Lifetime measurements in ^{102}Mo

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Lifetime measurements in ^{102}Mo interpreted in the interacting boson model and the X(5) symmetry

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Outline

Motivation and Physics

Experimental Details

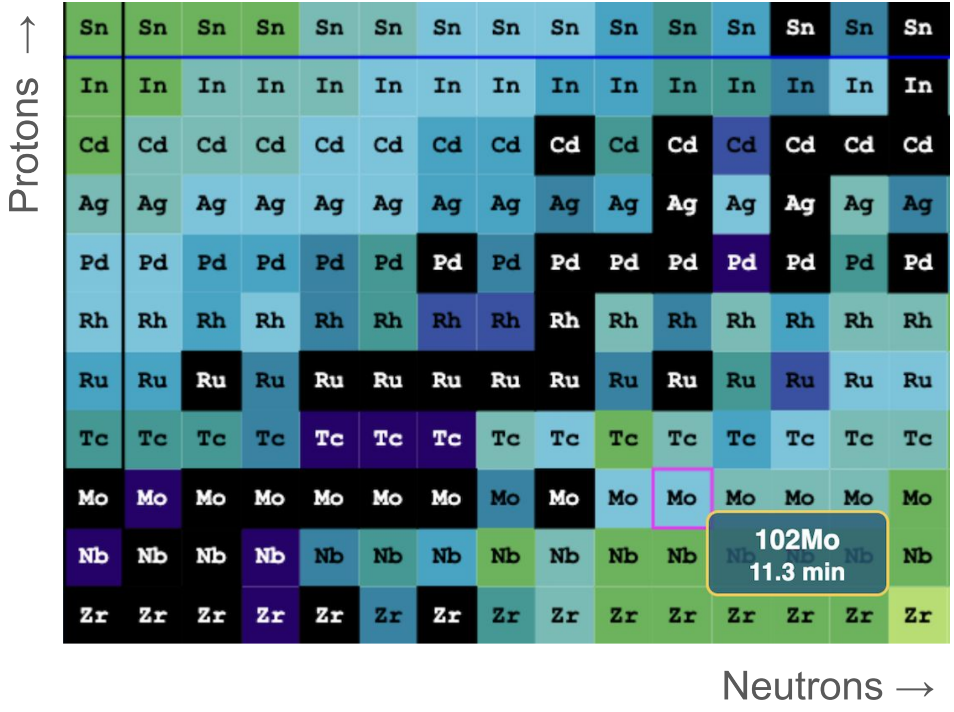
Results and Interpretation

Outlook

Motivation and Physics

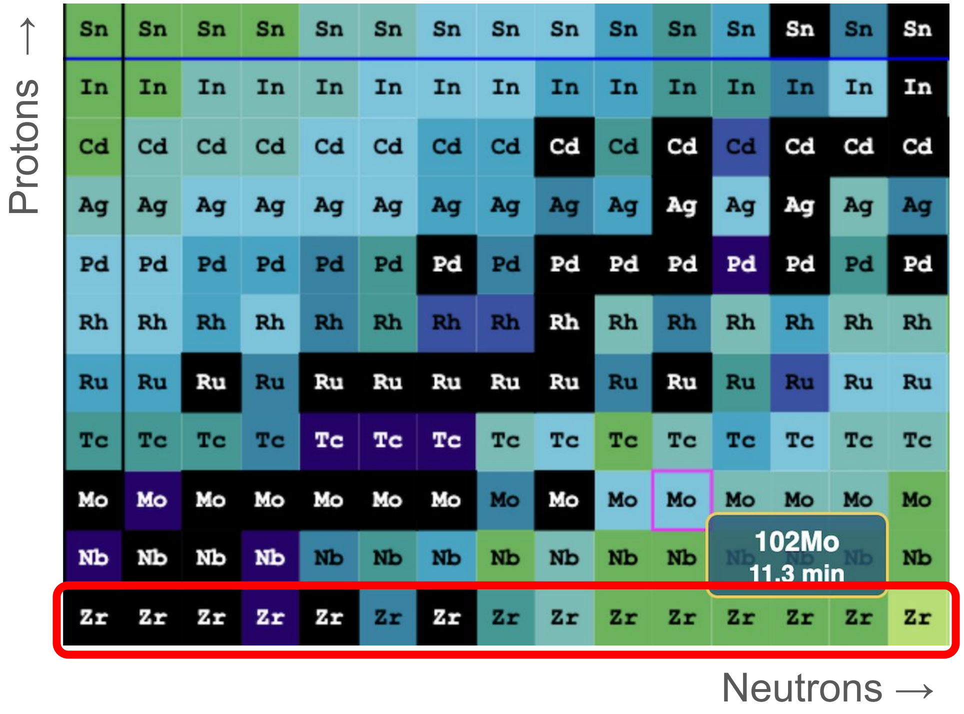
Even-Even Mass~100 Nuclei

- Neutron-rich even-even nuclei are interesting due to rapid structural evolution



Even-Even Mass~100 Nuclei

- Neutron-rich even-even nuclei are interesting due to rapid structural evolution
- Sudden onset quadrupole deformation at N=60 in Sr,Zr



Even-Even Mass~100 Nuclei

- Neutron-rich even-even nuclei are interesting due to rapid structural evolution
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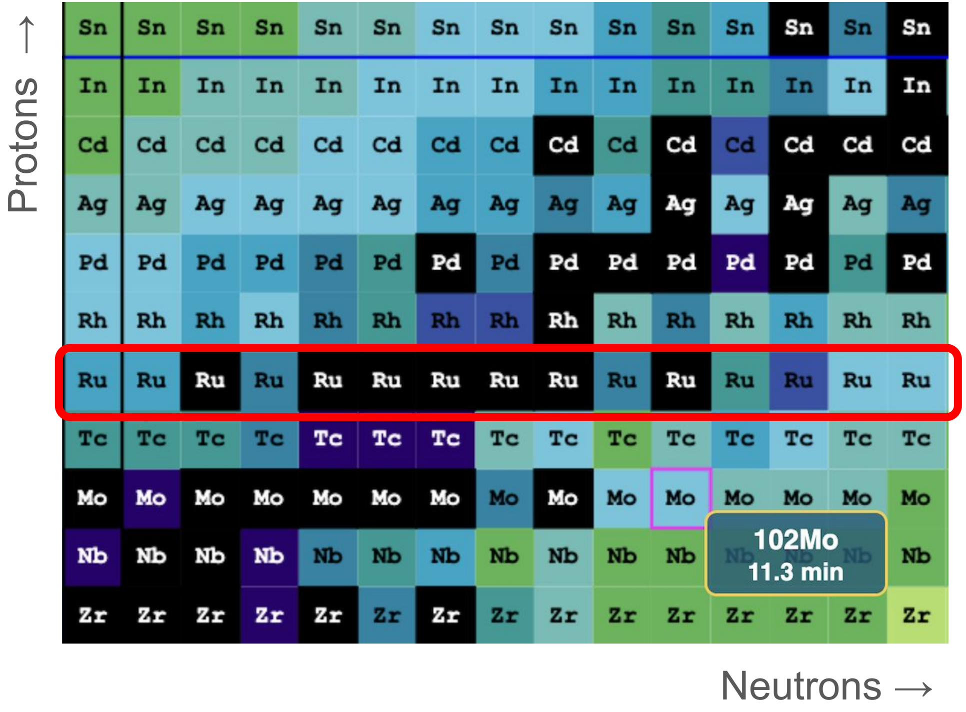
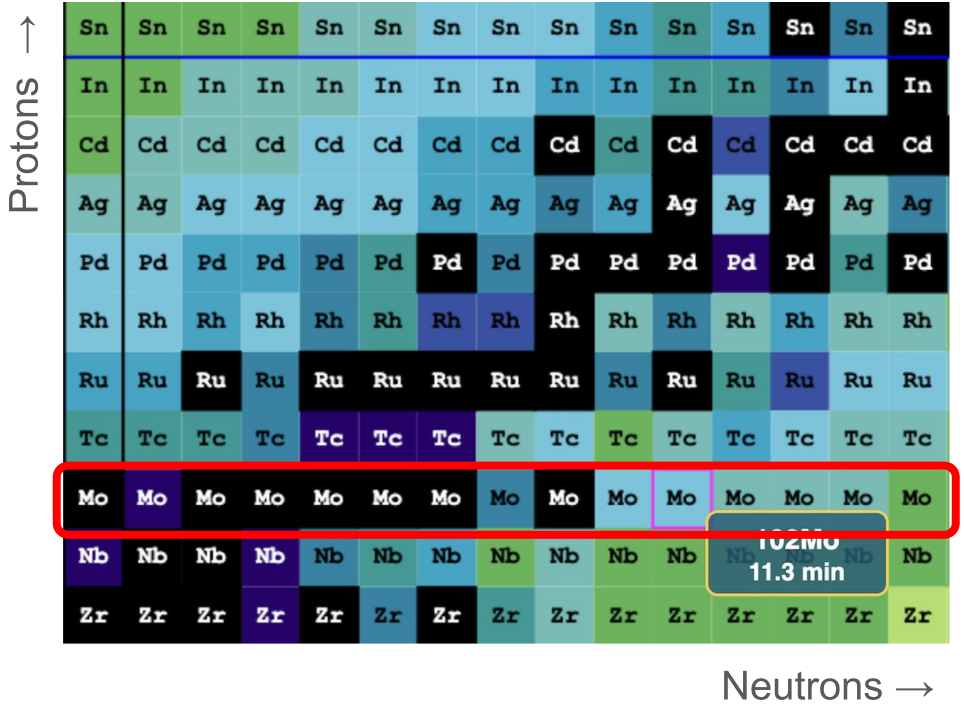
- Emergence of triaxiality in Ru



