Ion Source Characterization using Integrated Data Analysis

M. Showers, J.B. Titus, and C. Exton

***TAE Technologies, Inc.**, 19631 Pauling, Foothill Ranch, CA 92610*

mshowers@tae.com

www.tae.com

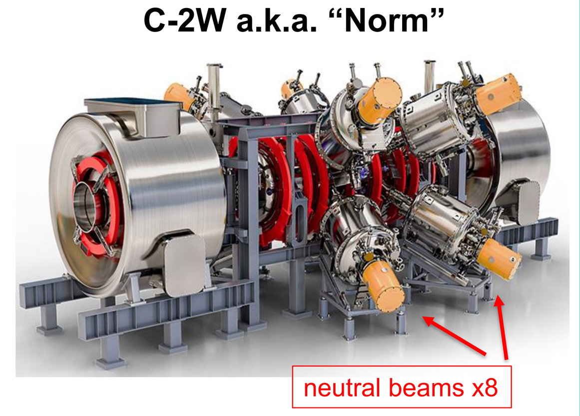
Abstract

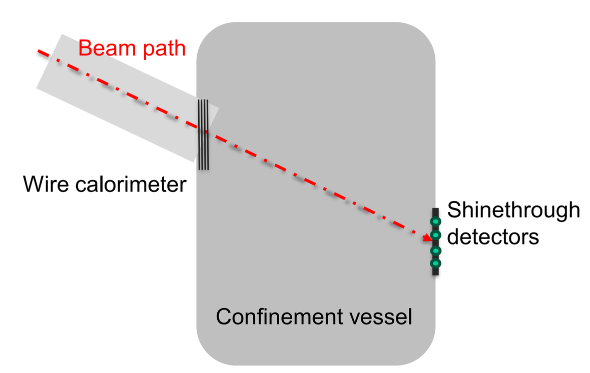
TAE's current experimental device, C-2W, utilizes edge-biasing, neutral beam injection, and plasma control to create and sustain a field-reversed configuration (FRC) plasma embedded in a mirror plasma. The eight positive ion-source neutral beams, four static energy (15 keV) and four tunable energy (15-40 keV), also stabilize, fuel, and heat the FRC. Injected beam power correlates non-linearly to plasma performance, and therefore, it is crucial to understand beam propagation from the ion source into the plasma. Ion sources are generally characterized by parameters such as divergence and focal length. Knowing the most probable values of these parameters assists in designing efficient beam lines and understanding the power injected into the confinement vessel. On short-pulsed, low-energy neutral beam systems, these quantities are estimated throughout the beam line by wire calorimeter and shinethrough detectors. A model has been developed to comprehensively combine the measurements and associated uncertainties with an integrated data analysis technique, based on Bayesian probability theory, to estimate the beam divergences and focal length.

C-2W and Neutral Beams

- C-2W field-reversed configuration (FRC) operations depend on **eight** neutral beams (NBs): four 15 keV, 150 A; four 15-40 keV, 150 A
- Ion sources characterized by parameters: (1) divergence; (2) focal length
- Need to know most likely parameter values to
  - Design efficient beam lines
  - Understand power injected into confinement vessel
- Previous analysis compared Gaussian fits of beam profile (determined from SEE detectors) to database of possible profiles, identifying the most likely divergence and focal length
  - **Diagnostics were considered separately**
- Actual geometry of beam path (from ion source to dump) with respect to confinement vessel increases the uncertainty of Gaussian fit profile modeling; diagnostic noise adds complexity
- **New analysis method to estimate NB ion source parameters must accurately accommodate both the geometry and synthetic data from multiple diagnostics**

C-2W a.k.a. "Norm"



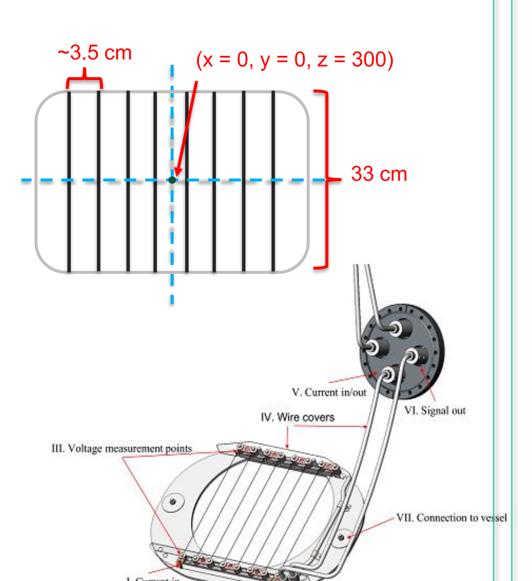


Diagnostics

WIRE CALORIMETER (WC):

