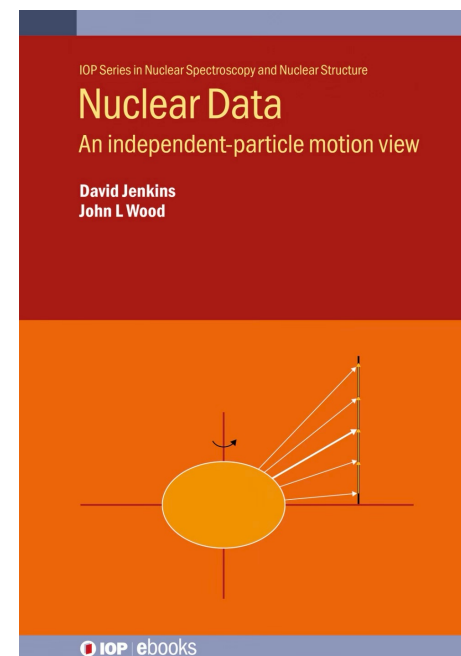
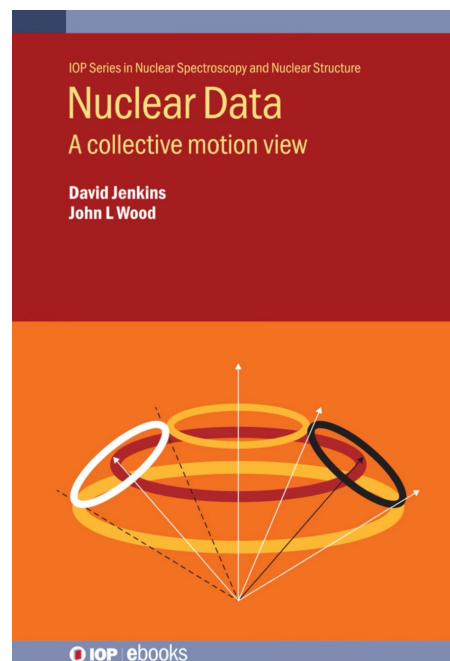
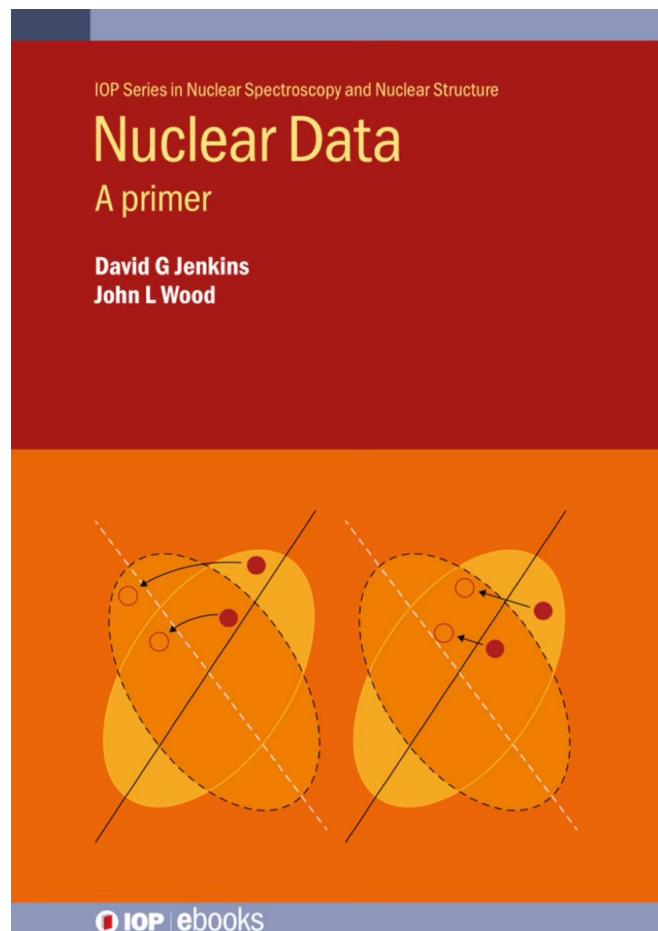


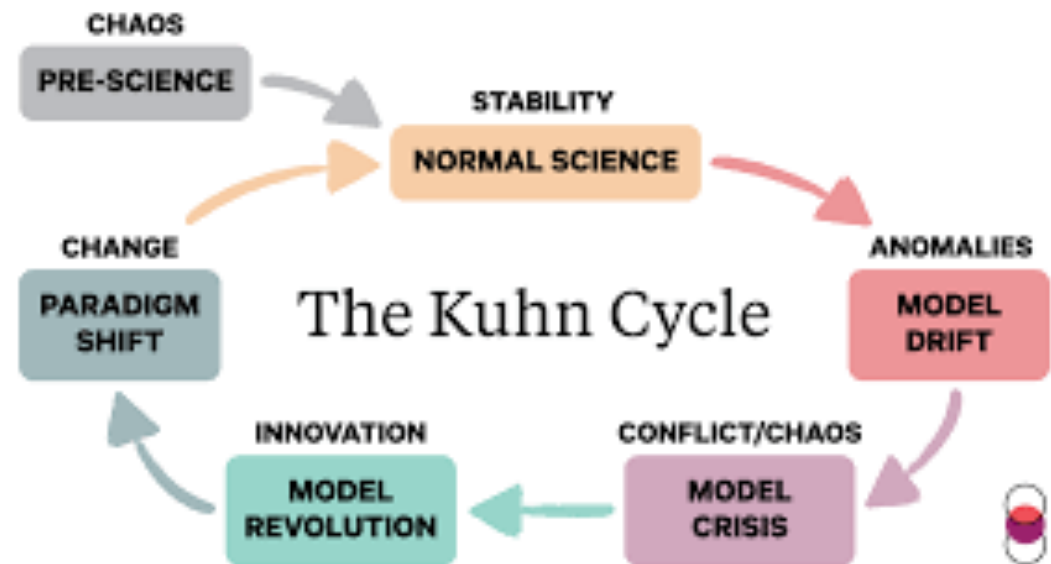
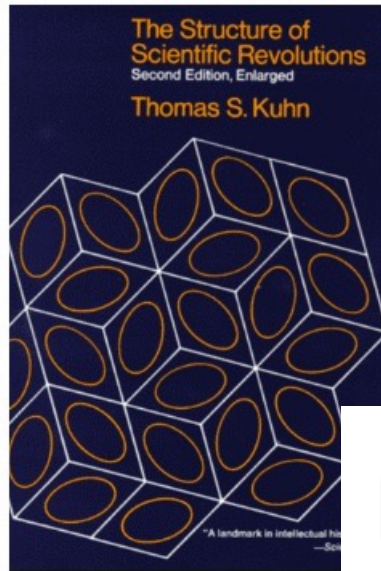