Even-Even Mass~100 Nuclei

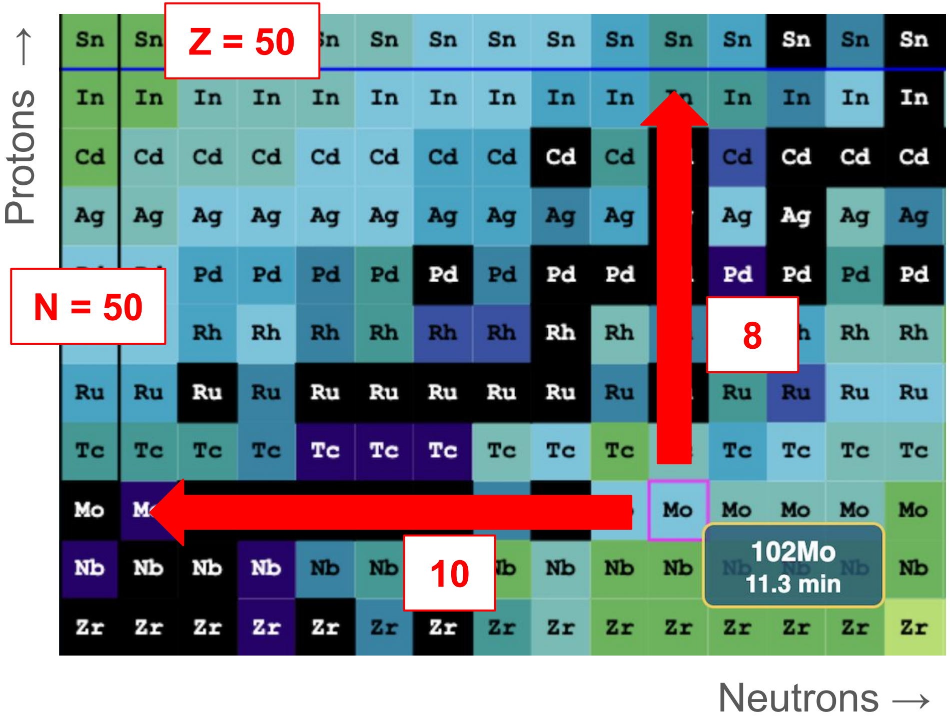
- Neutron-rich even-even nuclei are interesting due to rapid structural evolution
- Sudden onset quadrupole deformation at N=60 in Sr,Zr
- Emergence of triaxiality in Ru
- **Which effect dominates in Mo?**



Even-Even Mass~100 Nuclei

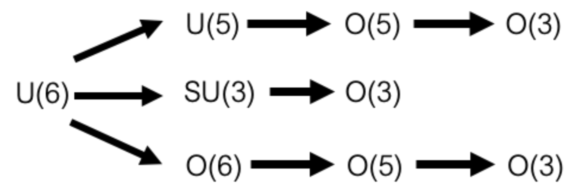
- Simplest form (IBM-1):
  - Pairs of valence nucleons considered effective bosons with J=0 (“s-bosons”) or J=2 (“d-bosons”)
  - Protons and neutrons are not differentiated
  - States built from boson interactions

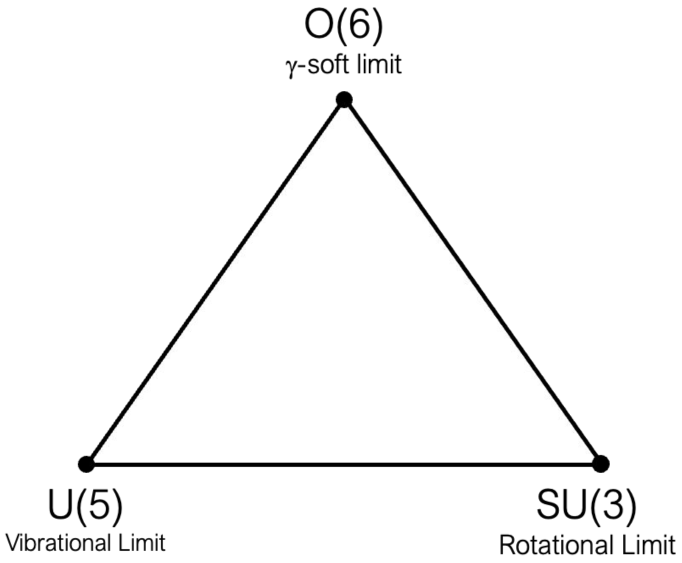
- Example - ^{102}Mo has 42 protons, 60 neutrons. From magic number 50: 8 valence proton-holes, 10 neutrons – pair for total of (8+10)/2 = **9 effective bosons**



Even-Even Mass~100 Nuclei

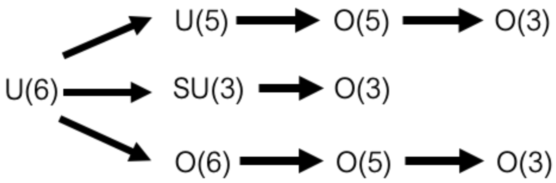
- Pairs of valence nucleons considered effective bosons with J=0 ("*s-bosons*") or J=2 ("*d-bosons*")
    - *s*- and *d*-bosons have 1 and 5 magnetic substates respectively, 6 degrees of freedom in total
    - Can be described by U(6) algebraic group
    - U(6) decomposes in 3 sequences that reach O(3) subgroup (*L* quantum number)

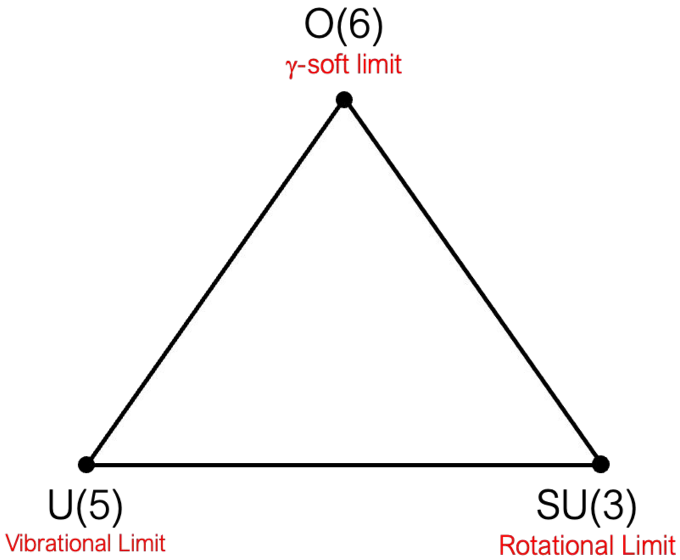




Even-Even Mass~100 Nuclei

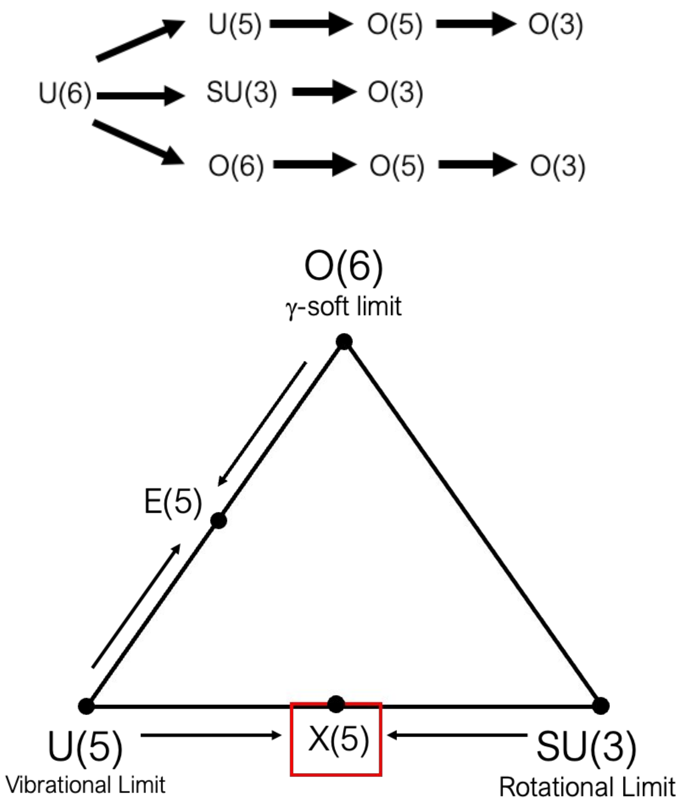
- Pairs of valence nucleons considered effective bosons with J=0 ("*s-bosons*") or J=2 ("*d-bosons*")
  - *s*- and *d*-bosons have 1 and 5 magnetic substates respectively, 6 degrees of freedom in total
  - Can be described by U(6) algebraic group
  - U(6) decomposes in 3 sequences that reach O(3) subgroup (*L* quantum number)
- Limits derived from boson algebra have **geometric analogues**