Infer deposited power on wires from change in wire resistance

- 8 tungsten wire array (+ 1 reference)
- Wire Diameter: 1 mm
- Wires vertically orientated
- **Center of diagnostic @ (x = 0, y = 0, z* = 300 cm)**
- Temperature-dependent material properties

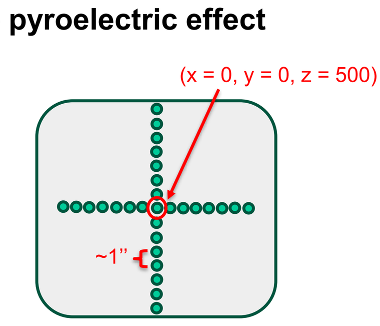


$$mc(T)\frac{\partial T}{\partial t} = A_c \frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) + \varepsilon(T)\sigma A_s(T^4 - T_{env}{}^4)$$

PYROBOLOMETERS (PB):

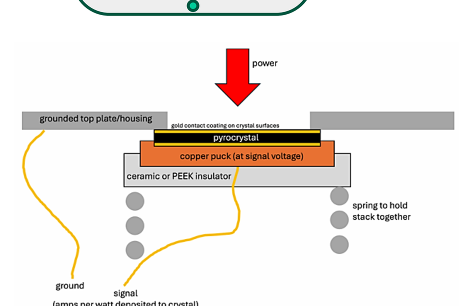
Infer deposited power on crystal surface from induced current due to pyroelectric effect

- 29 detector array: 15 horiz; 15 vert (shared center)
- **Center detector location: (x = 0, y = 0, z* = 500 cm)**
- Detector composition:
  - Crystal: LiTaO3 (Cr-Au coating [~200 nm])
  - Dimensions: 12 mm x 7 mm x 1mm
  - Pyroelectic coefficient: 60.73 [nA/W]
  - 4-layer attenuation grid:
    - 15 keV: 12.3% transparency
    - 60 keV: 3% transparency



$$I = \frac{\alpha\gamma Q}{c\rho d}$$

α = energy reflection coefficient
γ = pyroelectric coefficient
Q = deposited energy
c = crystal specific heat
ρ = crystal density
d = crystal thickness



*z-location measured with respect to the end of the IOS grids

Sources: [1], [2]

Beam Models

Sources: [1], [3]

Two methods to model NB to estimate beam parameters: x-divergence (div_x), y-divergence (div_y), focal length (foc)

1. Geometric beam propagation model, assuming Gaussian spread, combined with numerical analysis; "requested beam" (range of possible parameter values are inputs to model)

$$j(x,y,z) = \frac{J_0}{\pi^2 a^2 z^2 \alpha_x \alpha_y}\int_{-a}^{a} dx_0 \int_{-b}^{b} e^{-\left[\left(\frac{\theta_x}{\alpha_x}\right)^2 + \left(\frac{\theta_y}{\alpha_y}\right)^2\right]} dy_0$$

$$b = \sqrt{a^2 + x_0{}^2}; \quad \theta_l(l,z,R) = \left(\frac{1}{z} - l_0\left(\frac{1}{z} - \frac{1}{R}\right)\right)$$

J_0 : current
j : current density (at x, y, z-loc from grid)
a : beam radius
x_0, y_0 : horizontal, vertical points on ion source grid
α_x, α_y : x-divergence, y-divergence
R : focal length
l : x and y

2. Ion Beam Simulator **(IBsimU)** model, using CAD model of ion optical system (IOS) and Monte Carlo particle tracing. IOS designed to produce beam with parameters as close to requested values as possible.

