Characterising correlation from electromagnetic signatures of spacetime fluctuation

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Overview

- Motivation: What and why's of spacetime fluctuations
- Spacetime fluctuations and interferometric output
- Identifying gravity model signatures: Role of correlations in spacetime fluctuations
- Summary

Bosonic fields and vacuum fluctuations

- **Bosonic field vacuum:** Not a complete absence of bosons

Vacuum fluctuations: Random creation and annihilation of bosons.

Can this be observed?

Ex: Lamb-Retherford experiment measured the Lamb shift.

Via radio-frequency transitions between $2s_{1/2}$ and $2p_{1/2}$ levels of the Hydrogen atom.

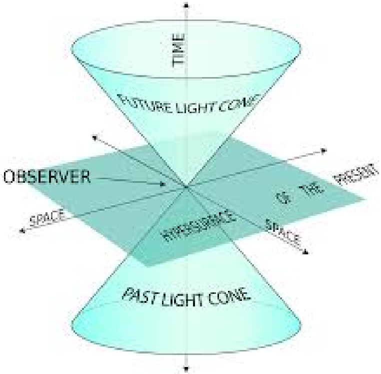
Lamb shift measurement laid the foundation for quantum electrodynamics.

Detecting vacuum fluctuations = Detecting "quantumness" of field.

Spacetime fluctuations

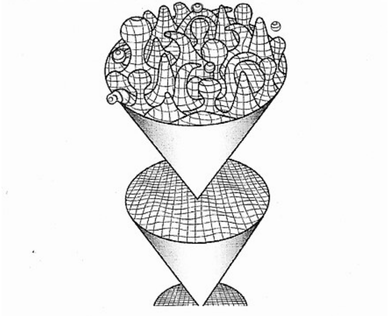
Gravitational vacuum fluctuations = Random fluctuations of spacetime manifold.

Flat Minkowski spacetime



$\longrightarrow$

Gravitational vacuum fluctuations



[Image source: Wikipedia]

Method...

Detecting vacuum fluctuations of gravitational field

1. Detecting effects of spacetime fluctuations in the dynamics of two masses.

[Ref: A. Malcolm, Z.-W. Wang, B. Sharmila, and A. Datta, arXiv preprint arXiv:2501.03886 (2025).]

2. Characterising the role of correlations in spacetime fluctuations in optical high-precision interferometers.

[B. Sharmila, S. Vermeulen, and A. Datta, Manuscript under preparation.]

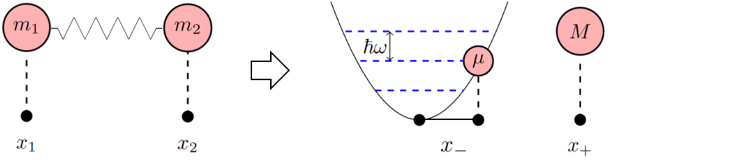
Dynamics of masses in gravitational vacuum

Dynamics is very sensitive to the choice of the position observable!

Free particle:

Frame-dependent co-ordinate separation: Unitary dynamics | *Frame-independent geodesic separation:* Dissipative dynamics

Harmonically bound:



Frame-dependent co-ordinate separation: Frequency shift $\propto \log \Omega_{\text{MAX}}$ | *Frame-independent geodesic separation:* Frequency shift $\propto \Omega^3_{\text{MAX}}$

Ω_{MAX}: UV cut-off introduced to regularise divergence.

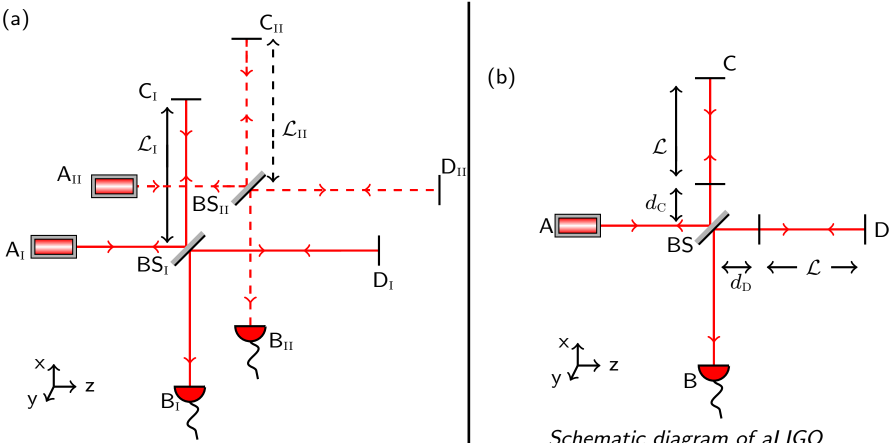
Results reported in arXiv:2501.03886!