# Data driven approaches to learning about nuclear structure

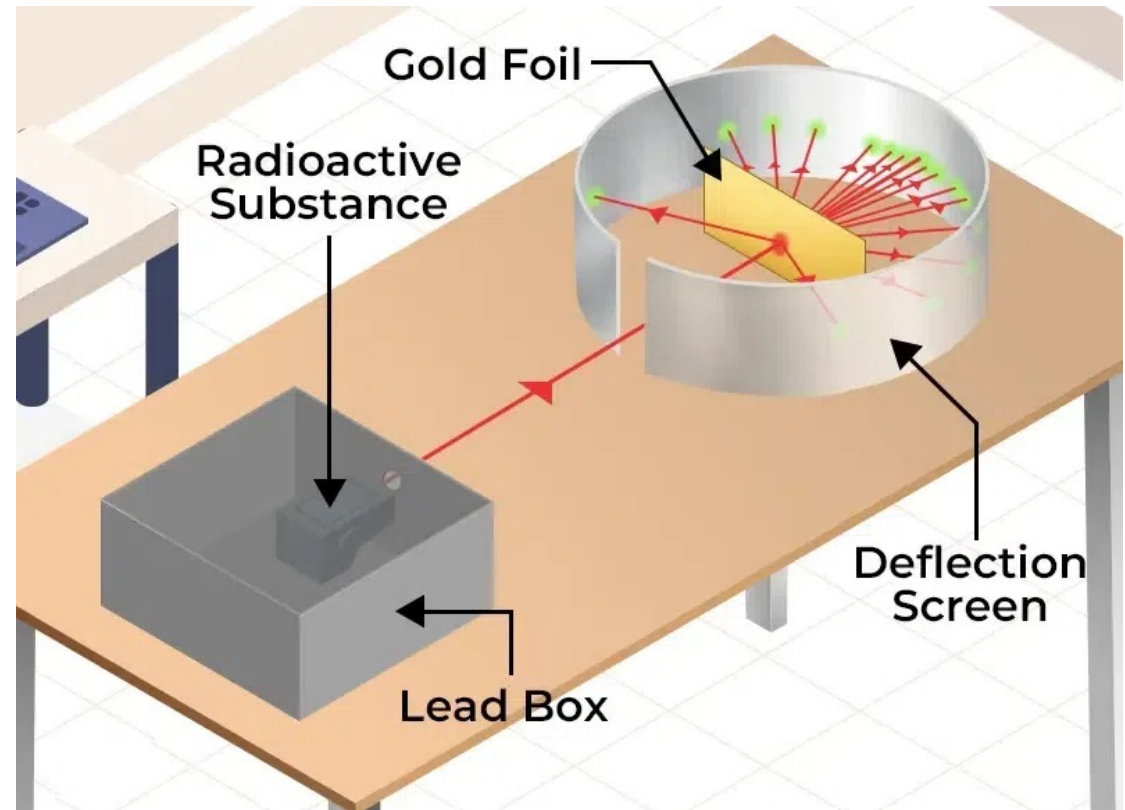
David Jenkins



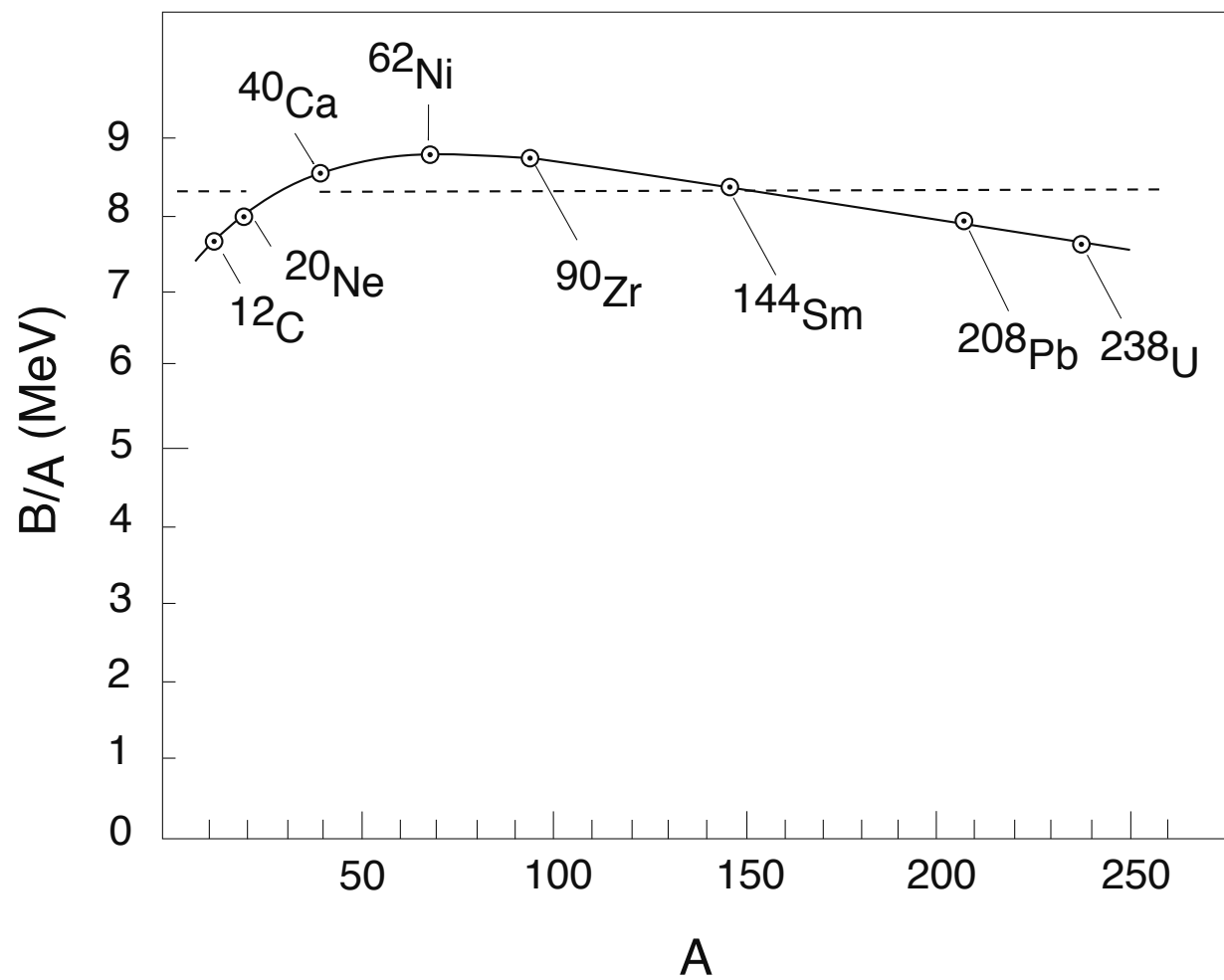




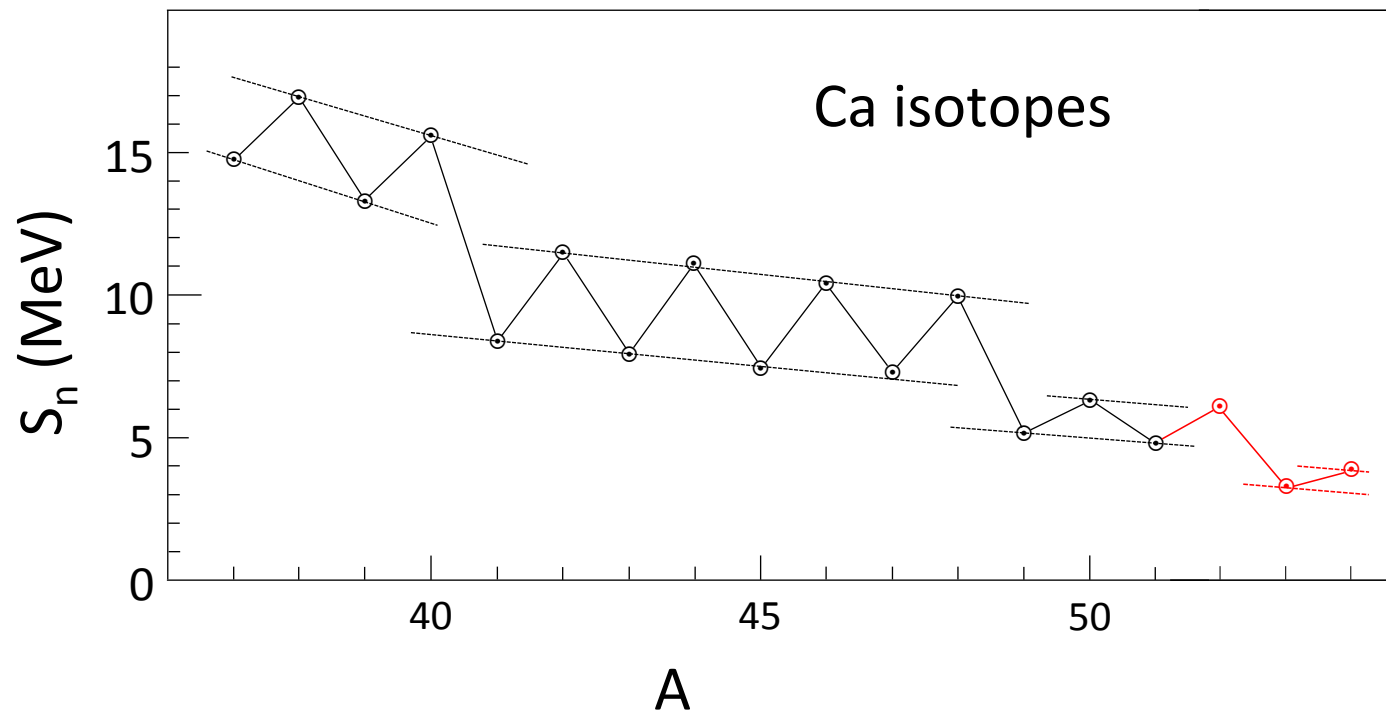
## Paradigm Shift Number 1: Rutherford Scattering



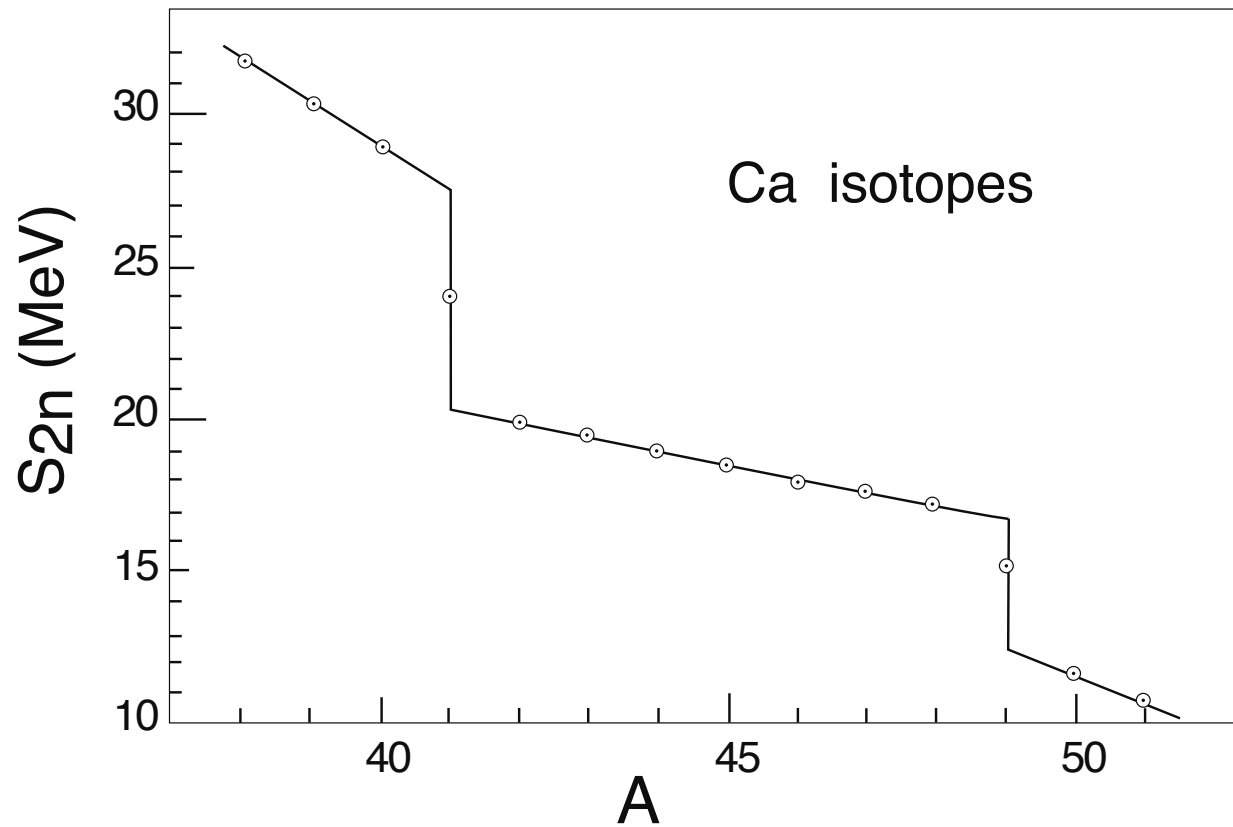


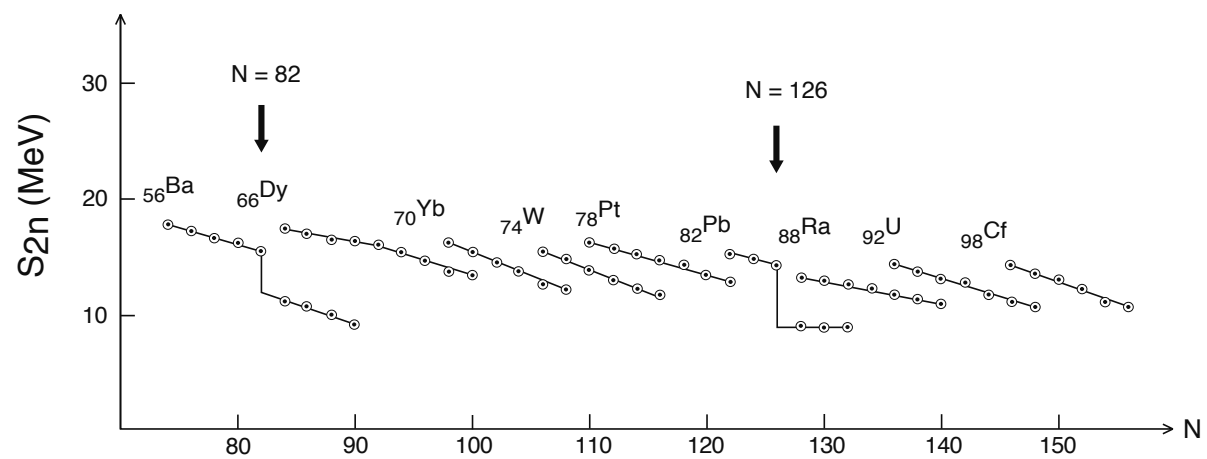
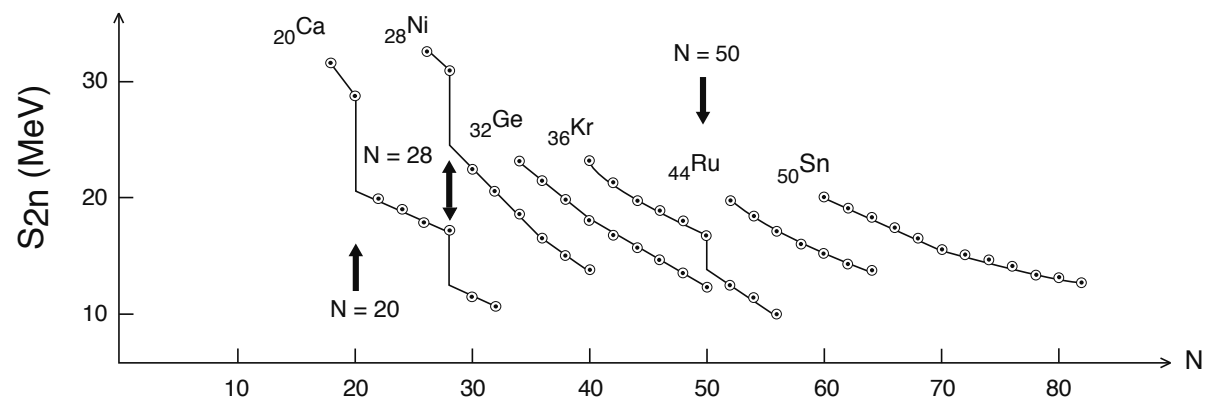


