Quantum Meets Quarks: towards developing quantum computation of the strong nuclear force

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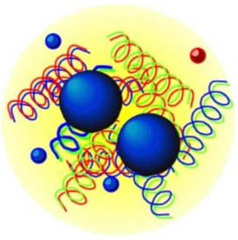
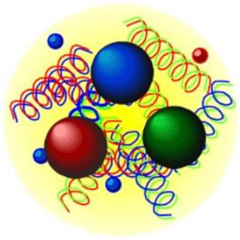
Quantum Field Theory (QFT) of the strong nuclear force – Quantum Chromodynamics (QCD)

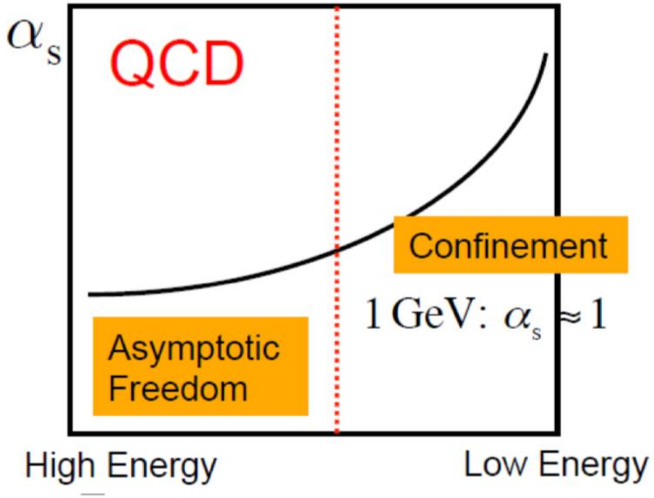
$$\mathcal{L}_{QCD}=\sum_{q}\left(\bar{\psi}_{qi}i\gamma^{\mu}\left[\delta_{ij}\partial_{\mu}+ig\left(G_{\mu}^{\alpha}t_{\alpha}\right)_{ij}\right]\psi_{qj}-m_{q}\bar{\psi}_{qi}\psi_{qi}\right)-\frac{1}{4}G_{\mu\nu}^{\alpha}G_{\alpha}^{\mu\nu}$$