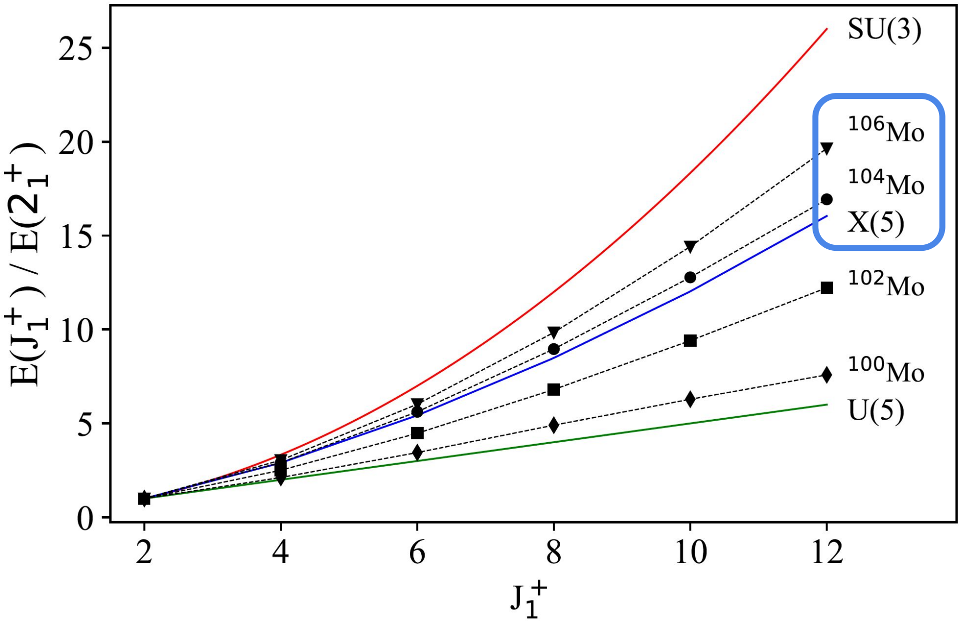
Even-Even Mass~100 Nuclei

- Vast majority of nuclei do not conform precisely to limits…
- ”Critical points” in transitions between limits can be investigated
- **X(5)** represents critical point between:
  - U(5) (vibrator),
  - SU(3) (axially symmetric rotor)
- X(5) is an approximate analytical solution to Bohr Hamiltonian – β and γ decoupled



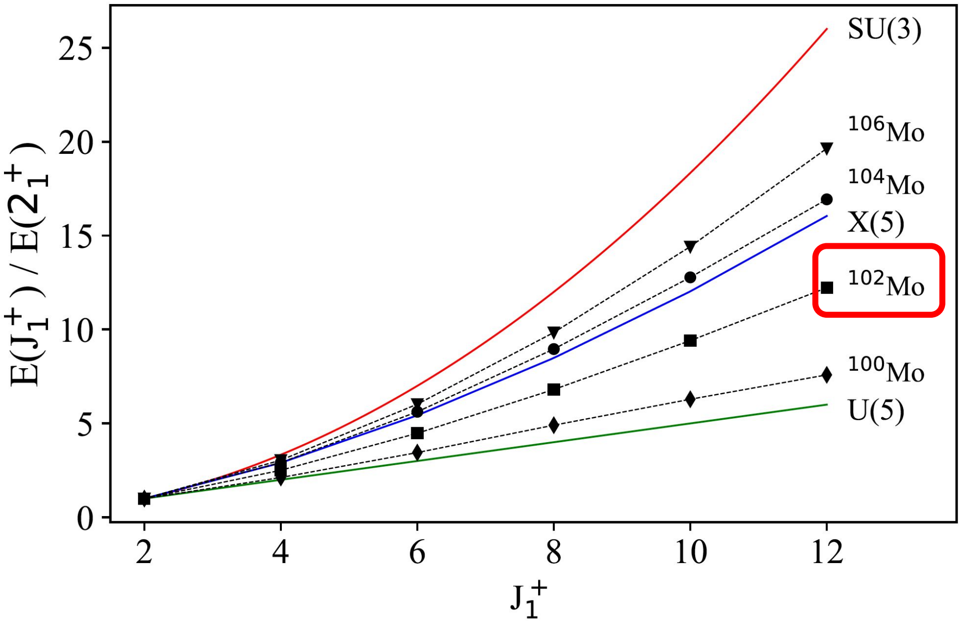
X(5) Symmetry

- **Hutter *et al*** considered some A~100 isotopes as potential candidates of an X(5) nucleus
- Performed RDDS measurements on ^{104}**Mo** and ^{106}**Mo** to search for this symmetry



X(5) Symmetry

- **Hutter *et al*** considered some A~100 isotopes as potential candidates of an X(5) nucleus
- Performed RDDS measurements on ^{104}**Mo** and ^{106}**Mo** to search for this symmetry
- **In this work we** have measured states in ^{102}**Mo**
- U(5) and X(5) should be useful frameworks for comparison



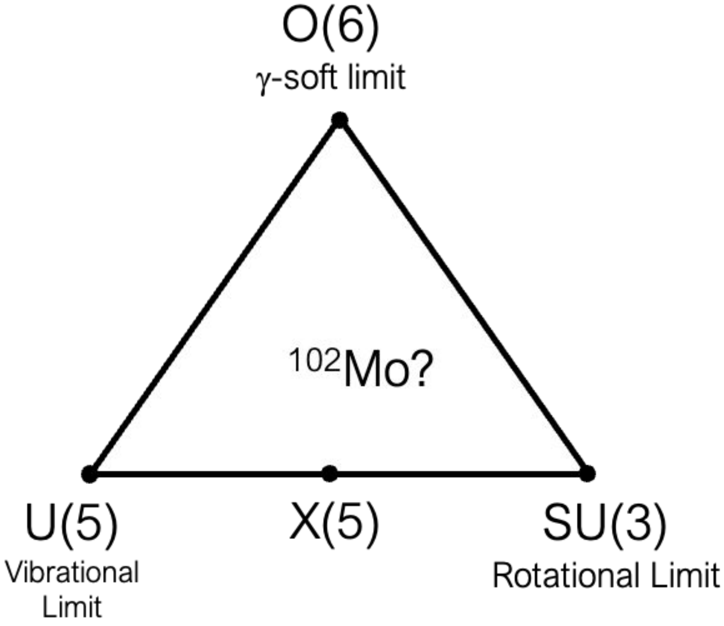
Project Aims

1. Measure transition energies to construct a level scheme

2. Observe transitions thought to depopulate 0^+_3 state at 1334(5) keV, and 3^-_1 at 1881(2) keV

3. Measure lifetimes of the triplet states (0^+, 4^+, 2^+) above first 2^+, and anything else that comes free

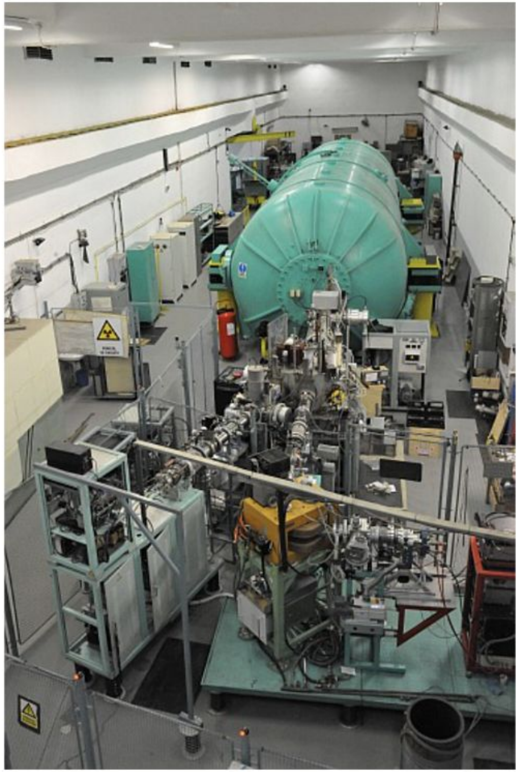
4. Compare to previous experimental data and IBM calculations



Experimental Details

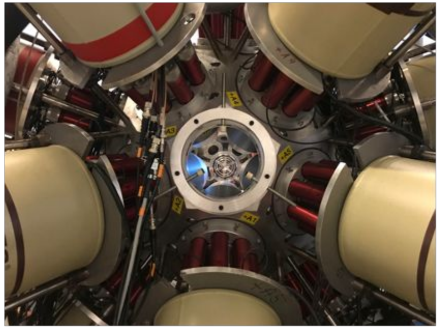
SECTOR 6
Bucharest
SECTOR 4
Bragadiru
Măgurele
401A
BUCHAREST
ILFOV COUNTY

Experimental Setup

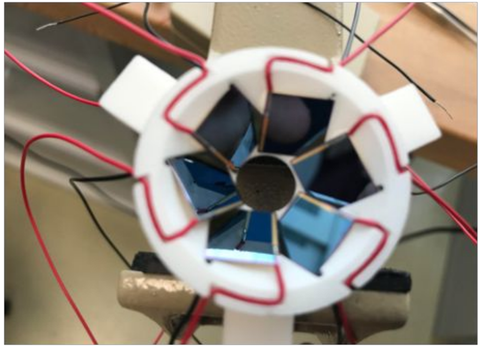

9 MV Pelletron Tandem accelerator

ROSPHERE

- 25 HPGes in 5 rings at angles 37°, 70°, 90°, 110°, 143° w.r.t. beam axis
- Compton-suppressed with BGOs


ROSPHERE

1 of 7 beamlines



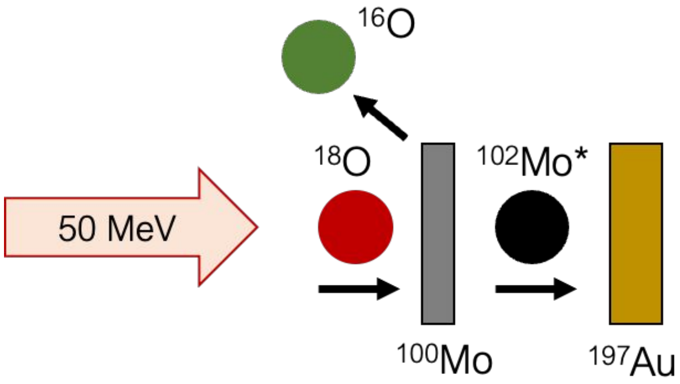
SORCERER

- Particle selection at backward angles, 130.4° to 165.5° w.r.t. beam axis
- ~11% solid angle coverage