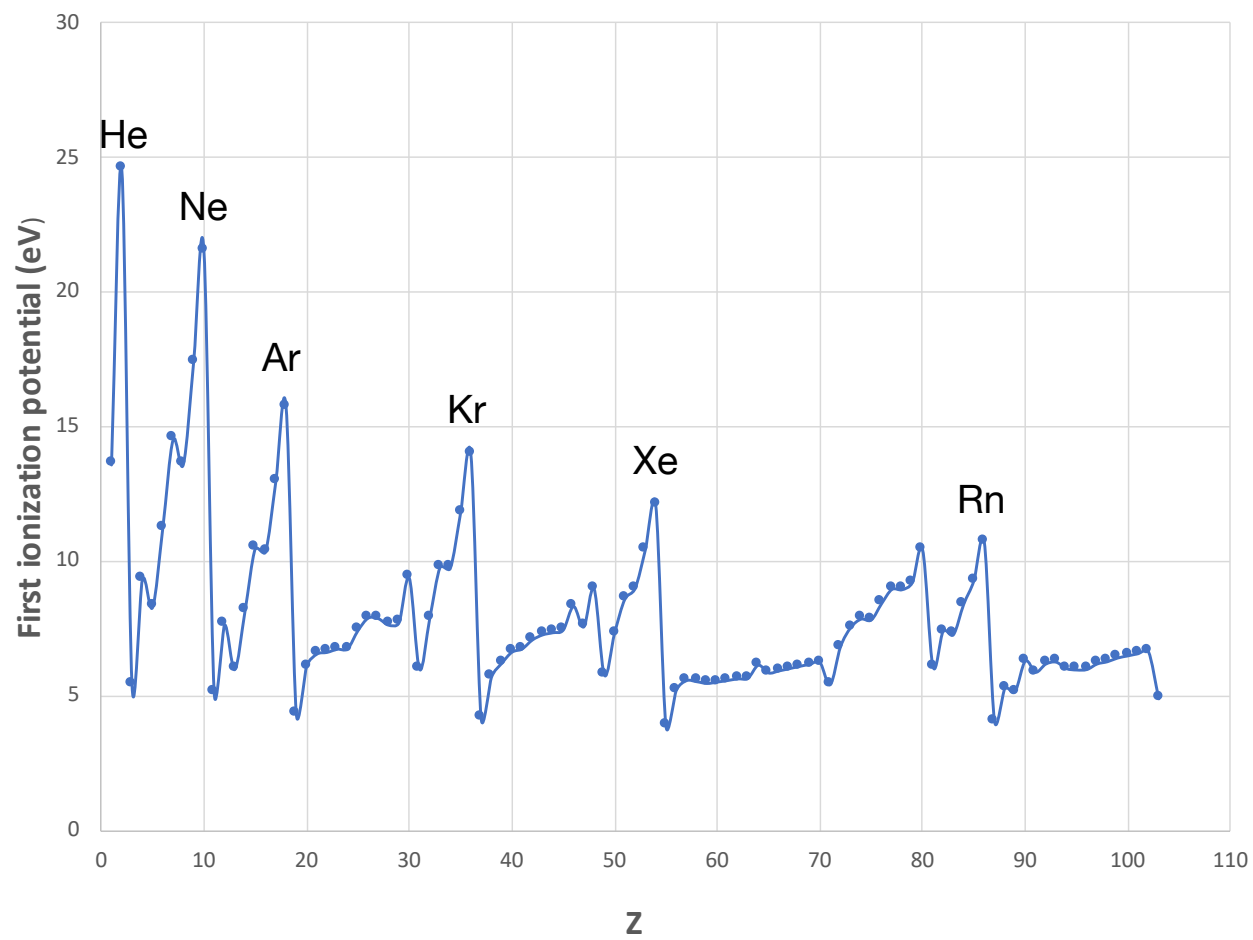
$$S_n = -M(A, Z, N) + M(A - 1, Z, N - 1) + m_n$$



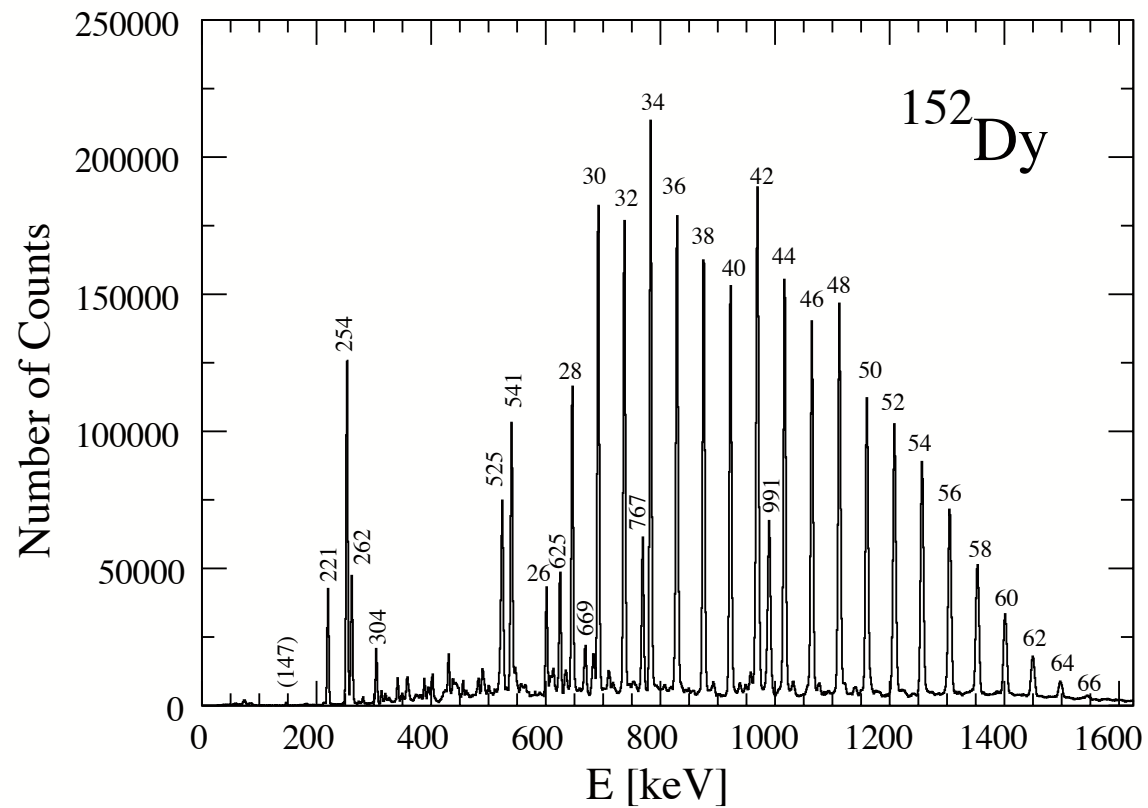
## Paradigm Shift Number 2: Shell structure and spin-orbit interaction





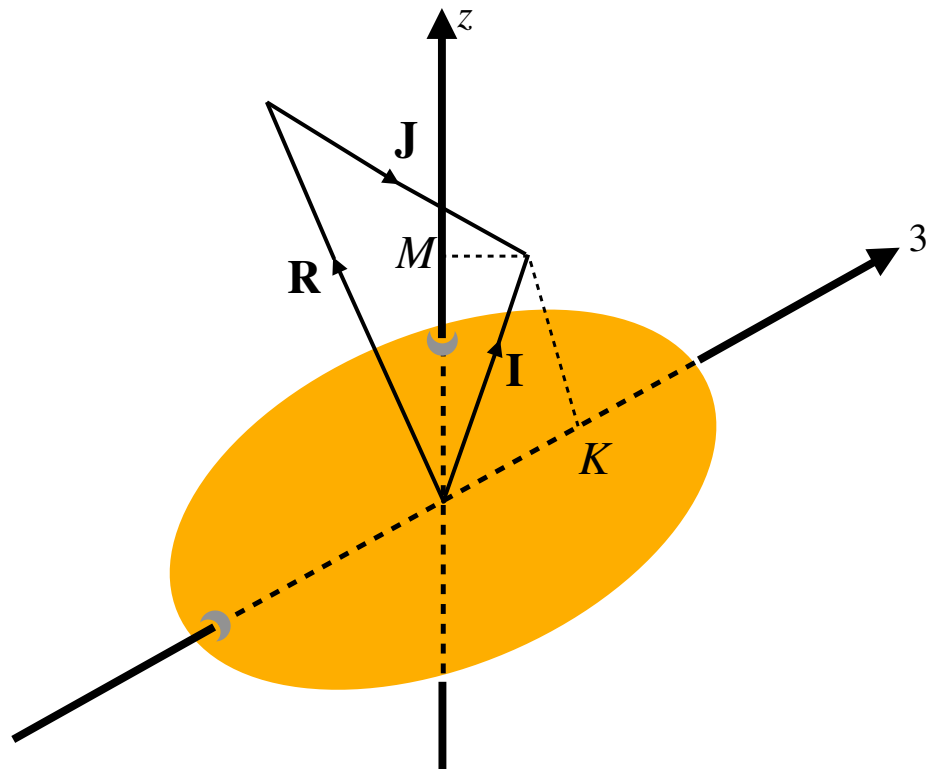


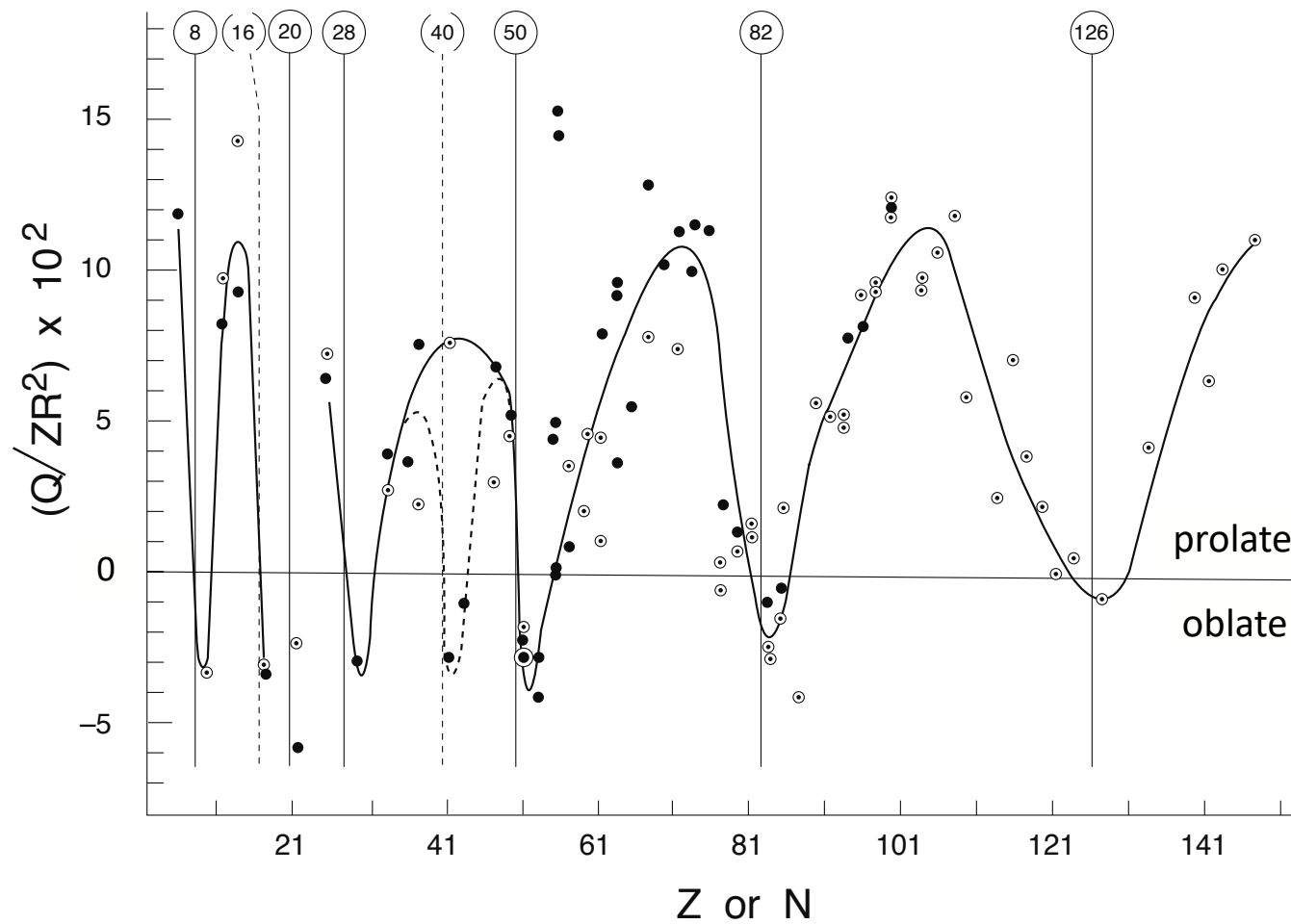
## Paradigm Shift Number 3: Superdeformation