Meson

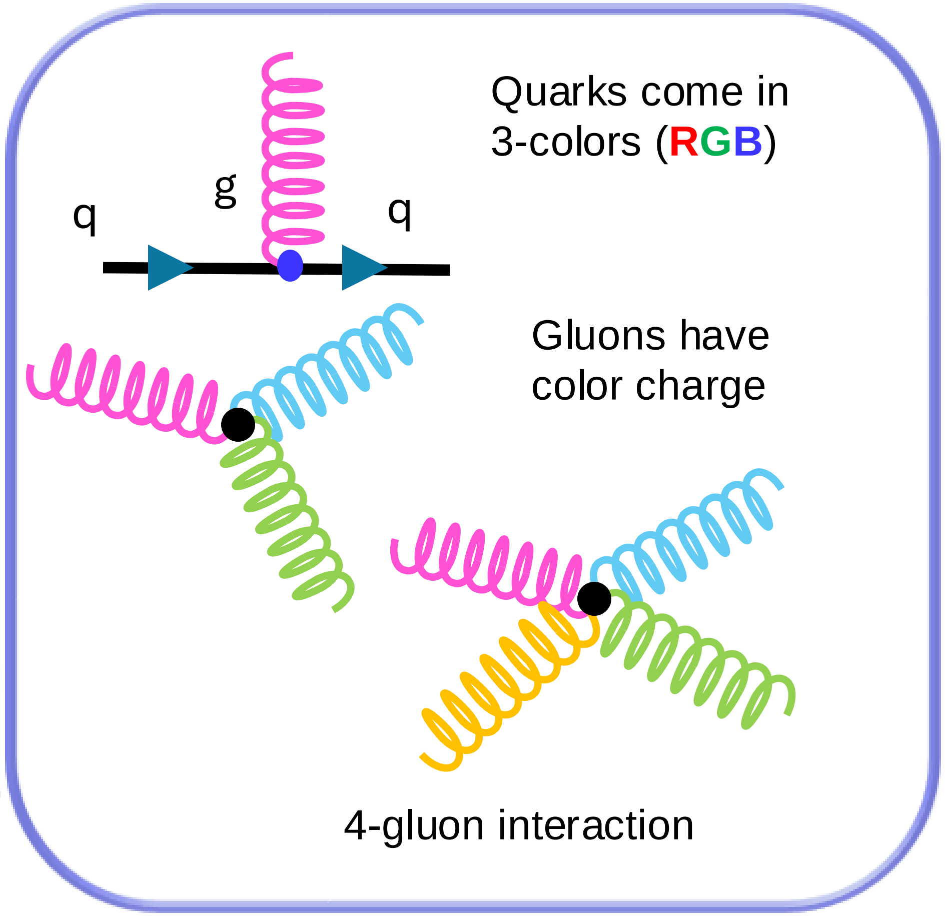


Baryon





$\sqrt{s}=100\ \text{GeV},\quad \alpha_s=0.12$



Quarks come in 3-colors (**R****G****B**)

Gluons have color charge

4-gluon interaction

First-principles non-perturbative method needed for studying hadrons in low energy – “Lattice QCD”

Classical computation of QFTS – example, lattice QCD

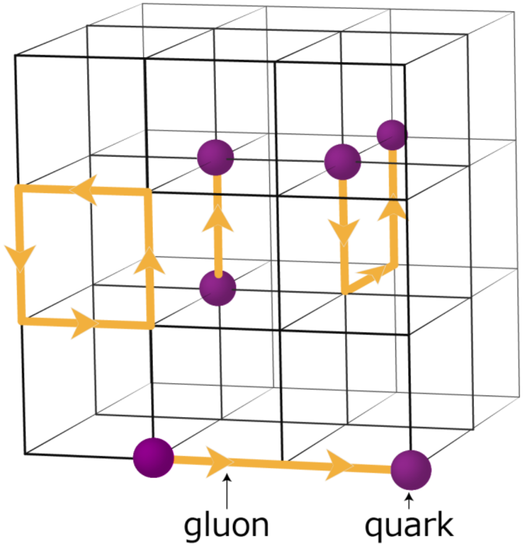
Feynman path integral - QFT

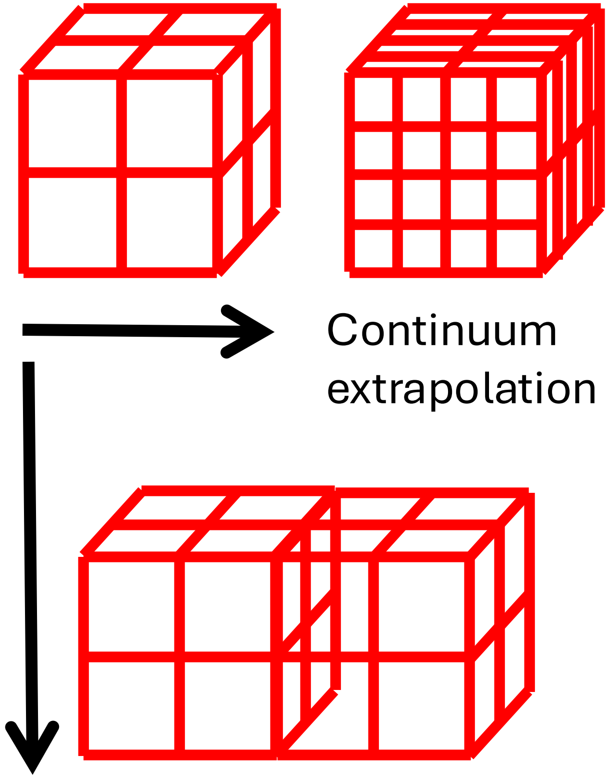
$$\langle 0|\mathcal{O}|0\rangle = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\,\mathcal{O}e^{iS_{\mathrm{QCD}}[\bar{\psi};\psi;A]}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\,e^{iS_{\mathrm{QCD}}[\bar{\psi};\psi;A]}}$$

$$\xrightarrow{x_0 \to -ix_4}$$

Kenneth G. Wilson

$$\langle 0|\mathcal{O}|0\rangle = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\,\mathcal{O}e^{-S_G^E[A]-S_F^E[\bar{\psi};\psi;A]}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\,e^{-S_G^E[A]-S_F^E[\bar{\psi};\psi;A]}}$$