Experimental Details

June 2019 – 11 days beamtime

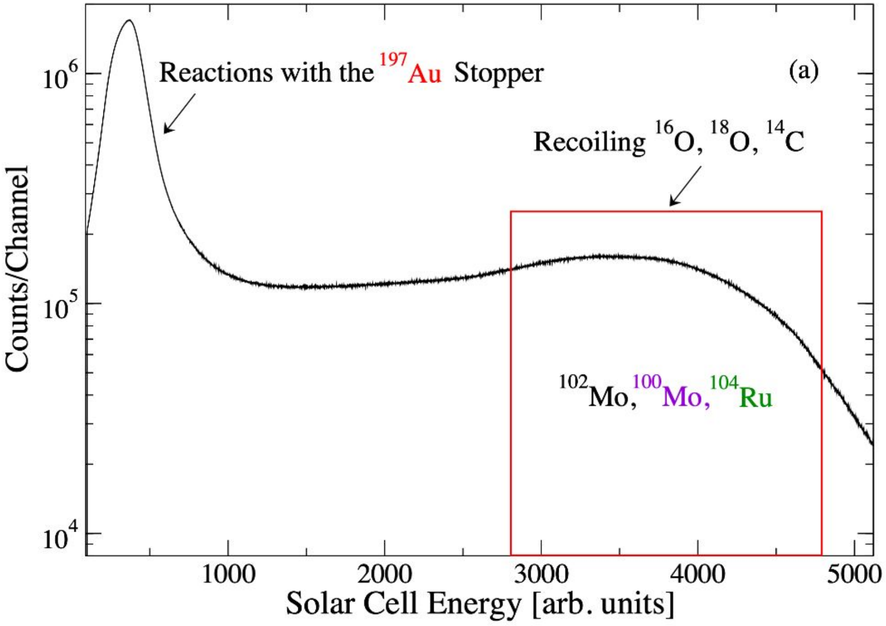
- 50 MeV ^{18}O beam, just below Coulomb barrier
- Impinged on ^{100}Mo for the 2n transfer reaction ^{100}Mo(^{18}O,^{16}O)^{102}Mo
- 1.0 mg cm^{-2} ^{100}Mo target (>99% enrichment), 6.0 mg cm^{-2} ^{197}Au stopper
- 8 target-stopper distances

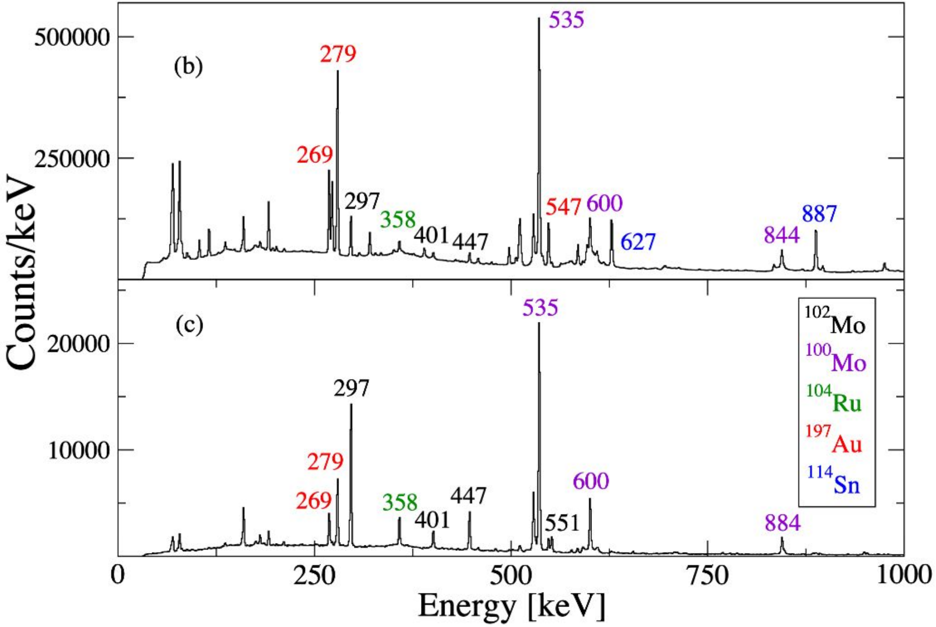


Left: ^{100}Mo target, *Right*: ^{197}Au stopper – after conclusion of beamtime.

SORCERER

- Counts are reduced by a factor of ~20, but ^{102}Mo enhanced over ^{197}Au by ~10



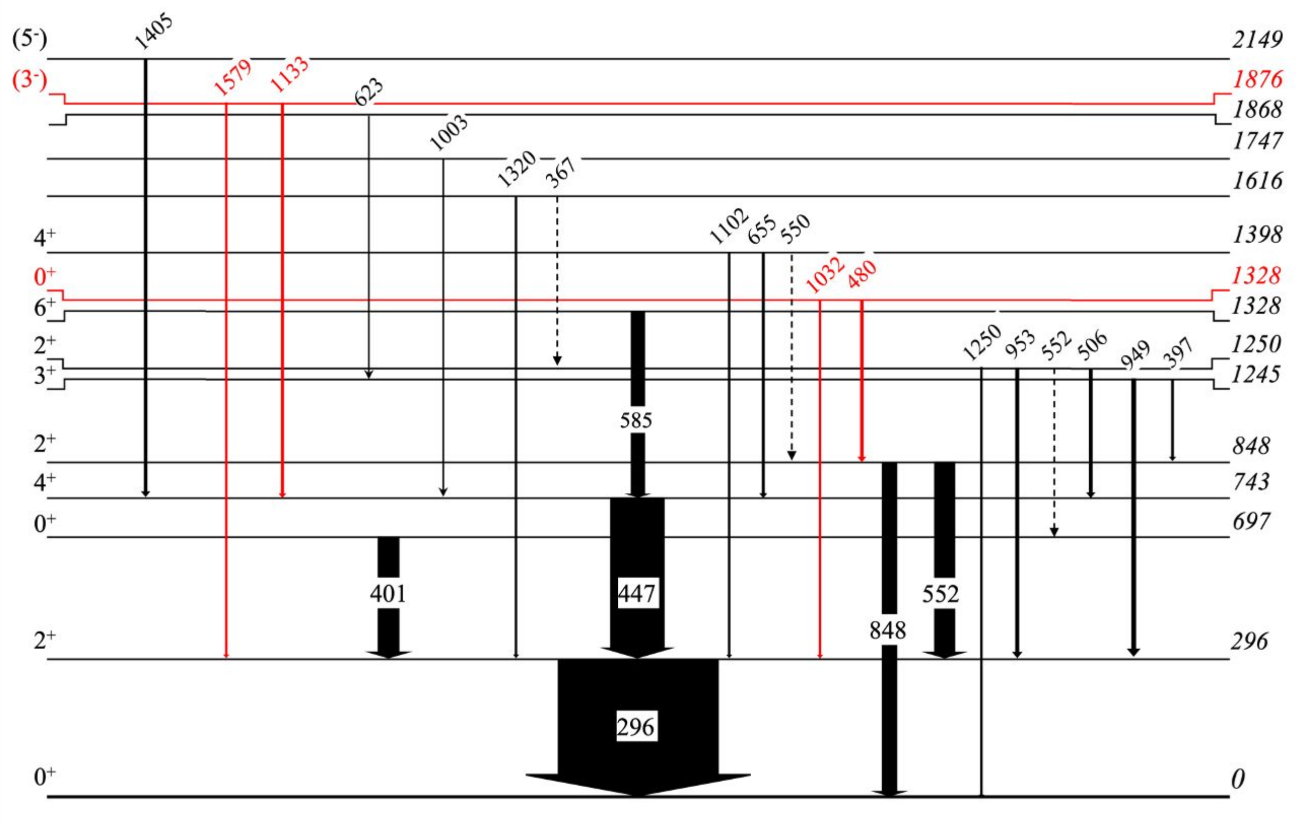


Results and Interpretation

Level Scheme

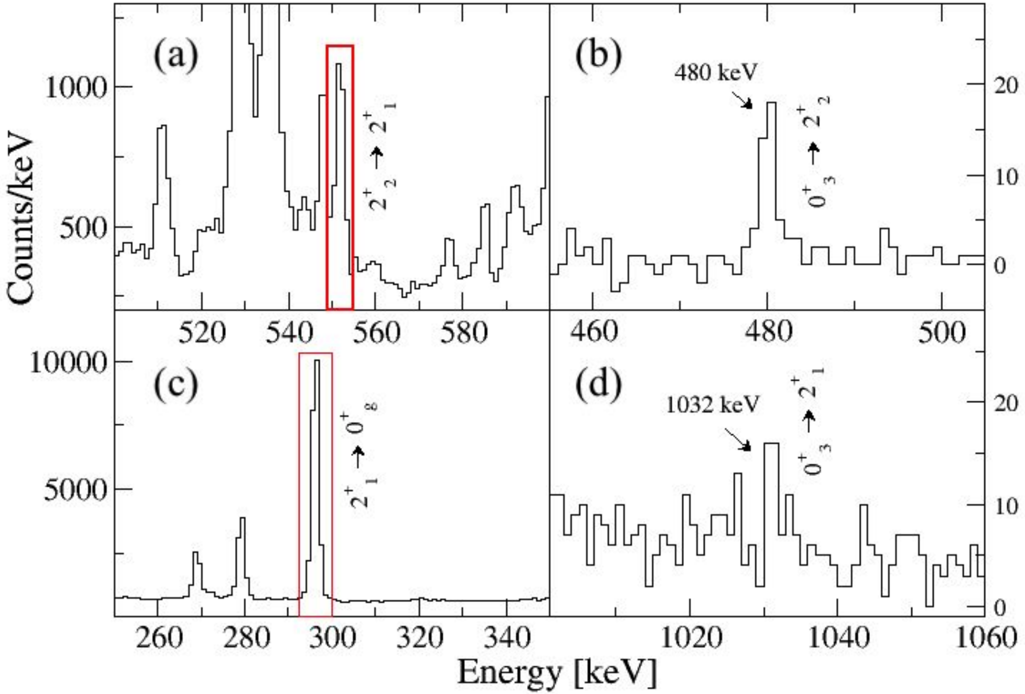
- 24 gamma-rays
(4 new)