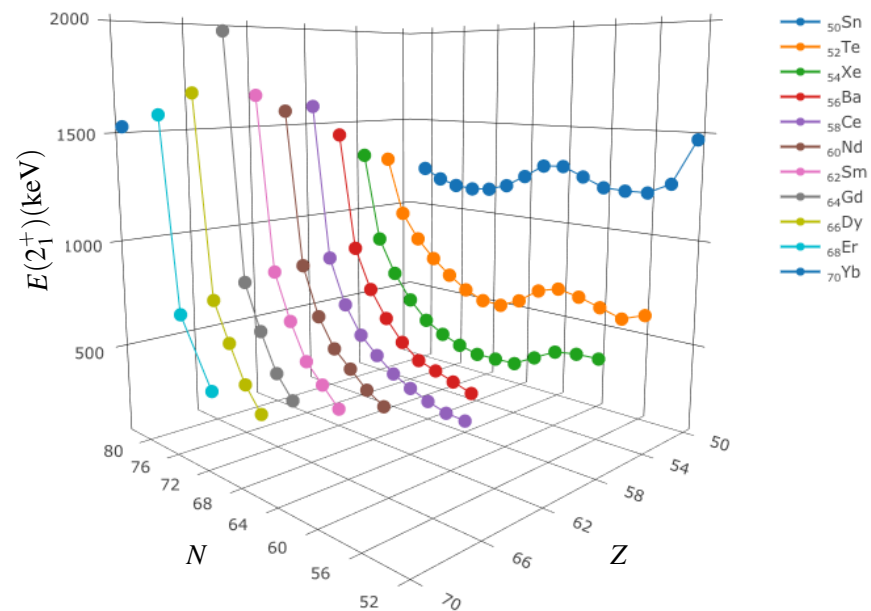
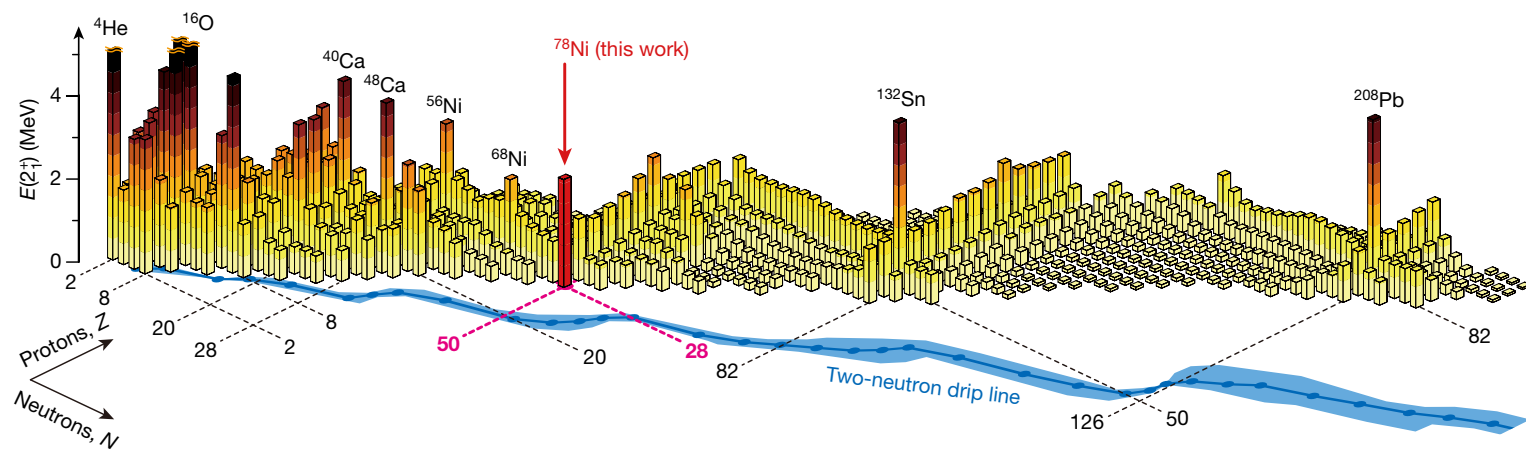


# How well defined are rotations in nuclei?





- NB  $Q$  is only defined for states with  $J \geq 1$



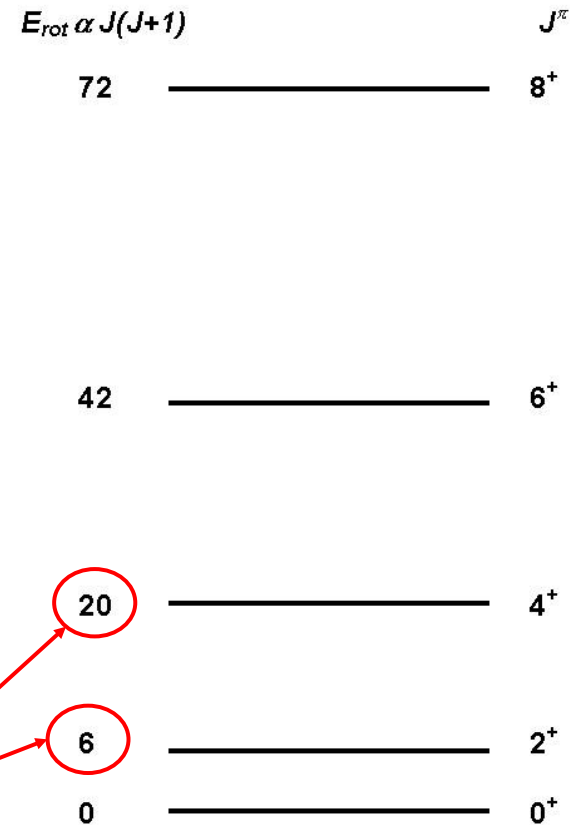
# Rotational bands even-even nuclei

**For the special case of even-even nuclei only even J values are allowed** in a rotational band built upon the  $J^\pi=0^+$  ground states (due to some symmetry properties)

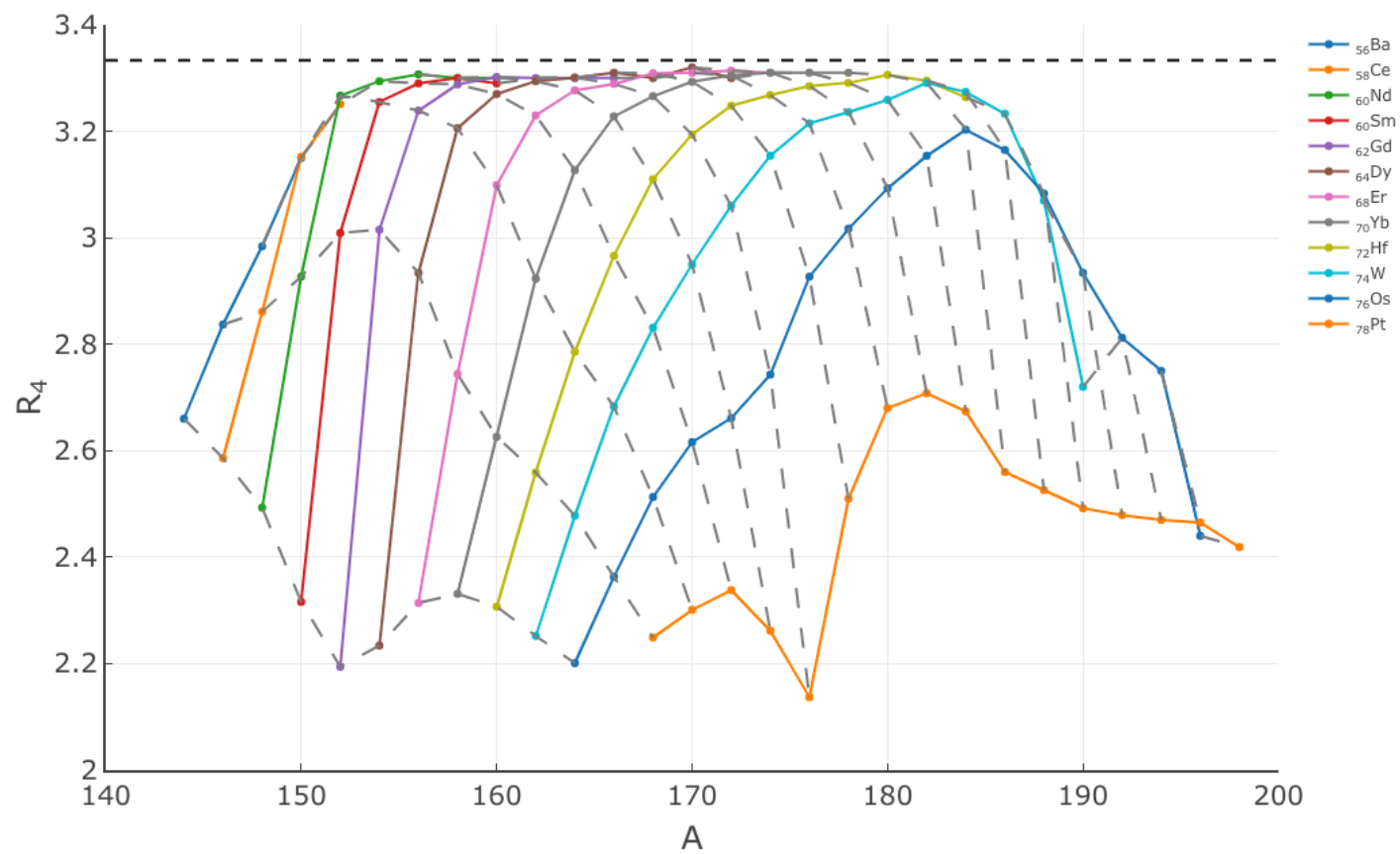
Thus, for an even-even nucleus with a fixed deformation (i.e. a fixed moment of inertia) we expect to find a level scheme like this

A structure of excited states such as this is known as a **rotational band, states within the band are known as “band members” and the lowest-E state is the “band head”**