Continuum extrapolation

Finite volume corrections

Science & Technology Facilities Council

DiRAC Distributed Research utilizing Advanced Computing

Quantum computation of quantum field theories – why do it?

$$\int d\phi\ \mathcal{O}(\phi)\underbrace{e^{-S(\phi)}}$$

Boltzmann factor as probability

"Sign problem" arises when (e.g.):

- topological term - complex action
- Finite Non-zero chemical potential - indefinite sign of fermion determinant
- real time formulation for collision
- Highly entangled many particle systems
- E.g. at the core of neutron stars

No "sign problem" by default in real time quantum computation

We can simulate chiral fermions

We could avoid "critical slowing down", provided we have quantum hardware

Quantum simulation for high energy physics

Potential for computationally-feasible approaches for many classically intractable problems

- collider physics
- Cosmology, early-universe physics, and dark matter
- Neutrino physics
- Quantum gravity
- More areas would be identified with time

Developed techniques will also be used in other areas of science, e.g.

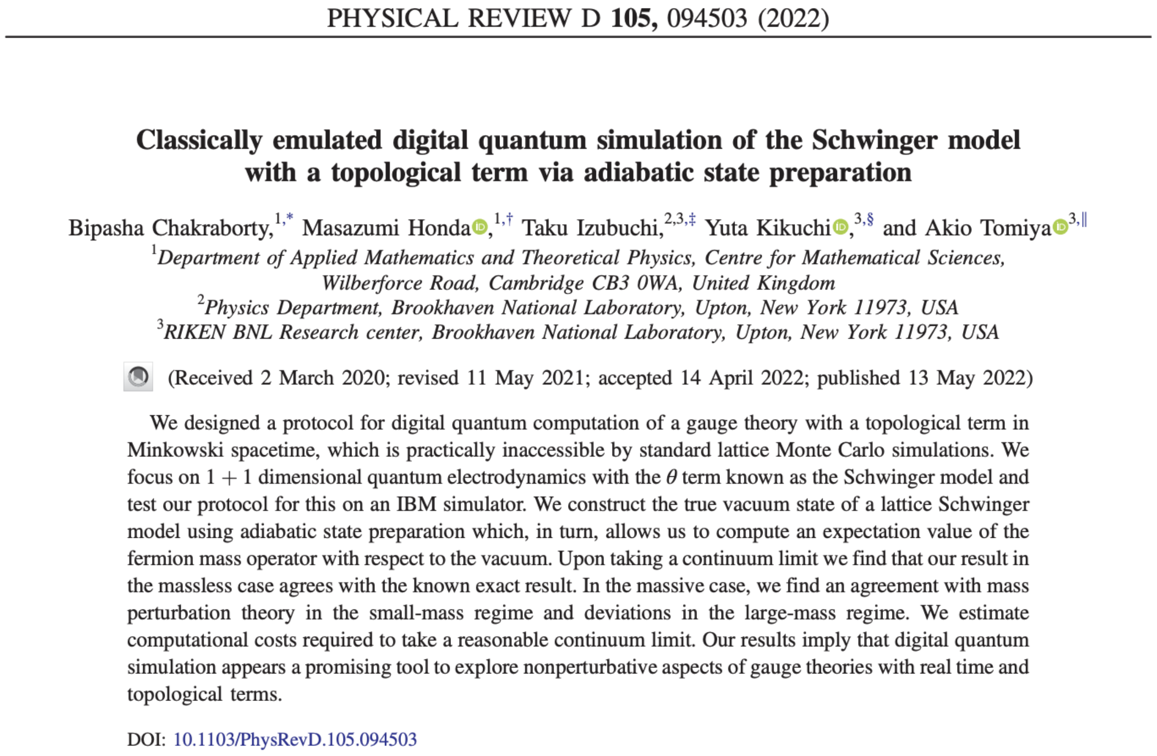
Both Quantum Chromodynamics (the quantum theory of the strong nuclear force) and quantum chemistry suffer from “sign problem”.

And quantum computation can remove them by default.