Integrated Data Analysis (IDA) with Bayesian Probability Theory

Integrated data analysis - technique to combine data from separate diagnostics (WC, PB) to jointly assess specific parameters (div_x, div_y, foc)

Our model (Bayes' Rule)

$$P(\vec{a}|\overrightarrow{WC},\overrightarrow{PB},I) = \prod_{i=1}^{8} P(WC_i|\vec{a},I)\prod_{j=1}^{29} P(PB_j|\vec{a},I)\prod_{k=1}^{3} P(a_k,I)$$

where:

$P(\vec{a}|\overrightarrow{WC},\overrightarrow{PB},I)$: probability of getting desired $\vec{a}$ = (div_x, div_y, foc) given our WC and PB experimental data sets and background information, I' "posterior probability"

$P(WC_i|\vec{a},I)$: likelihood function of the i^th wire calorimeter wire

$P(PB_j|\vec{a},I)$: likelihood function of the j^th pyrobolometer detector

$P(a_k,I)$: prior information about div_x, div_y, and foc; dependent on our beam model scenario; "prior probability"

Likelihood Functions:

$$P(WC_i|\vec{a},I) = \frac{1}{\sigma_{WC,i}\sqrt{2\pi}} e^{\left[\frac{-(R_{m,i}-R_{c,i})^2}{2\sigma_{WC,i}{}^2}\right]}$$

$$P(PB_j|\vec{a},I) = \frac{1}{\sigma_{PB,j}\sqrt{2\pi}} e^{\left[\frac{-(V_{m,j}-V_{c,j})^2}{2\sigma_{PB,j}{}^2}\right]}$$

where:
σ: uncertainty associated with i^th WC wire or j^th PB detector
$R_{m,i}$, $R_{c,i}$: measured and calculated resistances for i^th WC wire
$V_{m,i}$, $V_{c,i}$: measured and calculated voltages for j^th PB detector

Uncertainty factors:
- **Ignored signal noise**
- Manufacturing error
- Material properties (crystal pyroelectric coefficient, temperature-variable tungsten)
- Beam error (energy, incident angle)

Prior Probabilities:

$$P(div_x|I) = \frac{1}{.045-.025} = \frac{1}{.02} = 50$$

for $0.025 \le div_x \le 0.045$, else = 0

Parameters	"C-2W"	"Next Gen"
div_x [min, max]	[25 mrad, 45 mrad]	[10 mrad, 30 mrad]
div_y [min, max]	[5 mrad, 15 mrad]	[10 mrad, 30 mrad]
foc [min, max]	[250 cm, 450 cm]	[500 cm, 700 cm]

Marginalization: probability of getting specific value of beam parameter in $\vec{a}$ regardless of the values of other parameters in $\vec{a}$

$$P(x|Data,I) = \int P(x,y|Data,I)dy \rightarrow P(div_x|Data,I) = \iint P(div_x,div_y,foc|Data,I)\,d(div_y)d(foc)$$

Compared two scenarios using 'geometric' beam model:
- Tested robustness of developed IDA tool
- Investigated ability of wire calorimeter and pyrobolometer diagnostics to infer divergences and focal length
  - (i.e.) How does focal length affect diagnostic effectiveness?

Parameters	"C-2W"	"Next Gen"
Voltage	15 keV	60 keV
Current	150 A	160 A
Pulse	20 ms	20 ms
Expected $\vec{a}$ (div_x, div_y, foc)	(35 mrad, 10 mrad, 350 cm)	(20 mrad, 20 mrad, 600 cm)

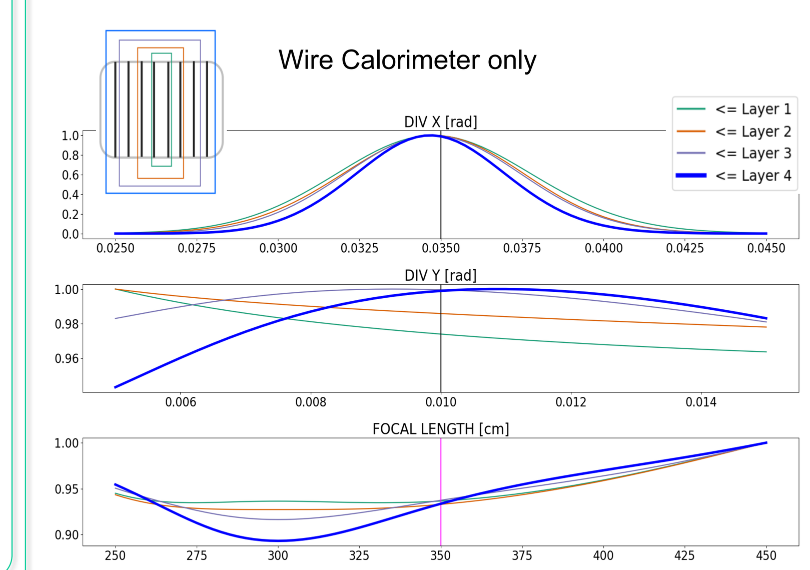
Sources: [4], [5]

