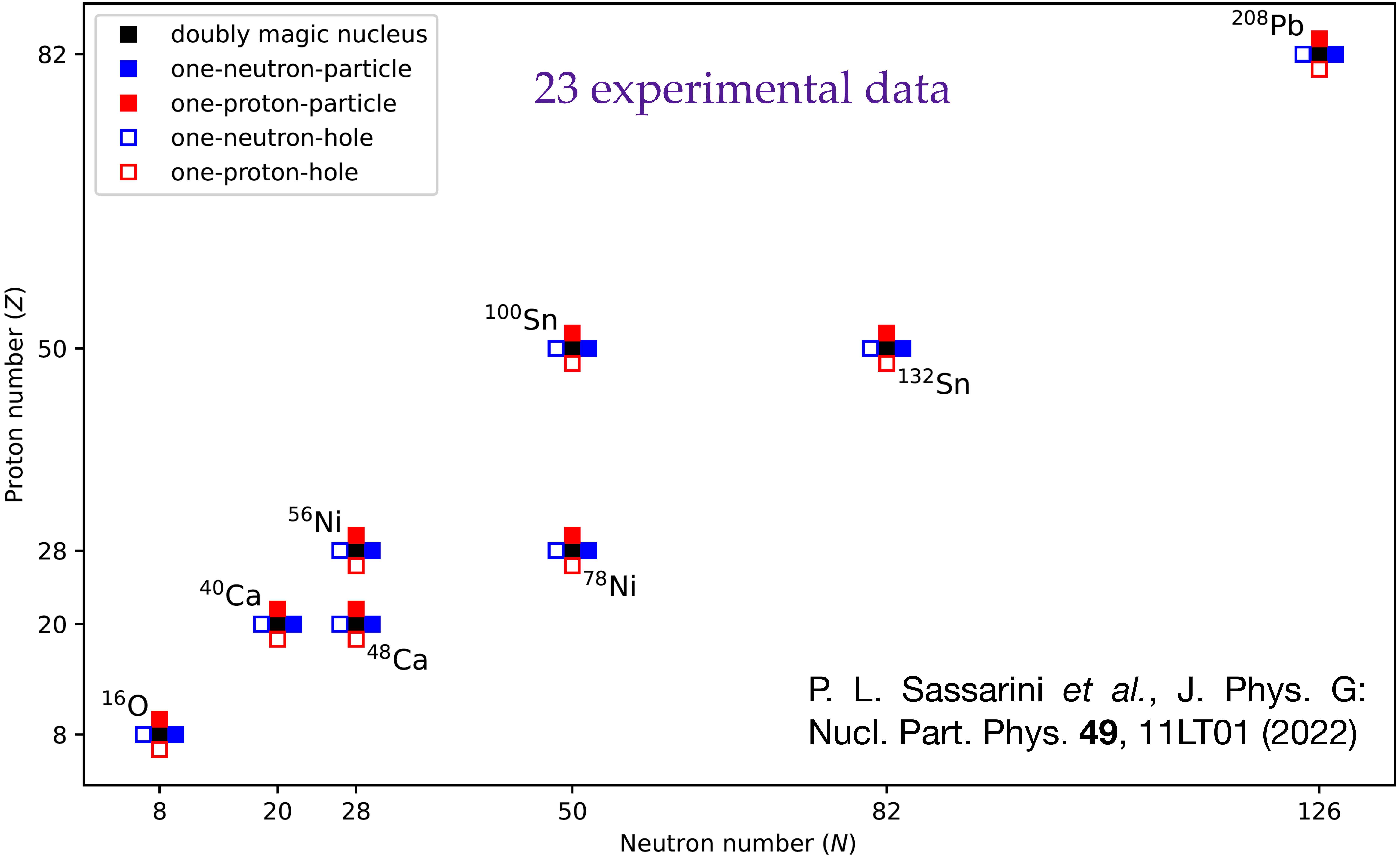
Since the self-consistent polarizations act in the full single-particle space, no effective g-factors are needed!

Odd near doubly magic nuclei



5

Optimum Landau parameter g_0'

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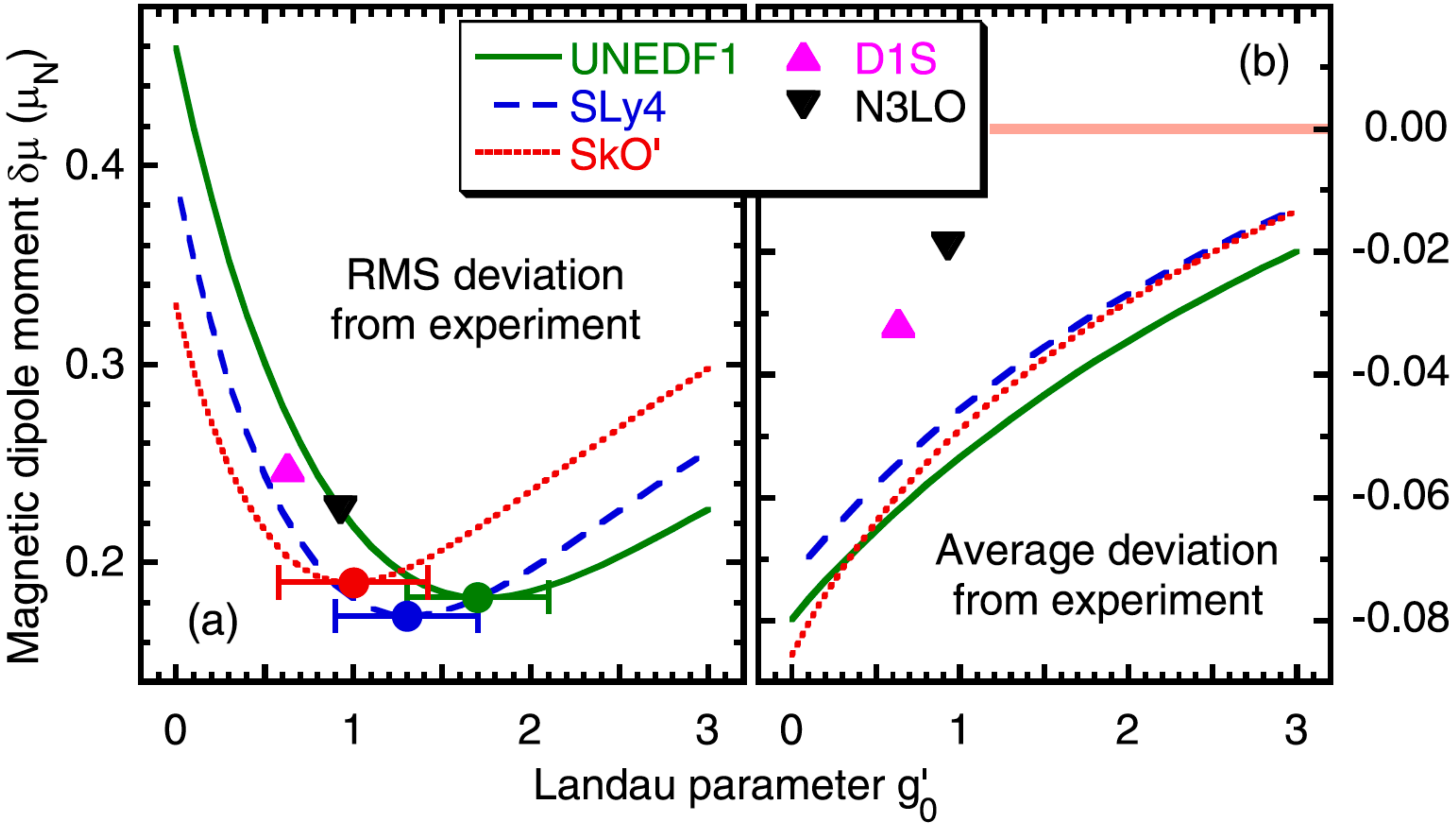

$g_0' = 1.7(4)$ (UNEDF1)