Recipes for digital quantum simulation of quantum QFTs and measuring an observable

- Write Hamiltonian of the system
- Discretise Fermion fields and gauge fields
- Remove gauge degrees of freedom *(Use Gauss Law and boundary conditions?)*
- Map fermions to qubits effeiciently *(using Jordan Wigner transformation?)*
- Prepare ground state (and excited states) of the Hamiltonian *(adiabetic theorem, VQE, VQA, QAOA, symmetry-based formulations?)*
- Use Suzuki-Trotter decomposition for the time evolution
- Take measurements and compute expectation values of Hermitian operators

Challenge: State preparation for QFTs



PHYSICAL REVIEW D **105,** 094503 (2022)

Classically emulated digital quantum simulation of the Schwinger model with a topological term via adiabatic state preparation

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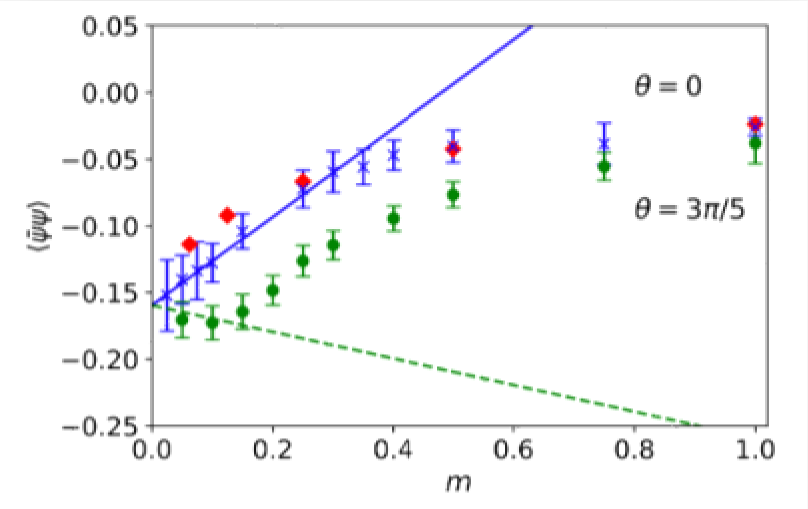
We designed a protocol for digital quantum computation of a gauge theory with a topological term in Minkowski spacetime, which is practically inaccessible by standard lattice Monte Carlo simulations. We focus on $1+1$ dimensional quantum electrodynamics with the θ term known as the Schwinger model and test our protocol for this on an IBM simulator. We construct the true vacuum state of a lattice Schwinger model using adiabatic state preparation which, in turn, allows us to compute an expectation value of the fermion mass operator with respect to the vacuum. Upon taking a continuum limit we find that our result in the massless case agrees with the known exact result. In the massive case, we find an agreement with mass perturbation theory in the small-mass regime and deviations in the large-mass regime. We estimate computational costs required to take a reasonable continuum limit. Our results imply that digital quantum simulation appears a promising tool to explore nonperturbative aspects of gauge theories with real time and topological terms.

DOI: 10.1103/PhysRevD.105.094503

Adiabatic state preparation (ASP) for Schwinger Model Hamiltonian with theta term in 1+1D

$$H = -\mathrm{i} \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger e^{\mathrm{i}\phi_n} \chi_{n+1} - \mathrm{h.c.} \right] + m\cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n + J \sum_{n=1}^{N-1} L_n^2, \qquad (5)$$

$$|\mathrm{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-\mathrm{i} \int_0^T dt\ H_{\mathrm{A}}(t) \right) |\mathrm{vac}\rangle_0$$



However, ASP requires unfeasibly large circuit depth for near term Digital quantum computation