Spacetime fluctuations

- Spacetime foam:
  - Proposed by Wheeler [Ref: J. A. Wheeler, *Ann. Phys.* (NY) **2**, 604 (1957)].
  - Effective quantum field theories [Ref: D. Carney *et al.*, *arXiv preprint arXiv:2409.03894* (2024)].
  - Holographic models [Ref: D. Li *et al.*, *Phys. Rev.* D **107**, 024002 (2023)].
- Semiclassical models [Ref: J. Oppenheim *et al.*, *Nat. Commun.* **14**, 7910 (2023)].
- *Reading off* signatures *specific* to various gravity models.
- **Interferometers**: high precision can be exploited to study spacetime fluctuations.

  [Ref: G. Amelino-Camelia, Nature 398, 216 (1999).]

In three interferometric setups...

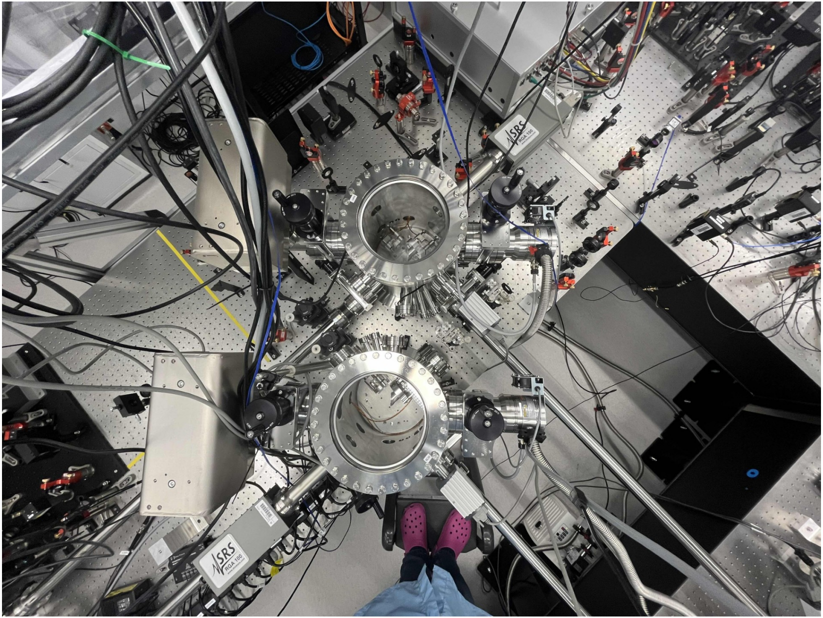


Schematic diagram of the Holometer and QUEST setups.

Schematic diagram of aLIGO.

Electric field E_{out} of the output light is obtained.

QUEST setup



Modelling metric fluctuations and its effects

$$g_{\mu\nu}: \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 2w(\mathbf{r}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ with } w \ll 1;$$

Visual repn.

$$\overline{w} = 0; \qquad \overline{w(\mathbf{r}_1)w(\mathbf{r}_2)} = \Gamma_{\mathrm{S}}\, \rho(\mathbf{r}_1 - \mathbf{r}_2);$$

Γ_{S}: Fluctuation strength; $\rho(\mathbf{r}_1, \mathbf{r}_2)$ with ℓ_r: Correlation scale.

$\Downarrow$ Using relativistic wave eqn.