P. L. Sassarini *et al.*, J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

5

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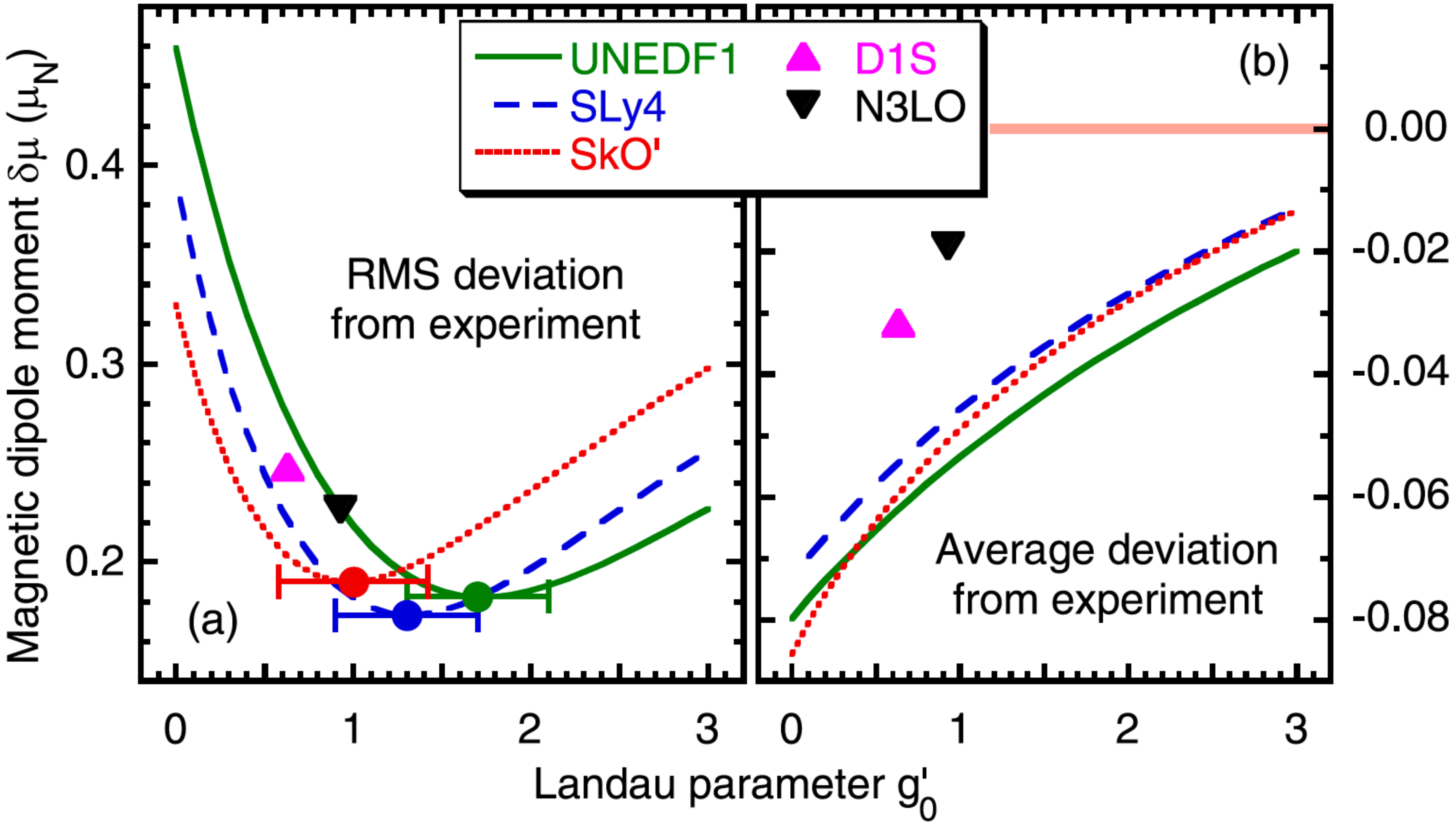

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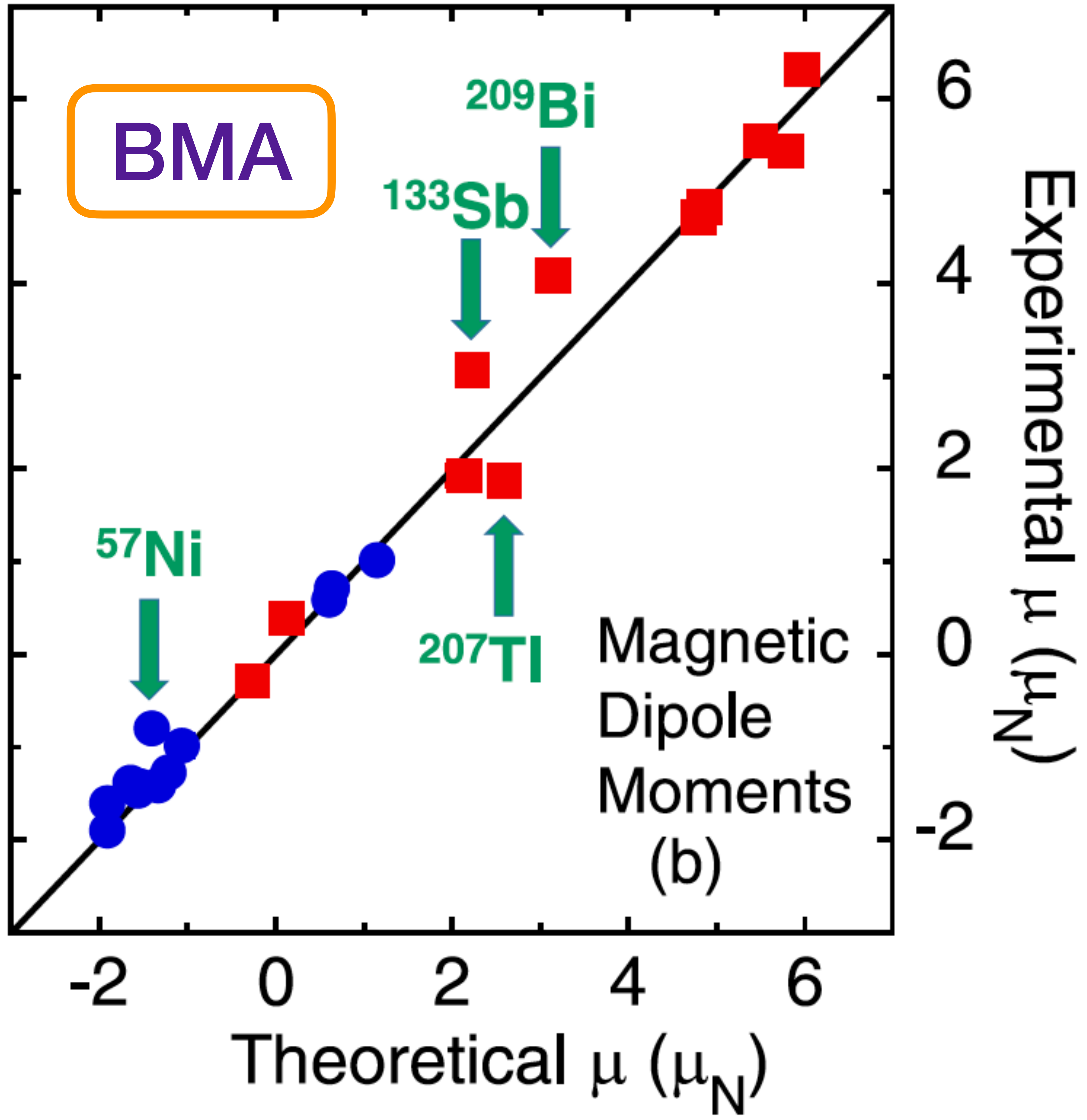
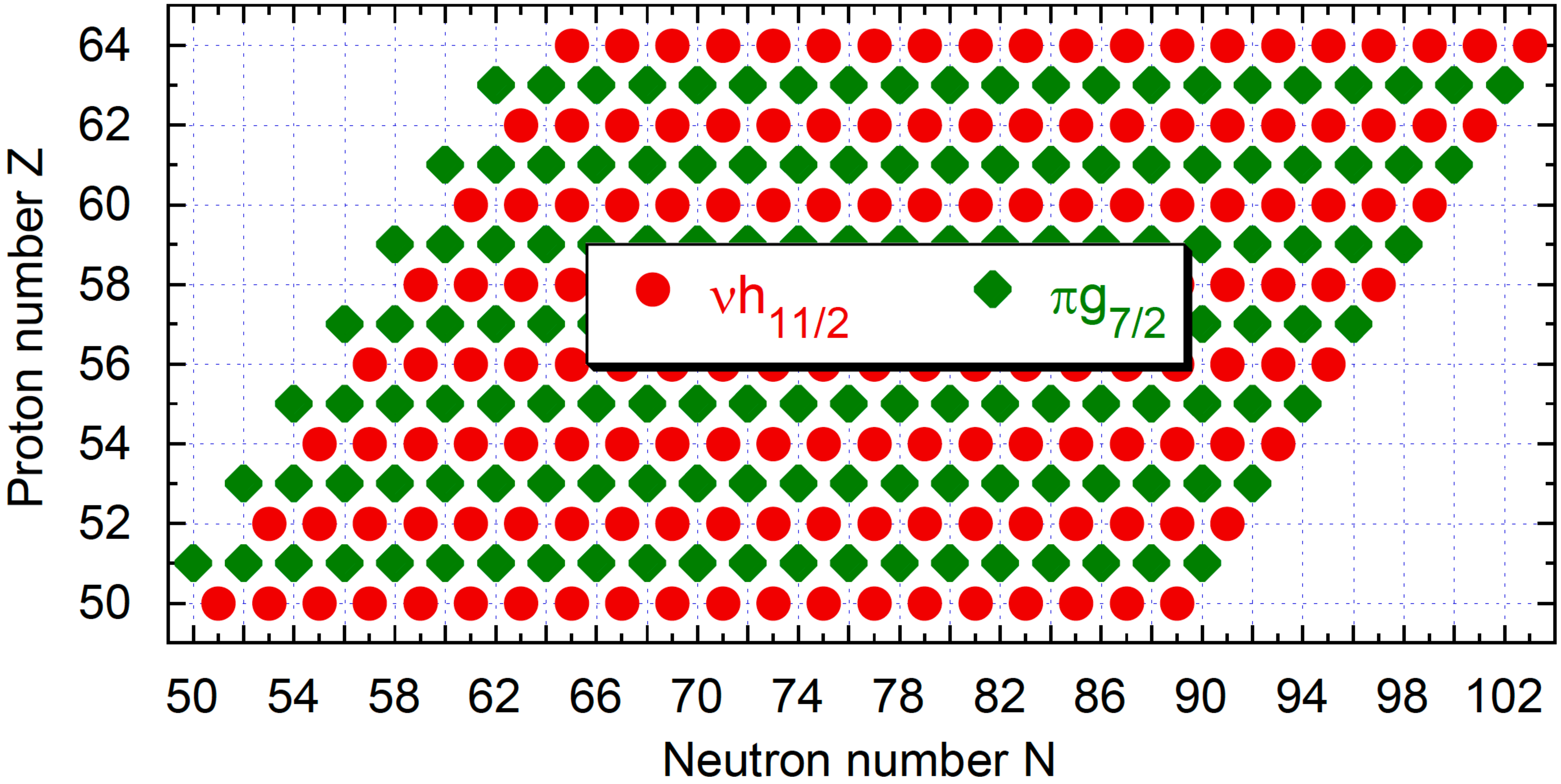
P. L. Sassarini *et al.*, J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)