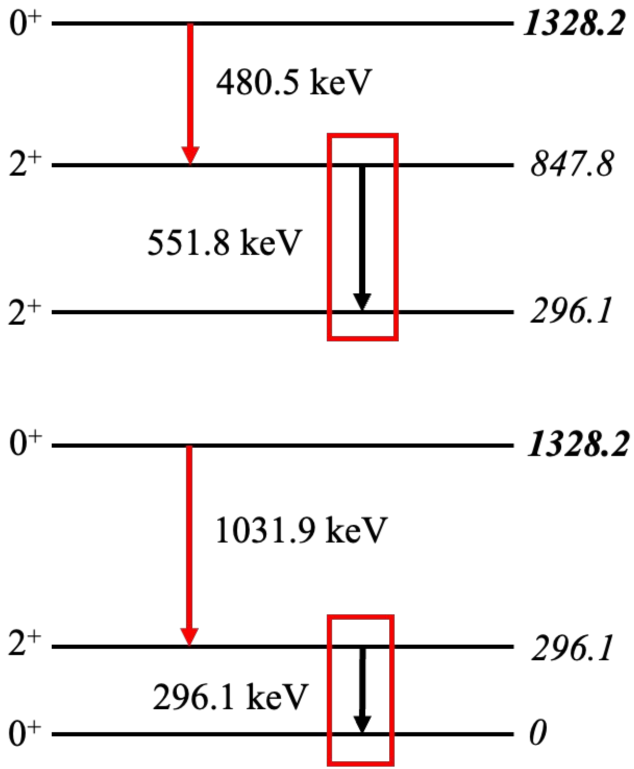
- 14 excited states
(2 new in γ-spec)



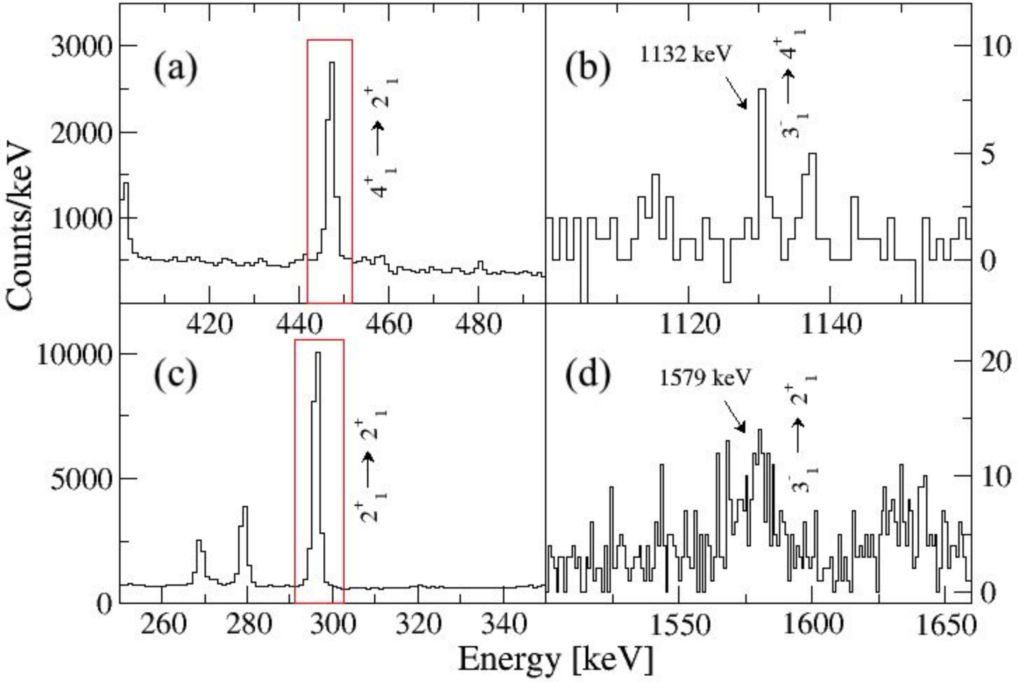
Experimental level scheme – arrow widths proportional to observed intensities