Spectral density $\propto \int_0^\infty \mathrm{d}\Delta_\tau \, \overline{E^i_{\mathrm{out}}(\vec{r}_1, \tau_0)E^j_{\mathrm{out}}(\vec{r}_2, \tau_0 + \Delta_\tau)} \, \cos 2\nu \frac{c\Delta_\tau}{\mathcal{L}}$

$i = j \implies$ Power spectral density $S(\nu)$. $\quad [i, j = \mathrm{I,II};\ \tau_0 = 2\mathcal{L}/c.]$

Assumptions involved

1. **Eikonal approximation:** $\lambda \to 0$, i.e., wavelength is the smallest length scale in the system, (Eg: Correlation scale $\ell_r \gg \lambda$).
2. **Slowly varying envelope approximation (SVEA):** The rate of metric-fluctuation-induced-phase fluctuation is much smaller than the light frequency.
3. **Stationarity of the metric fluctuation:** $\overline{w} = 0$; $\overline{w(\mathbf{r}_1)w(\mathbf{r}_2)} = \Gamma_{\mathrm{S}}\,\rho(\mathbf{r}_1 - \mathbf{r}_2)$, $\overline{w(\mathbf{r}_1)w(\mathbf{r}_2)}$: non-negative definite function, ...
4. **Correlation function ρ is isotropic in space**.
5. **Stationarity of the resulting random process in path difference:** Required for transforming from the autocorrelation function to the spectral densities. This is checked to be true in the cases considered.

PSDs: Correlation integrals and Response functions

Spectral densities computed using two approaches:

Holometer:

1. **Using correlation integrals:** Cosine transform of correlation integrals,
   $S(f) = \frac{c^2\Gamma_S}{2\pi} \int_0^\infty \mathrm{d}\Delta_\tau \left[\sigma(\Delta_\tau) - \xi(\Delta_\tau)\right] \cos 2\pi f \Delta_\tau$.
   σ: Intra-arm correlation integral, ξ: Inter-arm correlation integral.

2. **Using response function:** $S(f) = \int \mathrm{d}^3\vec{k}_1 \, \Gamma_S \, \widetilde{\rho}(2\pi f, \vec{k}_1) \, \widetilde{\chi}_I(f, \vec{k}_1)$.
   $\widetilde{\rho}$: 4-d Fourier transform of ρ, $\widetilde{\chi}_I$: interferometric response function.

More...

aLIGO:

Response function offers interesting insight: $S(f) = \int \mathrm{d}^3\vec{k}_1 \, \Gamma_S \, \widetilde{\rho}(2\pi f, \vec{k}_1) \, \widetilde{\chi}_I(f, \vec{k}_1) \, \widetilde{\chi}_{FP}(f, \vec{k}_1)$.

$\widetilde{\rho}$: 4-d Fourier transform of ρ, $\widetilde{\chi}_{FP}$: Fabry-Perot cavity response function.

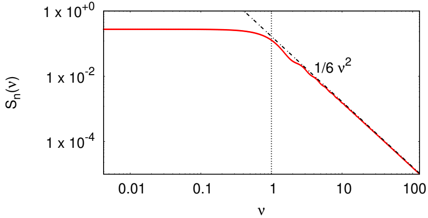
More...

Scaled PSDs $S_n(\nu) = \left(\frac{\ell_r}{\Gamma_S}\right)\frac{S(f)}{\mathcal{L}^3}$ *and* $S_N(\nu) = \left(\frac{1}{\Gamma_S \ell_r}\right)\frac{S(f)}{\mathcal{L}^2}$ *vs scaled frequency* $\nu = \pi f \mathcal{L}/c$

Semiclassical model:

$\rho(\mathbf{r}_1, \mathbf{r}_2) = \rho_s(|\Delta\vec{r}_{12}|)\rho_t(|\Delta t_{12}|)$

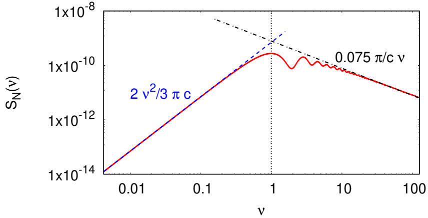
[J. Oppenheim *et al.*, *Nat. Commun.* **14**, 7910 (2023).]