Key challenges -

Algorithm development for QFTs with gauge degrees beyond one spatial dimension

Scalability of quantum hardware

High resource requirements

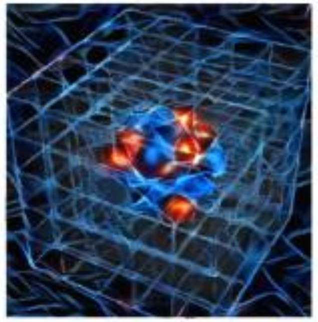
Quantum noise and decoherence

Interdisciplinary challenges

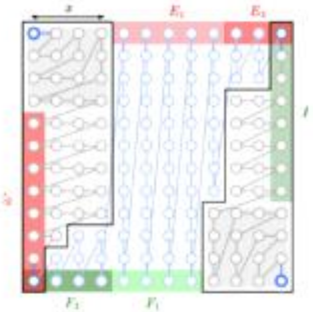
Verification and validation

Cost and accessibility

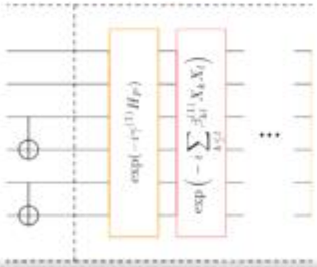
STFC QTFP 2022- **Quantum simulation algorithms for quantum chromodynamics**



Quantum Algorithms for solving Dirac equations



Efficient Fermion-Qubit mappings for QCD



Quantum-inspired Classical Simulation Algorithms for QCD

PhD students

Alex Tomlinson

Mitchell Chiew

Tejas Acharya

Project Leaders:

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(co-I)

Sergii Strelchuk
(PI, Cambridge/
Oxford)

Postdoctoral Research Associates:

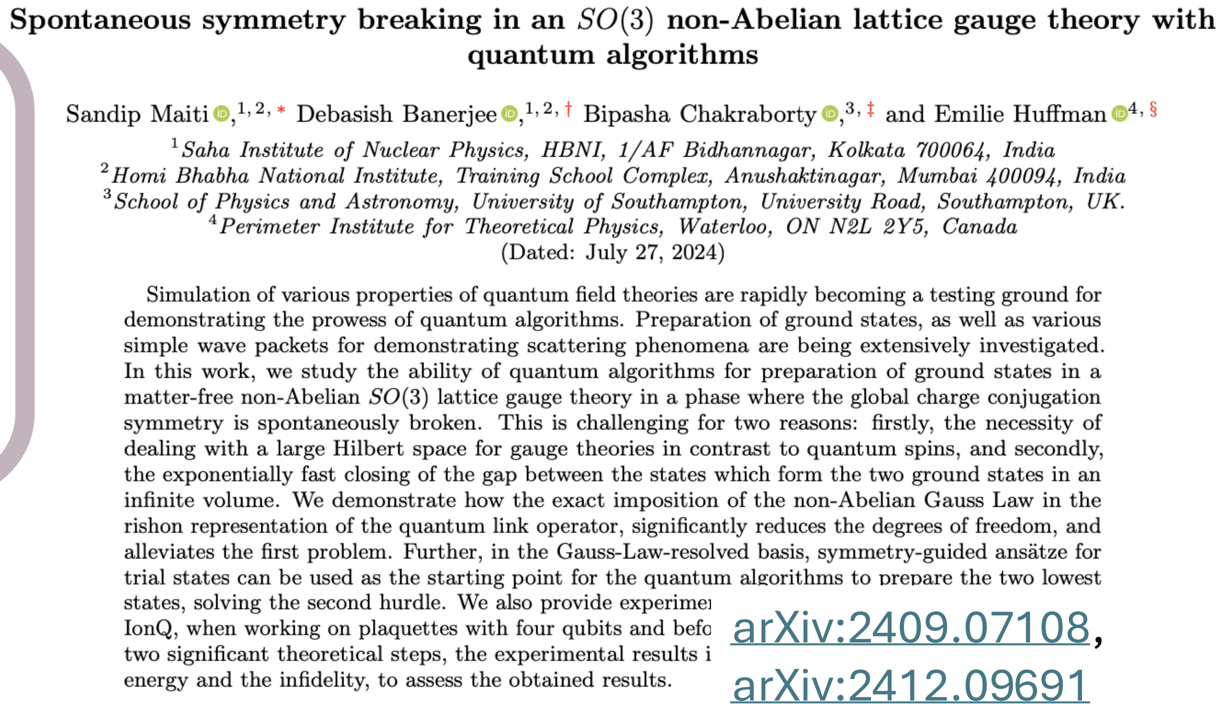
Graham Van
Goffrier

Subhayan Roy
Moulik

Challenge: Large Hilbert space of gauge theories

We demonstrate the ability of Variational algorithms and QAOA to prepare ground states and excited states in a matter free non-Abelian SO(3) lattice gauge theories on 2+1 D