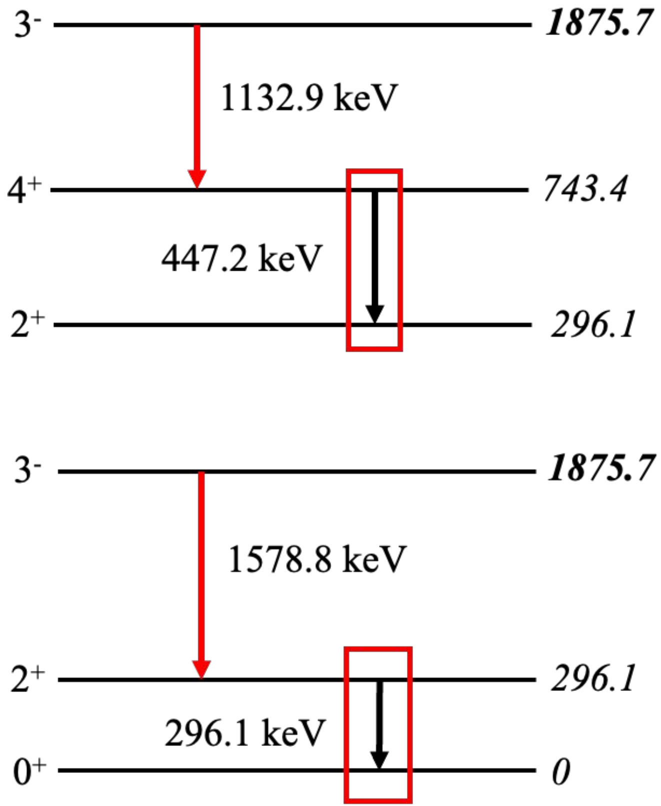
Level Scheme: 0+



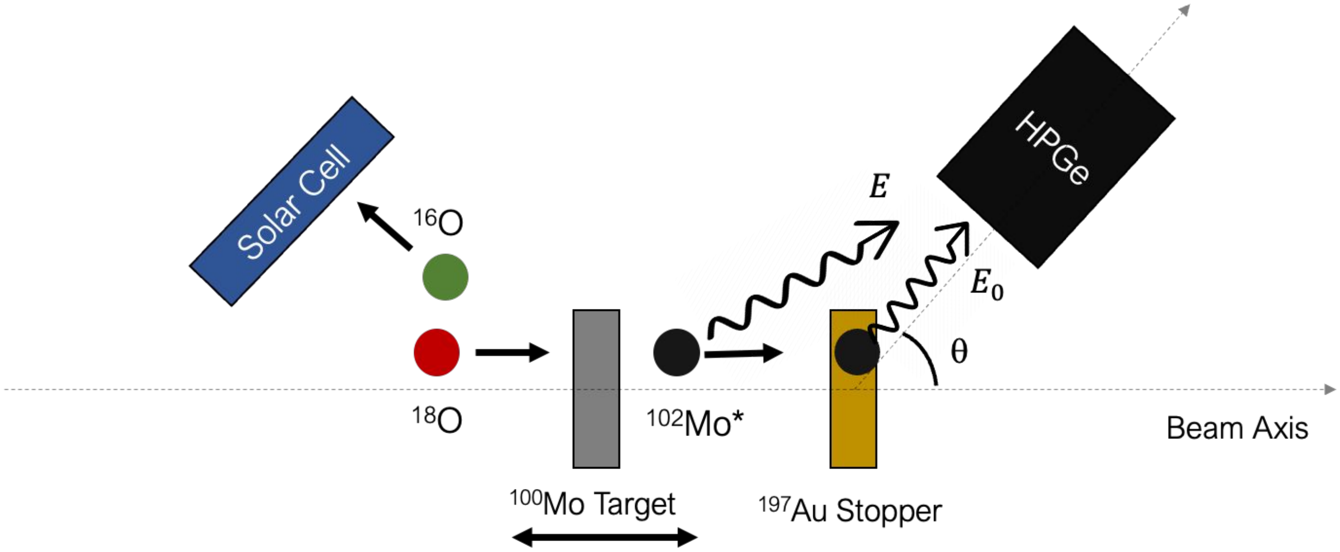


Level Scheme: 3-



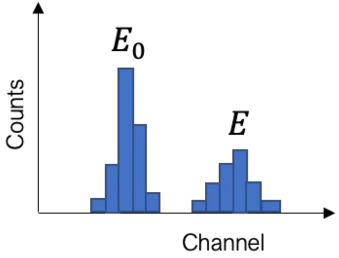


Lifetime Measurements: RDDS

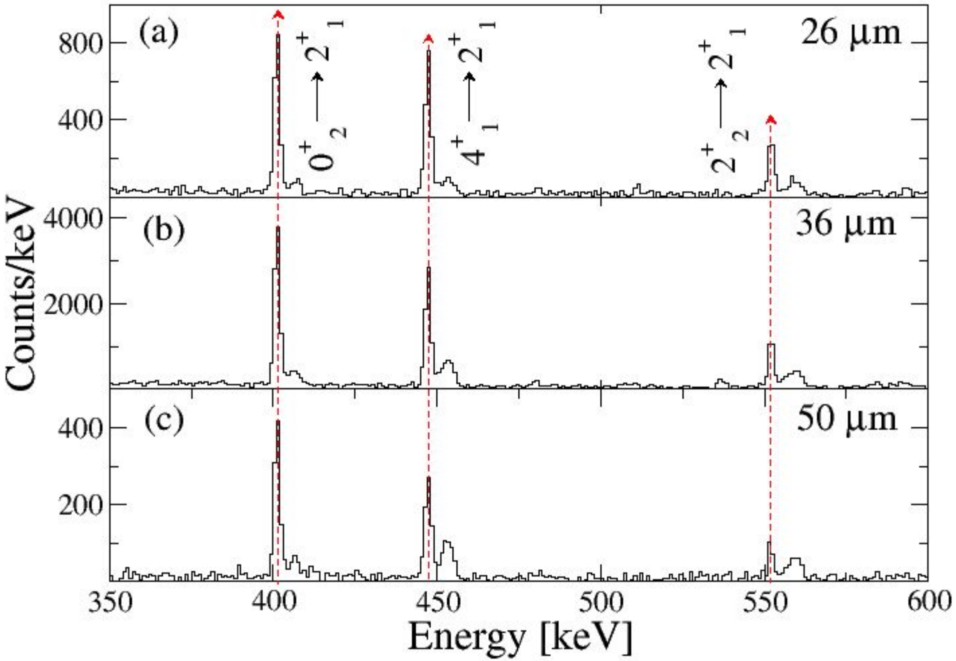


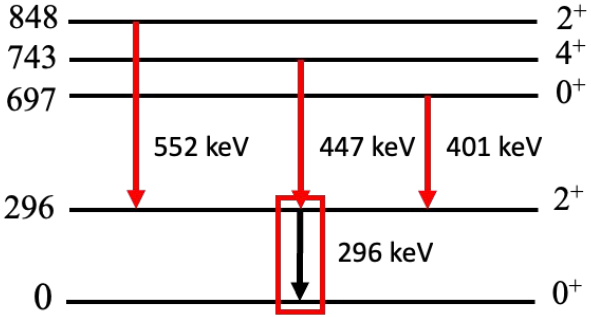
- γ-rays emitted from ^{102}Mo nuclei that decay in flight are Doppler shifted:

$$E = E_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$



Lifetime Measurements: Triplet States



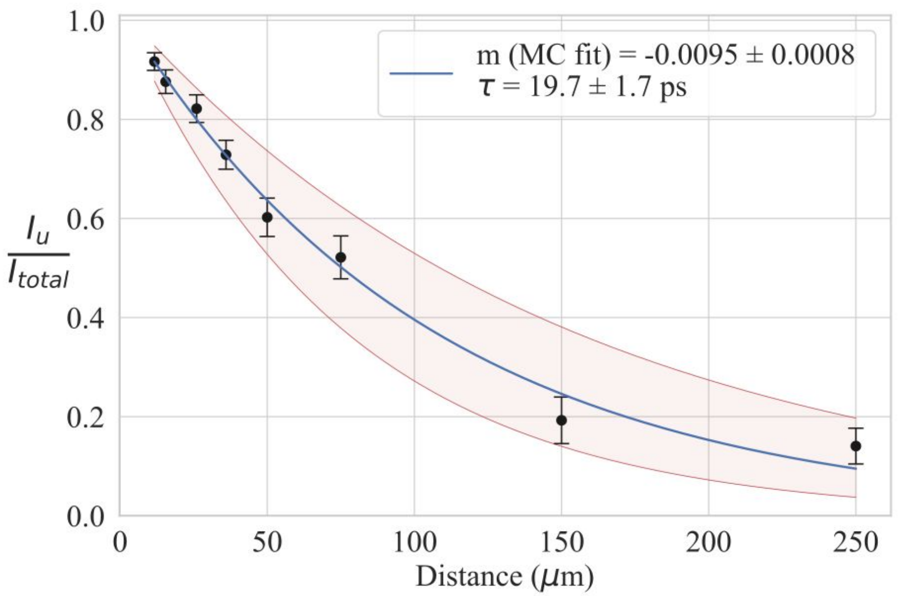


$$\frac{I_{Unshifted}}{I_{Total}} = e^{-\frac{x}{v\tau}}$$

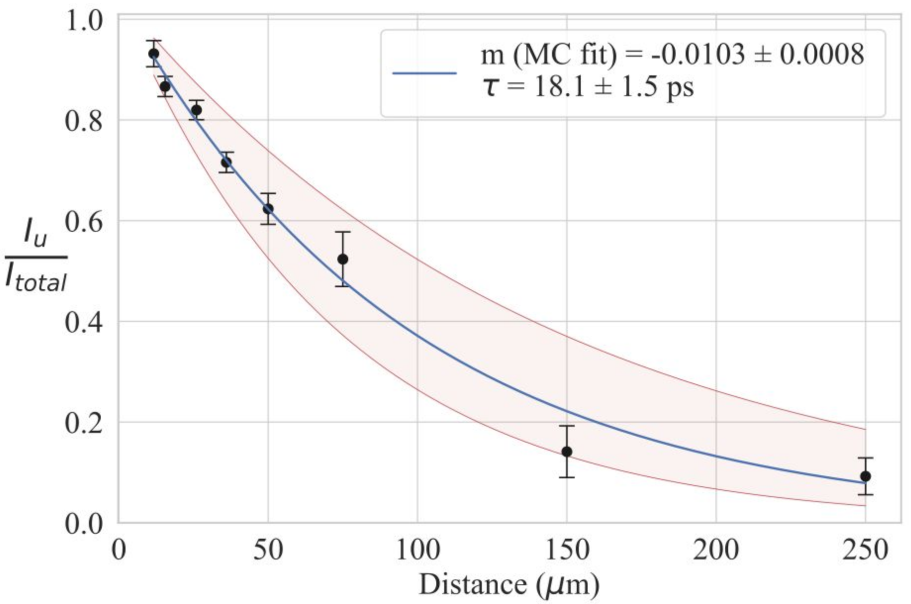
Lifetime Results: 4+1

$\tau(4^+_1)$ = 19 (1) ps

FW detectors (37° ring)



BW detectors (143° ring)



Lifetimes Measured

- 11 lifetimes measured, 4 for the **first time**
- In reasonable agreement with literature, most within 1 or 2σ
- Calculated reduced transition strengths for comparison with calculations

	This work					τ (ps), previous works		
E_{level}	τ (ps)	$J_i^\pi \rightarrow J_f^\pi$	$B(E2)\downarrow$	$B(M1)\downarrow$	$B(E1)\downarrow$	Ref. [24]	Ref. [25]	Ref. [26]
296.1	175(75)	$2_1^+ \rightarrow 0_g^+$	70^{+50}_{-20}			180(6)	$186.9^{+18.3}_{-18.7}$	150(10)
697.3	41(5)	$0_2^+ \rightarrow 2_1^+$	68(8)			40(16)		33(4)
743.4	19(1)	$4_1^+ \rightarrow 2_1^+$	85(4)			18(4)	$27.8^{+10.5}_{-8.3}$	15.9(12)
847.8	13(2)	$2_2^+ \rightarrow 2_1^+$	25(5)[a]	$3.0^{+1.3}_{-1.5}$[a]				10.3(12)
		$2_2^+ \rightarrow 0_g^+$	2.1(5)					
1245.1	9.2(17)	$3_1^+ \rightarrow 2_2^+$	⩽ 82(22)[b]	⩽ 254(69)[b]				$5.5^{+1.0}_{-3.5}$
		$3_1^+ \rightarrow 2_1^+$	3.0(6)[c]	$0.7^{+0.6}_{-0.4}$[c]				
1249.6	7.1(15)	$2_3^+ \rightarrow 4_1^+$	20(4)					
		$2_3^+ \rightarrow 0_2^+$	31(5)					
		$2_3^+ \rightarrow 2_1^+$	⩽ 1.8(3)[b]	⩽ 31(4)[b]				
		$2_3^+ \rightarrow 0_g^+$	0.14(3)					
1327.9	5.1(8)	$6_1^+ \rightarrow 4_1^+$	83(13)				3.2(7)	$6.7^{+0.7}_{-3.1}$
1328.2	3.8(8)	$(0_3^+) \rightarrow 2_2^+$	210^{+70}_{-50}					
		$(0_3^+) \rightarrow 2_1^+$	1.9(8)					
1398.3	7.2(16)	$4_2^+ \rightarrow 2_2^+$	24(6)					< 4
		$4_2^+ \rightarrow 4_1^+$	12(3)[d]	24(4)[d]				
		$4_2^+ \rightarrow 2_1^+$	0.67(19)					
1875.7	2.9(5)	$(3_1^-) \rightarrow 4_1^+$			6.8(20)			
		$(3_1^-) \rightarrow 2_1^+$			1.4(6)			
2148.6	6.0(42)	$(5_1^-) \rightarrow 4_1^+$			2.7^{+63}_{-11}			