Quantum pixellon model:

$\rho(\mathbf{r}_1, \mathbf{r}_2) \propto \frac{1}{|\Delta\vec{r}_{12}|}$

[D. Li *et al.*, *Phys. Rev.* D **107**, 024002 (2023).]



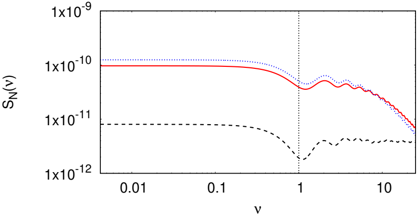
Exponential decay with $|\Delta\vec{r}_{12}|$**:**

[Quantum: M. Van Raamsdonk, *Int. J. Mod. Phys.* D **19**, 2429 (2010).;

Semiclassical: P. Pelliconi *et al.*, *arXiv preprint* arXiv:2409.13808 (2024).]

$\mathbf{r}_i = \{\vec{r}_i, t_i\}$ $(i = 1, 2)$; $\Delta\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$;

$\Delta t_{12} = t_1 - t_2$.



[Manuscript in preparation.]

Spatial vs Spacetime separation

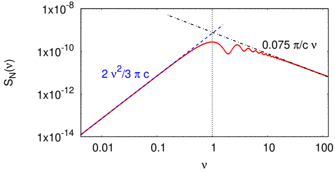
Scaled PSD $S_N(\nu) = \left(\frac{1}{\Gamma_S \ell_r}\right)\frac{S(f)}{\mathcal{L}^2}$ *vs scaled frequency* $\nu = \pi f \mathcal{L}/c$

Spatial sep.: $|\Delta\vec{r}_{12}|$

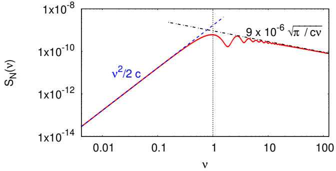
Spacetime sep.: $\Delta\mathbf{r}_{12} = \sqrt{|\Delta\vec{r}_{12}|^2 - c^2|\Delta t_{12}|^2}$

Inverse correlation functions:

$$\rho(\mathbf{r}_1, \mathbf{r}_2) = \frac{\ell_r}{|\Delta\vec{r}_{12}|}\Theta(|\Delta\vec{r}_{12}| - c|\Delta t_{12}|)$$

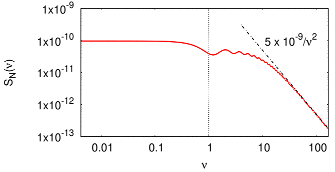


$$\rho(\mathbf{r}_1, \mathbf{r}_2) = \frac{\ell_r}{\Delta\mathbf{r}_{12}}\Theta(|\Delta\vec{r}_{12}| - c|\Delta t_{12}|)$$

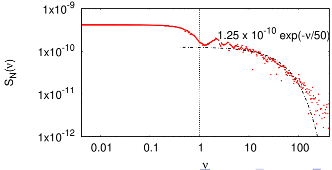


Exponential correlation functions:

$$\rho(\mathbf{r}_1, \mathbf{r}_2) = e^{-|\Delta\vec{r}_{12}|/\ell_r}\Theta(|\Delta\vec{r}_{12}| - c|\Delta t_{12}|)$$



$$\rho(\mathbf{r}_1, \mathbf{r}_2) = e^{-\Delta\mathbf{r}_{12}/\ell_r}\Theta(|\Delta\vec{r}_{12}| - c|\Delta t_{12}|)$$



Results

Summary of the spectral density behaviour for different correlation function classes.

Correlation type	Low frequency	High frequency	Scaled PSD overlaps for different choices of $\mathcal{L}$
(a) Factorised $\rho_{\rm F}$	$S_{\sf n}(\nu = 0) > 0$	$\propto 1/\nu^2$	Yes with $S_{\sf n}(\nu)$ (For Oppenheim model only)
(b) Inverse $\rho_{{\rm I}m}$	$S_{\rm N}(\nu = 0) = 0$ $S_{\rm N}(\nu) \propto \nu^2$	$\propto 1/\nu$ $(m = 1)$ or $\propto 1/\sqrt{\nu}$ $(m = 2)$	Yes with $S_{\rm N}(\nu)$
(c) Exponential $\rho_{{\rm E}m}$	$S_{\rm N}(\nu = 0) > 0$	$\propto 1/\nu^2$ $(m = 1)$ or $\propto e^{-\nu/50}$ $(m = 2)$	No

$m = 1$: *spatial separation and* $m = 2$: *spacetime separation.*

We need interferometers that at least have a frequency span of $0.1 \leqslant \nu \leqslant 10$.