Z-odd nuclei

N-odd nuclei

Systematic nuclear DFT calculations: Sn-Gd



H. Wibowo, *et.al.*, *arXiv:2503.15738*

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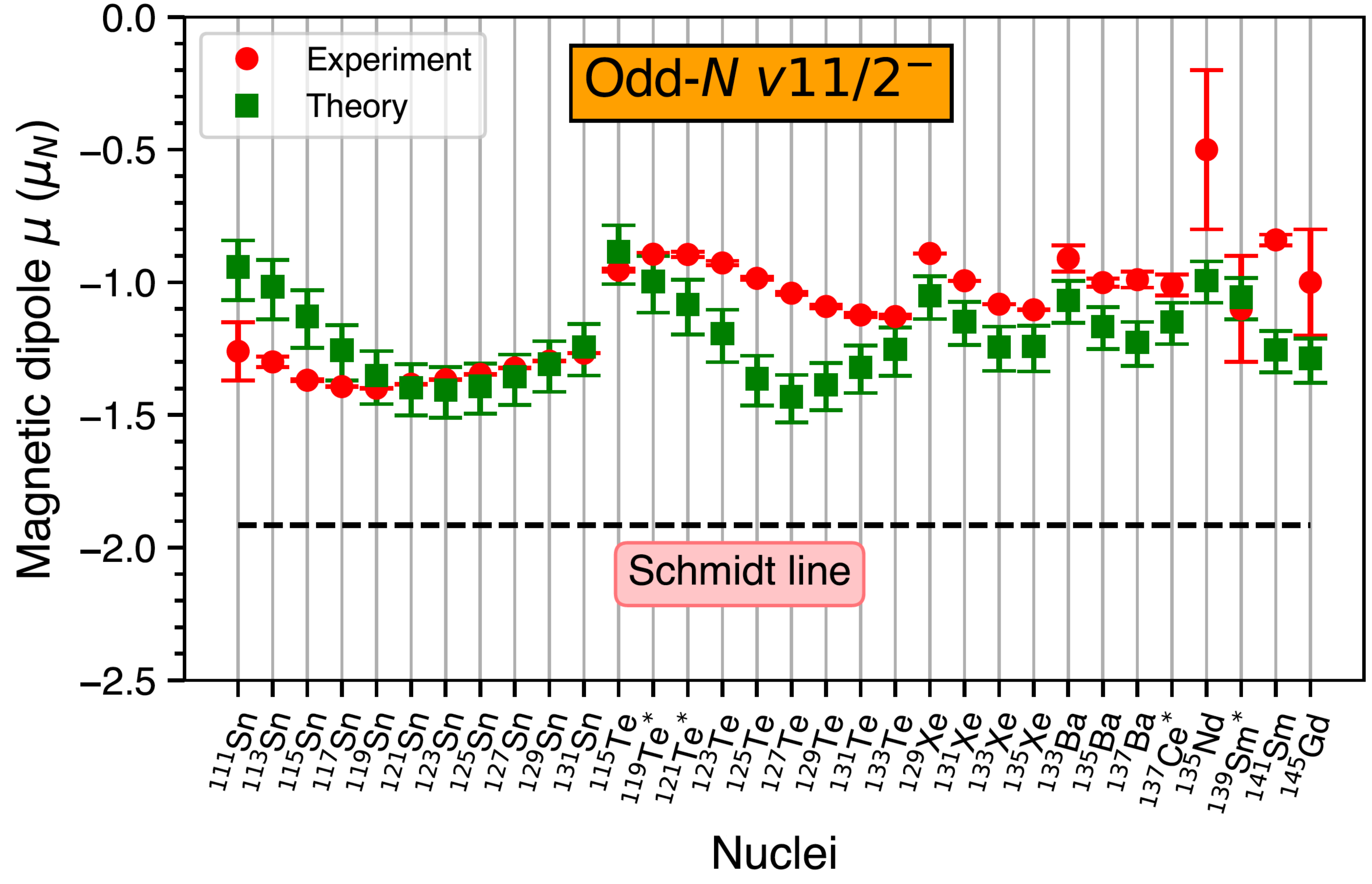
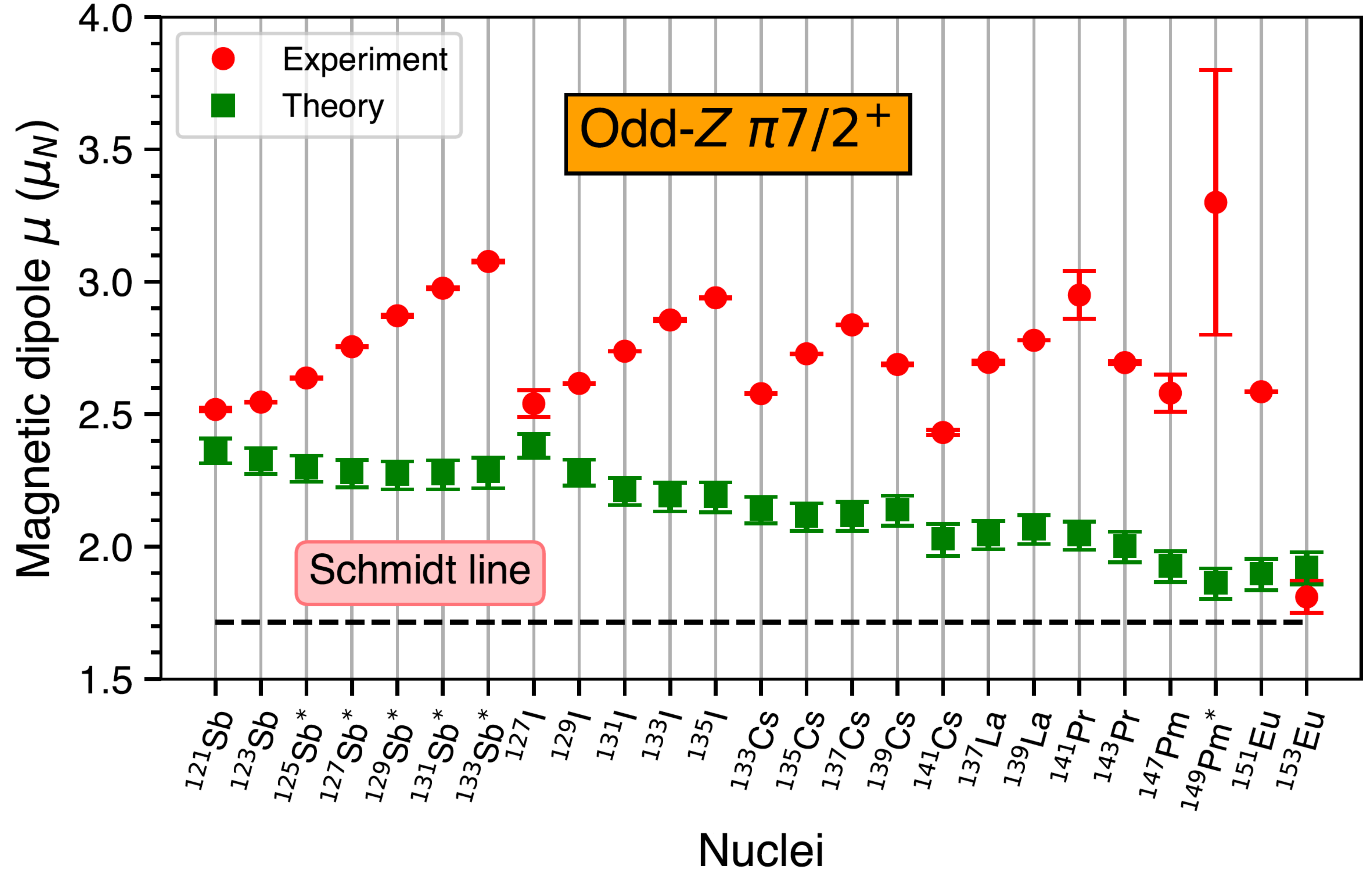
Theory vs. experiment: Sn-Gd