IBM Calculations

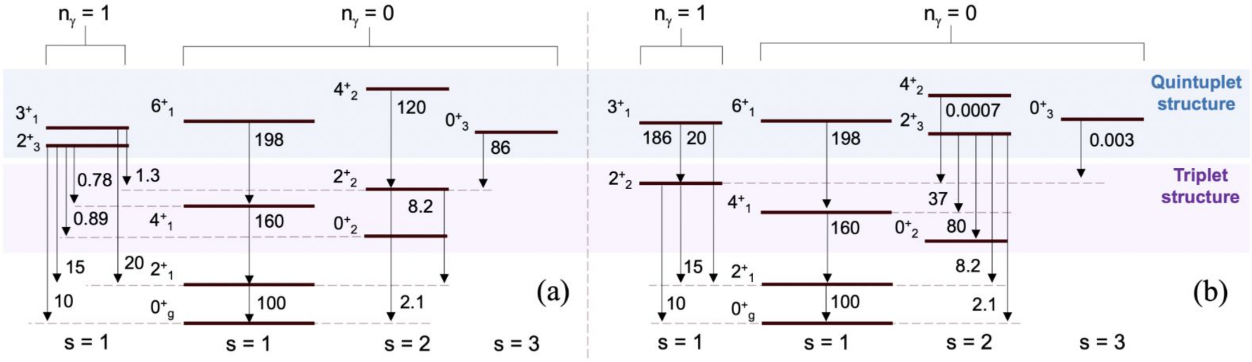
- We want to compare our data to several theoretical calculations

- In the U(5) limit, Hamiltonian reduces to…

$$\hat{H}_{\mathrm{U}(5)} = \epsilon n_d + a_1 \hat{L}\cdot\hat{L} + a_3 \hat{T}_3\cdot\hat{T}_3 + a_4 \hat{T}_4\cdot\hat{T}_4.$$

- Since we know 102Mo is strictly at a limit, it is useful to define a generalised "IBM" Hamiltonian…

$$\hat{H}_{\mathrm{IBM}} = \epsilon n_d + a_0 \hat{P}^\dagger \hat{P} + a_1 \hat{L}\cdot\hat{L} + a_2 \hat{Q}\cdot\hat{Q} + a_3 \hat{T}_3\cdot\hat{T}_3 + a_4 \hat{T}_4\cdot\hat{T}_4,$$

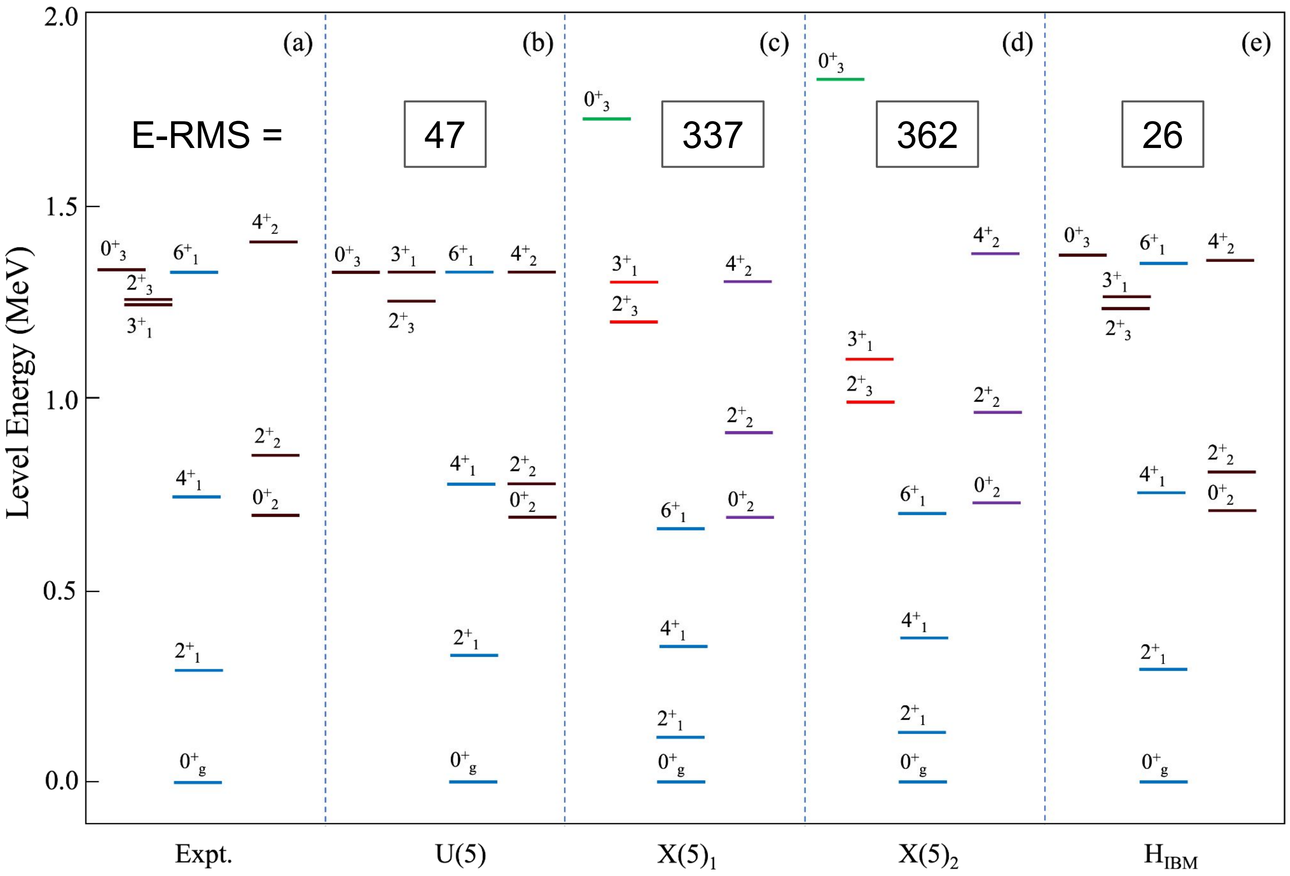


- An X(5) scheme could be configured in many different ways…we pick two

- Level energies given by this Eigenvalue equation

$$E(s, L, n_\gamma, K) = E_0 + B(x_{s,\nu})^2 + A(n_\gamma + 1),$$

IBM Calculations: Level Scheme



- U(5) does well because the lowest-lying structure is fairly quadrupole-vibration-like, but many states degenerate
- E-RMS is smallest for H_{IBM}.
- X(5) struggles to scale yrast and non-yrast schemes together

IBM Calculations: B(E2)s

$$\sigma(E2) = \left[\frac{1}{N} \sum_{i=1}^{N} \left(\frac{B(E2)_i^{\mathrm{expt}} - B(E2)_i^{\mathrm{theo}}}{B(E2)_i^{\mathrm{unc}}} \right)^2 \right]^{1/2},$$

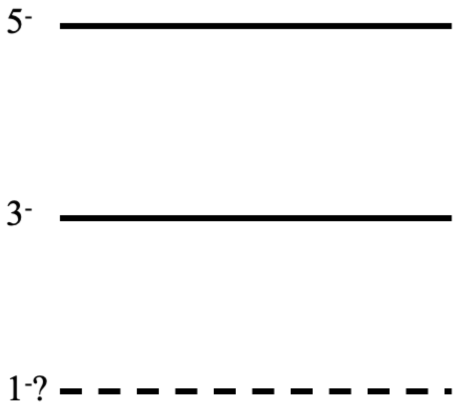
- Calculated B(E2) values using the parameterisations determined from level scheme best-fits
- Defined a value σ(E2) to quantify quality of calculation compared to experiment
- Clear hierarchy is seen, H_{IBM} > $X(5)_1$ > U(5) > $X(5)_2$
  - inclusion of gamma-soft term very important!

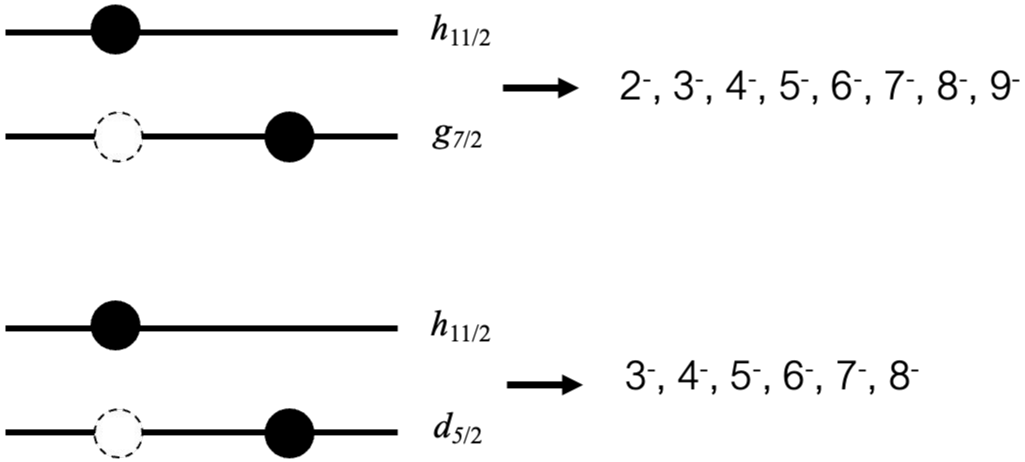
	B(E2)				
$J_i^\pi \rightarrow J_f^\pi$	Expt.	U(5)	$X(5)_1$	$X(5)_2$	IBM
$2_1^+ \rightarrow 0_g^+$	70^{+50}_{-20}	22	49	21	45
$4_1^+ \rightarrow 2_1^+$	85(4)	40	79	34	74
$6_1^+ \rightarrow 4_1^+$	83(13)	52	97	42	87
$0_2^+ \rightarrow 2_1^+$	68(8)	40	31	13	43
$2_2^+ \rightarrow 0_g^+$	2.1(5)	0	1.0	1.8	0.73
$2_2^+ \rightarrow 2_1^+$	25(5)	40	4.0	2.7	26
$4_2^+ \rightarrow 2_1^+$	0.67(19)	0	0.46	0.20	0.04
$4_2^+ \rightarrow 4_1^+$	12(3)	25	3.0	1.3	15
$4_2^+ \rightarrow 2_2^+$	24(6)	27	59	5×10^{-5}	36
$(0_3^+) \rightarrow 2_1^+$	1.9(8)	0	0.29	0.12	0.09
$(0_3^+) \rightarrow 2_2^+$	210^{+70}_{-50}	52	42	4×10^{-3}	27
$2_3^+ \rightarrow 0_g^+$	0.14(3)	0	0.20	0.44	0.24
$2_3^+ \rightarrow 4_1^+$	20(4)	18	0.01	7.7	14
$2_3^+ \rightarrow 0_2^+$	31(5)	24	0.02	17	28
$3_1^+ \rightarrow 2_1^+$	3.0(6)	0	0.31	3.5	1.4
		$\sigma(E2)$			
Triplet transitions		6.43	3.41	7.63	2.49
Quintuplet transitions		3.11	3.73	4.24	2.21
All transitions		4.19	3.52	5.25	2.22

Outlook and Summary

Future: Octupole Collectivity?