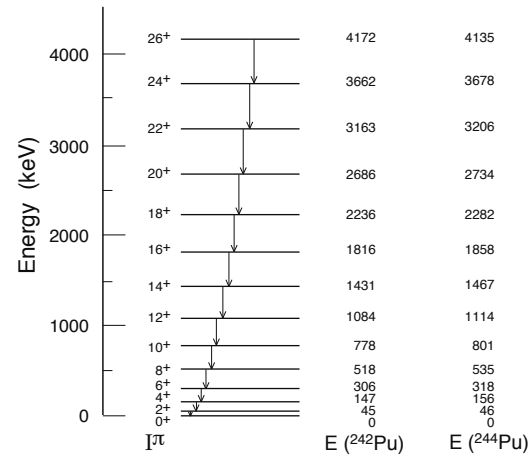
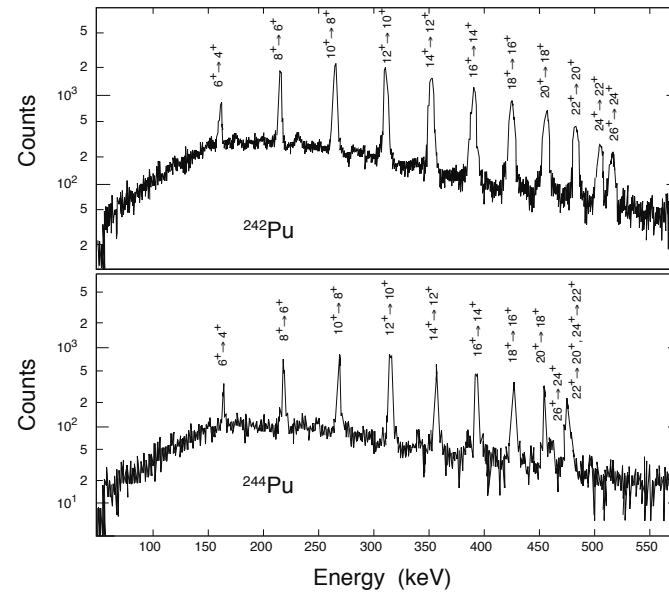
**So we expect  $E(4_1^+)/E(2_1^+) \approx 3.33$**



$$R_4 = E(4^+)/E(2^+)$$

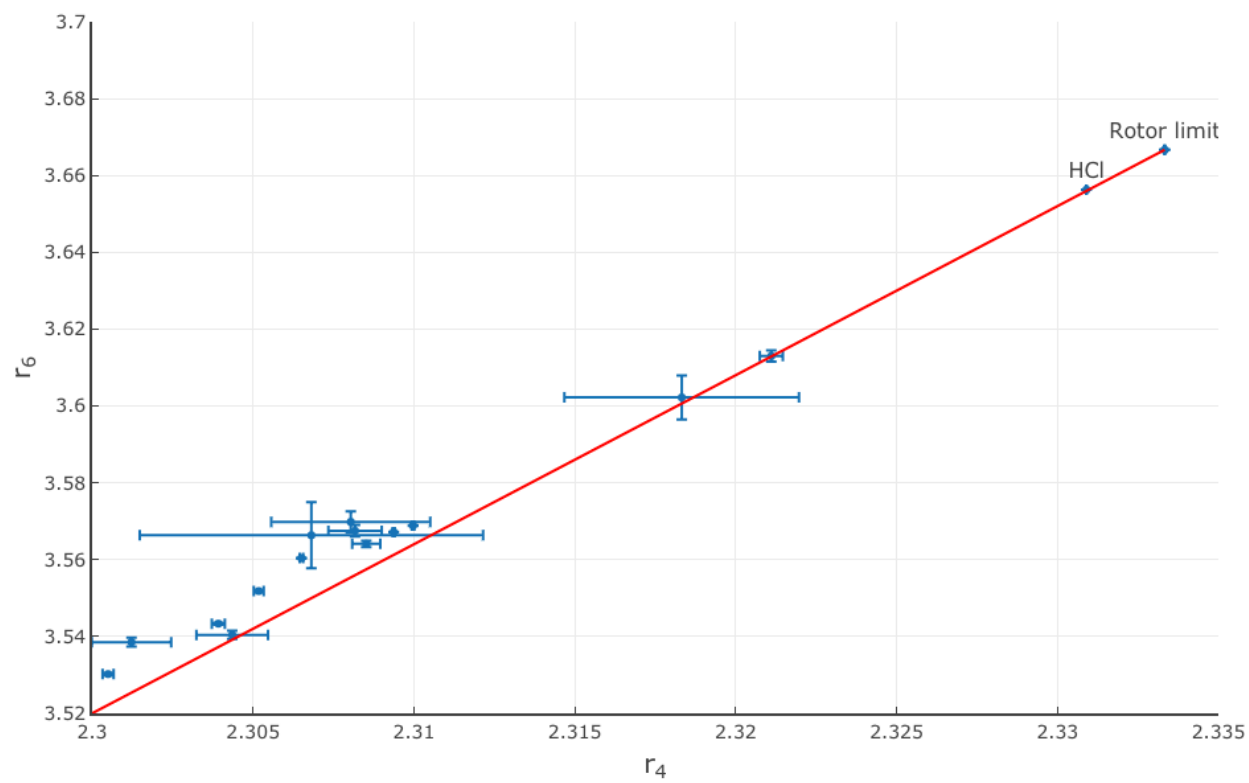


$$E = \frac{\hbar^2 I(I+1)}{2 \mathcal{J}}$$

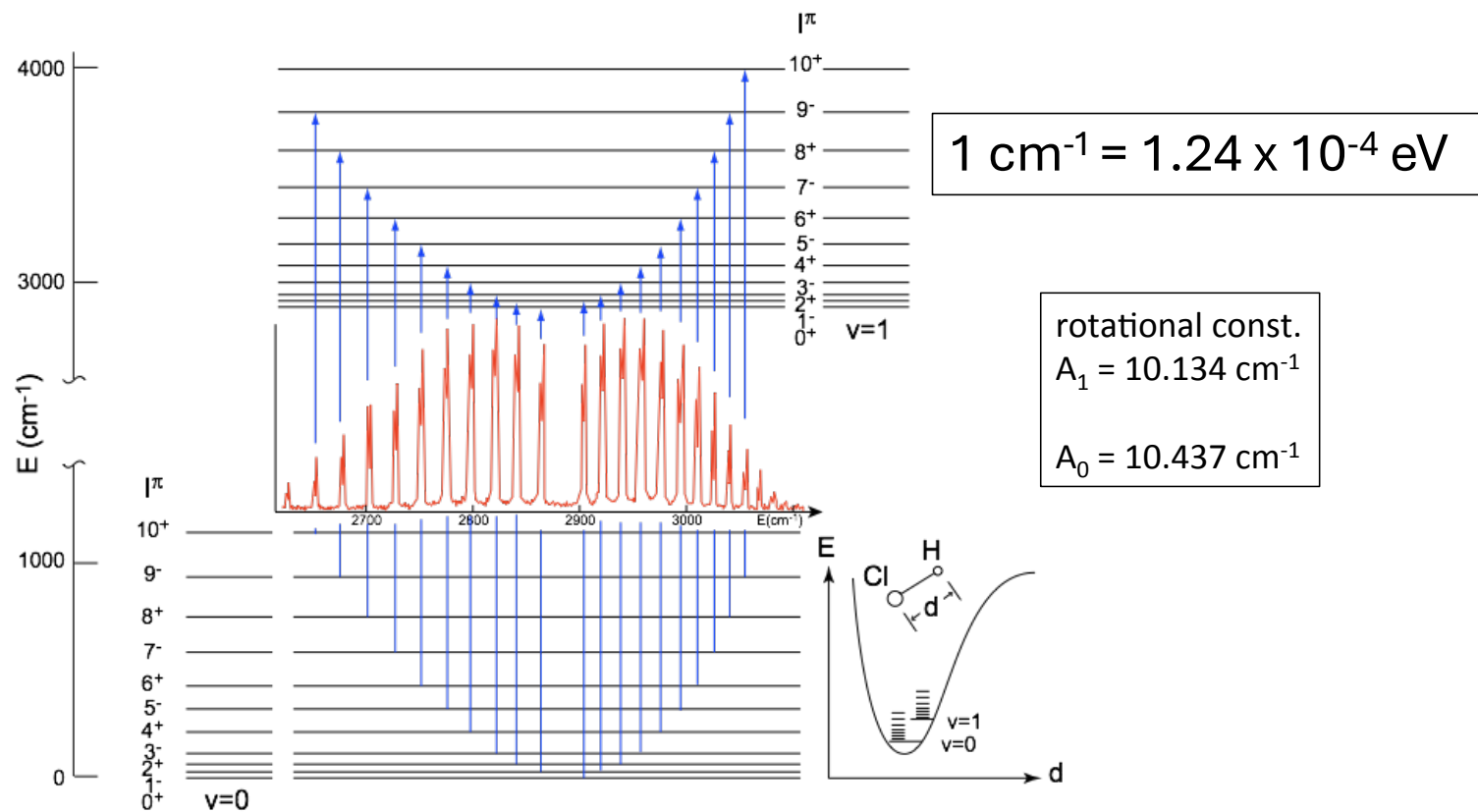




$$r_6 := \frac{E(6_1^+) - E(4_1^+)}{E(2_1^+)} \text{ vs. } r_4 := \frac{E(4_1^+) - E(2_1^+)}{E(2_1^+)},$$



The infrared absorption spectrum of HCl reveals molecular vibrations and rotations.

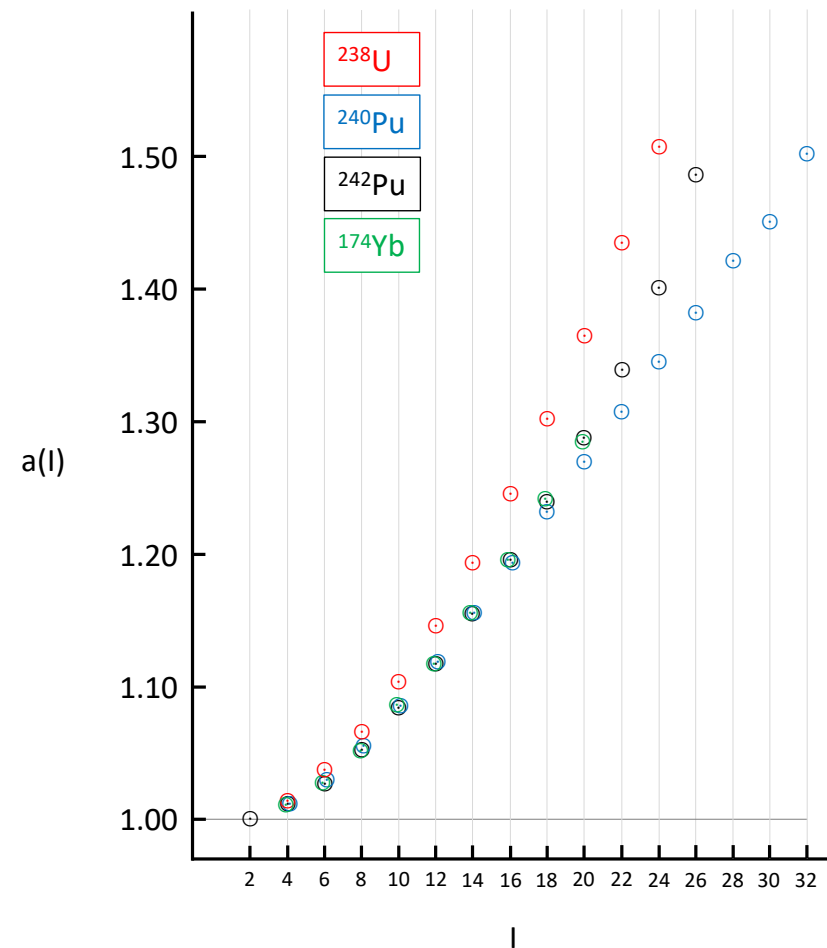


We can also explore the deviation from rotor behaviour as a function of  $I$  to very large values of angular momentum using the rotational parameter:

$$a(I) := [E_\gamma(I \rightarrow I - 2)/(4I - 2)]/[E_\gamma(2 \rightarrow 0)/6]$$

This parameter should be unit for all values of  $I$  if the rotor model were perfectly respected.

For discussion: what is fascinating is that the rotational parameter for  $^{174}\text{Yb}$  and  $^{242}\text{Pu}$  are almost identical despite the very large difference in number of protons and neutrons making up the nucleus!