QUEST is best suited as its proposed span of 1 MHz to 250 MHz corresponds to $0.03 \leqslant \nu \leqslant 78$**.**

Summary

- Role of the correlation functions assessed clearly.
- Clear characteristic signatures identified for classes of correlation function.
- Experimental setups most suitable for the detection of spacetime fluctuation identified.

Spacetime fluctuations: Single-parameter models

- Single-parameter models based on hypothesised holographic principle:

$$\Delta x \geqslant \ell_{\mathrm{P}} \sqrt{\mathcal{L}/R} \quad \text{and} \quad \Delta x \geqslant \mathcal{L}^{1-\alpha} \ell_{\mathrm{P}}^{\alpha}$$

  Δx: std. dev. in path difference, ℓ_{P}: Planck length, R: effective size of interferometer, $\mathcal{L}$: interferometer arm length, and $\alpha \in (0, 1]$.

  [Ref: Y. J. Ng *et al.*, *Mod. Phys. Lett. A* **9**, 335 (1994); Y. J. Ng, *Mod. Phys. Lett. A* **18**, 1073 (2003).]

- R, α decide extent of macroscopic coherence of spacetime fluctuations. [Ref: G. Amelino-Camelia, *Mod. Phys. Lett. A* **9**, 3415 (1994).]

Spacetime fluctuations: Macroscopic coherence

No coherence $R = \mathcal{L}$ or $\alpha = 1$:

$$\Delta x \geqslant \ell_{\mathrm{P}}$$

Macroscopic coherence $R = \ell_{\mathrm{P}}$ or $\alpha = \frac{1}{2}$:

$$\Delta x \geqslant \sqrt{\ell_{\mathrm{P}} \mathcal{L}}$$

Macroscopic coherence in **Holographic noise model**.

Interferometers: high precision can be exploited to study spacetime fluctuations.

[Ref: G. Amelino-Camelia, Nature 398, 216 (1999).]

PSDs: Correlation integrals and Response function

Using Correlation integrals:

$$S(f) = \frac{c^2\Gamma_{\rm S}}{2\pi}\int_0^\infty \mathrm{d}\Delta_\tau \left[\sigma(\Delta_\tau) - \xi(\Delta_\tau)\right]\cos 2\pi f\Delta_\tau,$$

where

$$\sigma(\Delta_\tau) = \int_0^{\tau_0}\mathrm{d}t_1\int_0^{\tau_0}\mathrm{d}t_2\,\rho(0,0,s(t_1)-s(t_2),c(t_1+\Delta_\tau-t_2))$$
$$= \int_0^{\tau_0}\mathrm{d}t_1\int_0^{\tau_0}\mathrm{d}t_2\,\rho(s(t_1)-s(t_2),0,0,c(t_1+\Delta_\tau-t_2)),$$

$$\xi(\Delta_\tau) = \int_0^{\tau_0}\mathrm{d}t_1\int_0^{\tau_0}\mathrm{d}t_2\,\rho(s(t_1),0,-s(t_2),c(t_1+\Delta_\tau-t_2)).$$

Here $s(t) = ct$ if $t \leqslant \tau_0/2$ and $s(t) = 2\mathcal{L} - ct$ if $t > \tau_0/2$.