References

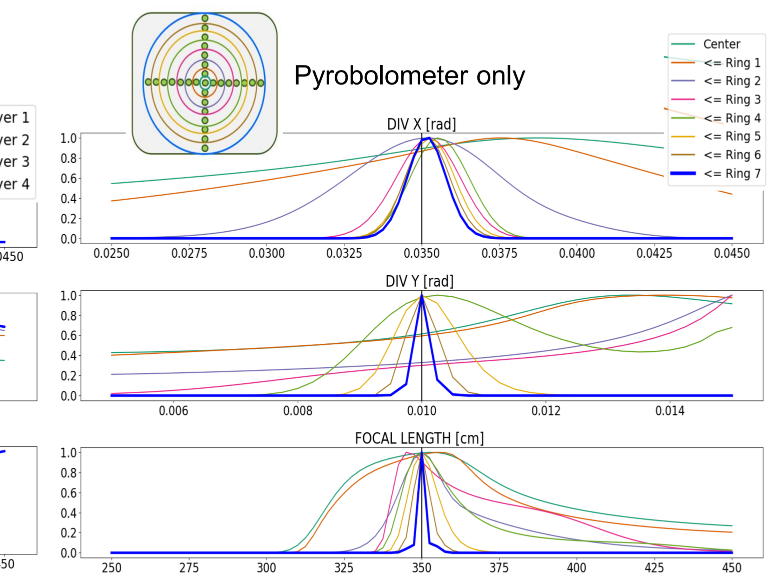
1. Titus, J. B., et al. (2018). Secondary electron emission detectors for neutral beam characterization on C-2W. *Review of Scientific Instruments*, 89 (10); https://doi.org/10.1063/1.5037099
2. Titus, J. B., et al. (2021). Wire calorimeter for direct neutral beam power measurements on C-2W. *Review of Scientific Instruments*, 92 (5); https://doi.org/10.1063/5.0043871
3. Kalvas, T., et al. (2010). IBSIMU: A three-dimensional simulation software for charged particle optics. *Review of Scientific Instruments,* 81 (2); https://doi.org/10.1063/1.3258608
4. Johnson, J. (2013). *Implementing Bayesian Statistics and a Full Systematic Uncertainty Propagation with the Soft X-Ray Tomography Diagnostic on the Madison Symmetric Torus* (thesis). University of Wisconsin – Madison, Wisconsin, U.S.A.
5. Reusch, L.M., et al. (2014). An integrated data analysis tool for improving measurements on the MST RFP. *Review of Scientific Instruments*, 85 (11); https://doi.org/10.1063/1.4886957