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H. Wibowo, *et.al.*, *arXiv:2503.15738*

N. J. Stone, INDC, report INDC(NDS)-0658
N. J. Stone, INDC, report INDC(NDS)-0794
N. J. Stone, INDC, report INDC(NDS)-0816
Yordanov D. T. *et al.*, Comm. Phys. 3, 107 (2020)
Lechner S. *et. al.*, Phys. Lett. B 847, 138278 (2023)

8

Meson-exchange contributions

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Magnetic moment operator:

$$\hat{\boldsymbol{\mu}} = \sum_{k}^{A} \hat{\boldsymbol{\mu}}_{1\mathrm{b},k} + \sum_{k<\ell}^{A} \hat{\boldsymbol{\mu}}_{2\mathrm{b},k\ell} + \cdots$$

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Two-body meson-exchange:

$$\hat{\boldsymbol{\mu}}_{2\text{b}}^{\text{NLO}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} \int d^3\mathbf{x}\, \mathbf{x} \times \mathbf{j}_{2\text{b}}^{\text{NLO}}(\mathbf{x}, \mathbf{r}_1, \mathbf{r}_2)$$

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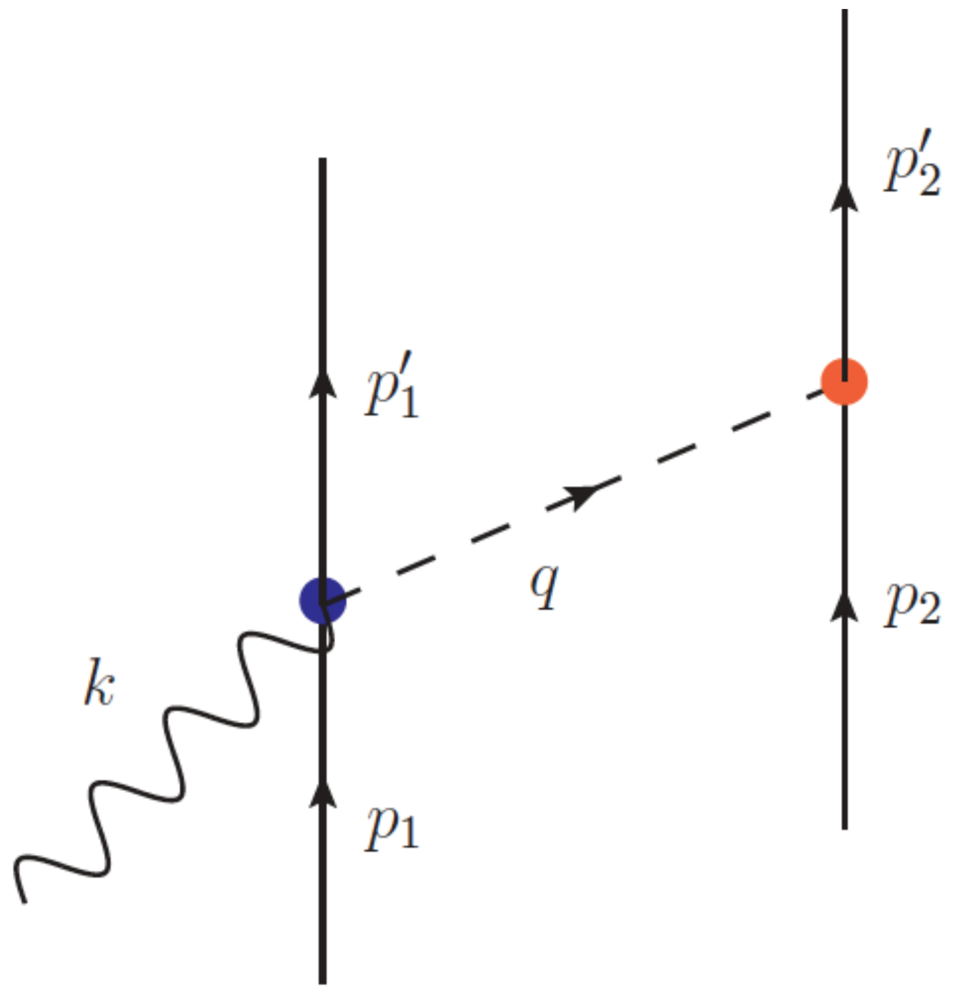
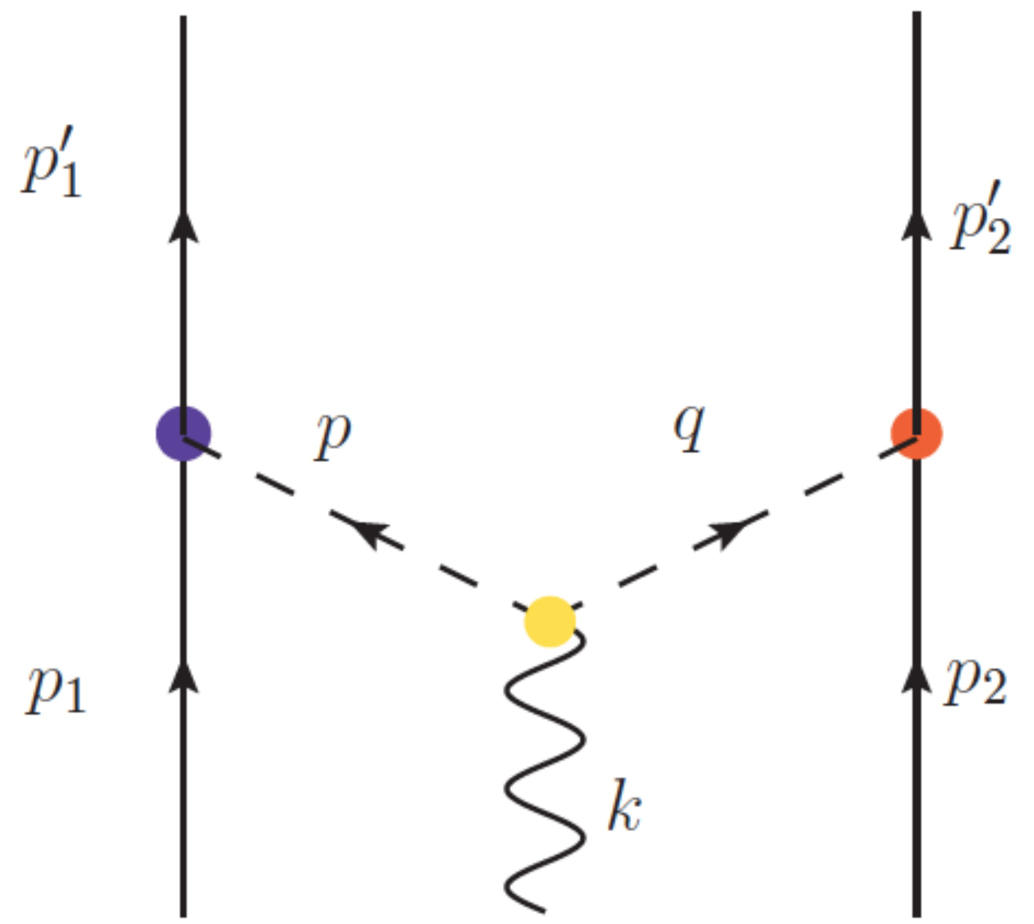
Next-to-leading order (NLO):