Back

Using Response function:

$$S(f) = \int \mathrm{d}^3\vec{k}_1\,\Gamma_{\rm S}\,\widetilde{\rho}(2\pi f,\vec{k}_1)\,\widetilde{\chi}(f,\vec{k}_1).$$

Here, the interferometeric response function

$$\widetilde{\chi}(f,\vec{k}_1) = \left(\frac{\mathcal{L}}{2}\right)^2\left|C_x(f,\vec{k}_1) - C_z(f,\vec{k}_1)\right|^2,$$

with

$$C_i(f,\hat{n}) = e^{ifT_+^{(i)}}\left\{\mathrm{Sinc}\left(fT_+^{(i)}\right) + e^{\frac{2\pi i f\mathcal{L}}{c}}\,\mathrm{Sinc}\left(fT_-^{(i)}\right)\right\},$$

$$T_\pm^{(i)}(\hat{n}) = \frac{\pi\mathcal{L}}{c}\left(1 \pm \frac{c}{2\pi f}\,\vec{k}_1\cdot\hat{\mathbf{e}}_i\right),\quad (i = x, z).$$

The transformed correlation

$$\rho(\vec{r}_{12}, ct_{12}) = 2\pi\int \mathrm{d}^3\vec{k}_1\int_{-\infty}^{\infty}\mathrm{d}f\,\widetilde{w}(2\pi f,\vec{k}_1)\,e^{i\left(2\pi f t_{12} + \vec{k}_1\cdot\vec{r}_{12}\right)}.$$

PSD in aLIGO

Using Correlation integrals:

$$S(f) = \frac{c^2 \Gamma_{\rm S} T_{\rm M}^4}{2} \left(\frac{1}{1-\sqrt{R_{\rm M}}} \right)^2 \sum_{q_1, q_2 = 1}^{\infty} (\sqrt{R_{\rm M}})^{q_1+q_2-2}$$

$$\int_0^{\infty} \mathrm{d}\Delta_\tau \left[\sigma^{(q_1,q_2)}(\Delta_\tau) - \xi^{(q_1,q_2)}(\Delta_\tau) \right] \cos 2\pi f \Delta_\tau ,$$

where

$$\sigma^{(p,q)}(\Delta_\tau)$$

$$= \int_0^{p\tau_0} \mathrm{d}t_1 \int_0^{q\tau_0} \mathrm{d}t_2 \, \rho(0,0,s(t_1)-s(t_2), t_1+\Delta_\tau - t_2)$$

$$= \int_0^{p\tau_0} \mathrm{d}t_1 \int_0^{q\tau_0} \mathrm{d}t_2 \, \rho(s(t_1)-s(t_2), 0, 0, t_1+\Delta_\tau - t_2),$$

$$\xi^{(p,q)}(\Delta_\tau)$$

$$= \int_0^{p\tau_0} \mathrm{d}t_1 \int_0^{q\tau_0} \mathrm{d}t_2 \, \rho(s(t_1), 0, -s(t_2), t_1+\Delta_\tau - t_2).$$

Using Response function:

$$S(f) = \int \mathrm{d}^3\vec{k}_1 \, \Gamma_{\rm S} \, \widetilde{\rho}(2\pi f, \vec{k}_1) \, \widetilde{\chi}_{\rm L}(f, \vec{k}_1)$$

Here, the interferometeric response function

$$\widetilde{\chi}_{\rm L}(f,\vec{k}_1) = \widetilde{\chi}_{\rm I}(f,\vec{k}_1)\, \widetilde{\chi}_{\rm FP}(f,\vec{k}_1),$$

$$\widetilde{\chi}_{\rm I}(f,\vec{k}_1) = \left(\frac{\mathcal{L}}{2}\right)^2 \left| C_x(f,\vec{k}_1) - C_z(f,\vec{k}_1) \right|^2$$

$$\widetilde{\chi}_{\rm FP}(f,\vec{k}_1) = T_{\rm M}^4 \left(\frac{1}{1-\sqrt{R_{\rm M}}} \right)^2$$

$$\left| \left(\frac{1}{1-e^{4\pi i f \mathcal{L}/c}} \right) \left(\frac{1}{1-\sqrt{R_{\rm M}}} \right) - \right.$$

$$\left. \left(\frac{e^{4\pi i f \mathcal{L}/c}}{1-e^{4\pi i f \mathcal{L}/c}} \right) \left(\frac{1}{1-\sqrt{R_{\rm M}} e^{4\pi i f \mathcal{L}/c}} \right) \right|^2 .$$

Back