Additionally, we handle the exponentially decreasing mass gap due to the spontaneously broken global charge conjugation symmetry



Spontaneous symmetry breaking in an $SO(3)$ non-Abelian lattice gauge theory with quantum algorithms

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(Dated: July 27, 2024)

Simulation of various properties of quantum field theories are rapidly becoming a testing ground for demonstrating the prowess of quantum algorithms. Preparation of ground states, as well as various simple wave packets for demonstrating scattering phenomena are being extensively investigated. In this work, we study the ability of quantum algorithms for preparation of ground states in a matter-free non-Abelian $SO(3)$ lattice gauge theory in a phase where the global charge conjugation symmetry is spontaneously broken. This is challenging for two reasons: firstly, the necessity of dealing with a large Hilbert space for gauge theories in contrast to quantum spins, and secondly, the exponentially fast closing of the gap between the states which form the two ground states in an infinite volume. We demonstrate how the exact imposition of the non-Abelian Gauss Law in the rishon representation of the quantum link operator, significantly reduces the degrees of freedom, and alleviates the first problem. Further, in the Gauss-Law-resolved basis, symmetry-guided ansätze for trial states can be used as the starting point for the quantum algorithms to prepare the two lowest
states, solving the second hurdle. We also provide experime
IonQ, when working on plaquettes with four qubits and befo
two significant theoretical steps, the experimental results i
energy and the infidelity, to assess the obtained results.

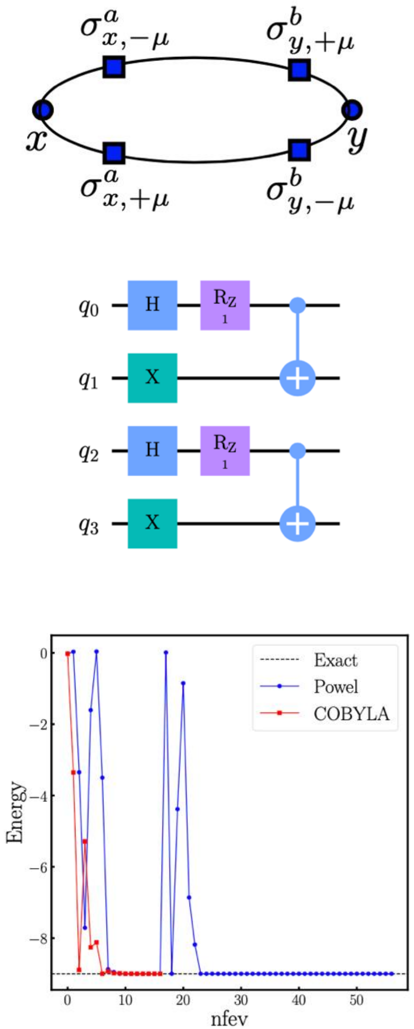
arXiv:2409.07108,
arXiv:2412.09691

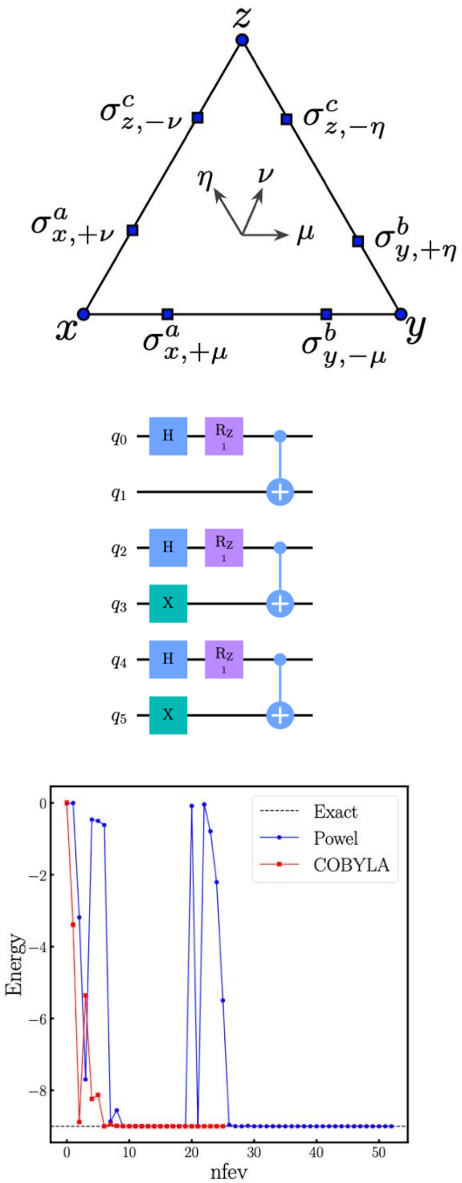
SO(3) shares fundamental properties with QCD