Seagull graph



Pion-in-flight graph

R. Seutin, *et al.*, PRC **108**, 054005 (2023)
T. Miyagi, *et al.*, PRL **132**, 232503 (2024)

NLO magnetic moment operators

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The NLO intrinsic and Sachs contributions to the magnetic moment operator are given by

$$\hat{\boldsymbol{\mu}}_{2b}^{\text{NLO, int}}(\mathbf{r}) = -\frac{g_A^2 m_\pi}{32\pi F_\pi^2}(\hat{\boldsymbol{\tau}}_1 \times \hat{\boldsymbol{\tau}}_2)_z \left\{\left(1 + \frac{1}{m_\pi r}\right)[(\hat{\boldsymbol{\sigma}}_1 \times \hat{\boldsymbol{\sigma}}_2)\cdot\hat{\mathbf{r}}]\hat{\mathbf{r}} - (\hat{\boldsymbol{\sigma}}_1 \times \hat{\boldsymbol{\sigma}}_2)\right\} e^{-m_\pi r}$$

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and $(g_A = 1.27; \quad F_\pi = 92.3 \text{ MeV}; \quad m_\pi = 138.039 \text{ MeV})$

$$\hat{\boldsymbol{\mu}}_{2\text{b}}^{\text{NLO, Sachs}}(\mathbf{r}) = -\frac{1}{2}(\hat{\boldsymbol{\tau}}_1 \times \hat{\boldsymbol{\tau}}_2)_z V_{1\pi}(r) \mathbf{R}_{\text{NN}} \times \mathbf{r},$$

relative coordinate

COM

respectively, where

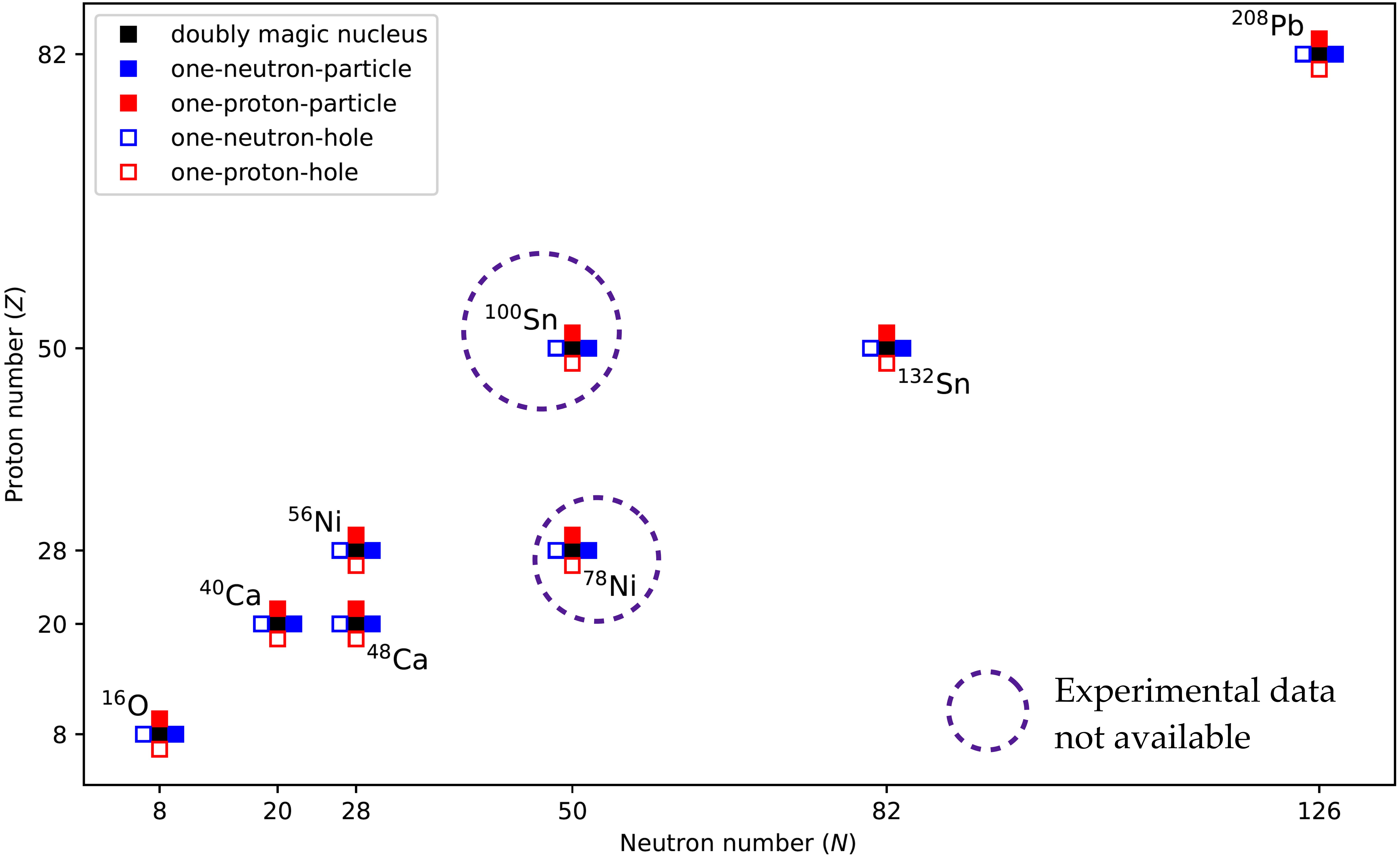
$$V_{1\pi}(r) = \frac{m_\pi^2 g_A^2}{48\pi F_\pi^2} \left[\hat{S}_{12} \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) + \hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2\right] \frac{e^{-m_\pi r}}{r}.$$

Tensor operator: $\hat{S}_{12} = 3(\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\sigma}}_1)(\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\sigma}}_2) - \hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2.$

T. Miyagi, *et al.*, PRL **132**, 232503 (2024)

R. Seutin, *et al.*, PRC **108**, 054005 (2023)

10 Odd near doubly magic nuclei (two-body currents)



11

Optimum Landau parameter g_0' (two-body currents)

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$$\delta\mu(g_0') = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{\text{calc}}(i, g_0') - \mu_{\text{exp}}(i, g_0')\right]^2}$$

i = odd nucleus

N = number of odd nuclei

11

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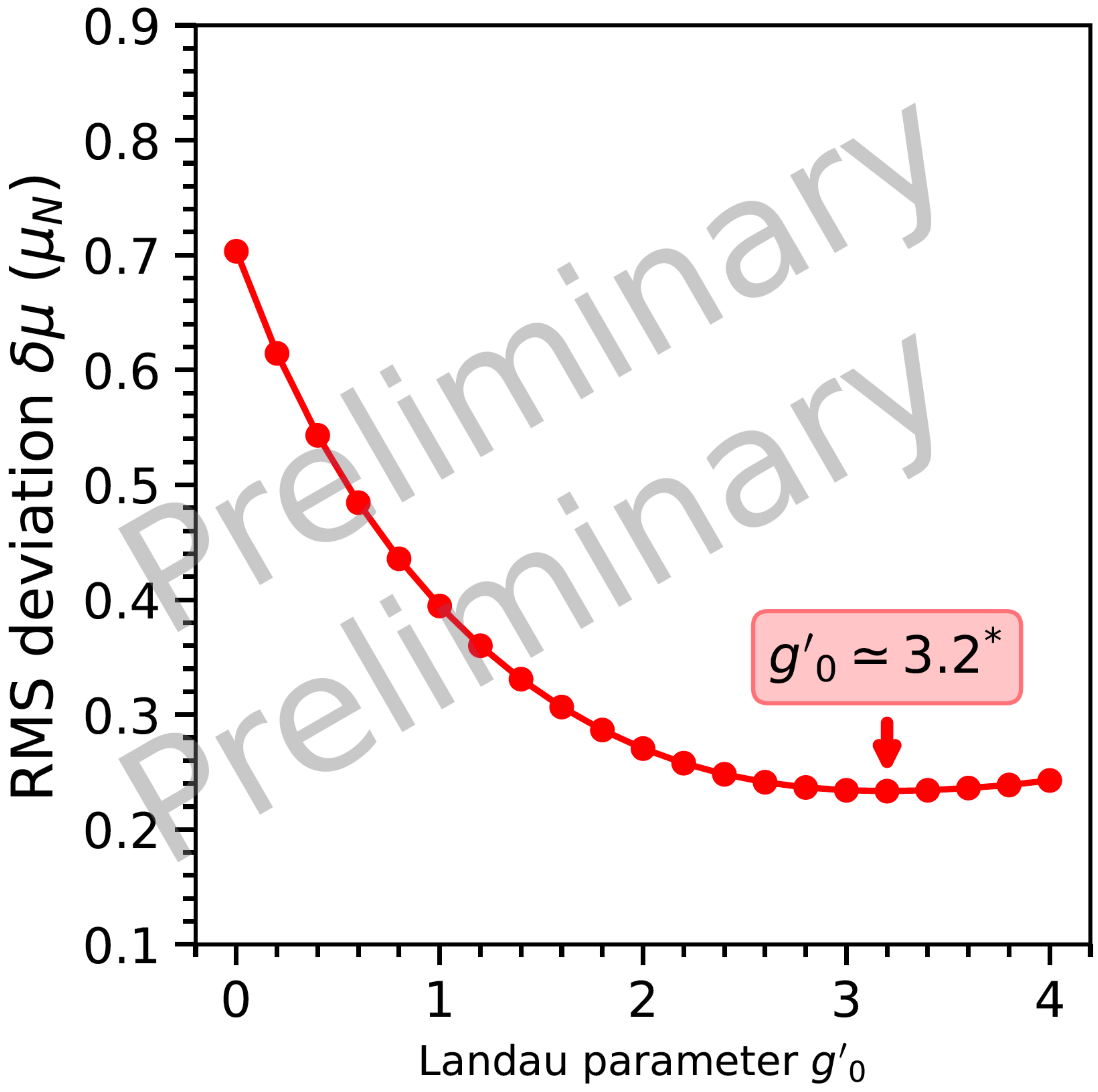