See just how similar  $^{242}\text{Pu}$  and  $^{174}\text{Yb}$  are when energies are scaled

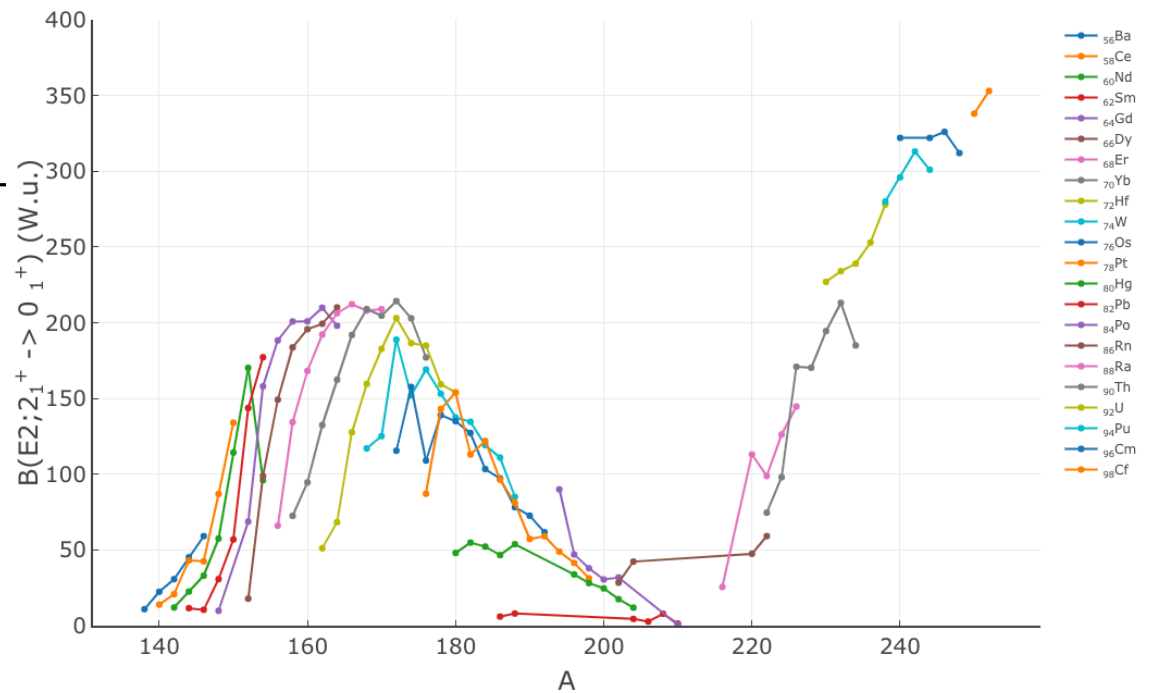
$I_i$	$E(^{242}\text{Pu})$ (keV)	$E(^{174}\text{Yb})$ (keV) $\times 0.5824$	$E(^{174}\text{Yb})$ (keV)	% dev.
2	44.54 <sup>2</sup>	44.54 [norm.]	76.471 <sup>1</sup>	-
4	102.8 <sup>1</sup>	102.9	176.645 <sup>2</sup>	+0.098
6	159.0 <sup>1</sup>	158.9	272.918 <sup>6</sup>	-0.063
8	211.7 <sup>4</sup>	211.8	363.64 <sup>5</sup>	+0.047
10	260.5 <sup>6</sup>	260.4 <sup>6</sup>	447.2 <sup>10</sup>	-0.038
12	305.8 <sup>8</sup>	305.4 <sup>8</sup>	524.4 <sup>13</sup>	-0.131
14	347.3 <sup>10</sup>	347.1 <sup>10</sup>	595.9 <sup>17</sup>	-0.058
16	385.0 <sup>11</sup>	384.4 <sup>11</sup>	660 <sup>2</sup>	-0.156
18	419.3 <sup>12</sup>	418.7 <sup>17</sup>	719 <sup>3</sup>	-0.143
20	450.2 <sup>13</sup>	450.8 <sup>29</sup>	774 <sup>5</sup>	+0.133
				-0.035 (avg.)

Transition strengths indicate how probable an electromagnetic decay is. Weisskopf made an estimate for the decay strength for a single proton transition. This is our yardstick for transition strengths - the Weisskopf unit (W.u.).

We can calculate transition strengths in W.u. for E2 transitions from the following formula:

$$B(E2) = \frac{9527}{E_\gamma^5 T_{1/2}(\gamma) A^{4/3}}$$

where  $E_\gamma$  is in MeV and  $T_{1/2}$  in ps.



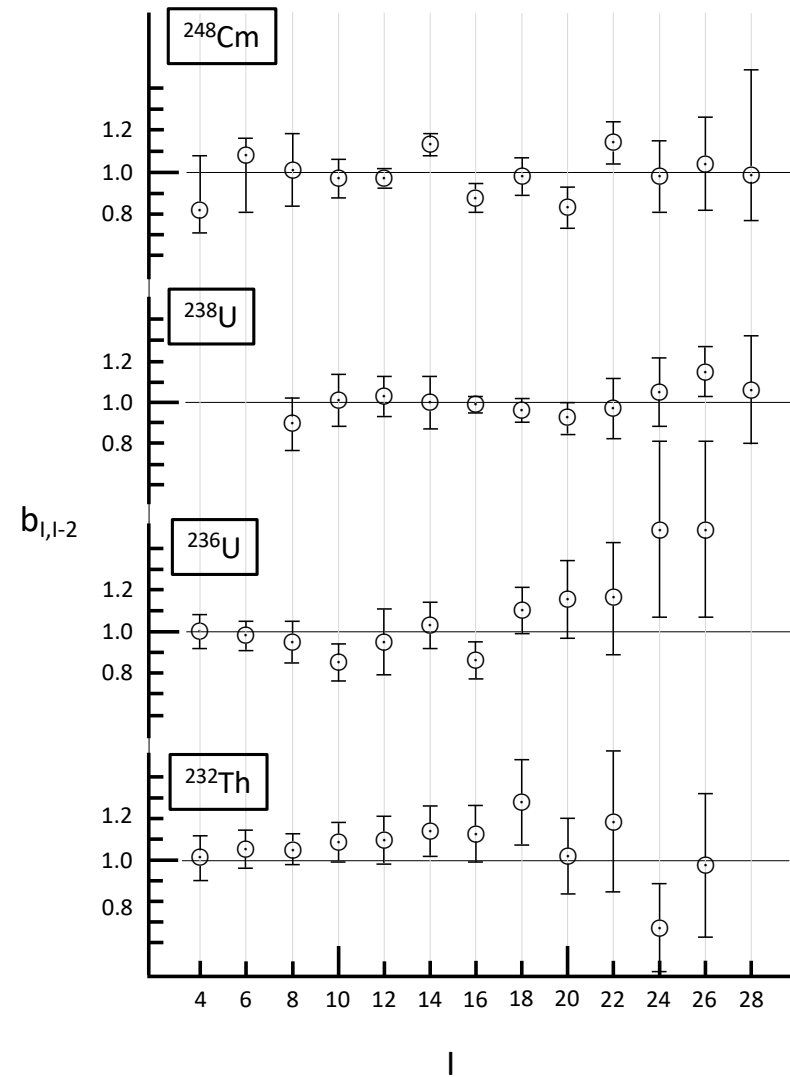
There should be a fixed relationship between B(E2) transition strengths up a rotational band

$$\frac{B_{I,I-2}}{B_{20}} = \frac{15I(I-1)}{2(2I-1)(2I+1)} := f(I)$$

We then define

$$b_{I,I-2} := B_{I,I-2}/B_{20}f(I)$$

which should be unity for all values of I if the rotor model is exactly observed. The plot of  $b_{I,I-2}$  shows that this is well reproduced for actinide nuclei.