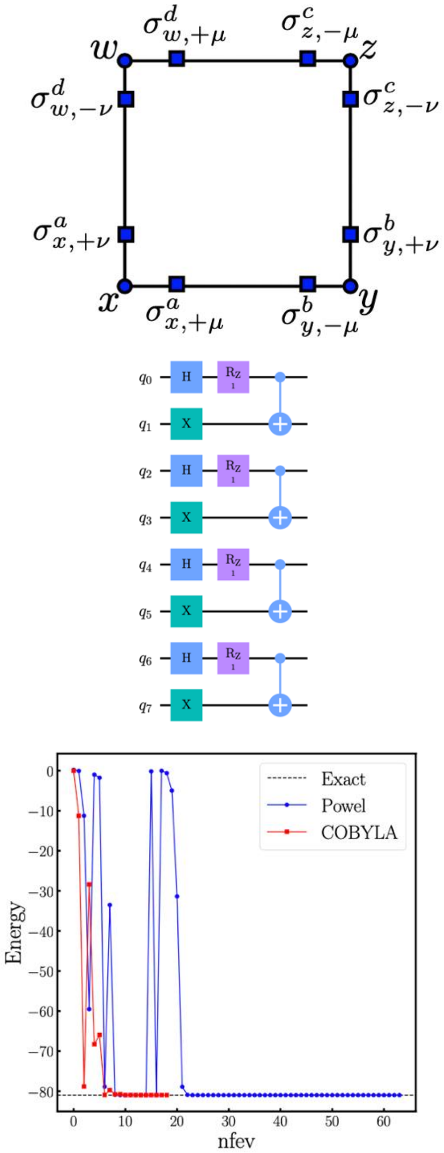
Many groups have been working on various gauge theories in 2+1D and 3+1D

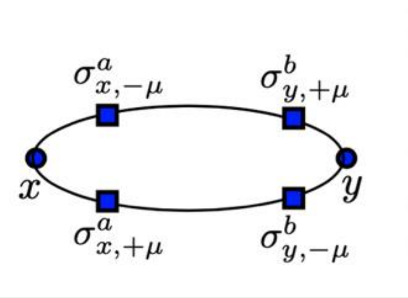
Rico et. al., Annals Phys. 393, 466-483 (2018)