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(*) 22 odd near doubly magic nuclei without Bayesian analysis (**for now!**).

H. Wibowo, *et.al.*, to be published

12

Magnetic dipole moments: theory vs. experiment

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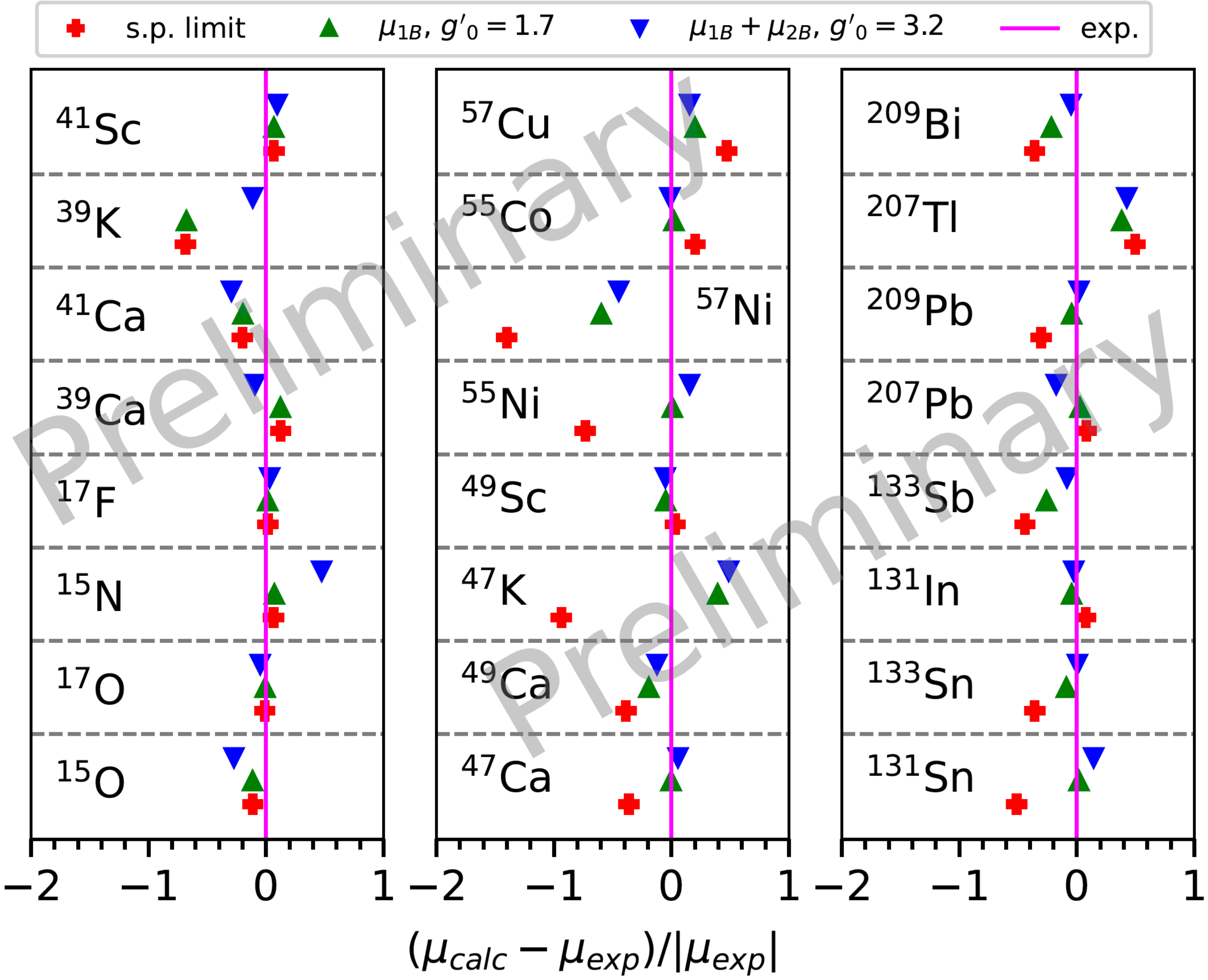

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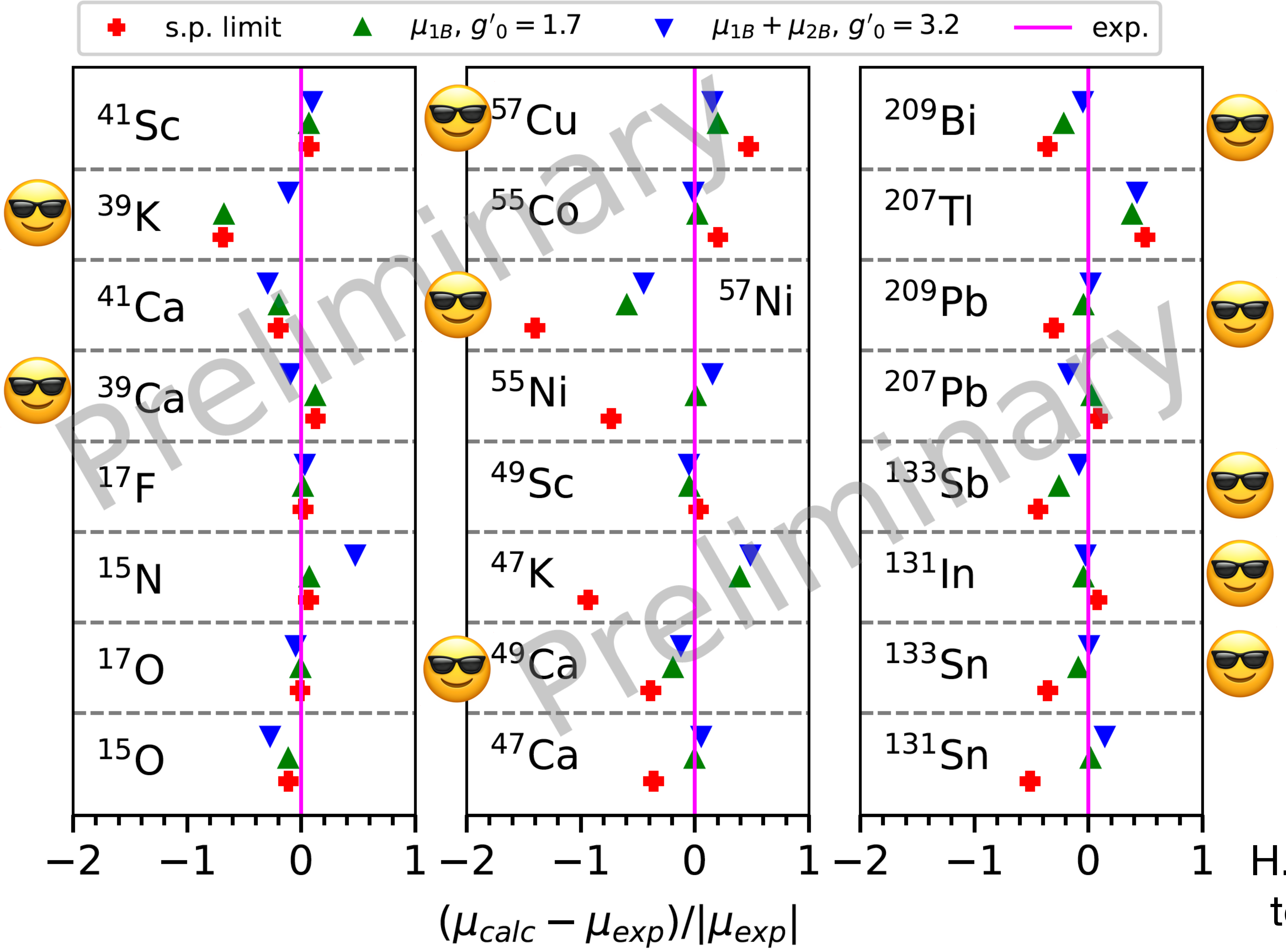

★ In 10/14 cases, the two-body-current corrections **improve**/**deteriorate** agreement with experimental data.