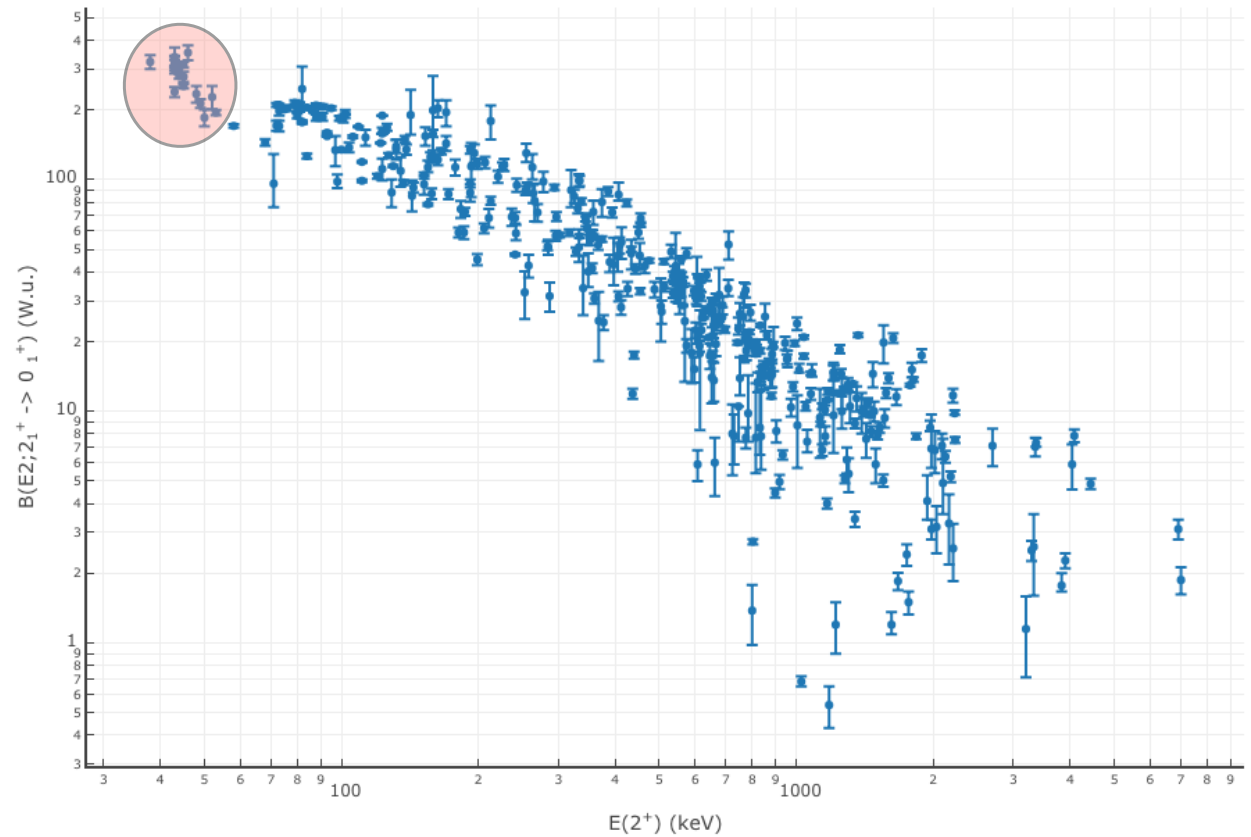


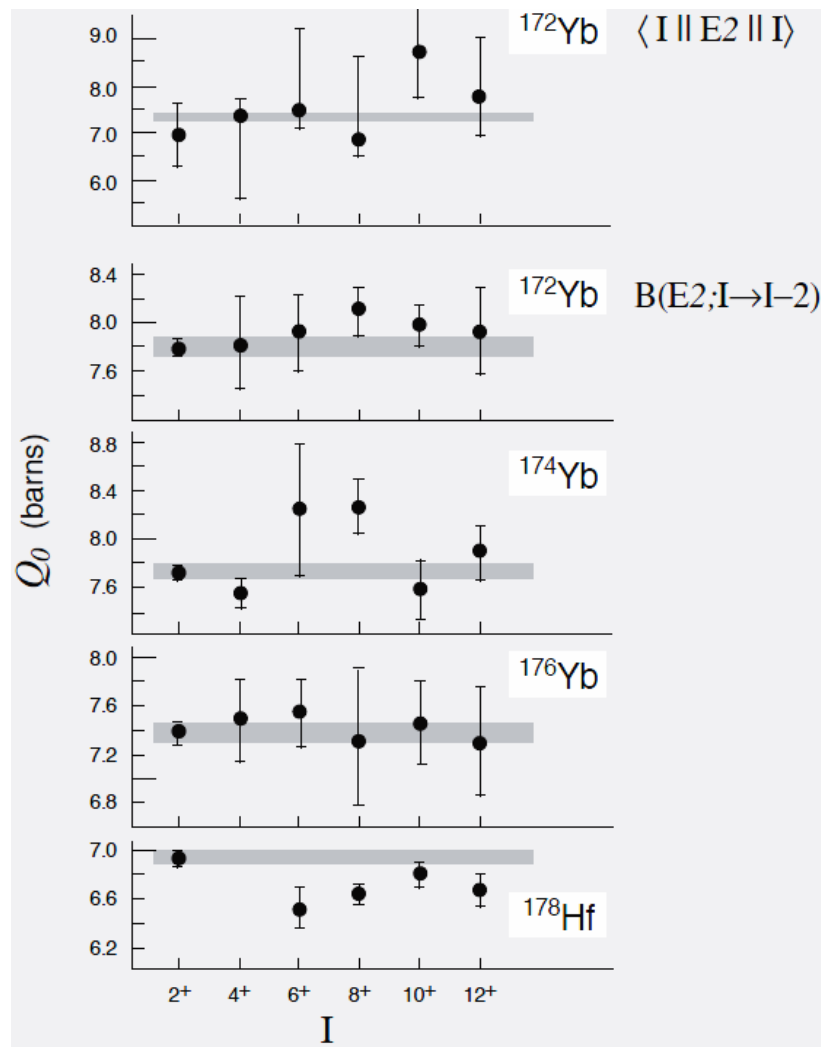


There is a general trend between transition strength and transition energy for  $2^+ \rightarrow 0^+$  transitions in even-even nuclei. NOTE: This is a log-log plot.

On this plot, our actinide nuclei have the highest transition strengths and lowest transition energies.

Q: Why should there be such a trend?





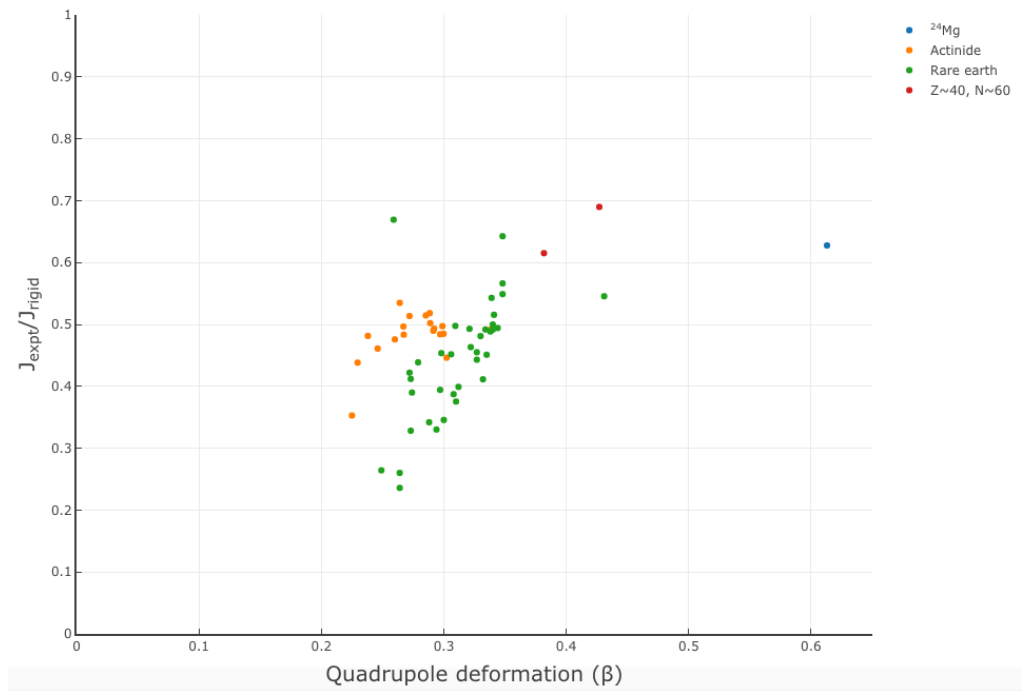
An experimental moment of inertia can be derived from the excitation energy of the 2+ state. Theoretical moments of inertia can be extracted for two different scenarios of rigid rotor and irrotational flow and compared to experiment.

$$\begin{aligned}\mathcal{I}_{\text{expt}} &= 0.2080 \times 10^{-54} E(2_1^+)^{-1} [\text{MeV}^{-1}], \\ \mathcal{I}_{\text{rigid}} &= 0.8864 \times 10^{-57} A^{5/3} (1 + 0.3154\beta + 0.44\beta^2), \\ \mathcal{I}_{\text{irrot}} &= 0.8864 \times 10^{-57} A^{5/3} 0.8951\beta^2,\end{aligned}$$

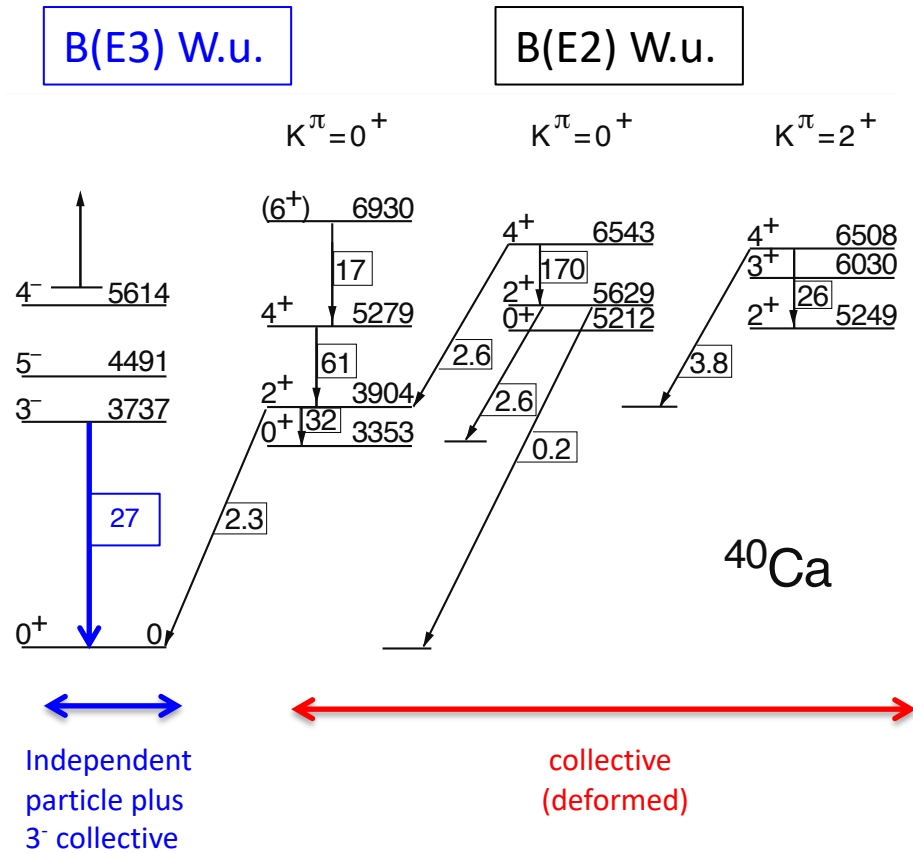
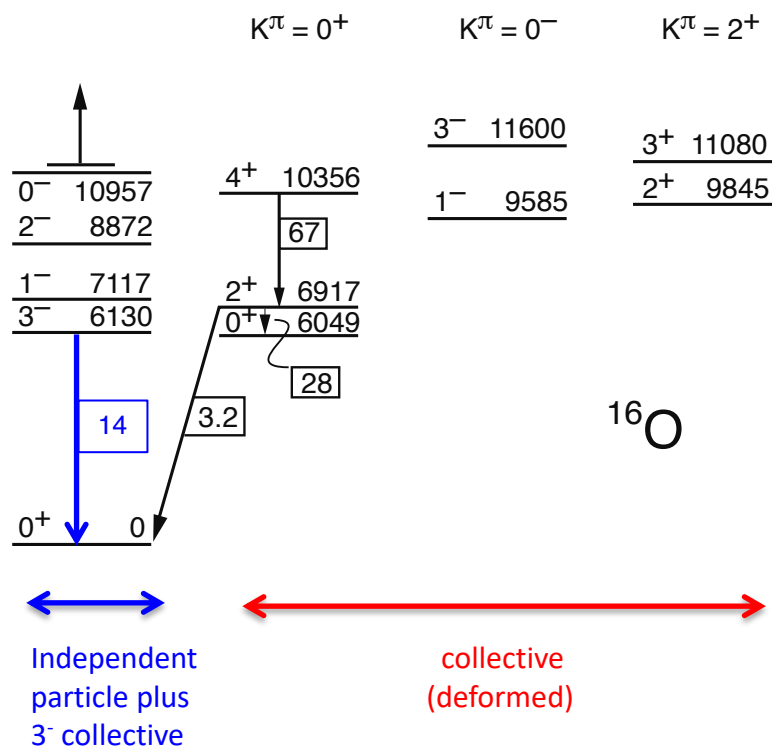
$$\beta = Q_0 \frac{\sqrt{5\pi}}{3ZR^2},$$

The figure shows that the experimental value is never more than about 0.3-0.7 of the rigid rotor value.

**CONCLUSION:** Everything looks like rotation but it's nothing like a classical rotating rigid body. What is actually rotating is a very good question!



	Z	$Q_0$ (b)	$A^{2/3}$	$\beta$	$\mathcal{I}_{\text{rigid}}$ $\times 10^{-54} \text{ (kg.m}^2\text{)}$	$E(2_1^+)$ (keV)	$\mathcal{I}_{\text{expt}}$ (kg.m <sup>2</sup> )	$\frac{\mathcal{I}_{\text{expt}}}{\mathcal{I}_{\text{rigid}}}$
$^{174}\text{Yb}$	70	$7.82^5$	31.167	0.3081	$5.955 \times 10^{-54}$	$76.471^1$	$2.723 \times 10^{-54}$	0.4573
$^{242}\text{Pu}$	94	$11.90^6$	38.834	0.2823	$10.184 \times 10^{-54}$	$44.54^2$	$4.675 \times 10^{-54}$	0.4591
$^{152}\text{Dy}$	66	$17.5^2$	28.482	0.7076	$6.025 \times 10^{-54}$	33.75	$6.170 \times 10^{-54}$	1.024





*A few parting questions....*

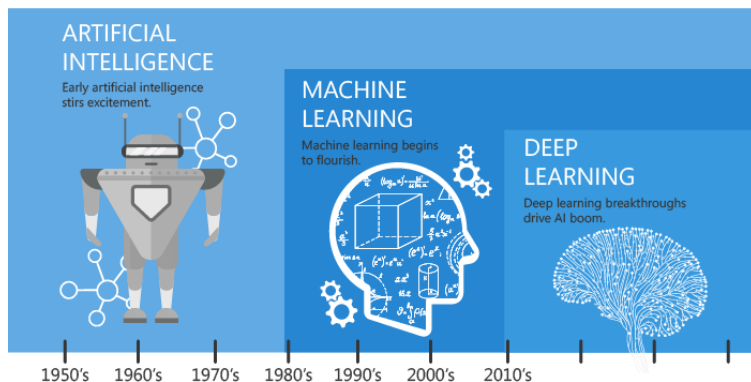
1) Do we need more data?