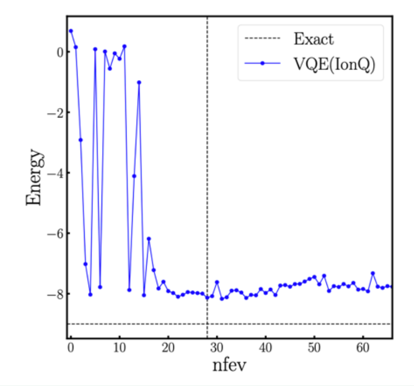
Variational algorithms: results

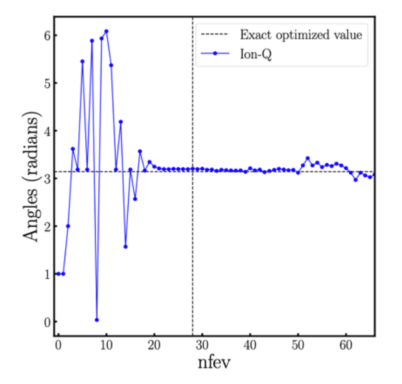




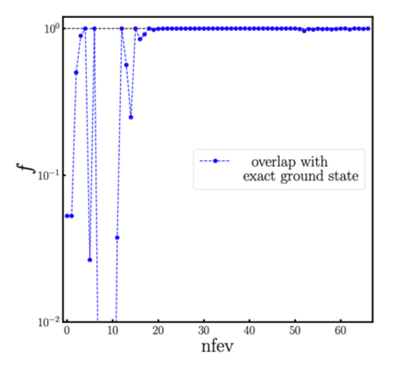








Results from real Hardware IonQ



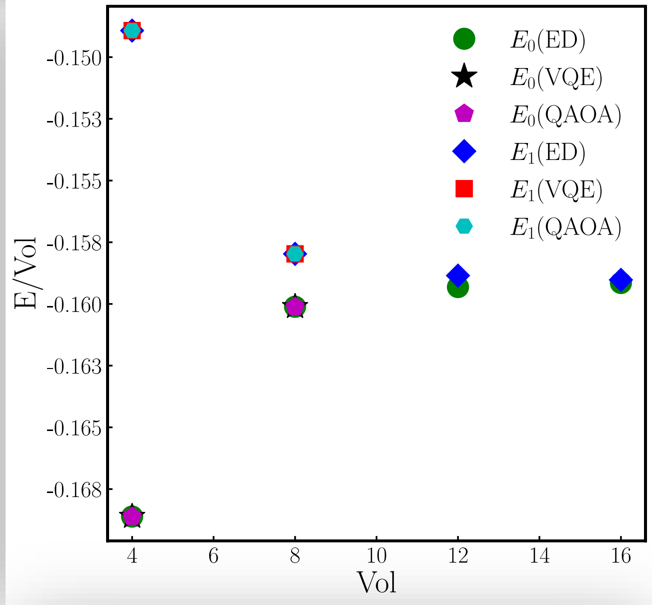
Summary of the results

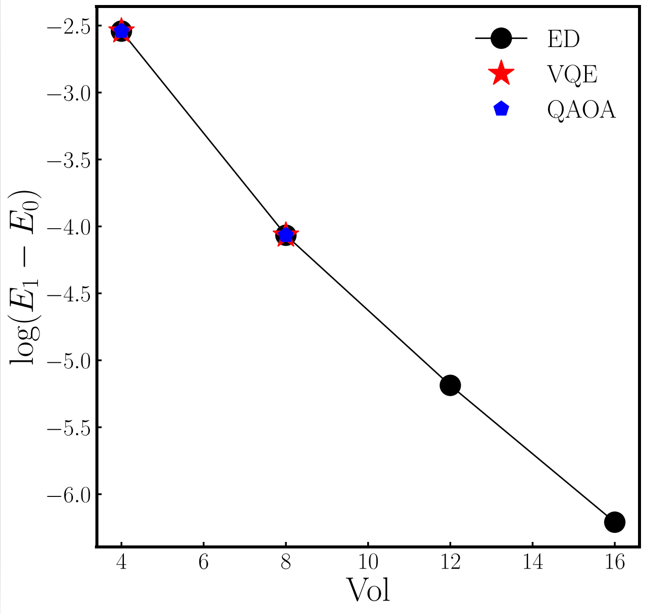
- VQE and QAOA prepared ground state and excited state with novel implementation for SO(3) in 2+1D

- A matter-free SO(3) gauge theory Hamiltonian in 2+1D has be written in gauge invariant basis using Quantum Link Model, and the Hilbert space was heavily reduced

- The spontaneous discrete symmetry breaking in SO(3) in 2+1D was established via quantum simulation,

- A simple but effective way of QEM is using symmetry constraints and post selection is underway





Outlook

What we have already achieved in QTFP program:

Algorithm development for up to two spatial dimensions &
meaningful results even with shallow circuits and without active error-correction on NISQ era trapped-ion technology IonQ.

What are underway: three papers will arrive soon

- Collaboration with computer scientist
- Optimization of our circuits
- Error mitigation
- Hardware comparison
- Benchmarking with classical computation
- Testing scalability of our algorithms
- Developing Hybrid classical-quantum approaches in both near and far terms.

Announcement:
Quantum computation for high energy physics workshop at Southampton, March 24-25, 2025. (details will be shared next week).

Open to collaboration opportunities!