H. Wibowo, *et.al.*, to be published

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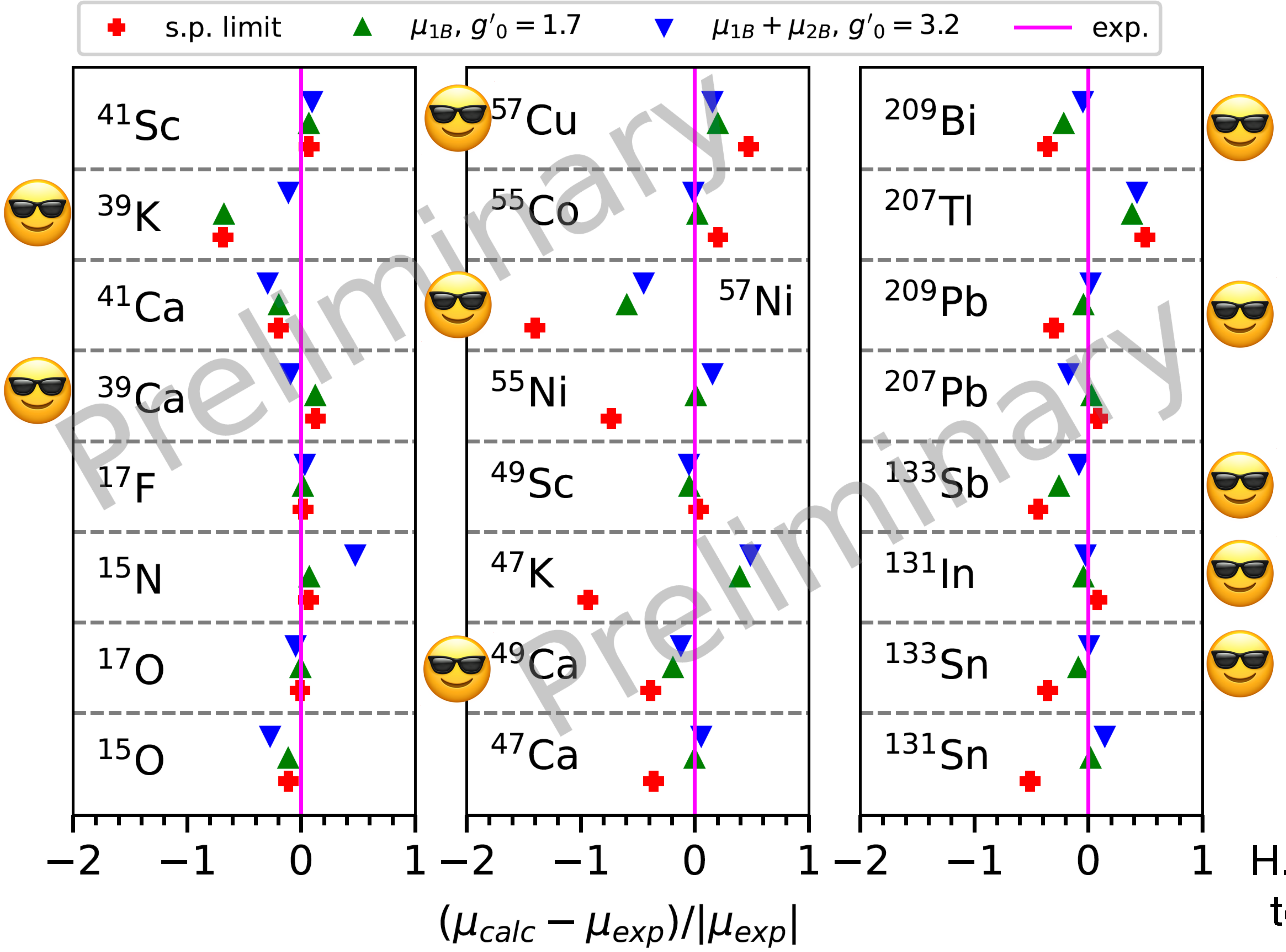

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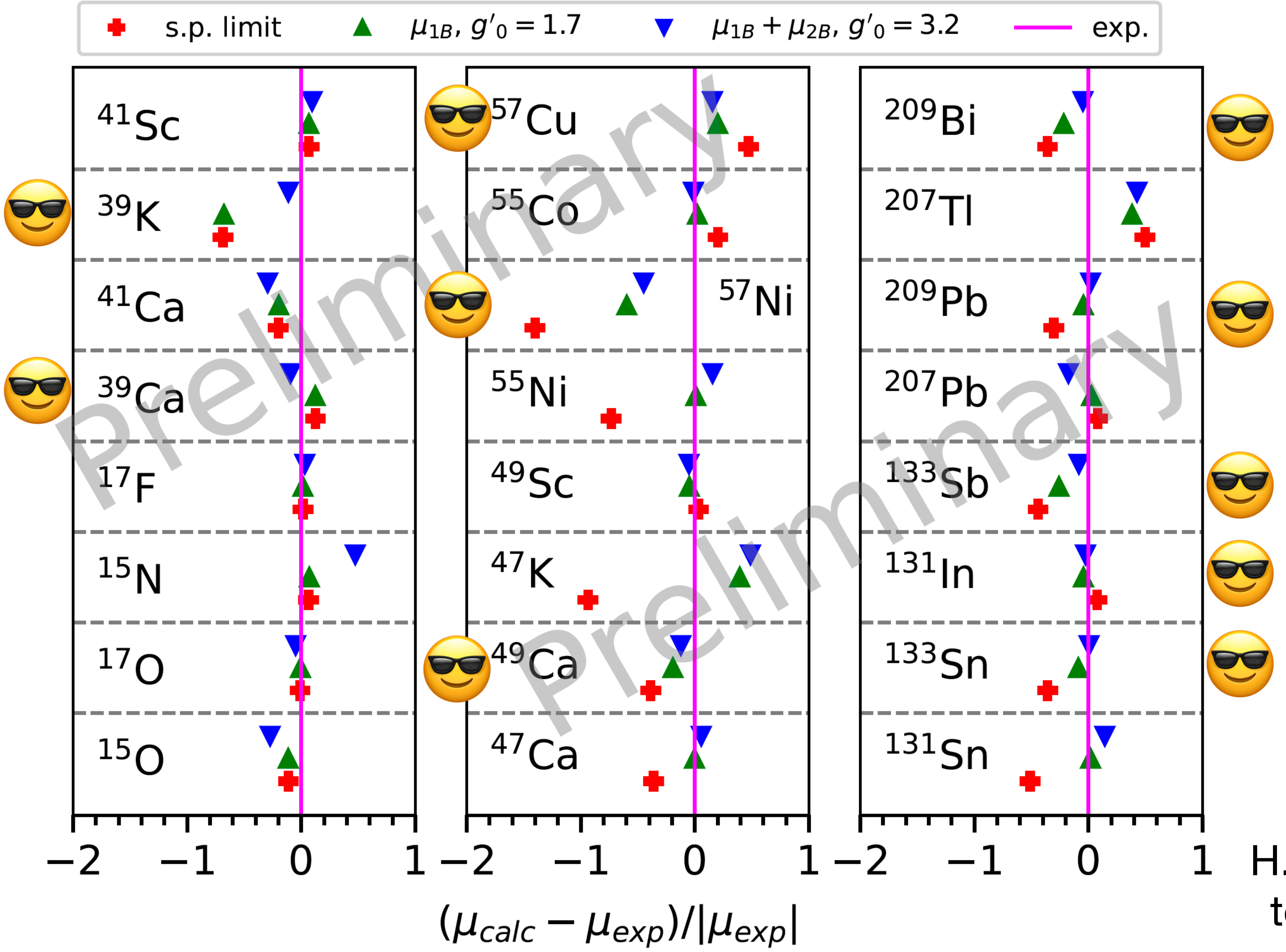

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- The complete statistical analysis will follow.