2) If so, what kind of data do we need?

3) Are we doing enough with the data we have?



# How to use the amazing resources on NNDC?



$^{24}_{12}\text{Mg}_{12-1}$

From ENSDF - Evaluated March 2022

$^{24}_{12}\text{Mg}_{12-1}$

## Adopted Levels, Gammas

Type	Author	History	Citation	Literature Cutoff Date
Full Evaluation	M. Shamsuzzoha Basunia, Anagha Chakraborty		NDS 186, 2 (2022)	31-Mar-2022

$Q(\beta^-) = -13884.77 \text{ 23}$ ;  $S(n) = 16531.22 \text{ 3}$ ;  $S(p) = 11692.69 \text{ 1}$ ;  $Q(\alpha) = -9316.56 \text{ 1}$  [2021Wa16](#)  
 $S(2n) = 29676.23 \text{ 16}$ ,  $S(2p) = 20486.805 \text{ 22}$  ([2021Wa16](#)).

Other reactions:

[2004Be18](#), [2004Be08](#):  $^{12}\text{C}(^{24}\text{Mg}, ^{12}\text{C})$ ,  $E = 130 \text{ MeV}$ ; measured  $E_\gamma$ , (particle) $\gamma$ -coin.

[2011Fr14](#):  $^{12}\text{C}(^{13}\text{C}, n)$   $E = 12, 13.5, 20 \text{ MeV}$ ; measured reaction products  $^{25}\text{Mg}$ ; deduced  $^{24}\text{Mg}$  excited states and reported resonance energies at  $13.25 \text{ MeV } 20$  and  $14.25 \text{ MeV } 20$ .

[2001Di12](#):  $^{11}\text{B}(^{13}\text{N}, X)$ ,  $(^{13}\text{N}, ^{12}\text{C})$ ,  $E = 29.5, 45 \text{ MeV}$ . Measured particle spectra, fusion  $\sigma$ . Deduced  $^{24}\text{Mg}$   $6-\alpha$  decay features, isospin purity/mixing in  $^{24}\text{Mg}$  at excitation energy  $\sim 47 \text{ MeV}$ , GDR  $\gamma$ -emission features.

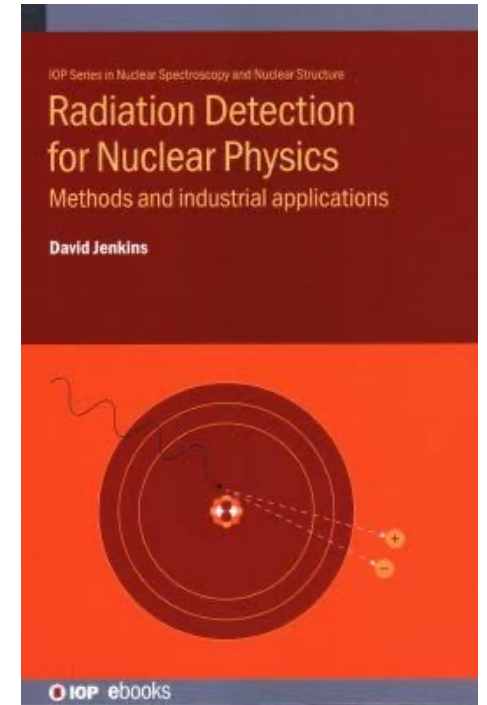
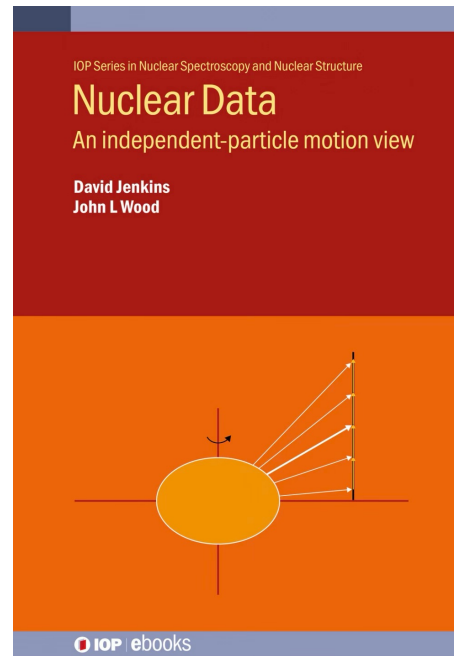
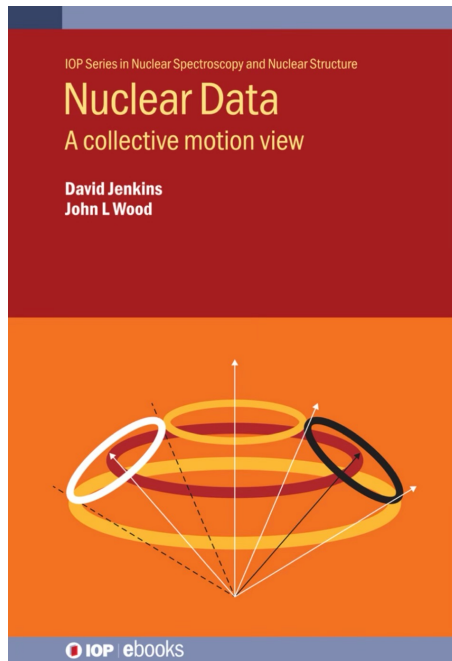
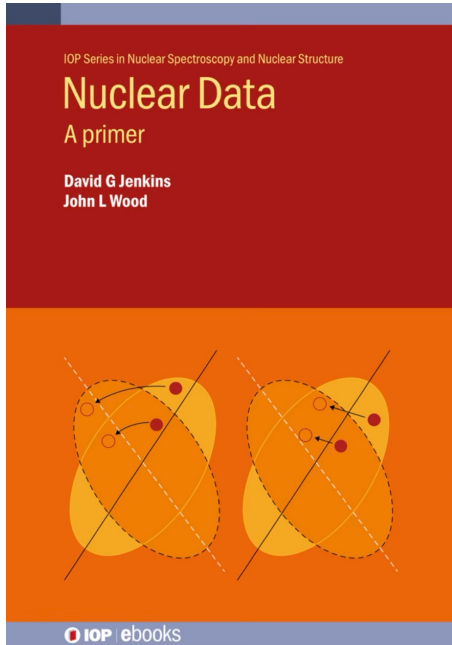
[2006Va20](#):  $^{28}\text{Si}(p, p')^{24}\text{Mg}$ ,  $E = 1 \text{ GeV}$ ; measured  $E_\gamma$ ; deduced  $\sigma$ .

## $^{24}\text{Mg}$ Levels

### Cross Reference (XREF) Flags

<b>A</b> $^{24}\text{Na } \beta^-$ decay (14.956 h)	<b>N</b> $^{20}\text{Ne}(\alpha, \gamma)$ :Resonances	Others:
<b>B</b> $^{24}\text{Na } \beta^-$ decay (20.18 ms)	<b>O</b> $^{20}\text{Ne}(\alpha, \alpha), (\alpha, \alpha')$ :Resonances	<b>AA</b> Coulomb excitation
<b>C</b> $^{24}\text{Al } \varepsilon$ decay (2.053 s)	<b>P</b> $^{20}\text{Ne}(^6\text{Li}, d), (^7\text{Li}, t)$	<b>AB</b> $^{24}\text{Mg}(\alpha, \alpha' \gamma)$
<b>D</b> $^{24}\text{Al } \varepsilon$ decay (130.7 ms)	<b>Q</b> $^{22}\text{Ne}(^3\text{He}, n)$	<b>AC</b> $^{24}\text{Mg}(^6\text{Li}, ^6\text{Li}')$
<b>E</b> $^{25}\text{Si } \varepsilon p$ decay	<b>R</b> $^{23}\text{Na}(p, \gamma), (p, p'), (p, X)$	<b>AD</b> $^{24}\text{Mg}(^{16}\text{O}, ^{16}\text{O}')$
<b>F</b> $^{26}\text{P } \varepsilon 2p$ decay	<b>S</b> $^{23}\text{Na}(^3\text{He}, d), (^3\text{He}, d\gamma)$	<b>AE</b> $^{25}\text{Mg}(p, d)$
<b>G</b> $^{28}\text{P } \varepsilon \alpha$ decay	<b>T</b> $^{24}\text{Mg}(\gamma, \gamma')$	<b>AF</b> $^{25}\text{Mg}(^3\text{He}, ^4\text{He})$
<b>H</b> $^{12}\text{C}(^{12}\text{C}, \gamma)$	<b>U</b> $^{24}\text{Mg}(e, e')$	<b>AG</b> $^{27}\text{Al}(\mu^-, \nu 3n\gamma)$
<b>I</b> $^{12}\text{C}(^{12}\text{C}, p)$ :Resonances	<b>V</b> $^{24}\text{Mg}(\pi^+, \pi^{+'}), (\pi^-, \pi^{-'})$	<b>AH</b> $^{27}\text{Al}(p, \alpha)$
<b>J</b> $^{12}\text{C}(^{14}\text{N}, d)$	<b>W</b> $^{24}\text{Mg}(p, p'), (\text{pol } p, p')$	<b>AI</b> $^{28}\text{Si}(d, ^6\text{Li})$
<b>K</b> $^{12}\text{C}(^{24}\text{Mg}, ^{12}\text{C}\gamma)$	<b>X</b> $^{24}\text{Mg}(n, n' \gamma)$	<b>AJ</b> $^{28}\text{Si}(^{28}\text{Si}, X\gamma)$
<b>L</b> $^{12}\text{C}(^{16}\text{O}, \alpha), (^{16}\text{O}, \alpha\gamma)$	<b>Y</b> $^{24}\text{Mg}(^3\text{He}, ^3\text{He}')$	
<b>M</b> $^{12}\text{C}(^{24}\text{Mg}, 2^{12}\text{C}), (^{20}\text{Ne}, 2^{12}\text{C})$	<b>Z</b> $^{24}\text{Mg}(\alpha, \alpha')$	

E(level) <sup>†</sup>	J <sup>π</sup>	T <sub>1/2</sub> or Γ <sup>j</sup>	XREF	Comments
0 <sup>P</sup>	0 <sup>+</sup>	stable	ABCDEFGHIJKL N PQRSTUWXYZ	XREF: Others: <a href="#">AA</a> , <a href="#">AB</a> , <a href="#">AD</a> , <a href="#">AE</a> , <a href="#">AF</a> , <a href="#">AG</a> , <a href="#">AH</a> , <a href="#">AI</a> , <a href="#">AJ</a> $\delta \langle r^2 \rangle > (^{26}\text{Mg}, ^{24}\text{Mg}) = +0.140 \text{ fm}^2 \text{ 5 (stat) 25 (syst)}$ <a href="#">(2012Yo01)</a> . $\langle r^2 \rangle^{1/2} (^{24}\text{Mg}) = 3.0570 \text{ 16 (charge radius)}$ <a href="#">(2013An02</a> evaluation). Others: $3.0570 \text{ fm 7 (stat) 48 (syst)}$ <a href="#">(2012Yo01)</a> , $3.030 \text{ fm 30 (1971Li26 - (e, e'))}$ . XREF: Others: <a href="#">AA</a> , <a href="#">AB</a> , <a href="#">AC</a> , <a href="#">AD</a> , <a href="#">AE</a> , <a href="#">AF</a> , <a href="#">AG</a> , <a href="#">AH</a> , <a href="#">AI</a> , <a href="#">AJ</a> $\mu = +1.08 \text{ 3}$ ; $Q = -0.29 \text{ 3}$
1368.667 <sup>P 5</sup>	2 <sup>+</sup>	1.36 ps 3	A CDEF H JKL N PQRSTUWXYZ	



# Finis