"C-2W" Beam Model Results

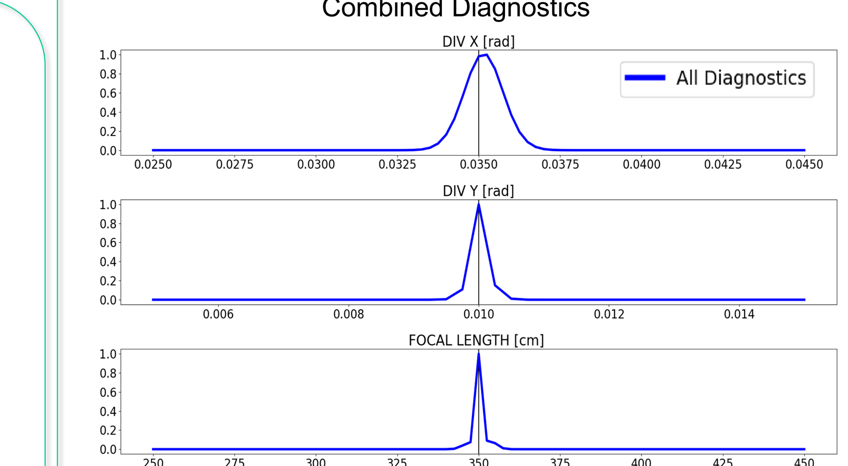
Wire Calorimeter only



Pyrobolometer only



Combined Diagnostics



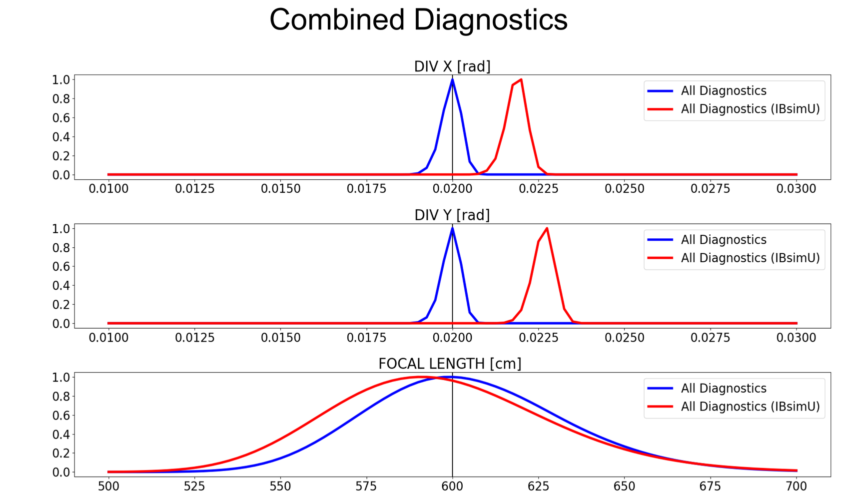
Parameters	WC only	PB only	Combined	Expected
div_x [mrad]	34.75 ± 2.35	35.25 ± 0.64	35.25 ± 0.61	35
div_y [mrad]	10.75 ± 2.94	10.0 ± 0.13	10.0 ± 0.13	10
foc [cm]	450 ± 58.8	350 ± 1.87	350 ± 1.85	350

We have confirmed:
- Accuracy and uncertainty improved when PB and WC data considered together
- Accuracy and uncertainty improved as number of detectors in PB array and number of wires in WC array increase

Incidentals:
- WC provides good prediction of x-divergence
- Vertically-oriented WCs cannot accurately estimate div_y
- Type of diagnostic measurement (line-integrated WC vs. point-like PB) affects accuracy
- WC is installed near the focal length – cannot provide a good focal length estimate (beam generally symmetric with respect to focal length)

"Next Generation" Beam Model & Simulation Results

Combined Diagnostics



- **Compared 'geometric' model with IBsimU model for 'next gen' beam**
  - IBsimU model provides more realistic beam
- Amplitude and uncertainty of parameter profiles of geometric model and IBsimU model approximately the same
- IBsimU profiles shifted "off-center", suggesting larger divergences in 'x' and 'y' and closer focal length vs. 'geometric' model profiles

Parameters	WC only	PB only	Combined	Combined (IBsimU)	Expected
div_x [mrad]	20.0 ± 2.52	20.25 ±0.45	20.0 ± 0.29	22 ± 0.31	20
div_y [mrad]	21.25 ± 5.57	20.25 ± 0.45	20.0 ± 0.28	22.75 ± 0.32	20
foc [cm]	602.5 ± 51.8	700 ± 58.5	600 ± 28.6	590 ± 31.5	600

Summary and future work

- **Successfully created IDA tool for TAE neutral beam systems**
  - Good agreement between IDA predicted results and expected parameter values
  - Good agreement between 'next generation' geometric model and IBsimU model
- Installed location of diagnostic is critical to predict focal length
  - If diagnostic is too close to the anticipated focal length, it cannot provide an accurate estimation
- Line-integrated diagnostics (i.e. wire calorimeter) do not estimate focal length as accurately as point-measured diagnostics (pyrobolometers)
- **Increasing number of detectors in a diagnostic array increases the accuracy of the beam parameter predictions and reduces uncertainty**
- **Increasing the number of diagnostics available to measure beam parameters improves the accuracy and uncertainty of the beam parameter predictions**
- **Future work includes applying the IDA analysis tool :** (1) using C-2W diagnostic data; (2) optimizing diagnostic type and installation location to measure desired beam parameters; (3) determining beam parameters (divergence, focal length) for other beam configurations (applied voltage, current); exploring more geometrically complex scenarios (beam angle, shading).