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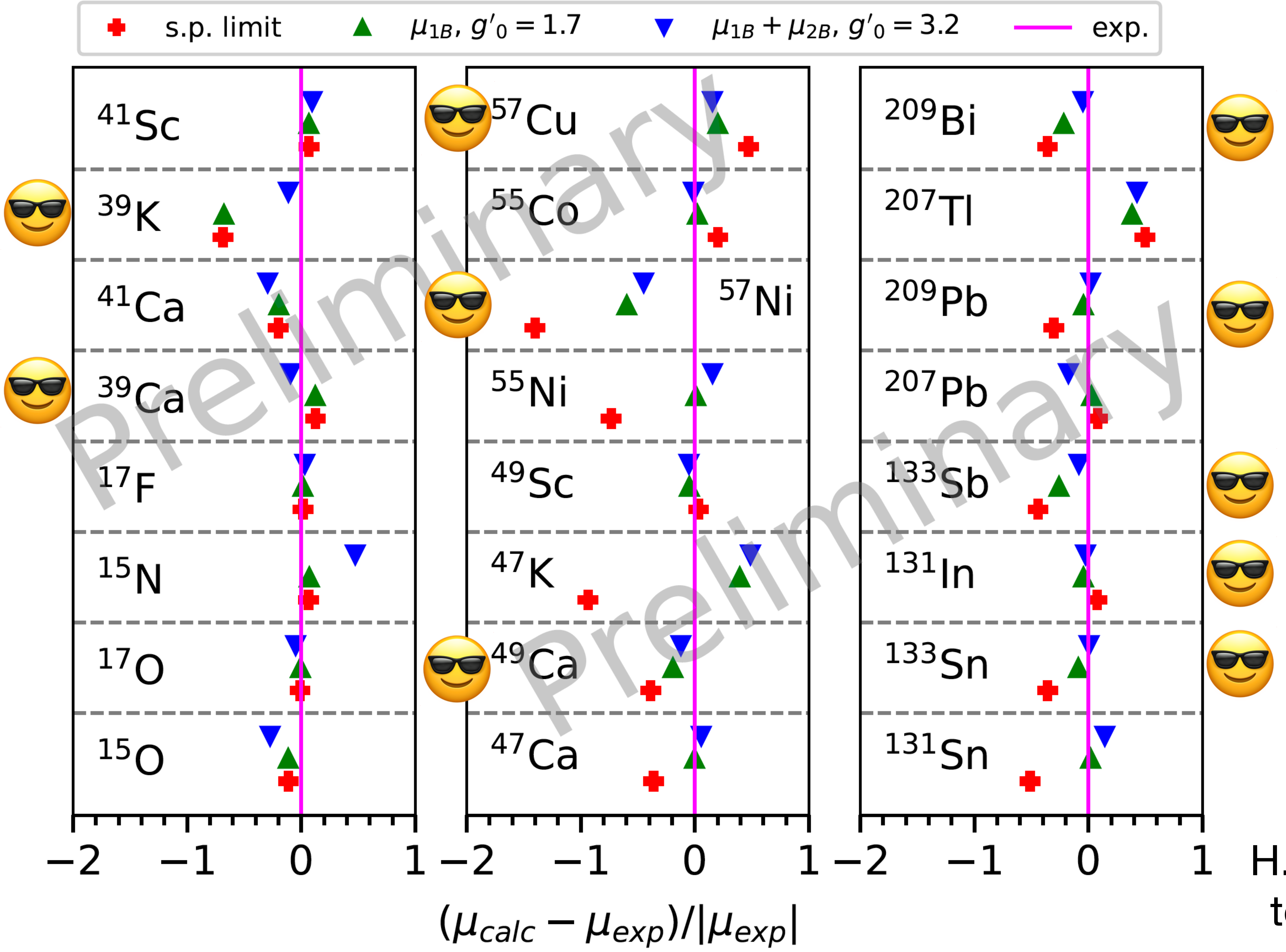

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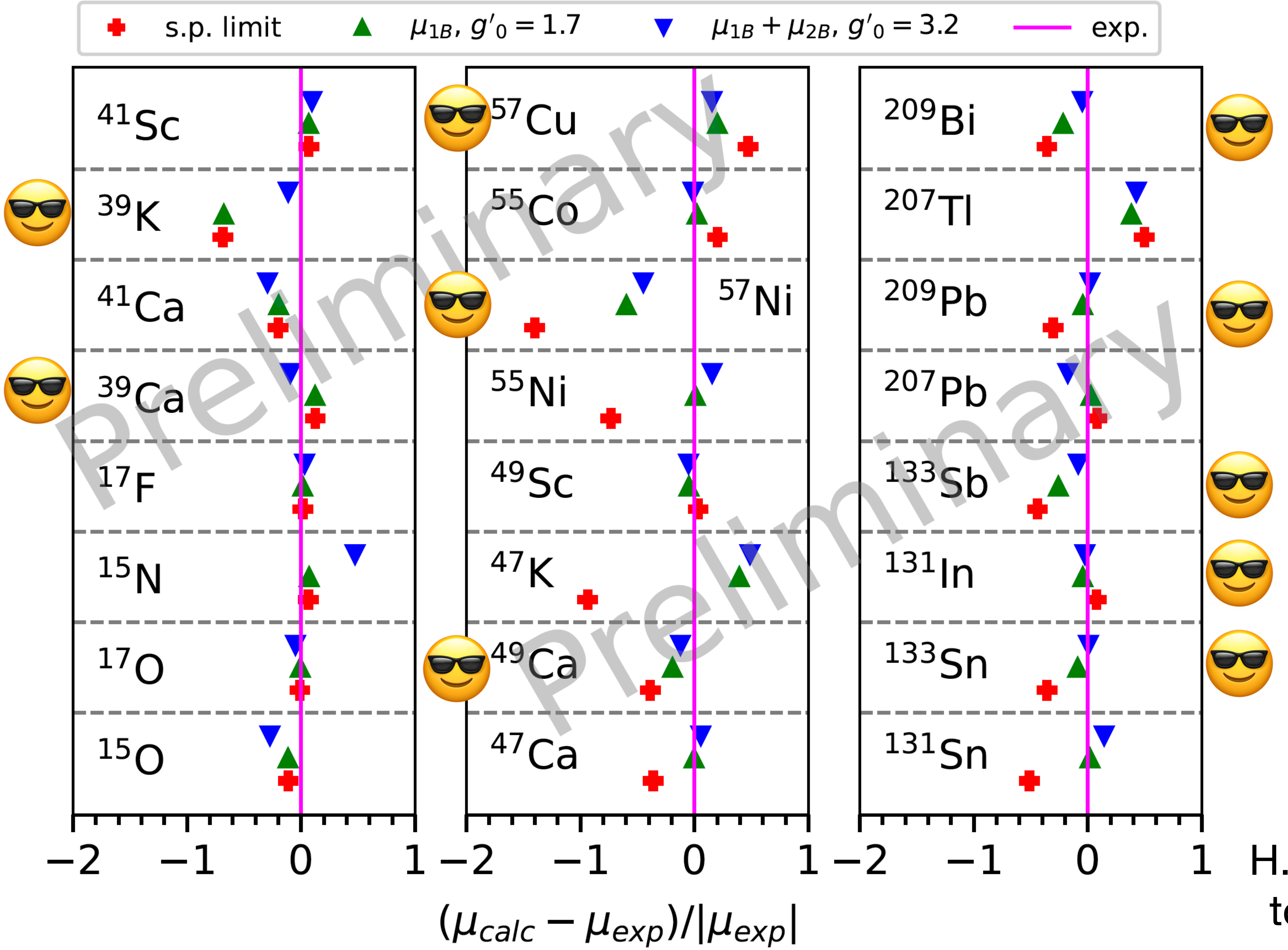

- In 10 / 14 cases, the two-body-current corrections **improve** / **deteriorate** agreement with experimental data.
- The complete statistical analysis will follow.
- Results in deformed nuclei will follow.

H. Wibowo, *et.al.*, to be published

Summary and Outlook

Summary

- ★ The obtained value of the optimum Landau parameter g_0' is around 3.2.
- ★ The inclusion of the meson-exchange currents improves the predictions of magnetic dipole moments in ^{39}K, ^{39}Ca, ^{57}Cu, ^{57}Ni, ^{49}Ca, ^{209}Bi, ^{209}Pb, ^{133}Sb, ^{131}In, and ^{133}Sn.

Outlook

- ★ Bayesian analysis to determine precisely the optimum Landau parameter g_0'.
- ★ Systematic nuclear DFT calculations of magnetic dipole moments across the nuclear chart with the inclusion of meson-exchange currents.

Acknowledgements

We acknowledge the support from a **Leverhulme Trust Research Project Grant**. This work was partially supported by the **STFC Grant** Nos. ST/P003885/1 and ST/V001035/1 and by the **Polish National Science Centre** under Contract No. 2018/31/B/ST2/02220. We acknowledge the **CSC-IT Center for Science Ltd., Finland**, for the allocation of computational resources. This project was partly undertaken on the **Viking Cluster**, which is a high performance compute facility provided by the **University of York**. We are grateful for computational support from the **University of York High Performance Computing service, Viking and the Research Computing team**.