- Negative parity states - what is their structure?





- Octupole vibrational band?
- Pair-breaking states?

Summary

- RDDS measurements were carried out in ^{102}Mo following the reaction $^{100}Mo(^{18}O,^{16}O)^{102}Mo$
- 4 new gammas reported for the first time, depopulating the 0^+_3 and 3^-_1 states of interest
- 11 lifetimes have been measured, 4 for the first time - in reasonable agreement with previous experimental data
- Excited states and B(E2) values suggest nucleus lies between U(5) and X(5) symmetries, with possible indications of γ-softness
- In the future…
  - Explorations of Octupole collectivity in the region with 3^- and 5^- lifetimes in neighboring nuclei

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Thank you!

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IFIN-HH

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Themba ABS boratory for Accelerator sed Sciences

Spare

Side-feeding

SIMPLE MONTE-CARLO METHODS IMPLEMENTED IN ROOT FOR FITS USING PARAMETERS WITH ERRORS

Lucian Stan[1] and Cristina Clisu[3]

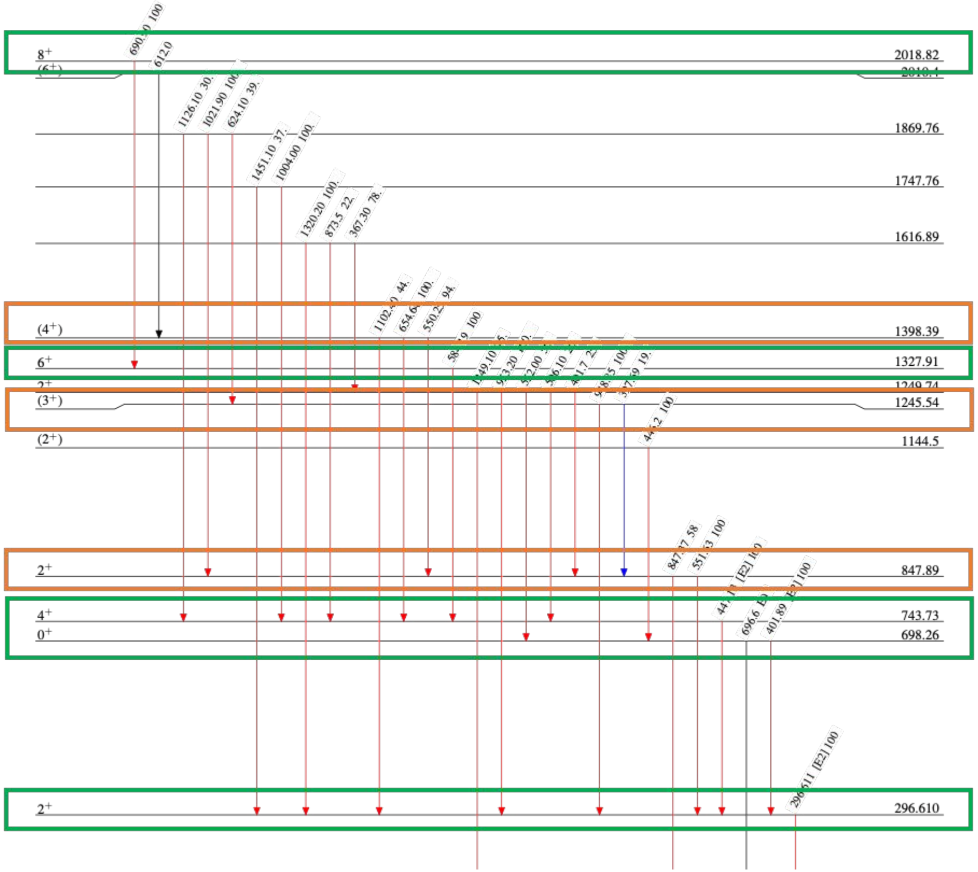
$$N_1(t_f) = N_1(0) * e^{-\frac{t_f}{\tau_1}} + N_0(0) * \frac{\tau_1}{\tau_0 - \tau_1} * (e^{-\frac{t_f}{\tau_0}} - e^{-\frac{t_f}{\tau_1}}) \qquad (2)$$

$$N_2(t_f) = N_2(0) * e^{-\frac{t_f}{\tau_2}} + N_1(0) * \frac{\tau_2}{\tau_1 - \tau_2} * (e^{-\frac{t_f}{\tau_1}} - e^{-\frac{t_f}{\tau_2}}) +$$
$$+ N_0(0) * \frac{\tau_2}{(\tau_1 - \tau_2)(\tau_0 - \tau_2)(\tau_0 - \tau_1)} * (\tau_0(\tau_1 - \tau_2) e^{-\frac{t_f}{\tau_0}} - \tau_1(\tau_0 - \tau_2) e^{-\frac{t_f}{\tau_1}} + \tau_2(\tau_0 - \tau_1) e^{-\frac{t_f}{\tau_2}}) \qquad (3)$$

Molybdenum 102

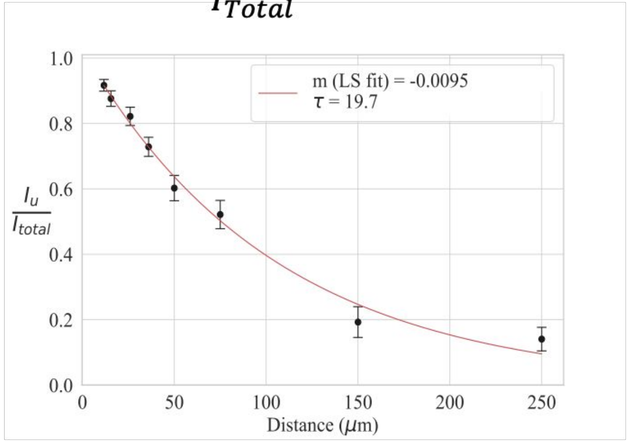
- Relatively little lifetime information **(less when we started!)**

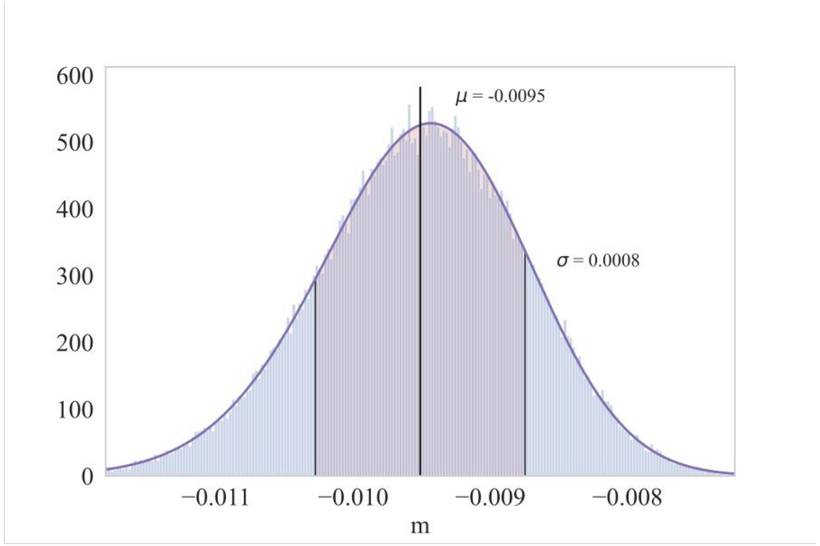
E_{level} (keV)	τ (ps)
2_1^+: 296.61	180(6) [2], $186.9^{+18.3}_{-18.7}$ [3], 150(10) [6]
0_2^+: 698.26	40(16) [2], 33(4) [6]
4_1^+: 743.73	18(4) [2], $27.8^{+10.5}_{-8.3}$ [3], 15.9(12) [6]
2_2^+: 847.89	10.3(12) [6]
3_1^+: 1245.54	$5.5^{+1.0}_{-3.5}$ [6]
6_1^+: 1327.91	3.2(7) [3], $6.7^{+0.7}_{-3.1}$ [6]
4_2^+: 1398.39	<4 [6]
8_1^+: 2018.82	2.7(4) [4]
(10_1^+): 2790.3	1.5(2) [4]
(12_1^+): 3632.3	0.95(13) [4]



Lifetime Results: Monte Carlo simulation

$$\frac{I_{Unshifted}}{I_{Total}} = e^{-\frac{x}{v\tau}}$$





- At each distance x_i, a value for y is sampled from a normal distribution with $\mu = y_i$, $\sigma = dy_i$
- Least-squares fit is performed for each set of values, determining m (= - 1 / vτ)
- Repeated 10^5 times – set of *m* values fitted with a gaussian, we take $\mu_m \pm \sigma_m$ to determine τ